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Lagrangian and Hamiltonian BRST structures of the antisymmetric tensor gauge theory

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We study the Becchi-Rouet-Stora-Tyutin (BRST) structure of a self-interacting antisymmetric tensor gauge field, which has an on-shell null-vector gauge transformation. The Batalin-Vilkovisky covariant general formalism is briefly reviewed, and the issue of on-shell nilpotency of the BRST transformation is elucidated. We establish the connection between the covariant and the canonical BRST formalisms for our particular theory. Finally, we point out the similarities and differences with Witten's string field theory.

I. INTRODUCTION

As is well known,[1,2] the gauge transformation of the free string field theory admits an infinite tower of null vectors, leading to the appearance of states of an arbitrary ghost number in the quantum gauge-fixed action, coming from the ghost-for-ghost mechanism. In contrast, Witten's interacting string field theory has a gauge transformation with null vectors only on shell, that is to say, using the equations of motion of the classical field with a ghost number $-\frac{1}{2}$ (Refs. 3 and 4). This fact prevents one from using the naive Becchi-Rouet-Stora-Tyutin- (BRST-)covariant gauge-fixing procedure. In Ref. 3, this problem was solved by inspection and the quantum gauge-fixed action was constructed demanding that it be invariant under a guessed BRST transformation, which turned out to be nilpotent only on the equations of motion of the quantum action. This allowed us to introduce the whole tower of fields with arbitrary ghost number, making the transition to the free theory understandable. As was already mentioned in Ref. 3 and completely worked out in Refs. 5 and 6, the final result can be obtained in the framework of the Batalin-Vilkovisky covariant general formalism under the only assumption that the naive measure of the functional integral is BRST invariant, which has been shown to hold in Ref. 7.

In Ref. 8 the similarity between Witten's theory and the theory of a self-interacting antisymmetric tensor field[9] was traced, and the problem for the latter was solved along the lines of Ref. 3. In this paper we want to clarify the relation between the on-shell nilpotency of the BRST transformation of the quantum action and the existence of on-shell null vectors of the gauge transformation of the original theory, specializing our results to the antisymmetric tensor theory mentioned above. For this theory we also study the relation between the BRST generator obtained from the gauge-field action using Noether's first theorem and the BRST generator obtained from the knowledge of the canonical constraints and their relations. This is of current interest because it could give some insight into the canonical formalism of Witten's theory.

The paper is organized as follows. In Sec. II we review the Batalin-Vilkovisky formalism, and present the general result concerning the nilpotency of the BRST transformation. In Sec. III we apply the general results to the antisymmetric tensor, whose canonical structure is studied in Sec. IV, where the relation to the covariant (Lagrangian) formalism is commented upon. Finally, in Sec. V we set our conclusions and the relation to Witten's theory is discussed.

II. BATALIN-VILKOVISKY FORMALISM

As we have already said, the quantum actions of both the self-interacting antisymmetric tensor and Witten's string field can be written down using the covariant general formalism developed by Batalin and Vilkovisky.[10-12] Here we just want to set the notations and specialize the results to the particular kind of theory we are interested in. The general result concerning the nilpotency of the BRST transformation is also presented.

The fields appearing in the classical action $\mathcal{S}$ are denoted by ϕ^i:

$$\mathcal{S}=\mathcal{S}[\phi^i],\quad i=1,\ldots,n \tag{2.1}$$

and they are included in a larger set of quantum fields generally denoted by Φ^A:

$$\{\phi^i\}\subset\{\Phi^A\},\quad A=1,\ldots,N\ . \tag{2.2}$$

To each field we associate an antifield Φ^*_A with opposite statistics,

$$\epsilon(\Phi^A)\equiv\epsilon_A,\quad \epsilon(\Phi^*_A)=\epsilon_A+1\ (\mathrm{mod}\ 2)\ , \tag{2.3}$$

and this allows us to define an antibrackets in the phase space of fields and antifields:

$$(F,G)\equiv\frac{\partial_r F}{\partial\Phi^A}\frac{\partial_l G}{\partial\Phi^*_A}-\frac{\partial_r F}{\partial\Phi^*_A}\frac{\partial_l G}{\partial\Phi^A}\ . \tag{2.4}$$

The functional integral of the theory is given by

$$Z_\Psi=\int\exp\left[\frac{i}{\hbar}W_\Sigma[\Phi]\right]\prod_A d\Phi^A\ , \tag{2.5}$$

where $W_\Sigma[\Phi]$ is the restriction of $W[\Phi,\Phi^*]$ to a surface determined by

$$\Phi^* = \frac{\partial \Psi[\Phi]}{\partial \Phi} \ .$$

The functional $\Psi[\Phi]$, with odd statistics, is called the gauge-fixing fermion, and the functional integral Z_Ψ does not depend on its form, provided that Ψ satisfies some admissibility conditions and W is the solution of the equation

$$\tfrac{1}{2}(W,W) = i\hbar \Delta W \ , \tag{2.6}$$

where

$$\Delta \equiv \frac{\partial_r}{\partial \Phi^A}\frac{\partial_l}{\partial \Phi^*_A} \ . \tag{2.7}$$

