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# Truncation of the operator-product expansion for the $\langle q\bar{q}\rangle$-condensate component of the quark mass

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Quark-condensate contributions to $m_{\rm dyn}$, the quark mass generated by the dynamical breakdown of chiral symmetry, are shown to arise from only the first two terms of the operator-product expansion (OPE) for the appropriate $O(g^2)$ nonperturbative vacuum expectation value. Since the $m/|p|$ expansion parameter of the OPE is unity on the gauge-invariant $\not p = m_{\rm dyn}$ propagator pole, such truncation of OPE contributions is an essential support to the calculation of $m_{\rm dyn}$.

A persistent problem in elementary-particle physics is to find field-theoretical support for the discrepancy between the small nonstrange "current"-quark masses, motivated by PCAC (partial conservation of axial-vector current) and deep-inelastic scattering, and the relatively large ($\sim m_{\rm nucleon}/3$) constituent-quark masses, motivated most convincingly by baryonic magnetic moments. To account for this discrepancy, a large component of the nonstrange-quark mass must be dynamical in origin, dominating the quark mass near the $m_{\rm nucleon}/3$ "mass shell" and falling off sharply with $p^2$ away from this mass shell.[1]

When expressed in terms of condensates arising from nonperturbative vacuum expectation values of quark and gluon fields, dynamical contributions to the quark mass necessarily involve inserting *all* orders of the operator-product expansion (expansion parameter $m/|p|$) into *each* order of perturbation theory (expansion parameter $g$). For example, order-$g^2$ quark-condensate contributions to the dynamical quark mass[1-3] are obtained by substituting the short-distance operator-product expansion (OPE) for the nonperturbative vacuum expectation value $\langle 0|:\psi(y)\bar{\psi}(z):|0\rangle^{\rm NP}$ into the quark self-energy (Fig. 1). These contributions are expressed in terms of gauge-invariant condensates through use of the Fock-Schwinger gauge (FSG) condition[4]

$$x_\mu B^\mu(x)=0\ , \tag{1}$$

in which case derivatives within the Taylor-series expansion about $y=z=0$ for the nonperturbative matrix element may be replaced with covariant derivatives evaluated at zero:[5]

$$\begin{aligned}
\langle 0|:\psi_n^\alpha(y)\bar{\psi}_r^\beta(z):|0\rangle =& \langle 0|:\psi_n^\alpha(0)\bar{\psi}_r^\beta(0):|0\rangle + y^\omega\langle 0|:[\partial_\omega\psi_n^\alpha(0)]\bar{\psi}_r^\beta(0):|0\rangle \\
&+z^\omega\langle 0|:\psi_n^\alpha(0)[\partial_\omega\bar{\psi}_r^\beta(0)]:|0\rangle+\tfrac{1}{2}y^{\omega_1}y^{\omega_2}\langle 0|:[\partial_{\omega_1}\partial_{\omega_2}\psi_n^\alpha(0)]\bar{\psi}_r^\beta(0):|0\rangle \\
&+\tfrac{1}{2}z^{\omega_1}z^{\omega_2}\langle 0|:\psi_n^\alpha(0)[\partial_{\omega_1}\partial_{\omega_2}\bar{\psi}_r^\beta(0)]:|0\rangle+y^{\omega_1}z^{\omega_2}\langle 0|:[\partial_{\omega_1}\psi_n^\alpha(0)][\partial_{\omega_2}\bar{\psi}_r^\beta(0)]:|0\rangle+\cdots \\
\xrightarrow[\partial^\omega \underset{\rm FSG}{\to} D^\omega(0)]{}& -\delta^{\alpha\beta}\Bigg(\frac{\delta_{nr}\langle 0|:\bar{\psi}_a^\rho(0)\psi_a^\rho(0):|0\rangle}{12}+\frac{[\gamma\cdot(y-z)]_{nr}}{48}\langle 0|:\bar{\psi}_a^\rho(0)\not{D}_{ab}^{\rho\sigma}(0)\psi_b^\sigma(0):|0\rangle \\
&+\tfrac{1}{2}(y^{\omega_1}y^{\omega_2}+z^{\omega_1}z^{\omega_2}-2y^{\omega_1}z^{\omega_2})\langle 0|:\bar{\psi}_r^\rho(0)D_{\omega_1}^{\rho\sigma}(0)D_{\omega_2}^{\sigma\tau}(0)\psi_n^\tau(0):|0\rangle+\cdots\Bigg) \\
=& -\delta^{\alpha\beta}\{[\delta_{nr}/12-im_{\rm dyn}[\gamma\cdot(y-z)]_{nr}/48+[-m_{\rm dyn}{}^2(y-z)^2\delta_{nr}/32+O(m_{\rm dyn}{}^3)]]\langle\bar{\psi}\psi\rangle \\
&+[-i(y-z)^2\delta_{nr}/64-y_\alpha z_\beta\sigma_{nr}^{\alpha\beta}/96+O(m_{\rm dyn})]\langle 0|:\bar{\psi}_a^\alpha(0)F_{\mu\nu}^I(0)(ig\lambda_{\alpha\beta}^I/2)\sigma_{ab}^{\mu\nu}\psi_b^\beta(0):|0\rangle \\
&+\text{higher-dimensional condensates}\}
\end{aligned} \tag{2}$$

