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# **Quality Polarization and International Trade**

Till Ferdinand Hollstein Kristian Estevez



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**Abstract**: This paper builds a model that examines firm heterogeneity across two dimensions: productivity and quality. We show that when firms are able to choose their input quality and there exists a negative relationship between a firm's product quality and their marginal cost, this can lead to a non-unimodal distribution of quality across firms. Trade liberalization, represented by reductions in trade costs, and stronger vertical linkages, represented by an increase in the cost share of intermediate goods, shift the distribution of firms towards the modes of the distribution, which we call quality polarization. With this approach, we are able to explain empirical trade patterns relating to firm size, prices, and quality of exported goods.

JEL Codes: F12, L11, L16,O14.

Keywords: Heterogeneous firms, international trade, quality.

Till Ferdinand Hollstein Universitat de Barcelona

Kristian Estevez Universitat de Barcelona

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## 1 Introduction

Based on the increasing availability of firm-level data, the new trade literature has documented substantial firm heterogeneity in export performance. Exporting firms are more profitable than non-exporters, have a larger market share, and enter more markets.<sup>1</sup> The Melitz (2003) model provides a theoretical framework for these stylized facts. Firms enter a market with Dixit-Stiglitz monopolistic competition and draw their productivity from a common distribution. Facing fixed entry cost and production costs, firms surpassing a productivity threshold survive in the domestic market. When export costs are present, only the more productive firms will find it profitable to export. Reducing trade costs allows new entrants in the export market and increases competition in the domestic market, forcing the least productive firms out of the market and reallocating market share towards the more productive firms.

Recent empirical literature has revealed a price puzzle: there is no conclusive evidence whether exporting firms charge higher or lower prices than non-exporters. Studies by Roberts and Supina (1996), Roberts and Supina (2000), and Syverson (2007) report a negative correlation between output price and firm size. This correlation is predicted by the Melitz (2003) trade model of heterogeneous firms. The higher a firm's productivity the lower its marginal cost. Therefore, more productive firms charge a lower price, produce more output and export to foreign markets. In contrast to these findings, recent empirical work by Verhoogen (2008), Iacovone and Javorcik (2008), Manova and Zhang (2011), Kugler and Verhoogen (2012), and Crozet et al. (2012) show that exporters charge higher prices, have larger market shares and pay higher input prices. To account for these findings, economists have incorporated quality heterogeneity across firms into the Melitz (2003) model.<sup>2</sup> In these models, firms with higher productivity choose to use more expensive, higher-quality inputs to produce high-quality goods. Consumer demand for quality allows these firms to charge a higher price so that they are more profitable than firms producing lower-quality goods.

Models incorporating quality sorting predict the positive correlation between prices and firm size reported in some sectors, while models with only

<sup>&</sup>lt;sup>1</sup>See Eaton et al. (2004), Eaton et al. (2008), and Bernard et al. (2011) for a review of the literature. Manova and Zhang (2011) also provide stylized facts on Chinese exporters.

 $<sup>^{2}</sup>$ See Verhoogen (2008), Baldwin and Harrigan (2011), Dinopoulos and Unel (2011), and Kugler and Verhoogen (2012).

productivity heterogeneity predict the negative correlation between prices and firm size reported in other sectors. Antoniades (2015) addresses this by allowing for sector-specific variation in the scope for quality differentiation. In sectors with a higher scope of quality differentiation, firms with high productivity choose to produce high-quality goods and therefore prices increase with firm size.

Besides these cross-sectoral variations, Antoniades (2015) follows the existing quality heterogeneous firm literature and assumes that firms producing either high or low quality are the most profitable within a single sector. Therefore, the quality of traded goods is predicted to be unimodally distributed around high or low quality levels. In Section 2 we draw from a rich data set (U.S. import data from fifty-six countries in 1990, 2000, and 2005), collected and aggregated by Amiti and Khandelwal (2013), and show that the quality of goods exported to the U.S. appears to follow a non-unimodal distribution in 14 out of the 25 sectors (HS 6-digit classification level) with over 1,000 observations. This evidence suggests that firms within a sector may find it profitable to export different quality levels and therefore the correlation between price and market size would be ambiguous.

Section 3 describes our model. There is an intermediate input sector that produces inputs with two discrete quality levels under perfect competition with labor being the only factor of production. In the final goods sector, there is a continuum of firms that produce under monopolistic competition. Physical output is generated using a a Cobb-Douglas production function with labor and the intermediate input as the input factors of production.

We address our findings from the data analysis by extending the standard heterogeneous firms trade model by incorporating endogenous quality choice of an intermediate input and assuming quality complementarity between a firm's capability and their choice of intermediate input quality.<sup>3</sup> Standard heterogeneous firms models with endogenous quality choice emphasize the substitutability of input quality and firm productivity. We assume the representative consumer values the quality of the final good by the lowest quality component. Therefore, output quality is determined in a Leontief-type production function with the quality of an intermediate good and a firm spe-

<sup>&</sup>lt;sup>3</sup>We follow the ideas of Kremer (1993) who developed the O-ring theory with the story of the space shuttle Challenger in mind. The shuttle was destroyed due to the failure of a single, low-cost rubber O-ring. While the model by Kremer (1993) is focused on the quality of individual workers' tasks, we directly determine the output quality of a final good in a Leontief-type production function using similar reasoning.

cific quality parameter that can be interpreted as the quality of the firm's blueprint, design, self-produced parts, etc. The firm specific quality parameter (referred to as capability) is randomly drawn from a common distribution and is assumed to be positively correlated with the firm's marginal cost indicating that higher quality is costly.

Firms are sorted along the quality axis, but firm profits are bimodally distributed due to three effects: First, higher quality firms have higher marginal cost through two channels: production costs are assumed to increase with the firm specific quality parameter and firms with higher quality are more likely to purchase more expensive, higher quality inputs. Second, firms are able to charge a higher price for higher output quality due to a positive demand for quality. Output quality increases with firm capability until the firm specific quality parameter surpasses the quality of intermediate inputs, then, output quality stagnates at the quality level of the intermediate input. The combination of both effects leads to firm profits initially increasing with capability until output quality stagnates due to perfect quality complementarity where profits decline with capability since marginal cost increase while output prices do not. And third, above an endogenous threshold level, firms find it profitable to switch from the low-quality to the high-quality intermediate input where profits once again increase with capability. The final result is a bimodal profit distribution where low- and high-quality firms may find it profitable to export.

We find that the shape of this distribution is determined by the intensity of consumers' desire for quality, the strength of vertical linkages within a sector, the price of the high-quality intermediate input relative to the lowquality input, and the relationship between firm capability and marginal cost. Our model can explain the findings in Hallak and Schott (2011) that the level of quality produced by a country is linked to on its level of income. Furthermore, we find this effect to be stronger in sectors that exhibit stronger vertical linkages.