Expanding W in powers of $\hbar$,

$$W = S + \sum_{n=1}^{\infty} \hbar^n W_n \ , \tag{2.8}$$

Eq. (2.6) gives, for the lowest term,

$$(S,S) = 0 \ . \tag{2.9}$$

This is called the master equation and its solution S is the main object in the Batalin-Vilkovisky formalism. If the naive measure $\prod_A d\Phi^A$ is invariant under the BRST transformations

$$\delta_\Psi \Phi^A = (-1)^{\epsilon_A} \frac{\partial_l S}{\partial \Phi^*_A}\Bigg|_\Sigma \tag{2.10}$$

then one can show that it is possible to set $W_n = 0$, $n \geq 1$. The solution to the master equation, when restricted to Σ, gives the full quantum action of the theory, whereas the W_n, $n \geq 1$, take care of the noninvariance of the naive measure under (2.10). What the Batalin-Vilkovisky formalism shows is that it is always possible to restore the BRST invariance of the whole functional integral by adding new terms to S, provided that now the BRST transformation is defined using W instead of S. Anyway, the measure is not well defined in the covariant formalism, because BRST-invariant factors, which would give new Feynman diagrams, can always be considered.[10,11,3] This issue can only be addressed in the framework of canonical formalism, where unitarity fixes everything.

The solution to the master equation has to meet two basic requirements. The first is the correctness of the classical limit, which essentially means that

$$S[\Phi,\Phi^*]\,|_{\Phi^*=0} = \mathcal{S}[\phi] \ . \tag{2.11}$$

The second is the nondegeneracy of the functional integral. This dictates the minimal content of the set of quantum fields as well as the admissibility conditions on the gauge fermion. All this depends on the stage of reducibility of the theory, which we are going to define now.

The classical action is supposed to have at least one stationary point ϕ_0,

$$\frac{\partial_r \mathcal{S}[\phi]}{\partial \phi^i}\Bigg|_{\phi_0} = 0 \ , \tag{2.12}$$

and to be regular in a neighborhood of ϕ_0. As we are dealing with a gauge theory, m_0 (bosonic and fermionic) Noether identities hold in a neighborhood of the stationary point:

$$\frac{\partial_r \mathcal{S}[\phi]}{\partial \phi^i} R^i_{\alpha_0}[\phi] = 0, \quad \alpha_0 = 1, \ldots, m_0 \ , \tag{2.13}$$

where $R^i_{\alpha_0}$ are regular functionals and $\epsilon(R^i_{\alpha_0}) = \epsilon_{\alpha_0} + \epsilon_i$. If they are independent the theory is called irreducible. Otherwise, there exist m_1 null vectors $Z^{\alpha_0}_{(1)\alpha_1}[\phi]$:

$$R^i_{\alpha_0} Z^{\alpha_0}_{(1)\alpha_1}\,|_{\phi_0} = 0, \quad \epsilon(Z^{\alpha_0}_{(1)\alpha_1}) = \epsilon_{\alpha_0} + \epsilon_{\alpha_1}, \quad \alpha_1 = 1, \ldots, m_1 \ . \tag{2.14}$$

Notice that, unlike (2.13), it is sufficient that these relations hold on shell. If the $Z^{\alpha_0}_{(1)\alpha_1}$ are independent, the theory is said to be a first-stage-reducible theory. This is the case we are interested in. From (2.13) and (2.14) it follows that

$$\operatorname{rank} \frac{\partial_l \partial_r \mathcal{S}}{\partial \phi^i \partial \phi^j}\Bigg|_{\phi_0} = n - (m_0 - m_1) \tag{2.15}$$

which is called the condition of completeness for first-stage theories. The set of quantum fields for these kinds of theories is

$$\phi^i, C^{\alpha_0}_{(0)}, C^{\alpha_1}_{(1)}, \bar{C}_{(0)\alpha_0}, \Pi_{(0)\alpha_0}, \bar{C}_{(1)\alpha_1}, \Pi_{(1)\alpha_1}, C'^{\alpha_1}_{(1)}, \Pi'^{\alpha_1}_{(1)} \ . \tag{2.16}$$

All these fields are assigned a ghost number according to

$$\begin{aligned} &\mathrm{gh}(\phi^i) = \mathrm{gh}(\Pi_{(0)\alpha_0}) = \mathrm{gh}(C'^{\alpha_1}_{(1)}) = 0 \ , \\ &\mathrm{gh}(C^{\alpha_0}_{(0)}) = -\mathrm{gh}(\bar{C}_{(0)\alpha_0}) \\ &\qquad = -\mathrm{gh}(\Pi_{(1)\alpha_1}) = \mathrm{gh}(\Pi'^{\alpha_1}_{(1)}) = 1 \ , \\ &\mathrm{gh}(C^{\alpha_1}_{(1)}) = -\mathrm{gh}(\bar{C}_{(1)\alpha_1}) = 2 \ . \end{aligned} \tag{2.17}$$

The gauge fermion has $\mathrm{gh}(\Psi) = -1$, and all the antifields are given ghost numbers consistently with $\Phi^*_A = \partial\Psi/\partial\Phi^A$. The statistics of a field (or antifield) are given by the statistics of its index plus the absolute value of its ghost number.

