BRST-invariant path integral for a spinning relativistic particle

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The propagator of a relativistic spinning particle is calculated using the Becchi-Rouet-Stora-Tyutin- (BRST-)invariant path-integral formalism of Fradkin and Vilkovisky. The spinless case is considered as an introduction to the formalism.

I. INTRODUCTION

The Becchi-Rouet-Stora-Tyutin (BRST) transformations in gauge-invariant field theories were first discovered as symmetries of the quantum Lagrangians.[1] The Hamiltonian version was constructed later.[2,3] The BRST formalism has also been used in the first-quantized approach to particles[4] and strings,[5] as well as in the construction of associated field theories.[6] The unification of BRST, Parisi-Sourlas, and space-time symmetries has recently been achieved.[7]

The propagators of spinless and spinning relativistic particles have been derived.[8] In this paper we want to compute the same propagators using the BRST-invariant path-integral method of Batalin, Fradkin, and Vilkovisky[2] in order to further understand the BRST formalism in the first-quantization approach. The spinning particle contains four first-class constraints associated with reparametrization and supersymmetry invariance and five second-class constraints brought in by the first-order character of the equations of motion of the Grassmann odd variables. The organization of the paper is as follows. In Sec. II we consider the case of the spinless relativistic particle as an introduction to the formalism. Then in Sec. III we deal with the more involved case of the spinning relativistic particle. We summarize our results in Sec. IV.

II. THE BRST PATH INTEGRAL FOR THE SPINLESS RELATIVISTIC PARTICLE

The action for the free relativistic scalar particle is

$$S[x]=-m\int_{\tau_a}^{\tau_b}d\tau\sqrt{\dot{x}^2}\ . \tag{1}$$

The canonical momenta

$$p_\mu=-\frac{\partial L}{\partial \dot{x}^\mu}=\frac{m\dot{x}_\mu}{\sqrt{\dot{x}^2}},\quad \mu=0,1,\ldots,D-1 \tag{2}$$

give rise to a vanishing canonical Hamiltonian and a first-class constraint

$$\phi=p^2-m^2 \tag{3}$$

which are consequences of the reparametrization invariance of (1).

The canonical action is then

$$S[x,p,\lambda]=\int_{\tau_a}^{\tau_b}d\tau[-\dot{x}p-\lambda(p^2-m^2)]\ . \tag{4}$$

Using the BRST-invariant canonical formulation of Batalin, Fradkin, and Vilkovisky we consider the action[2,3]

$$S_{\rm eff}=\int_{\tau_a}^{\tau_b}d\tau(-\dot{x}p+\dot{\lambda}\pi+\dot{\eta}^aP_a-H_{\rm eff})\ , \tag{5}$$

where

$$H_{\rm eff}=H_c-\{\Psi,\Omega\}=-\{\Psi,\Omega\}\ . \tag{6}$$

Ψ is the so-called gauge fermion, which has a ghost number -1 and Ω is the BRST charge.

Let us rewrite the ghost variables as

$$\eta^a=(-iP,C),\quad P_a=(+i\bar{C},\bar{P})\ ,$$

with nonvanishing Poisson brackets

$$\{\bar{P},C\}=\{C,\bar{P}\}=\{P,\bar{C}\}=\{\bar{C},P\}=-1\ .$$

The expression for the BRST charge Ω is

$$\Omega=-iP\pi+C\phi\ . \tag{7}$$

As we are interested in the relativistic gauge[9] $\dot{\lambda}=0$, the gauge fermion Ψ is chosen to be

$$\Psi=\bar{P}\lambda\ . \tag{8}$$

Plugging all this into (6), the effective action (5) becomes

$$S_{\rm eff}=\int_{\tau_a}^{\tau_b}[-\dot{x}p+\dot{\lambda}\pi+\dot{P}\bar{C}+\dot{C}\bar{P}-i\bar{P}P -\lambda(p^2-m^2)]d\tau\ . \tag{9}$$

The transition amplitude for a particle going from x_a at τ_a to x_b at τ_b, with the BRST-invariant boundary conditions[3]

$$\pi(\tau_b)=\pi(\tau_a)=\bar{C}(\tau_b)=\bar{C}(\tau_a)=C(\tau_b)=C(\tau_a)=0 \tag{10}$$

is given by

$$G=\int[d\mu]\exp(iS_{\rm eff})\ , \tag{11}$$

where

$$[d\mu]=[dx][dp][d\lambda][d\pi][d\bar{P}][dP][d\bar{C}][dC] .$$

Let us compute the bosonic part of the path integral:

$$G_b=\int [dp][dx][d\pi][d\lambda]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau[-\dot{x}p+\dot{\lambda}\pi-\lambda(p^2-m^2)]\right] . \tag{12}$$

After integration over p_μ and π we are left with

$$G_b=\int [dx][d\lambda]\delta[\dot{\lambda}(\tau)]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau\left[\frac{\dot{x}^2}{4\lambda}+\lambda m^2\right]\right] . \tag{13}$$