In Section 4, we extend our model and examine trade between two symmetric economies to analyze the role of a reduction in trade costs. Like Foster et al. (2008), we find that firms displace less profitable but not necessarily less productive or lower quality businesses. Trade liberalization leads to quality polarization, a reallocation of market share and resources towards the modes of the profit distribution. An important result of our model is that it may explain why evidence on the correlation between price and market share of traded goods are inconsistent across many sectors, as well as the existence

of non-unimodal distributions of export quality that we first illustrate in the next section. Introducing quality complementarity between firm capability and discrete input quality in a heterogeneous firms trade model may help to explain international patterns of trade. We conclude this paper in Section 5 with final remarks.

## 2 Data Analysis

In order to examine the distribution of quality across a sector, we use a sample of 10,000 products across fifty-six countries collected and aggregated by Amiti and Khandelwal (2013). They used U.S import data from 1990, 2000 and 2005 at the HS 10-digit level and estimated the quality of each product exported to the U.S. by using both price and quantity information. Higher quality is assigned to products with higher market shares conditional on price. Many studies only use price data to infer product quality, but Khandelwal (2010) finds that prices are not always a good proxy for quality. An advantage of the standard disaggregate trade data used is that the sector-specific quality measure, that is otherwise unobserved, is derived by product market shares. Additionally to the trims of the data by Amiti and Khandelwal (2013),<sup>4</sup> we combine the product data at the HS 6-digit level to highlight the distribution of quality at the sectoral level. Furthermore, we analyze only sectors with more than 1,000 observations.

The resulting data shows the product quality of exported goods to the U.S. for 25 sectors. Plotting a kernel density of quality for each sector, we find that a majority of the 25 sectors feature a wide range of quality differences with its distribution tending to be non-unimodal, as shown in Figure 1. We run Hartigans' dip test to statistically test the null hypothesis that the distribution of quality is unimodal. With p-values of less than 0.05, we reject the null hypothesis that the quality distribution in that sector is unimodal in 14 of the 25 sectors. The dip test results are shown in Table 1.

These results contradict the models by Baldwin and Harrigan (2011) and Kugler and Verhoogen (2012) that assume a unimodal quality distribution.

<sup>&</sup>lt;sup>4</sup>The authors dropped variety-year observations above or below the 1st and 99th percentile of unit values, excluded varieties with annual price increases of more than 200 percent or price declines of more than 66 percent, and dropped varieties with export quantities of fewer than ten. They also trimmed the quality estimates at the 5th and 95th percentiles, respectively.

HS6	Description	Dip Statistic	p-value
420292	Cases and containers	0.0089	0.6905
490199	Books, brochures, leaflets, etc.	0.0747	0.0000*
610910	T-shirts, singlets, etc.; of cotton	0.0222	0.0000*
610990	T-shirts, singlets, etc.; of textile	0.0148	0.0564
611020	Jerseys, pullovers, etc.; of cotton	0.0177	$0.0027^{*}$
611030	Jerseys, pullovers, etc.; of fibres	0.0143	$0.0296^{*}$
611120	Garments and accessories	0.0074	0.9631
620342	Trousers, etc.; men's, of cotton	0.0276	0.0000*
620343	Trousers, etc.; men's, of synthetic	0.0189	$0.0156^{*}$
620462	Trousers, etc.; women's, of cotton	0.0307	0.0000*
620469	Trousers, etc.; women's, of textile	0.0543	0.0000*
620520	Shirts; men's or boys', of cotton	0.0352	0.0000*
630231	Bed linen; of cotton	0.0162	0.0621
640299	Footwear; no. 6402, (other)	0.0127	0.1346
640391	Footwear; no. 6403, covering ankle	0.0399	0.0000*
640399	Footwear; no. 6403, (other)	0.0171	$0.0008^{*}$
640419	Footwear; (other than sportswear)	0.0046	0.9970
650590	Hats and other headgear	0.0062	0.9505
691110	Household and toilet articles	0.0078	0.9768
691200	Ceramic tableware, etc.	0.0141	0.1900
731210	Iron or steel; stranded wire, etc.	0.0095	0.7669
731700	Iron or steel; nails, tacks, etc.	0.0189	$0.0132^{*}$
731815	Iron or steel; threaded screws, etc.	0.0070	0.9751
848180	Taps, cocks, valves and similar	0.0197	0.0000*
910211	Wrist-watches; electrically operated	0.0221	0.0000*

Table 1: Dip test results

Report of the dip tests result for each sector at the HS 6-digits level. The second row reports the dip statistic value. The third column shows the corresponding p-value. The asterisks (\*) represent that the p-value is less than 0.05, so that we reject the null hypothesis that the distribution of quality in that sector is unimodal.

Figure 1: Quality distribution - HS 620469



The graph shows the kernel density distribution of product quality in the HS 6-digit sector "Women's or girls trousers, overalls, and shorts" exported to the U.S.. Quality is distributed around the 0-value, describing average product quality in the sector. Negative values represent below-average and positive values represent above-average quality levels.

Products exported to the U.S. show a wide variety of quality levels that often show higher densities in different quality ranges. In other words, firms that export goods to the U.S. are not necessarily the ones that produce the highest quality.<sup>5</sup> We therefore present a new model in which plant productivity and input quality are complements in generating output quality, conceptually following the O-ring theory of Kremer (1993). With this new extension of the standard heterogeneous firm models, we are able to explain empirical trade patterns as it relates to the product quality of exported goods as well as address inconsistencies in the correlations between export prices and firm size.

<sup>&</sup>lt;sup>5</sup>The non-unimodal distribution in many sectors cannot be explained with income differences across exporting economies. There are 44 single-country single-sector cases when the null-hypothesis of unimodality was rejected using Hartigan's dip test. Single countries export goods with a wide variety of quality to the U.S.

## 3 The Closed Economy

In this section, we extend the heterogeneous firm models of Kugler and Verhoogen (2012) and Baldwin and Harrigan (2011) by incorporating an endogenous quality choice of an intermediate good for a closed economy. Our variant incorporates quality complementarity with the limitation of two discrete input quality choices.

#### 3.1 Demand

The preferences of a representative consumer are given by a standard C.E.S. utility function over final goods indexed by  $\omega$ :

$$U = \left[ \int_{\omega \in \Omega} (q(\omega)^{\delta} x(\omega))^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where  $\Omega$  represents the mass of available final goods. The parameter  $\sigma$  captures the elasticity of substitution between varieties and is assumed to be greater than one,  $\sigma > 1$ . The quality of variety  $\omega$  is denoted by  $q(\omega)$  and  $x(\omega)$  is the quantity of good  $\omega$  consumed. Following Hallak (2006), the intensity of consumers' desire for quality is given by  $\delta$ . If  $\delta = 0$ , the model reverts back to a standard Melitz formulation since consumers do not value quality differences and therefore all firms would choose the lower cost intermediate input. Following Kugler and Verhoogen (2012), we interpret product quality as any attribute that the representative consumer values in a differentiated good.