The solution to the master equation is written as

$$\begin{aligned} S[\Phi,\Phi^*] = {} & S[\Phi_{\mathrm{min}},\Phi^*_{\mathrm{min}}] + \bar{C}^{*\alpha_0}_{(0)} \Pi_{(0)\alpha_0} \\ & + \bar{C}^{*\alpha_1}_{(1)} \Pi_{(1)\alpha_1} + C'^*_{(1)\alpha_1} \Pi'^{\alpha_1}_{(1)} \ , \end{aligned} \tag{2.18}$$

where the "minimal" set of quantum fields is

$$\Phi^A_{\mathrm{min}} = \{\phi^i, C^{\alpha_0}_{(0)}, C^{\alpha_1}_{(1)}\} \ , \tag{2.19}$$

whereas the other fields are necessary for the gauge fixing. It is apparent that, if $S[\Phi_{\mathrm{min}},\Phi^*_{\mathrm{min}}]$ is a solution of the master equation, then $S[\Phi,\Phi^*]$ is a solution also. One can expand $S[\Phi_{\mathrm{min}},\Phi^*_{\mathrm{min}}]$ in powers of the antifields keeping a zero total ghost number. The lowest terms are[11]

$$S[\Phi_{\rm min},\Phi^*_{\rm min}]=\mathcal{S}+\phi^*_i R^i_{\alpha_0}C^{\alpha_0}_{(0)}+C^*_{(0)\alpha_0}(Z^{\alpha_0}_{(1)\alpha_1}C^{\alpha_1}_{(1)}+T^{\alpha_0}_{\beta_0\gamma_0}C^{\gamma_0}_{(0)}C^{\beta_0}_{(0)})$$
$$+C^*_{(1)\alpha_1}(A^{\alpha_1}_{\beta_1\alpha_0}C^{\alpha_0}_{(0)}C^{\beta_1}_{(1)}+F^{\alpha_1}_{\alpha_0\beta_0\gamma_0}C^{\gamma_0}_{(0)}C^{\beta_0}_{(0)}C^{\alpha_0}_{(0)})+\phi^*_i\phi^*_j(B^{ji}_{\alpha_1}C^{\alpha_1}_{(1)}+E^{ji}_{\alpha_0\beta_0}C^{\beta_0}_{(0)}C^{\alpha_0}_{(0)})$$
$$+2C^*_{(0)\alpha_0}\phi^*_i(G^{i\alpha_0}_{\alpha_1\beta_0}C^{\beta_0}_{(0)}C^{\alpha_1}_{(1)}+D^{i\alpha_0}_{\beta_0\gamma_0\delta_0}C^{\delta_0}_{(0)}C^{\gamma_0}_{(0)}C^{\beta_0}_{(0)})+\cdots\ . \tag{2.20}$$

For the usual first-stage theories no more terms are needed. All the coefficients are supposed to be functionals of the classical fields ϕ^i. Let us briefly comment on the meaning of the various terms appearing in (2.20). $\mathcal{S}$, $R^i_{\alpha_0}$, and $Z^{\alpha_0}_{(1)\alpha_1}$ have been previously defined. The $T^{\alpha_0}_{\beta_0\gamma_0}$ are the structure functions of the gauge algebra. The $F^{\alpha_1}_{\alpha_0\beta_0\gamma_0}$ take into account the modification of the Jacobi identity brought in by the presence of the null vectors $Z^{\alpha_0}_{(1)\alpha_1}$, and the $A^{\beta_1}_{\alpha_1\alpha_0}$ appear when the $Z^{\alpha_0}_{(1)\alpha_1}$ are field dependent. The remaining coefficients extend the previous relations off shell. For instance, the $E^{ji}_{\alpha_0\beta_0}$ appear when it is necessary to use the equations of motion in order to close the commutator of two gauge transformations (the theory is said to have an open algebra). The $D^{i\alpha_0}_{\beta_0\gamma_0\delta_0}$ plays a similar role with respect to the Jacobi identity and the $G^{i\alpha_0}_{\alpha_1\beta_0}$ are needed to close a higher-order relation. In the case of the antisymmetric tensor which we will consider, the most important coefficients are the $B^{ji}_{\alpha_0}$, which appear when the nonindependence of the $Z^{\alpha_0}_{(1)\alpha_1}$ holds only on shell. Thus, let us consider the case when all the higher corrections are zero except for the $B^{ji}_{\alpha_0}$:

$$S[\Phi_{\rm min},\Phi^*_{\rm min}]=\mathcal{S}+\phi^*_i R^i_{\alpha_0}C^{\alpha_0}_{(0)}+C^*_{(0)\alpha_0}Z^{\alpha_0}_{(1)\alpha_1}C^{\alpha_1}_{(1)}+\phi^*_i\phi^*_j B^{ji}_{\alpha_1}C^{\alpha_1}_{(1)}\ . \tag{2.21}$$

After substitution in the master equation, we get the relations

$$\frac{\partial_r\mathcal{S}}{\partial\phi^i}R^i_{\alpha_0}C^{\alpha_0}_{(0)}=0\ , \tag{2.22}$$

$$R^i_{\alpha_0}Z^{\alpha_0}_{(1)\alpha_1}C^{\alpha_1}_{(1)}-2\frac{\partial_r\mathcal{S}}{\partial\phi^j}B^{ji}_{\alpha_1}C^{\alpha_1}_{(1)}(-1)^{\epsilon_i}=0\ . \tag{2.23}$$

In the next section we will solve Eq. (2.23) in the case of the antisymmetric tensor.

Next, let us give the general form of the gauge fermion. It has to satisfy several conditions concerning its second derivatives in order to remove the degeneracy of the functional integral.[11]

For first-stage theories, the simplest form is

$$\Psi=\bar{C}_{(0)\alpha_0}\chi^{\alpha_0}[\phi]+\bar{C}_{(1)\alpha_1}\omega^{\alpha_1}_{\alpha_0}C^{\alpha_0}_{(0)}+\bar{C}_{(0)\alpha_0}\sigma^{\alpha_0}_{\alpha_1}C'^{\alpha_1}_{(1)}\ , \tag{2.24}$$

where the $\chi^{\alpha_0}[\phi]$ are the gauge conditions on the classical fields, and the $\omega^{\alpha_1}_{\alpha_0}$ and $\sigma^{\alpha_0}_{\alpha_1}$ are some maximal rank matrices which remove the degeneracy of the kinetic term of the first generation ghosts $C^{\alpha_0}_{(0)}$ and $\bar{C}_{(0)\alpha_0}$. This form of the gauge fermion gives rise to the so-called degenerate gauges, where the gauge conditions are enforced through δ functionals. Other forms are available in order to obtain Gaussian gauges.