 

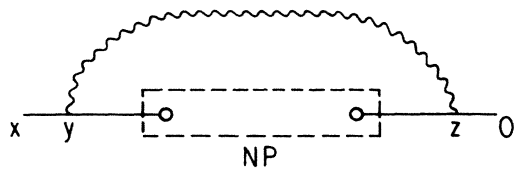


FIG. 1. The graph for $O(g^2\langle\bar{\psi}\psi\rangle)$ contributions to the dynamical component of the quark mass.

Insertion into Fig. 1 of the OPE terms in (2), involving at most one covariant derivative (i.e., those terms proportional to $\langle\bar{\psi}\psi\rangle$ containing at most one power of $m_{\rm dyn}$), leads to an order-$g^2$ correction to the fermion propagator which is independent of the gauge parameter $(a)$ at the $\not{p}=m_{\rm dyn}$ pole position:[3]

$$\Delta S_F(p)=\frac{g^2|\langle\bar{\psi}\psi\rangle|}{9(p^2)^2}[(3+a)-am_{\rm dyn}\not{p}/p^2]. \tag{3}$$

This on-shell gauge independence is not merely a coincidence; it is also seen to occur (trivially) in gluon-gluon-condensate contributions[3] to $\Delta S_F(p)$, and (nontrivially) in mixed quark-antiquark-gluon-condensate contributions[6] to $\Delta S_F(p)$.

Equation (3) can therefore be used to show that the quark mass obtained from chiral-symmetry breakdown is

$$m_{\rm dyn}=|g^2\langle\bar{\psi}\psi\rangle/3|^{1/3}, \tag{4}$$

regardless of gauge.[3,7] True confidence in this result (for which reasonable phenomenological support exists[7]), however, is contingent on there being no further alterations from OPE terms in (2) arising from two or more covariant derivatives. Note that the "one-covariant-derivative term" in (2) led to a contribution to $\Delta S_F(p)$ involving a factor of $m_{\rm dyn}\not{p}/p^2$ relative to the lead term in (3). This factor exhibits $m/|p|$ as the expansion parameter in momentum space of the OPE given by (2), a standard OPE property. The second-derivative terms in (2) are therefore expected to yield contributions to $\Delta S_F(p)$ comparable to $(m_{\rm dyn}{}^2/p^2)$ times the lead term. Moreover, third-derivative contributions should differ in magnitude from the lead term by $m_{\rm dyn}{}^3\not{p}/(p^2)^2$, etc.

This $m\not{p}/p^2$ expansion parameter is necessarily *unity* on the $\not{p}=m_{\rm dyn}$ mass shell. Consequently, the only way to extract a meaningful dynamical quark mass from (2) is to show that the full set of higher-order OPE contributions to $\Delta S_F(p)$ truncates to a finite number of terms. We prove below that such truncation does indeed occur. Specifically, we show that no nonvanishing $O(g^2\langle\bar{\psi}\psi\rangle)$ contributions to $\Delta S_F(p)$ occur from OPE terms in (2) arising from two or more covariant derivatives.

To obtain this result, we first note that Fig. 1 corresponds to the second-order interaction-picture corrections to the fermion propagator generated from the quark-quark-gluon interaction of QCD:

$$\begin{aligned}[\Delta S_F(p)]_{ij}^{\rho\sigma}&=-i\int d^4x e^{ip\cdot x}\Big\langle T\psi_i^\rho(x)\bar{\psi}_j^\sigma(0)\Big(-\tfrac{1}{2}\int d^4y\int d^4z\,\mathcal{L}_{\rm int}(y)\mathcal{L}_{\rm int}(z)\Big)\Big\rangle_0\\&=\frac{ig^2}{4}\int d^4(x-y)e^{ip\cdot(x-y)}\langle T\psi_i^\rho(x)\bar{\psi}_k^\tau(y)\rangle_0\lambda_{\tau a}^a\gamma_{kn}^\mu\Big[\int d^4(y-z)e^{ip\cdot(y-z)}\langle:\psi_n^\alpha(y)\bar{\psi}_r^\beta(z):\rangle_0^{\rm NP}\langle TB_\mu^a(y)B_\nu^b(z)\rangle_0\Big]\\&\quad\times\gamma_{rs}^\nu\lambda_{\beta\eta}^b\int d^4z e^{ip\cdot z}\langle T\psi_s^\eta(z)\bar{\psi}_j^\sigma(0)\rangle_0\,.\end{aligned} \tag{5}$$