The aggregate quality-adjusted price index is given by:

$$P = \left[ \int_{\omega \in \Omega} \left( \frac{p(\omega)}{q(\omega)^{\delta}} \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},$$
(2)

where  $p(\omega)$  is the price of variety  $\omega$ . From the maximization problem, we have a constant elasticity of demand function for any variety produced:

$$x(\omega) = Aq(\omega)^{\delta(\sigma-1)} p(\omega)^{-\sigma}, \qquad (3)$$

where  $A = IP^{\sigma-1}$  and I denotes aggregate income.

#### 3.2 Production

We assume an inelastic labor supply, denoted by L, earning a common wage that is normalized to one. There are two sectors: an intermediate input sector and a final goods sector with differentiated varieties.

The intermediate input sector produces inputs of two types of quality, indexed by  $\kappa = \{L, H\}$  for low and high, respectively, under perfect competition.<sup>6</sup> The production function for a given quality  $\kappa$  is given by the following constant returns to scale production function:

$$y_I^{\kappa}(l_I^{\kappa},\theta^{\kappa}) = \frac{l_I^{\kappa}}{\theta^{\kappa}},\tag{4}$$

where  $l_I^{\kappa}$  is the amount of labor producing input goods of quality  $\kappa$  and  $1/\theta^{\kappa}$  is labor productivity. Producing a low-quality intermediate input is assumed to entail lower costs than producing a high-quality intermediate input. Since what matters is the relative productivity of the two types of input goods, we normalize the labor productivity of low-quality inputs to one. Moreover, the labor productivity of high-quality inputs is assumed to be smaller,  $1/\theta^{H} < 1/\theta^{L} = 1$ . We follow Kugler and Verhoogen (2012) by assuming that final goods producers are price-takers and the price of the intermediate input equal the marginal cost of production:  $p^{H} = \theta^{H} > 1$  and  $p^{L} = \theta^{L} = 1$ .

Production in the final goods sector is characterized by two functions: one describing the production of physical output and the other describing the quality of the final good produced. Like in Melitz (2003), there is a continuum of firms producing physical output under monopolistic competition, each producing a different variety represented by subscript i. Firms combine labor and the intermediate input in a Cobb-Douglas production function:

$$y_i(l_i, x_{Ii}^{\kappa}, \lambda_i) = \frac{l_i^{1-\alpha}(x_{Ii}^{\kappa})^{\alpha}}{\lambda_i^{\beta}},\tag{5}$$

where  $l_i$  represents labor employed and  $x_{Ii}^{\kappa}$  is the quantity of intermediate input of quality  $\kappa$  used by the firm. Like in Nocco (2012),  $\alpha \in (0, 1)$  is the intermediate input share and represents a measure of the strength of vertical

 $<sup>^{6}</sup>$ The assumption of only two quality levels is a simplification to better demonstrate the effect of the Leontief-type production function of the final good quality. See Rogerson (1983) and Kranton (2003) for other models that differentiate between high- and low-quality producers.

linkages. Stronger vertical linkages in a sector implies that intermediate inputs are more important in the production of the final good. It can also be seen as a measure of product complexity. Complex products feature a higher number of intermediate inputs relative to labor in production.

Firms differ in their capabilities indexed by  $\lambda_i \in (0, 1]$ .<sup>7</sup> This capability parameter represents a firm's ability to implement the intermediate input in the production process. This can be interpreted as the quality of the firm's blueprint, design, self-produced parts, assembly, etc. Higher capabilities are assumed to affect a firm's marginal cost in a nonpositive way:

$$MC_i(\lambda_i) = \frac{\lambda_i^{\beta}(p^{\kappa})^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}},\tag{6}$$

where  $\beta$  represents the elasticity of quality, as it is referred to in Baldwin and Harrigan (2011), and  $p^{\kappa}$  is the price of intermediate input quality  $\kappa = \{L, H\}$ . Setting  $\beta = 0$  reduces the model to the standard heterogeneous firm trade model with firms being sorted by exogenous quality differences. We follow the empirical evidence provided by Kugler and Verhoogen (2012) and assume firms producing higher quality goods have higher marginal cost, all else equal, which implies that  $\beta > 0$ . marginal cost are also affected by the choice of intermediate input quality by each firm. Sectors with stronger vertical linkages,  $\alpha$ , will tend to have larger cost discrepancies between firms that employ low-quality and high-quality intermediate inputs.

With CES preferences, firms will choose the same profit maximizing markup such that the price they charge is equal to:

$$p_i(\lambda) = \frac{\sigma}{\sigma - 1} M C_i(\lambda).$$
(7)

Taking  $A = IP^{\sigma-1}$  as given in equation (3), the profit of a firm can be written as:

$$\pi_i(\lambda) = Bq_i^{\delta(\sigma-1)} \lambda_i^{\beta(1-\sigma)} (p^{\kappa})^{\alpha(1-\sigma)} - F_P, \qquad (8)$$

where  $B = (1/\sigma - 1)(\sigma - 1/\sigma)^{\sigma} [\alpha^{\alpha}(1 - \alpha)^{1-\alpha}]^{\sigma-1} A$  and  $F_P$  is a fixed production cost. Following Falvey et al. (2005), A and B represent market size and the

<sup>&</sup>lt;sup>7</sup>Melitz (2003) refers to this parameter as productivity, while models with quality heterogeneity across firms differ in its denotation. We follow Sutton (2007) and Kugler and Verhoogen (2012) and interpret  $\lambda_i$  as capability in order to account for its effect on both production cost and output quality.

extent of competition and are taken as given by individual firms. Firms that have higher quality goods,  $q_i$ , or lower marginal cost (lower  $\lambda_i$ ) will earn higher profits, all else equal.

#### 3.3 Output Quality and Input Choice

The quality of the final good is characterized by a Leontief-type production function:

$$q_i(\lambda_i) = \min\left[\lambda_i, \lambda_I^{\kappa}\right],\tag{9}$$

where  $\lambda_I^k$  is the quality of the intermediate input chosen by firm *i* and it is assumed that  $0 < \lambda_I^L < \lambda_I^H < 1$ .

Capability,  $\lambda_i$ , and intermediate input quality,  $\lambda_I^k$ , are perfect complements in generating output quality,  $q_i$ . This follows the O-ring theory by Kremer (1993) by assuming that the representative consumer values the quality of the final product by the lowest quality component, the quality of the intermediate input or the quality of the production process. Figure 2 shows how output quality varies over the range  $\lambda_i \in [0, 1]$ , where  $\hat{\lambda}$  is the firm that is indifferent between using the low and high-quality input.<sup>8</sup> A final product generated by a high-quality production process (high  $\lambda_i$ ) will be perceived as low-quality product if it contains a low-quality intermediate input. Similarly, a high-quality input in a low-quality production process will not improve the final product's quality perceived by the representative consumer.