To end this section, we report two general results concerning the BRST transformation. In the space of fields and antifields, we can define a "gauge-independent" BRST transformation in the following way. If $F=F[\Phi,\Phi^*]$ then

$$\delta F\equiv(-1)^{\epsilon_F}(F,S)\ , \tag{2.25}$$

where the $(-1)^{\epsilon_F}$ factor has been set to enforce the property

$$\delta(FG)=(\delta F)G+(-1)^{\epsilon_F}F(\delta G)\ . \tag{2.26}$$

This transformation is nilpotent due to the master equation, which also shows that S is BRST invariant. However, one finally eliminates the antifields and deals with the "gauge-dependent" BRST transformation (2.10) and the gauge-fixed quantum action

$$S_{\rm gf}[\Phi]=S\left[\Phi,\Phi^*=\frac{\partial\Psi}{\partial\Phi}\right] \tag{2.27}$$

which depends on the election of the gauge fermion Ψ. Now two questions must be posed: (1) is $S_{\rm gf}$ invariant under δ_Ψ and (2) is δ_Ψ nilpotent? The answer to the first question is affirmative:

$$\delta_\Psi S_{\rm gf}[\Phi]=0\ , \tag{2.28}$$

whereas with respect to the nilpotency we have the following result:

$$\delta^2_\Psi\Phi^A=(-1)^{\epsilon_B(\epsilon_A+1)}\frac{\partial_r S_{\rm gf}}{\partial\Phi^B}\left[\frac{\partial_l\partial_l S}{\partial\Phi^*_A\partial\Phi^*_B}\right]\Bigg|_\Sigma\ . \tag{2.29}$$

Both demonstrations are quite straightforward, the only nontrivial point being to realize that

$$\frac{\partial_r S}{\partial\Phi^*_B}\frac{\partial_r\partial_r\Psi}{\partial\Phi^A\partial\Phi^B}\frac{\partial_l S}{\partial\Phi^*_A}=0 \tag{2.30}$$

due to an antisymmetry property of its indices. These results are totally general and do not depend on the stage of reducibility of the theory. They rely entirely on (2.9), (2.10), and (2.27). Equation (2.29) can be understood in terms of the consistency of the gauge fixing $\Phi^*_A=\partial\Psi/\partial\Phi^A$ with the gauge-dependent BRST transformations $\delta\Psi\Phi^A$ and $\delta_\Psi\Phi^*_A$, in the sense of demanding

$$\delta_\Psi\left[\frac{\partial\Psi}{\partial\Phi^A}\right]=(-1)^{\epsilon_A}\frac{\partial_l S}{\partial\Phi^A}\bigg|_\Sigma . \qquad (2.31)$$

If the transformation was nilpotent before the gauge-fixing procedure, it has to remain nilpotent if the relations introduced by the gauge fixing do not contradict the transformation. It turns out that (2.31) is exactly equivalent to

$$\frac{\partial_r S_{\mathrm{gf}}}{\partial\Phi^A}=0 . \qquad (2.32)$$

Thus, the first factor in (2.29) is understood, and so is the second one, because if $S[\Phi,\Phi^*]$ is at most linear in the antifields, then $\delta\Phi^A=\delta_\Psi\Phi^A$.

The above results tell us that one can always define a BRST transformation depending only on the quantum fields (classical fields, ghost fields, and auxiliary fields) and construct a gauge-fixed action invariant under it. However, in general the nilpotency of that BRST transformation is assured to hold only on shell, using the equations of motion provided by S_{gf}. Nipotency off shell will be kept only if the solution to the master equation contains terms at most linear in the antifields. Terms at least quadratic in the antifields are brought in when the relations of the classical gauge algebra need the equations of motion of the classical action to close.

III. THE ANTISYMMETRIC TENSOR FIELD

Consider a self-interacting tensor field $B^a_{\mu\nu}$ in four dimensions, described by the Lagrangian density[8,9]

$$\mathcal{L}_B=-\tfrac{1}{2}B^a_{\mu\nu}F^{a\mu\nu}+\tfrac{1}{4}A^a_\mu A^{a\mu} , \qquad (3.1)$$

where A^a_μ is an auxiliary vector field with field strength

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+f^{abc}A^b_\mu A^c_\nu . \qquad (3.2)$$

The action is invariant under the gauge transformation

$$\delta B^{a\mu\nu}=\epsilon^{\mu\nu\lambda\sigma}(D_\lambda\mathcal{S}_\sigma)^a,\quad \delta A^{a\mu}=0 . \qquad (3.3)$$

The covariant derivative

$$D^{ab}_\mu=\delta^{ab}\partial_\mu+f^{acb}A^c_\mu \qquad (3.4)$$

satisfies

$$[D_\rho,D_\sigma]^{ab}=f^{adb}F^d_{\rho\sigma},\quad \epsilon^{\mu\nu\rho\sigma}(D_\nu F_{\rho\sigma})^a=0 . \qquad (3.5)$$