The nonperturbative vacuum expectation value within the large parentheses on the right-hand side (RHS) of (5) is precisely the same as the expression (2). Let us first consider within (5) the contributions to (2) arising from two covariant derivatives:

$$-\left(\frac{y^{\omega_1}y^{\omega_2}+z^{\omega_1}z^{\omega_2}-2y^{\omega_1}z^{\omega_2}}{2}\right)\langle 0|:\bar{\psi}_r(0)[D_{\omega_1}(0)D_{\omega_2}(0)\psi_n(0)]:|0\rangle=m_{\rm dyn}{}^2\delta_{nr}(y-z)^2\langle\bar{\psi}\psi\rangle/32+\cdots\,. \tag{6}$$

[The dots in (6) represent a mixed quark-antiquark-gluon condensate contribution; color indices in (6) are suppressed.] Upon insertion into the large parentheses on the RHS of (5), the quark condensate component of (6) leads to a contribution proportional to the following integral over $(y-z)$:

$$\begin{aligned}&\left[\int\frac{d^4(y-z)}{(2\pi)^4}e^{ip\cdot(y-z)}(y-z)^2\int\frac{d^4k}{(k^2)^2}e^{-ik\cdot(y-z)}[-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu]\right]\\&\qquad=-\int\frac{d^4k}{(k^2)^2}[-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu]\frac{\partial}{\partial p^\omega}\frac{\partial}{\partial p_\omega}\int\frac{d^4(y-z)}{(2\pi)^4}e^{i(y-z)\cdot(p-k)}\\&\qquad=-\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\left(\frac{[-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu]}{(p^2)^2}\right)=\frac{2(1-a)}{(p^2)^3}(g_{\mu\nu}p^2-4p_\mu p_\nu).\end{aligned} \tag{7}$$

The final expression of (7) vanishes upon contraction into the $\gamma_\mu$ and $\gamma_\nu$ adjacent to the large parentheses on the RHS in (5). Consequently, $O(m^2/p^2)$ contributions to $\Delta S_F(p)$ proportional to $\langle\bar{\psi}\psi\rangle$ do not occur.[3]

We now consider the contributions to (2) arising from terms involving three covariant derivatives:

$$\left[-\left(\frac{y^{\omega_1}y^{\omega_2}y^{\omega_3}-z^{\omega_1}z^{\omega_2}z^{\omega_3}+3z^{\omega_1}z^{\omega_2}y^{\omega_3}-3z^{\omega_1}y^{\omega_2}y^{\omega_3}}{6}\right)\langle 0|:\bar{\psi}_r(0)[D_{\omega_1}(0)D_{\omega_2}(0)D_{\omega_3}(0)\psi_n(0)]:|0\rangle\right]$$
$$\sim m_{\rm dyn}{}^3(y-z)^2\gamma_{nr}^\omega(y-z)_\omega\langle\bar{\psi}\psi\rangle+\cdots\,. \tag{8}$$

Substitution of (8) into the nonperturbative vacuum expectation value within the large parentheses on the RHS of (5) leads to the following integral over $(y-z)$:

$$\left[\gamma_{nr}^{\omega}\int\frac{d^4(y-z)}{(2\pi)^4}e^{ip\cdot(y-z)}(y-z)^2(y-z)_{\omega}\int\frac{d^4k}{(k^2)^2}e^{-ik\cdot(y-z)}[-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu]\right]$$

$$=i\left[\gamma\cdot\frac{\partial}{\partial p}\right]_{nr}\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]=8i(1-a)(p^2)^{-3}(-\not p g_{\mu\nu}-\gamma_\mu p_\nu-\gamma_\nu p_\mu+6p_\mu p_\nu\not p/p^2)_{nr}\ . \quad (9)$$

The result (9) vanishes when contracted into the $\gamma^\mu$ and $\gamma^\nu$ on opposite sides of the large parentheses on the RHS in (5):

$$\gamma^\mu\left[\gamma\cdot\frac{\partial}{\partial p}\right]\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]\left[\frac{[-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu]}{(p^2)^2}\right]\gamma^\nu=0\ . \quad (10)$$

Consequently, the OPE terms arising from three covariant derivatives within (2) also fail to contribute to $\Delta S_F(p)$, and $O(m^3\not p/p^4)$ contributions to $\Delta S_F(p)$ are similarly absent.