Choosing a high-quality intermediate input can potentially increase the demand for a particular variety but also will increase the firm's marginal cost. The capability parameter must be larger than the low-quality input level for a firm to benefit from choosing the high-quality input. The input choice is characterized by the capability threshold,  $\hat{\lambda}$ , and is derived by equalizing profits from equation (8) when firms use high and low-quality inputs, respectively:

$$\hat{\lambda} = (p^H)^{\frac{\alpha}{\delta}} \lambda_I^L. \tag{10}$$

Firms with a capability parameter equal to the threshold capability level,  $\lambda$ , are indifferent in their input quality choice. Firms with a higher capability

<sup>&</sup>lt;sup>8</sup>Using a CES production function for a final product's quality,  $q_i = \left[\frac{1}{2}(\lambda_i)^{\mu} + \frac{1}{2}(\lambda_I^{\kappa})^{\mu}\right]^{\frac{1}{\mu}}$ , does not qualitatively alter results but makes the model less tractable.

Figure 2: Final good quality over capability parameter



parameter than the threshold level  $(\lambda_i > \hat{\lambda})$  will choose the high-quality input. An increase in the price of the high-quality input,  $p_I^H$ , or an increase in the perceived quality of the low-quality input,  $\lambda_I^L$ , will increase the quality threshold,  $\hat{\lambda}$ .<sup>9</sup>

#### 3.4 Firm Entry and Exit

As in Melitz (2003), there is a continuum of prospective entrants into the final goods sector. Each firm has to make an irreversible investment of  $F_E > 0$  to enter the market. Only after entry do firms discover their capability,  $\lambda_i$ , from a uniform *ex ante* distribution,  $g(\lambda)$ . The distribution has positive support over (0, 1] and has a continuous cumulative distribution,  $G(\lambda) = \lambda / \lambda^{max}$ , where  $\lambda^{max}$  is normalized to 1. Upon entry, a firm will decide to stay or exit the industry depending on whether the capability draw allows operating profits to be non-negative. We let  $\lambda^*$  denote the cutoff level for the firm with the

<sup>&</sup>lt;sup>9</sup>The threshold level,  $\hat{\lambda}$ , rises above the high-quality input level,  $\lambda_I^H$ , if  $p^H > (\lambda_I^H/\lambda_I^L)^{\delta/\alpha}$ . In that case, all firms will use the low-quality input, resulting in a unimodal profit distribution across firms.

lowest capability for which  $\pi = 0$ :

$$\pi(\lambda^*) = B(q(\lambda^*))^{\delta(\sigma-1)}(\lambda^*)^{\beta(1-\sigma)} - F_P = 0.$$
(11)

Since firms with a lower capability draw than the cutoff level,  $\lambda^*$ , will not generate operating profits, they will exit the market. Firms with a higher capability draw may operate in the industry. For the remainder of this paper, we consider the more interesting case of the cutoff level being smaller than the low input quality ( $\lambda^* < \lambda_i^L$ ) and the existence of only one cutoff.<sup>10</sup> Combining equations (9) and (11), we find the Zero Cutoff Profit Condition to be:

$$\pi(\lambda^*) = B(\lambda^*)^{(\delta-\beta)(\sigma-1)} - F_P = 0.$$
(12)

And solving for B as a function of the cutoff capability, we have:

$$B = F_P(\lambda^*)^{(\delta - \beta)(1 - \sigma)}.$$
(13)

Combining (8) and (13), we can define the profit of any firm in relation to the cutoff capability:

$$\pi_i(\lambda) = \left[ \left(\frac{\lambda_i}{\lambda^*}\right)^{(\sigma-1)(\delta-\beta)} \left(\frac{q_i}{\lambda_i}\right)^{\delta(\sigma-1)} (p^{\kappa})^{\alpha(1-\sigma)} - 1 \right] F_P.$$
(14)

There are two opposing effects of a firm's capability on its profits. The higher a firm's capability draw, the higher their marginal cost. On the other hand, a higher capability draw can lead to higher output quality valued by consumers and greater demand. In contrast to other models, the final output quality may depend on input quality. Equation (9) lets us define four profit functions for four ranges of capability draws:

$$\pi(\lambda) = \begin{cases} \left[ \left(\frac{\lambda_i}{\lambda^*}\right)^{\Psi} - 1 \right] F_P & \text{if } \lambda_i \in [\lambda^*, \lambda_I^L), \\ \left[ \left(\frac{\lambda_i}{\lambda^*}\right)^{\Gamma} \left(\frac{\lambda_I^L}{\lambda^*}\right)^{\delta(\sigma-1)} - 1 \right] F_P & \text{if } \lambda_i \in [\lambda_I^L, \hat{\lambda}), \\ \left[ \left(\frac{\lambda_i}{\lambda^*}\right)^{\Psi} \left(p^H\right)^{\alpha(1-\sigma)} - 1 \right] F_P & \text{if } \lambda_i \in [\hat{\lambda}, \lambda_I^H), \\ \left[ \left(\frac{\lambda_i}{\lambda^*}\right)^{\Gamma} \left(\frac{\lambda_I^H}{\lambda^*}\right)^{\delta(\sigma-1)} \left(p^H\right)^{\alpha(1-\sigma)} - 1 \right] F_P & \text{if } \lambda_i \in [\lambda_I^H, 1), \end{cases}$$
(15)

where  $\Psi = (\sigma - 1)(\delta - \beta)$  and  $\Gamma = \beta(1 - \sigma) < 0$ .

<sup>&</sup>lt;sup>10</sup>Up to four cutoff levels may be possible when  $0 < \beta < \delta$ . All possible types of distribution are discussed in Appendix A.

#### 3.5 Equilibrium

Since firms are free to enter the market, they will continue to enter until expected profits, net of entry costs, are driven to zero,  $E(\pi) = F_E$ . From equations (9) and (14), we can write this free entry condition as:

$$E(\pi) = \int_{0}^{1} \pi(\lambda) dG(\lambda) = \left( \int_{\lambda^{*}}^{\lambda_{I}^{L}} \left[ \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Psi} - 1 \right] dG(\lambda) + \int_{\lambda_{I}^{L}}^{\hat{\lambda}} \left[ \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Gamma} \left( \frac{\lambda_{I}}{\lambda^{*}} \right)^{\delta(\sigma-1)} - 1 \right] dG(\lambda) + \int_{\hat{\lambda}}^{\lambda_{I}^{H}} \left[ \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Psi} \left( p^{H} \right)^{\alpha(1-\sigma)} - 1 \right] dG(\lambda) + \int_{\lambda_{I}^{H}}^{1} \left[ \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Gamma} \left( \frac{\lambda_{I}}{\lambda^{*}} \right)^{\delta(\sigma-1)} \left( p^{H} \right)^{\alpha(1-\sigma)} - 1 \right] dG(\lambda) \right] F_{P} = F_{E}.$$
(16)

Equation (16) determines the cutoff capability level,  $\lambda^* < \lambda_I^L$ , when it is the unique cutoff.