The equations of motion of $B^{a\mu\nu}$ give

$$F^{a\mu\nu}=0 \qquad (3.6)$$

while the rest of the equations relate $A^{a\mu}$ and $B^{a\mu\nu}$,

$$A^a_\mu+2(D^\nu B_{\nu\mu})^a=0 . \qquad (3.7)$$

The gauge transformation has a null vector on shell:

$$\mathcal{S}^a_\sigma=(D_\sigma\mathcal{S})^a\Longrightarrow\delta B^{a\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma}f^{adb}F^d_{\lambda\sigma}\mathcal{S}^b \qquad (3.8)$$

and the reducibility stops here, so we are dealing with a first-stage, on-shell reducible theory. In spite of the presence of the structure constants f^{abc}, our theory has an Abelian gauge algebra due to the fact that the vector field, which appears in the covariant derivative, does not transform. Therefore, we can use the expression (2.21) to find the solution to the master equation. The set of classical fields is

$$\phi^i=\{B^{a\mu\nu}(x),\,A^{a\mu}(x)\} \qquad (3.9)$$

while the $R^i_{\alpha_0}$, which are nonzero only when they refer to $B^{a\mu\nu}(x)$ are, according to (3.3),

$$R^{a\mu\nu(x);b\sigma(y)}=\epsilon^{\mu\nu\lambda\sigma}D^{ab}_\lambda(x)\delta(x-y) . \qquad (3.10)$$

From (3.8) we get $Z^{\alpha_1}_{(1)\alpha_0}$:

$$Z^{b(x);c(z)}_{(1)\sigma}=D^{bc}_\sigma(x)\delta(x-z) . \qquad (3.11)$$

Now we can substitute in (2.23) and obtain the $B^{ji}_{\alpha_1}$:

$$B^{b\lambda\sigma(x);a\mu\nu(z)}_{c(y)}=-\tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma}f^{abc}\delta(x-y)\delta(y-z) . \qquad (3.12)$$

The solution $S[\Phi,\Phi^*]$ to the master equation is thus

$$S[\Phi,\Phi^*]=\int dx[\mathcal{L}_B+B^*_{a\mu\nu}\epsilon^{\mu\nu d\sigma}(D_\lambda C_{(0)\sigma})^a+C^{*a\mu}_{(0)}(D_\mu C_{(1)})^a-\tfrac{1}{2}B^*_{a\mu\nu}B^*_{b\alpha\beta}\epsilon^{\mu\nu\alpha\beta}f^{abc}C^c_{(1)}+\bar{C}^{*a\mu}_{(0)}\Pi^a_{(0)\mu}+\bar{C}^{*a}_{(1)}\Pi^a_{(1)}+C'^{*a}_{(1)}\Pi'^a_{(1)}] . \qquad (3.13)$$

The gauge-independent BRST transformations produced by S are

$$\begin{aligned}&\delta B^{a\mu\nu}=\epsilon^{\mu\nu\lambda\sigma}(D_\lambda C_{(0)\sigma})^a-\epsilon^{\mu\nu\lambda\sigma}f^{abc}B^*_{b\lambda\sigma}C^c_{(1)} ,\\&\delta C^a_{(0)\mu}=-(D_\mu C_{(1)})^a,\quad \delta\bar{C}^a_{(0)\mu}=-\Pi^a_{(0)\mu},\\&\delta C^a_{(1)}=0,\quad \delta\bar{C}^a_{(1)}=\Pi^a_{(1)},\quad \delta C'^a_{(1)}=\Pi'^a_{(1)} ,\\&\delta\Pi^a_{(0)\mu}=\delta\Pi^a_{(1)}=\delta\Pi'^a_{(1)} ,\end{aligned} \qquad (3.14)$$

and

$$\begin{aligned}&\delta B^*_{a\mu\nu}=-\tfrac{1}{2}F^a_{\mu\nu},\quad \delta C^{*a}_{(0)\mu}=-(D_\lambda B^*_{\alpha\beta})^a\epsilon^{\alpha\beta\lambda}{}_\mu ,\\&\delta\bar{C}^{*a}_{(0)\mu}=0 ,\\&\delta C^{*a}_{(1)}=-\tfrac{1}{2}B^*_{c\mu\nu}B^*_{b\alpha\beta}\epsilon^{\mu\nu\alpha\beta}f^{cba}-(D_\mu C^{*\mu}_{(0)})^a ,\\&\delta\bar{C}^{*a}_{(1)}=0=\delta C'^{*a}_{(1)} ,\\&\delta\Pi^{*a}_{(0)\mu}=\bar{C}^{*a\mu}_{(0)},\quad \delta\Pi^{*a}_{(1)}=-\bar{C}^{*a}_{(1)},\quad \delta\Pi'^{*a}_{(1)}=-C'^{*a}_{(1)} .\end{aligned} \qquad (3.15)$$

In order to get the full quantum action, we have to choose a gauge fermion. The simplest Ψ having the form (2.24) one can use is

$$\Psi=\int dx\,(\bar{C}^{\,a}_{(0)\mu}\partial_\nu B^{a\nu\mu}+\bar{C}^{\,a}_{(1)}\partial^\mu C^a_{(0)\mu}+\bar{C}^{\,a}_{(0)\mu}\partial^\mu C'^a_{(1)})\ . \tag{3.16}$$

However, we have in mind to write down the BRST charge Q using Noether's first theorem, and then express the result in canonical terms. In order to do that, we have to be able to substitute all the velocities appearing in Q using the Legendre transformation provided by $\mathcal{L}_{\rm gf}$. It turns out that not all the admissible Ψ's allow us to carry out this program, and this is the case for Ψ in (3.16). Had we worked with the dual of our $B^{a\mu\nu}$, (3.16) would have been a good election.