One can now see easily that no higher-than-first-derivative terms within (2) contribute to $\Delta S_F(p)$. Consider, for example, the $n$th-derivative contribution to (2), where $n\geq 2$:

$$\left[-\left[\frac{y^{\omega_1}y^{\omega_2}\cdots y^{\omega_n}-nz^{\omega_1}y^{\omega_2}\cdots y^{\omega_n}+\cdots+(-1)^n z^{\omega_1}z^{\omega_2}\cdots z^{\omega_n}}{n!}\right]\langle 0|:\bar\psi_r(0)[D_{\omega_1}(0)D_{\omega_2}(0)\cdots D_{\omega_n}(0)\psi_n(0)]:|0\rangle\right]$$

$$\sim[\gamma\cdot(y-z)]^n(-im_{\text{dyn}})^n\langle\bar\psi\psi\rangle+\cdots\ . \quad (11)$$

[Equation (11) is easily obtained as a straightforward generalization of (6) and (8).] Upon substituting (11) for the nonperturbative vacuum expectation value in (5), one obtains a result proportional to

$$\gamma^\mu\left[\int\frac{d^4(y-z)}{(2\pi)^4}e^{ip\cdot(y-z)}[\gamma\cdot(y-z)]^n\int d^4k\,e^{-ik\cdot(y-z)}\left[\frac{-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu}{(k^2)^2}\right]\right]\gamma^\nu$$

$$\sim\gamma^\mu\left[\gamma\cdot\frac{\partial}{\partial p}\right]^n\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu\ . \quad (12)$$

If $n$ is even, the right-hand side of (12) is seen to vanish via the last line of (7):

$$\gamma^\mu\left[\gamma\cdot\frac{\partial}{\partial p}\right]^n\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu=\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]^{(n-2)/2}\left[\gamma^\mu\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu\right]=0\ . \quad (13)$$

If $n$ is odd, the right-hand side of (12) is seen to vanish via (10):

$$\gamma^\mu\left[\gamma\cdot\frac{\partial}{\partial p}\right]^n\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu=\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]^{(n-3)/2}\left[\gamma^\mu\left[\gamma\cdot\frac{\partial}{\partial p}\right]\left[\frac{\partial}{\partial p}\cdot\frac{\partial}{\partial p}\right]\left[\frac{-g_{\mu\nu}p^2+(1-a)p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu\right]=0\ . \quad (14)$$

Thus, there are *no* $O(g^2\langle\bar\psi\psi\rangle)$ contributions to $\Delta S_F(p)$ from the terms within (2) arising from two or more covariant derivatives. Consequently, (3) is correct to *all* orders of the OPE, and the resulting expression for the on-shell quark mass [Eq. (4)] does not suffer successive order-unity corrections from higher-order OPE contributions.

It is worth noting that there appear to be no $O(g^3\langle\bar\psi\psi\rangle)$ contributions to $\Delta S_F(p)$. Three-vertex self-energy graphs, as considered in Ref. 6, necessarily involve the nonperturbative vacuum expectation value $\langle 0|:\psi(x)\bar\psi(y)B(z):|0\rangle$. When this quantity is expanded [analogous to Eq. (2)] we see that there are no $\langle\bar\psi\psi\rangle$ components for lead and first-derivative OPE terms; i.e., the lowest-order condensate contributions are mixed quark-gluon condensates.

We further note that truncation of dynamical contributions to the quark mass in (2) does not appear to be restricted to the quark-antiquark condensate. In Ref. 6, the mixed quark-antiquark-gluon condensate [as in the last line of Eq. (2)] is shown to contribute in part to the dynamical quark mass through second-order (i.e., two-derivative) OPE contributions to the nonperturbative portion of Fig. 1. However, *both third- and fourth*-order OPE contributions proportional to this mixed condensate *vanish explicitly* when inserted into Fig. 1, suggesting a truncation of mixed-condensate contributions beyond two-derivative terms of the OPE.

As a final remark concerning the $\lambda=m_{\text{dyn}}\not p/p^2$ expansion parameter, truncation of the OPE, upon substitution into the quark self-energy (Fig. 1), justifies application of the OPE in this $\lambda\sim 1$ region. Such "precocious asymptotic freedom" is reminiscent of the known precocious scaling in deep-inelastic processes at low $q^2(\sim 1\ \text{GeV}^2)$. Indeed, in earlier work[7] we have regarded the renormalization point $M^2\cong 1\ \text{GeV}^2$ as deep Euclidean, and have extrapolated down to $\lambda=1$, $p^2\cong(m_N/3)^2$, by dimensional arguments alone to obtain (4) from (3).

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