In the extreme case where the capability draw does not affect a firm's marginal cost,  $\beta = 0$ , profits increase with output quality as shown in Figure 3. For low-quality producers, profits increase with the capability draw as long as  $\lambda_i < \lambda_I^L$  and remains constant over the range between  $\lambda_I^L$  and  $\hat{\lambda}$  as the quality of the final good does not change. Similarly, for high-quality producers profits increase for  $\lambda_i < \lambda_I^H$ . When the capability draw exceeds  $\lambda_I^H$ , profits once again remain constant as output quality does not change.

The more interesting case is when  $\beta \in (0, \delta)$ . A higher capability draw increases marginal cost, reducing firm profits. Output quality increases in the capability parameter if  $\lambda \in [\lambda^*, \lambda_I^L)$  and  $\lambda \in [\hat{\lambda}, \lambda_I^H)$ . Since  $\Psi > 0$ , we have  $\partial \pi / \partial \lambda_i > 0$  over those ranges.<sup>11</sup> For  $\lambda \in [\lambda_I^L, \hat{\lambda})$  or  $\lambda \in [\lambda_I^H, 1)$ , profits are decreasing with firm capability,  $\partial \pi_i / \partial \lambda_i < 0$ , given that  $\Gamma < 0$ . The increase of marginal cost cannot be compensated by an increase in output quality. The result is a bimodal profit distribution, as shown in Figure 4. For the remainder of the paper, we will focus on the more interesting case where  $\beta \in (0, \delta)$  is assumed.

<sup>&</sup>lt;sup>11</sup>If  $\beta > \delta$ , the exponent  $\Psi$  turns negative such that  $\partial \pi_i / \partial \lambda_i > 0$  for all  $\lambda_i$ . If  $\beta = \delta$ , the profit increase from higher capability cancels out with higher marginal cost when  $\lambda \in [\lambda^*, \lambda_I^L)$  and  $\lambda \in [\hat{\lambda}, \lambda_I^H)$ .

Figure 3: Profits over capability parameter  $\beta = 0$ 



Figure 4: Profits over capability parameter  $\beta \in (0, \delta)$ 



In contrast to other quality-heterogeneous firms models, we do not find firms to be more profitable with higher capability. Instead, firms with a capability draw close to their chosen input quality are the most profitable. The firms with the highest capability tend to be smaller and less profitable. This

view is supported by the findings of Holmes and Stevens (2014) who analyzed the North Carolina wood furniture industry. Large plants typically specialize on a high degree of standardization with mass-production techniques while small firms employ skilled labor to craft specialty products that are of higher quality and command a higher price.

#### 3.6 Intermediate Input Quality

In contrast to Kugler and Verhoogen (2012) where they assume a continuum of intermediate input quality levels, we limit the number of intermediate input quality levels,  $\lambda_I^{\kappa}$ , to a binary choice between low-quality and high-quality inputs. This limitation to discrete quality levels is an important aspect of our model. Firms will often have the choice between a limited number of suppliers offering discrete quality levels of intermediate goods. This may be interpreted as firms producing different goods that share identical inputs.<sup>12</sup>

The shape of the profit curve is affected by the location of the input quality levels, where  $0 < \lambda_I^L < \lambda_I^H < 1$ . An increase in  $\lambda_I^L$  increases both the cutoff level,  $\lambda^*$ , and the threshold level,  $\hat{\lambda}$ . Firms with the lowest capability draw lose market share to other low-quality firms due to the increase in their final output quality. Furthermore, previous high-quality producers may find it more profitable to reduce their marginal cost by choosing the new lowquality input. Resources and market share shift to low-quality producers, reducing profits of high-quality producers and forcing firms with the lowest capability to exit.

An increase in  $\lambda_I^H$ , meanwhile, increases profits of all high-quality producers without affecting the threshold level,  $\hat{\lambda}$ . This reduces profits of all low-quality producers, forcing the least capable firms to exit the market and increasing the cutoff level,  $\lambda^*$ . The gap between both intermediate input quality levels also affects the shape of the profit curve. The smaller the gap

<sup>&</sup>lt;sup>12</sup>We argue that in sectors with a non-unimodal distribution of output quality, there may be a continuum of input quality for a number of intermediate inputs. However, some intermediate inputs required for the production of the final good may be produced by a limited number of specialized suppliers. Thus, these intermediate inputs may only be available in a limited number of discrete quality levels. (As an example we considered the smart phone market in 2015, when high-cost phones prominently featured scratchresistant glass that was rarely implemented in low-cost phones.) Then, firms would match their input quality choice of intermediate inputs from a continuum of quality with the distinct quality levels. Therefore, all used intermediate inputs share the same quality that is determined by discrete quality levels.

between  $\lambda_I^L$  and  $\lambda_I^H$ , the more the distribution of profits appear unimodal and the variance of quality decreases. Increasing the gap strengthens the bimodal shape and increases the variance of quality among producing firms.

#### 3.7 Demand for Quality and Vertical Linkages

In contrast to the model in Melitz (2003), the equilibrium production cutoff is not the only indicator of industry structure and the highest profits are not generated by firms with the highest productivity. The shape of the profit curve is determined by three parameters. The demand side is represented by the intensity of consumers' desire for quality,  $\delta$ . If  $\delta = 0$ , the model falls back to the Melitz setup. If  $\delta > \beta > 0$ , the profit distribution becomes bimodal because there is higher demand for high-quality products. Firms that use high-quality intermediate inputs become more profitable when the demand for high-quality goods increases.

The supply side is represented by the sector-specific degree of vertical linkage,  $\alpha$ , and the price of the high-quality intermediate input,  $p^H$ . The more complex a product, represented by the strength of vertical linkages, the more inputs are required to produce the final good. If  $\alpha = 0$ , the model falls back to the setup of Baldwin and Harrigan (2011). A higher degree of vertical linkages leads to higher intermediate input requirements in the production process and, thus, the marginal cost in (6) increases only for firms using high-quality inputs. Therefore, the higher the degree of vertical linkages, the lower the profit of firms using high-quality inputs. Therefore, the higher degree of the high-quality intermediate input,  $p^H$ , can be thought of as being related to the level of development. The less developed an economy, the lower the technology and the labor productivity,  $1/\theta^H$ , in producing higher-quality inputs. Higher prices for the high-quality input leads to lower profits of high-quality firms. The effect of changes in any one of these parameters is shown in Figure 5.