Instead of (3.16) we choose

$$\Psi=\int dx\,(\epsilon^{\mu\nu\lambda\sigma}\bar{C}^{\,a}_{(0)\mu}\partial_\nu B^a_{\lambda\sigma}+\bar{C}^{\,a}_{(1)}\partial^\mu C^a_{(0)\mu}+\bar{C}^{\,a}_{(0)\mu}\partial^\mu C'^a_{(1)})\ . \tag{3.17}$$

The quantum gauge-fixed action is then

$$S_{\rm gf}[\Phi]=\int dx[-\tfrac{1}{2}B^a_{\mu\nu}F^{a\mu\nu}+\tfrac{1}{4}A^a_\mu A^{a\mu}-2\partial^\alpha\bar{C}^{\,a\beta}_{(0)}(D_\alpha C_{(0)\beta}-D_\beta C_{(0)\alpha})^a-\partial^\mu\bar{C}^{\,a}_{(1)}(D_\mu C_{(1)})^a$$
$$+2\epsilon_{\mu\nu\alpha\beta}f^{abc}\partial^\mu\bar{C}^{\,a\nu}_{(0)}\partial^\alpha\bar{C}^{\,b\beta}_{(0)}C^c_{(1)}+\epsilon^{\mu\nu\lambda\sigma}\partial_\nu B^a_{\lambda\sigma}\Pi^a_{(0)\mu}+\partial^\mu C'^a_{(1)}\Pi^a_{(0)\mu}+\partial^\mu C^a_{(0)\mu}\Pi^a_{(1)}-\partial^\mu\bar{C}^{\,a}_{(0)\mu}\Pi'^a_{(1)}] \tag{3.18}$$

and the gauge-dependent BRST transformation is the same as that in (3.14) except for

$$\delta_\Psi B^{a\mu\nu}=\epsilon^{\mu\nu\lambda\sigma}(D_\lambda C_{(0)\sigma})^a+2f^{abc}(\partial^\mu\bar{C}^{\,b\nu}_{(0)}-\partial^\nu\bar{C}^{\,b\mu}_{(0)})C^c_{(1)}\ . \tag{3.19}$$

We can check that $\delta_\Psi S_{\rm gf}=0$, and that

$$\delta^2_\Psi B^{a\mu\nu}=f^{abc}\epsilon^{\mu\nu\lambda r}(-\tfrac{1}{2}F^b_{\lambda\sigma}+\epsilon_{\alpha\beta\lambda\sigma}\partial^\alpha\Pi^{b\beta}_{(0)})C^c_{(1)}=f^{abc}\epsilon^{\mu\nu\lambda\sigma}\frac{\delta_r S_{\rm gf}}{\delta B^{b\lambda r}}C^c_{(1)} \tag{3.20}$$

according to (2.28) and (2.29).

Next we compute the BRST charge Q, associated to δ_Ψ and $S_{\rm gf}$ using Noether's first theorem. To this end we need

$$\delta_\Psi\mathcal{L}_{\rm gf}=\partial^\mu[\tfrac{1}{2}\epsilon_{\mu\nu\lambda\sigma}C^{ar}_{(0)}F^{a\lambda\nu}+2\Pi^a_{(0)\nu}(D_\mu C^\nu_{(0)}-D^\nu C_{(0)\mu})^a-\Pi^a_{(1)}(D_\mu C_{(1)})^a-4\epsilon_{\mu\nu\alpha\beta}f^{abc}\Pi^{a\nu}_{(0)}\partial^\alpha\bar{C}^{\,b\beta}_{(0)}C^c_{(1)}+\Pi^a_{(0)\mu}\Pi'^a_{(1)}]\ . \tag{3.21}$$

A straightforward calculation gives $(i,j=1,2,3)$

$$Q=\int d\mathbf{x}\{\epsilon^{0ijk}[F^a_{ij}+2(D_i\Pi_{0j})^a]C^a_{(0)k}-(D_iC_{(1)})^a\bar{\mathcal{P}}^{\,ai}_{(0)}+f^{abc}\bar{\mathcal{P}}^{\,bi}_{(0)}C^c_{(1)}\Pi^a_{0i}-\tfrac{1}{4}\epsilon^{0ijk}\mathcal{P}^a_{(0)k}\Pi^a_{ij}-\mathcal{P}'^a_{(1)}\mathcal{P}^{a0}_{(0)}+\mathcal{P}^a_{(1)}\bar{\mathcal{P}}^{\,a0}_{(0)}\}\ , \tag{3.22}$$

where

$$\Pi^a_{ij}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial B^{aij}}=-2\epsilon_{0ijk}\Pi^{ak}_{(0)},\quad i<j\ ,$$
$$\Pi^a_{0i}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial\dot{B}^{a0i}}=0\ ,$$
$$\bar{\mathcal{P}}^{\,a\mu}_{(0)}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial\dot{C}^{\,a}_{(0)\mu}}=2(\dot{\bar{C}}^{\,a\mu}_{(0)}-\partial^\mu\bar{C}^{\,a0}_{(0)})+g^{0\mu}\Pi^a_{(1)}\ ,$$
$$\mathcal{P}^{a\mu}_{(0)}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial\dot{C}^{\,a}_{(0)\mu}}$$
$$=-2(D_0C^\mu_{(0)}-D^\mu C^0_{(0)})^a$$
$$+4\epsilon^{0\mu\alpha\beta}f^{abc}\partial_\alpha\bar{C}^{\,b}_{(0)\beta}C^c_{(1)}-g^{0\mu}\Pi'^a_{(1)}\ ,$$
$$\mathcal{P}^a_{(1)}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial\dot{\bar{C}}^{\,a}_{(1)}}=-(D_0C_{(1)})^a\ ,$$
$$\mathcal{P}'^a_{(1)}=\frac{\partial_l\mathcal{L}_{\rm gf}}{\partial\dot{C}'^a_{(1)}}=\Pi^a_{(0)0}\ . \tag{3.23}$$