In a closed economy, the degree of vertical linkage and the price of the high-quality intermediate input have opposite effects on the firm profit distribution compared to the intensity of consumers' demand for quality. If we consider a sector with strong vertical linkages, the higher  $\alpha$ , the larger the share of intermediate goods in the production costs. Therefore, firms using high-quality intermediate inputs face higher production costs and generate lower profits in a sector with strong vertical linkages. While vertical linkages are sector-specific, the price of the high-quality intermediate input can be considered country-specific. The less developed an economy, the higher is

Figure 5: Change in profits due to decrease in quality demand, increase in the price of the high-quality input, or strengthening of vertical linkages



the price of high-quality intermediate inputs. Thus, firm producing highquality goods would be less profitable in less developed economies.

Following Linder (1961), Fan (2005), and Hallak (2006) and assuming that consumers' demand for quality increases with income, we have that in a lowincome country, firms using high-quality intermediate inputs generate lower profits and have smaller market shares relative to high-quality firms in a highincome country. As a result, high-income countries would tend to specialize in high-quality products and low-income countries would tend to specialize in low-quality products. This is consistent with the empirical findings in Schott (2004), who assigns higher quality to higher unit price values, and Khandelwal (2010), who assigns higher quality to a larger product market share conditional on the price. Since the intensity of consumers' demand for quality and the price of the high-quality intermediate input correlate with income, the model predicts that high-quality firms in rich economies would be more profitable relative to those in poorer economies. This effect would be stronger in sectors with strong vertical linkages.

### 4 The Open Economy

In this section, we allow for trade between two symmetric economies. This symmetry implies that wages will be equalized across the countries.

If we were to assume that there are no trade costs on exported differentiated products, all firms will sell their products in both markets and face identical costs and demand. In this case, two open economies can be modeled as a closed economy with an increase in country size, L. As in Melitz (2003), there is no effect on firm level outcome. The capability cutoff, capability threshold, and profit function would all stay the same while the mass of firms will increase proportionally to the increase in country size L.

Assuming the existence of trade costs in the final goods sector will only allow a subset of firms to increase their profits by exporting. Trade costs are modeled as per-unit iceberg trade costs,  $\tau > 1$ , where  $\tau$  units of the final good must be shipped for every unit sold at its destination, as well as a fixed export cost,  $F_X > F_P$ . Iceberg trade costs increase the marginal cost of exporting firms so they generate exporting profits of:

$$\pi_{iX}(\lambda) = Bq_i^{\delta(\sigma-1)} \lambda_i^{\beta(1-\sigma)} \tau^{1-\sigma} (p^{\kappa})^{\alpha(1-\sigma)} - F_X, \qquad (17)$$

where  $\pi_{iX}(\lambda)$  are profits from exporting. Setting exporting profits equal to zero ( $\pi_{iX} = 0$ ) and using equation (13) to substitute for B, we can derive the general exporting capability cutoff as a function of  $\lambda^*$ :

$$\lambda_X^* = \left(\frac{F_X}{F_P}\right)^{\frac{1}{\Gamma}} \left(\frac{q(\lambda_X^*)^{\delta}}{\tau(p^{\kappa})^{\alpha}}\right)^{\frac{1}{\beta}} (\lambda^*)^{\frac{\beta-\delta}{\beta}}.$$
 (18)

Combining (13) and (17), we can rewrite exporting profits as:

$$\pi_{iX}(\lambda) = \left(\frac{\lambda_i}{\lambda^*}\right)^{\Gamma} \left(\frac{q_i}{\lambda^*}\right)^{\delta(\sigma-1)} \tau^{1-\sigma} (p^{\kappa})^{\alpha(1-\sigma)} F_P - F_X.$$
(19)

#### 4.1 Multiple Export Cutoffs

When  $\beta \in (0, \delta)$ , there may exist up to four export cutoffs. Due to the assumption that trade costs are relatively larger than the overhead production costs  $(\tau^{(\sigma-1)}F_X > F_P)$ , there are fewer firms exporting than serving the domestic market. We derive these four potential export cutoff levels by solving equation (18) for each of the previously defined ranges of firms:

$$\lambda_{X1}^{*} = \left(\frac{F_X}{F_P}\right)^{\frac{1}{\Phi}} \tau^{\frac{1}{\delta-\beta}} \lambda^{*} \qquad \text{if} \quad \lambda_i \in [\lambda^*, \lambda_I^L),$$

$$\lambda_{X2}^{*} = \left(\frac{F_X}{F_P}\right)^{\frac{1}{\Gamma}} \left(\frac{(\lambda_I^L)^{\delta}}{\tau}\right)^{\frac{1}{\beta}} (\lambda^*)^{\frac{\beta-\delta}{\beta}} \qquad \text{if} \quad \lambda_i \in [\lambda_I^L, \hat{\lambda}),$$

$$\lambda_{X3}^{*} = \left(\frac{F_X}{F_P}\right)^{\frac{1}{\Phi}} \left(\tau(p^H)^{\alpha}\right)^{\frac{1}{\delta-\beta}} \lambda^{*} \qquad \text{if} \quad \lambda_i \in [\hat{\lambda}, \lambda_I^H),$$

$$\lambda_{X4}^{*} = \left(\frac{F_X}{F_P}\right)^{\frac{1}{\Gamma}} \left(\frac{(\lambda_I^H)^{\delta}}{\tau(p^H)^{\alpha}}\right)^{\frac{1}{\beta}} (\lambda^*)^{\frac{\beta-\delta}{\beta}} \qquad \text{if} \quad \lambda_i \in [\lambda_I^H, 1).$$
(20)

The first export cutoff level,  $\lambda_{X1}^*$ , exists iff  $\pi_X(\lambda_I^L) > 0$ . If the most profitable low-quality firm  $(\lambda_I^L)$  exports, then  $\lambda_{X1}^* > \lambda^*$  exists. The second export cutoff level,  $\lambda_{X2}^*$ , exists iff  $\pi_X(\lambda_I^L) > 0$  and  $\pi_X(\hat{\lambda}) < 0$ . In this case, there are low-quality exporters but firms that are indifferent between using low or high-quality inputs only serve the domestic market. The third export cutoff,  $\lambda_{X3}^*$ , exists iff  $\pi_X(\lambda_I^H) > 0$  and  $\pi_X(\hat{\lambda}) < 0$ . There are high-quality exporters using high-quality inputs and, similarly to the second export cutoff, firms indifferent between low and high-quality inputs only serve the domestic market. Consequently, if  $\pi_X(\hat{\lambda}) > 0$ ,  $\lambda_{X2}^*$  and  $\lambda_{X3}^*$  cannot exist. Finally, the fourth export cutoff,  $\lambda_{X4}^*$ , only exists if  $\pi_X(\lambda_I^H) > 0$  and  $\pi_X(1) < 0$ . There are firms profitably exporting goods with high-quality inputs but firms with the highest capability draw only serve the domestic market.<sup>13</sup>

#### 4.2 Equilibrium

Similarly to the closed economy, we determine the production cutoff level,  $\lambda^*$ , by setting expected profits equal to zero. In an open economy, expected profits are larger if firms are allowed to export. We determine expected profits in an open economy by combining expected profits in a closed economy,  $\pi_D(\lambda)$ , from (16) and add expected profits of exporters,  $\pi_X(\lambda)$ :

<sup>&</sup>lt;sup>13</sup>Appendix B graphically shows the firm distributions for all potential cases when  $\tau > 1$ and  $F_X > F_P$ .

$$E(\pi) = \int_{0}^{1} \pi_{D}(\lambda) dG(\lambda) + \int_{0}^{1} \pi_{X}(\lambda) dG(\lambda)$$

$$= \int_{\lambda^{*}}^{1} \pi_{D}(\lambda) dG(\lambda) + \int_{\lambda^{*}_{X1}}^{\lambda^{L}_{I}} \left( \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Psi} \tau^{1-\sigma} F_{P} - F_{X} \right) dG(\lambda)$$

$$+ \int_{\lambda^{L}_{I}}^{\lambda^{*}_{X2}} \left( \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Gamma} \left( \frac{\lambda^{L}_{I}}{\lambda^{*}} \right)^{\delta(\sigma-1)} \tau^{1-\sigma} F_{P} - F_{X} \right) dG(\lambda)$$

$$+ \int_{\lambda^{*}_{X3}}^{\lambda^{*}_{I}} \left( \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Psi} \left( p^{H} \right)^{\alpha(1-\sigma)} \tau^{1-\sigma} F_{P} - F_{X} \right) dG(\lambda)$$

$$+ \int_{\lambda^{H}_{I}}^{\lambda^{*}_{X4}} \left( \left( \frac{\lambda_{i}}{\lambda^{*}} \right)^{\Gamma} \left( \frac{\lambda^{H}_{I}}{\lambda^{*}} \right)^{\delta(\sigma-1)} \left( p^{H} \right)^{\alpha(1-\sigma)} \tau^{1-\sigma} F_{P} - F_{X} \right) dG(\lambda) = F_{E}.$$
(21)

#### 4.3 Quality Polarization

As in Melitz (2003), compared to a closed economy, all firms incur a loss in domestic sales when we allow for bilateral trade. Only exporting firms can increase their sales, leading to higher revenues and a larger market share. Similarly, non-exporting firms incur a loss in profits. This leads to a rise in the cutoff level,  $\lambda^*$ , in an open economy compared to a closed economy. The least profitable firms, with a capability draw near the production cutoff in a closed economy, do not generate profits in an open economy and exit the market.<sup>14</sup>

Exporting firms generate higher sales but face additional trade costs. Therefore, many exporters will also incur a profit loss if their gains in exporting sales are not sufficient to counter the loss of profits from domestic sales. The subset of exporting firms with quality draws near their chosen input quality will be able to increase their profits from trade. This results in a new profit distribution as shown in Figure 6.

We refer to this result as quality polarization: a reallocation of market share and resources towards the modes of the distribution due to a change

<sup>&</sup>lt;sup>14</sup>The least profitable firms may also be located around the capability threshold level,  $\hat{\lambda}$ , and the maximum capability draw,  $\lambda^{max}$ . Their profit loss due to trade may be so high that they also exit the domestic market and multiple production cutoff levels are created, as shown in Appendix A. This shows that opening up for trade may also negatively impact on some high-quality producers.

Figure 6: Quality polarization



from autarky to free trade. Quality polarization strengthens previously existing patterns of quality and market share in a closed economy. The range of produced varieties declines and the range of traded varieties increases.

Traded goods are characterized by a range of product quality that is a fraction of the total range of quality produced in the domestic market. This may explain the differences in empirical evidence about the correlation between prices and firms size in trade data. While a wide range of quality might be produced within an economy, exported goods are allocated around the modes of the profit distribution. We identify eight possible cases in Appendix B that describe open economy equilibria with different ranges of quality of exported goods. If there exist only low-quality exporters, trade data would suggest a negative correlation between export prices and firm size. If there are only high-quality exporters, the correlation would be positive. In other cases with the presence of both high- and low-quality exporting firms, the correlation would be ambiguous and depend on the mass of firms in each group.

In previous heterogeneous firms trade models, the correlation between price and firm size in trade data has been explained with exporters having a larger market share than non-exporting firms. When  $\beta \in (0, \delta)$ , we find that the input quality choice affects the market share of firms as well. Holding profits equal, firms that use high-quality intermediate inputs have a smaller market share than firms that choose low-quality intermediate inputs due to the higher price they charge not being offset by the higher demand for quality. Choosing an equal input quality level, more profitable firms have a larger market share. The effect of input quality on market share strengthens with the price of the high-quality intermediate input and the degree of vertical linkages,  $\alpha$ . Therefore, we expect high-quality firms to be smaller in sectors with high product complexity or in less developed economies.

Like Foster et al. (2008), we find that firms self-select to export by their profitability that is, in turn, determined by the characteristics of the industry and the economy, as shown in Section 3. This aspect is equal to the findings by Antoniades (2015) which explains inconsistencies in trade data with sector-specific variations in the scope for quality differentiation. However, our approach can not only explain the inconsistencies in the trade data across sectors but can also explain the presence of bimodal distributions of quality in some sectors. Furthermore, income differences across countries, which can affect consumer quality preferences, may explain why high-income countries tend to produce goods with higher quality and higher complexity, as found in Kremer (1993).

#### 4.4 Trade Liberalization

Following Melitz (2003), we investigate the effect of trade liberalization through a reduction in trade costs. The notation of the open economy remains and we add primes (') to all variables in the new equilibrium.

A decrease in trade costs to  $\tau' < \tau$  induces a reduction of the export cutoffs  $\lambda_{X1}^*$  and  $\lambda_{X3}^*$  and an increase of the export cutoffs  $\lambda_{X2}^*$  and  $\lambda_{X4}^*$ . Simultaneously, the production cutoff,  $\lambda^*$ , increases to  $\lambda^{*'}$ . The increased exposure to trade forces the least profitable firms to exit while allowing more firms to export. All firms incur a loss in domestic sales and firms that do not export earn less profit. The decrease in profits for non-exporters may generate additional production cutoffs (see Appendix A). In this case, highquality firms that either charge too high a price or produce very low quality do not generate profits and exit the market. Exporting firms increase their revenue through international sales. The most profitable of these firms are able to earn higher profits.

The production and export cutoffs are similarly affected by a decrease in the fixed export cost,  $F_X$ . Domestic firms face stronger competition due to importing firms and the least profitable firms leave the market ( $\lambda^*$  rises). New firms enter the export market, resulting in smaller changes in the export cutoffs as described when iceberg trade cost are reduced. However, a decrease in  $F_X$  will not increase the market share or profits of already exporting firms. The market share of existing firms are reallocated to firms who find it profitable to export. The change in quality polarization is only caused by a selection effect of firms entering the export market.