Q is the canonical generator of the transformation δ_Ψ, in the sense that

$$\delta_\Psi\Phi^A=FL^*\{\Phi^A,Q\}\ , \tag{3.24}$$

where $\{\ ,\ \}$ are graded Poisson brackets[13,14] and FL^* stands for the use of relations (3.23).

To what extent does expression (3.22) for Q depend on gauge fixing? As we have already said, one must be careful when choosing Ψ, because in general one is not able to express the resultant Q in terms of canonical variables, at least if one does not want to work in a submanifold of the whole phase space. However, we know[14–16] that Ω, the BRST charge of a given gauge theory, can be directly written in canonical formalism without any gauge fixing, taking only into account the Hamiltonian constraints of the original gauge-invariant theory and their relations. In the next section, we will see that Q has the same form as the canonical Ω.

IV. HAMILTONIAN ANALYSIS

In order to study the canonical structure of the gauge-invariant theory (3.1), we have to define the canonical momenta

$$\Pi^a_{\mu\nu}=\frac{\partial\mathcal{L}_B}{\partial\dot{B}^{a\mu\nu}},\quad \Pi^a_\mu=\frac{\partial\mathcal{L}_B}{\partial\dot{A}^{a\mu}} \tag{4.1}$$

which bring in a set of primary constraints,

$$\Pi^a_{0i}=0,\quad \Pi^a_{ij}=0,\quad \Pi^a_0=0,\quad \rho^a_i\equiv\Pi^a_i+B^a_{0i}=0\ , \tag{4.2}$$

and a Hamiltonian density

$$\mathcal{H}_c=\tfrac{1}{2}B^a_{ij}F^{aij}-\tfrac{1}{4}A^a_0A^{a0}-\tfrac{1}{4}A^a_iA^{ai}-B^a_{0i}\partial^iA^a_0 +f^{abc}B^a_{0i}A^b_0A^{ci}\ . \tag{4.3}$$

We are considering only the independent variables B^a_{0i}, B^a_{ij}, $i<j$, A^a_0, and A^a_i. However, whenever spatial indexes are summed, all of them are supposed to run from 1 to 3.

We consider the primary Hamiltonian density[17]

$$\mathcal{H}_p=\mathcal{H}_c+\lambda^{a0i}\Pi^a_{0i}+\lambda^{aij}\Pi^a_{ij}+\lambda^{ai}\rho^a_i+\lambda^{a0}\Pi^a_0 \tag{4.4}$$

and apply the stability algorithm. The primary constraints Π^a_{0i} and ρ^a_i form a second-class subset and their stability determines the up to now canonically unknown functions λ^{a0i} and λ^{ai}:

$$\lambda^a_{0i}=-\tfrac{1}{2}A^a_i+f^{abc}B^b_{0i}A^c_0-(D_kB_{ik})^a\ , \tag{4.5}$$

$$\lambda^a_i=(D_iA_0)^a\ . \tag{4.6}$$

Now we can compute $\dot{\Pi}^a_{ij}$ and $\dot{\Pi}^a_0$ using $\mathcal{H}^{*1}_c$, which is obtained[18] by adding to $\mathcal{H}_c$ the primary pieces (4.4) which have been canonically determined in (4.5) and (4.6). We get

$$\dot{\Pi}^a_{ij}=-\tfrac{1}{2}F^a_{ij}-(D_i\Pi_{0j})^a,\quad i,j=1,2,3\ , \tag{4.7}$$

$$\dot{\Pi}^a_0=\tfrac{1}{2}A^a_0+f^{abc}B^b_{0i}\Pi^c_{0i}-(D_i\Pi_i)^a\ . \tag{4.8}$$

We notice that $\dot{\Pi}^a_0$ is second class,

$$\{\dot{\Pi}^a_0(x),\Pi^b_0(y)\}=\tfrac{1}{2}\delta^{ab}\delta(\mathbf{x}-\mathbf{y})\ ,$$

and its stability $\ddot{\Pi}^a_0=0$ will determine λ^{a0}, which we do not need anyhow. Instead, $\dot{\Pi}^a_{ij}$ is first class because its brackets with all the other constraints produce, at most, terms proportional to Π^a_{0i}, which is a constraint. In addition, the algebra of first-class constraints Π^a_{ij} and $\dot{\Pi}^a_{ij}$ is Abelian, which was already noticed in the Lagrangian analysis.

One can see that

$$\dot{\Pi}^a_{ij}=-f^{abc}A^b_0\dot{\Pi}^c_{ij}\ , \tag{4.9}$$

so we end with a set of first-class constraints,

$$\Pi^a_{ij},\quad -\dot{\Pi}^a_{ij}=\tfrac{1}{2}F^a_{ij}+(D_i\Pi_{0j})^a\ , \tag{4.10}$$

and a set of second-class ones: Π^a_{0i}, ρ^a_i, Π^a_0, $\dot{\Pi}^a_0$. Let us introduce the notation

$$T^{ak}\equiv-\epsilon^{0kij}\dot{\Pi}^a_{ij} \tag{4.11}$$

which is a combination of first-class secondary constraints. We observe that

$$Z^{ab}_kT^{bk}\equiv(D^{ab}_k-f^{abc}\Pi^c_{0k})T^{bk} =-\epsilon^{0kij}f^{abc}(D_k\Pi_{0i})^b\Pi^c_{0j}\ , \tag{4.12}$$

so we get an on-shell relation between the constraints, because the right-hand side is (quadratically) vanishing on shell. This is the canonical counterpart of the on-shell null vectors of the gauge transformation, Eq. (3.8).