Trade liberalization increases the degree of quality polarization. Domestic firms produce over a smaller range of quality and the range of quality of exporting firms increases with reductions in trade costs. The market share and profits of the most profitable firms increase while the least profitable firms exit the market.

## 5 Conclusions

In this paper, we examine inconsistencies in trade data regarding the correlation between output price and market share of traded goods. Previous research found this correlation to be positive in some sectors and negative in others. This lead to the development of two alternative explanations in theoretical literature: either the most productive or the highest output quality firms export. We draw from a rich data set of U.S. imports collected and aggregated by Amiti and Khandelwal (2013) and find the distribution of quality to be bimodal in 14 of 25 sectors with over 1,000 observations at the HS 6-digit classification. This evidence suggests that firms find it profitable to export different quality levels within sectors.

We address these findings by extending the heterogeneous firms trade model of Melitz (2003) with endogenous quality choice and quality complementarity inspired by the O-ring theory by Kremer (1993). Output quality is determined in a Leontief-type production function with the quality of the intermediate input and a firm specific quality parameter serving as inputs. Firms have a binary choice between two quality levels of intermediate inputs. When  $\beta \in (0, \delta)$ , firm profits increase with capability until output quality stagnates due to perfect complementarity. Profits then decline with capability since marginal cost continue to increase while the output price does not. We derive a threshold level above which firms find it profitable to switch to the high-quality intermediate input. The resulting distribution of profits is bimodal.

We find the intensity of consumers' desire for quality, product complexity,

and the economy's technology to determine the shape of the distribution of profits. This paper examines two symmetric open economies. Firms displace less profitable but not necessarily less productive or lower quality businesses. Trade liberalization leads to quality polarization, a reallocation of market share and resources towards the modes of the profit distribution. Thus, empirical findings may be explained by the variable that determined the shape of the distribution of profits.

We conclude with a caveat about the trade model. Allowing trade between symmetric countries leads to a tractable model. In the real world, countries differ in labor productivity as well as in preferences for quality. We touch on how differences in income and technology affect the equilibrium between symmetric countries. A straightforward extension of the model would be the introduction of asymmetric countries in order to examine how income and productivity differences affect observed patterns of trade.

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## Appendix A Multiple Production Cutoffs

In the case of  $\beta \in (0, \delta)$ , there exists the possibility of up to four production cutoffs. This may happen if firms with a capability draw equal to the capability threshold  $(\lambda_i = \hat{\lambda})$  or equal to the maximum capability  $(\lambda_i = \lambda^{max})$ generate no profits. In this case, firms choose not to produce and exit the domestic market. The three additional production cutoff levels are derived by setting profits from equation (14) equal to zero for the capability ranges  $\lambda_i \in [\lambda_I^L, \hat{\lambda}), \lambda_i \in [\hat{\lambda}, \lambda_I^H)$ , and  $\lambda_i \in [\lambda_I^H, 1)$ :

$$\lambda_{2}^{*} = \lambda_{I}^{L\frac{\delta}{\beta}} \lambda^{*\frac{\beta-\delta}{\beta}} \quad \text{if} \quad \lambda_{i} \in [\lambda_{I}^{L}, \hat{\lambda}),$$
  

$$\lambda_{3}^{*} = p^{H\frac{\alpha}{\delta-\beta}} \lambda^{*} \quad \text{if} \quad \lambda_{i} \in [\hat{\lambda}, \lambda_{I}^{H}),$$
  

$$\lambda_{4}^{*} = \left(\frac{\lambda_{I}^{H}}{p^{H^{\alpha}}}\right)^{\frac{\delta}{\beta}} \lambda^{*\frac{\beta-\delta}{\beta}} \quad \text{if} \quad \lambda_{i} \in [\lambda_{I}^{H}, 1).$$
(22)

There are seven different possible cases for the existence of the production cutoff levels in a closed economy as shown in Figure 7. In Figure 7a there is the standard case used in this paper with only one production cutoff,  $\lambda^*$ . In Figure 7b, least productive firms that use high-quality inputs exit the market so that there exist the cutoff level  $\lambda^*$  and  $\lambda_4^*$ . In Figures 7c and 7d firms with a capability draw equal to the threshold level,  $\hat{\lambda}$ , do not generate profits and exit the market while both low-quality and high-quality inputs using firms exist. Therefore, there we can also find the cutoff levels  $\lambda_2^*$  and  $\lambda_3^*$ . In Figures 7e and 7f there are only firms that use high-quality inputs. This eliminates the existence of the first two production cutoff levels  $\lambda^*$  and  $\lambda_1^*$ . And, finally, in Figure 7f there are only low-quality input firms so that only firms with a capability draw between the first two production cutoff levels  $\lambda^*$  and  $\lambda_1^*$  and  $\lambda_1^*$ 

It is crucial to identify which case applies to an economy for determining the equilibrium. The free entry condition from section 3.5 is modeled after the simplest case shown in Figure 7a. Equation (16) must be altered in each of the other six cases. Additional production cutoff levels can also be created by opening up for trade. If firms with a capability draw equal to the threshold level,  $\hat{\lambda}$ , or maximum capability level,  $\lambda^{max}$ , do not export, their market share and profits will decline. The incurred profit loss can be so large that a firm exits the domestic market.

Figure 7: Seven closed economy cases



(g) Two low-quality cutoffs

## Appendix B Potential Export Cases

As shown in Figure 8, there are eight different possible cases in an open economy. In Figure 8a, trade costs are too large for any firm to export profitably. In Figure 8b, only firms using low-quality intermediate inputs are exporting ( $\pi_X(\lambda_I^L) > 0$ ) so that there exist the two export cutoff levels  $\lambda_{X1}^*$ and  $\lambda_{X2}^*$ . In Figures 8c and 8d, only firms using the high-quality intermediate input are exporters. In the first figure, there exist the two export cutoff levels  $\lambda_{X3}^*$  and  $\lambda_{X4}^*$ . In the former we find  $\pi_X(1) > 0$  and, therefore, firms of the whole range of  $\lambda \in (\lambda_I^H, 1)$  export profitably.

In Figures 8e and 8f, there are two ranges of firms exporting, each a proportion of firms using either high-quality or low-quality intermediate inputs. Therefore, all four export cutoff levels exist, though in the second figure we find all firms across the range of  $\lambda \in (\lambda_I^H, 1)$  exporting as in Figure 8d. In the last two Figures 8g and 8h, there is one range of exporting firms across both input quality choices. In Figure 8g, we find the first and fourth export cutoff levels  $\lambda_{X1}^*$  and  $\lambda_{X4}^*$ . In the last figure, all exporting firms have a capability draw that is higher than the first export cutoff.

Figure 8: Eight open economy cases