The knowledge of the first-class constraints (4.10) and their relations (4.9) and (4.12) allows us to construct the generators of the gauge transformations, applying an algorithm developed in Refs. 19 and 20. The ordinary generator is

$$G_{\text{ord}}=\int d\mathbf{x}[\epsilon^{0kij}\Pi^a_{ij}\dot{\mathcal{S}}^a_k-\epsilon^{0kij}(\dot{\Pi}^a_{ij}+f^{abc}A^b_0\Pi^c_{ij})\mathcal{S}^a_k] \tag{4.13}$$

which produces

$$\delta B^{aij}=\epsilon^{ij0k}(D_0\mathcal{S}_k)^a\ ,\qquad \delta B^{a0i}=\epsilon^{0ijk}(D_j\mathcal{S}_k)^a\ . \tag{4.14}$$

The part of δB^{aij} which is missing is brought in due to the existence of the relation (4.12), which allows us to consider

$$G_{\text{ext}}=\int d\mathbf{x}(Z_k\epsilon^{0kij}\Pi_{ij})^a\mathcal{S}^a_0 \tag{4.15}$$

which gives indeed

$$\delta B^{aij}=\epsilon^{ijk0}(D_k\mathcal{S}_0)^a\ . \tag{4.16}$$

The generator in (4.15) is an on-shell solution of the algorithm mentioned above.

Next we use the general canonical formalism of Fradkin and Vilkovisky[14,18,21] to construct the canonical BRST generator. Taking the first-class constraints (4.11) and their relations in (4.12) we write

$$\Omega=\int d\mathbf{x}[T^{ai}C^a_{(0)i}+\bar{\mathcal{P}}^{ai}_{(0)}(Z_iC_{(1)})^a +\epsilon^{0ijk}\mathcal{P}^a_{(0)k}\Pi^a_{ij}+\mathcal{P}'^a_{(1)}\mathcal{P}^{a0}_{(0)}+\mathcal{P}^a_{(1)}\bar{\mathcal{P}}^{a0}_{(0)}]\ . \tag{4.17}$$

Up to redefinitions of the ghosts by scaling factors, this has the same form as the BRST generator (3.22) coming from a gauge-fixed Lagrangian. However, one must not forget that the momenta in (3.22) have a meaning related to a given Legendre transformation, while those in (4.17) are just the coordinates in an extended phase space.

V. CONCLUSIONS

Using the Batalin-Vilkovisky covariant general formalism, we have recovered the result that a BRST transformation depending only on the quantum fields can always be defined and that a gauge-fixed BRST-invariant quantum action can always be constructed, see Eq. (2.28), no matter how involved the gauge algebra of the theory.

The BRST transformation is always nilpotent on shell, using the equations of motion of the gauge-fixed action, but it is nilpotent off shell only when the solution to the master equation is at most linear in the antifields, the relation being given by Eq. (2.29). This happens when the gauge algebra of the classical theory is verified off shell, as terms at least quadratic in the antifields appear when one needs the equations of motion of the classical theory to close the relations.

The antisymmetric tensor we have considered is an on-shell first-stage-reducible theory. We have performed the Lagrangian analysis and, after fixing the gauge, we have obtained the BRST generator using Noether's first theorem. This BRST generator can be given a canonical form by means of the Legendre transformation associated with the gauge-fixed action.

The canonical structure of the theory has been studied, and, in addition to first-class constraints associated with gauge invariance, second-class constraints have been found. The first-class constraints are on-shell nonindependent, which in this case means that a combination of them is identically zero up to a quadratic combination of second-class constraints. Using the general canonical formalism developed by Fradkin and Vilkovisky, we have constructed the BRST canonical generator out of the knowledge of the first-class constraints and their relations, with no gauge-fixing input. The result turns out to coincide formally with the generator obtained via the Lagrangian formalism. This relationship depends crucially on the election of the gauge fermion. For an arbitrary gauge fixing, one is not able to express the velocities appearing in the Lagrangian Q in terms of the canonical variables.

With respect to the relation to Witten's theory the similarity with the antisymmetric tensor holds as far as both theories are reducible on shell, but Witten's theory is infinitely reducible on shell (always with respect to the equations of motion of the classical field), besides being non-Abelian.[3]

In addition, Witten's theory in its usual form is an infinite-order action, with derivatives of arbitrary order.[22,23,24,4] A canonical theory for such singular systems is presently not at hand, in spite of some recent progress.[25,26] It has been argued[27] that Witten's theory can be reduced to an ordinary theory if one uses the time component of the midpoint as the evolution parameter, and a canonical analysis has been carried out in Ref. 28. This analysis shows up the presence of second-class constraints, tracing a parallelism to our antisymmetric tensor. The formulation of Witten's theory in midpoint coordinates has received a deeper analysis in Refs. 29 and 30.

Note added. After completion of this work, we received Ref. 31, where the Batalin-Vilkovisky formalism is also applied to the antisymmetric tensor, in agreement with our results when they overlap.

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