Since $\delta[\dot{\lambda}(\tau)]$ selects the constants paths $\lambda(\tau)=\lambda_0$, the functional integration becomes an ordinary one over λ_0. Thus

$$G_b=\int d\lambda_0[dx]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau\left[\frac{\dot{x}^2}{4\lambda_0}+\lambda_0 m^2\right]\right] . \tag{14}$$

The integration over x^μ is easily done by the classical shift procedure:

$$G_b=\int d\lambda_0[4\pi i\lambda_0(\tau_b-\tau_a)]^{-D/2}\exp\left[i\left[\frac{(x_b-x_a)^2}{4\lambda_0(\tau_b-\tau_a)}+\lambda_0(\tau_b-\tau_a)m^2\right]\right] . \tag{15}$$

Next, let us consider the ghost contribution

$$G_g=\int [d\bar{P}][dP][d\bar{C}][dC]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau(-P\dot{\bar{C}}+\dot{C}\bar{P}-i\bar{P}P)\right] . \tag{16}$$

The integration over $\bar{P}$ and P is Gaussian and leads to

$$G_g=\int [d\bar{C}][dC]\exp\left[-\int_{\tau_a}^{\tau_b} d\tau\dot{\bar{C}}\dot{C}\right] \tag{17}$$

which can also be computed by the shift method

$$G_g=-(\tau_b-\tau_a)\exp\left[-\frac{(\bar{C}_b-\bar{C}_a)(C_b-C_a)}{\tau_b-\tau_a}\right]=-(\tau_b-\tau_a) . \tag{18}$$

As it must be, the transition amplitude is independent of the unphysical parameter[10] $(\tau_b-\tau_a)$:

$$G(x_b-x_a)=G_g G_b=-\int \frac{dT}{(4\pi iT)^{D/2}}\exp\left[i\left[\frac{(x_b-x_a)^2}{4T}+m^2T\right]\right] , \tag{19}$$

where $T=\lambda_0(\tau_b-\tau_a)$. Although the ghosts are not coupled to the physical degrees of freedom, their contribution is essential in canceling the τ dependence. This last integration will be performed only over positive values of T (Ref. 9).

Next we pass to momentum space and get

$$\tilde{G}(p',p)=\delta^{(D)}(p'-p)\int d^Dx\, e^{ipx}G(x)=-\delta^{(D)}(p'-p)\int_0^\infty dT\exp[-iT(p^2-m^2)] \tag{20}$$

which gives us the well-known free scalar relativistic propagator

$$\tilde{G}(p',p)=-\frac{i\delta^{(D)}(p'-p)}{p^2-m^2-i\epsilon} . \tag{21}$$

III. THE BRST PATH INTEGRAL FOR THE SPINNING RELATIVISTIC PARTICLE

The action for the spinning relativistic particle[10] is

$$S=\tfrac{1}{2}\int_{\tau_a}^{\tau_b} d\tau\left[\frac{\dot{x}^2}{e}+em^2-i(\varepsilon^\mu\dot{\varepsilon}_\mu-\varepsilon_5\dot{\varepsilon}_5)-i\chi\left[\frac{\varepsilon^\mu\dot{x}_\mu}{e}-m\varepsilon_5\right]\right] , \tag{22}$$

where x_μ, e are even variables and ε_μ, ε_5, χ are odd. From their canonical momenta

$$\begin{aligned} p_\mu &= -\frac{\partial L}{\partial \dot{x}^\mu} = -\frac{1}{e}\left[\dot{x}_\mu - \frac{i}{2}\chi\varepsilon_\mu\right], \\ \pi_\mu &= \frac{\partial L}{\partial \dot{\varepsilon}^\mu} = \frac{i}{2}\varepsilon_\mu, \quad \pi_e = \frac{\partial L}{\partial \dot{e}} = 0, \\ \pi_5 &= \frac{\partial L}{\partial \dot{\varepsilon}_5} = -\frac{i}{2}\varepsilon_5, \quad \pi_\chi = \frac{\partial L}{\partial \dot{\chi}} = 0, \end{aligned} \tag{23}$$

we get the canonical Hamiltonian

$$H_c = \frac{e}{2}(p^2 - m^2) - \frac{i}{2}\chi(\varepsilon p + m\varepsilon_5), \tag{24}$$

five second-class constraints

$$\varphi_\mu = \pi_\mu - \frac{i}{2}\varepsilon_\mu, \quad \{\varphi_\mu, \varphi_\nu\} = ig_{\mu\nu}, \quad \varphi_5 = \pi_5 + \frac{i}{2}\varepsilon_5, \quad \{\varphi_5, \varphi_5\} = -i, \tag{25}$$

and four first-class constraints

$$\chi_1 = \pi_e, \quad \Phi_1 = -\frac{i}{2}(\varepsilon p + m\varepsilon_5) - \tfrac{1}{2}p^\mu\varphi_\mu + \tfrac{1}{2}m\varphi_5, \quad \chi_2 = \pi_\chi, \quad \Phi_2 = \frac{1}{2}(p^2 - m^2). \tag{26}$$

We use the graded symplectic structure[11]

$$\begin{aligned} -\{x_\mu, p_\nu\} &= \{p_\nu, x_\mu\} = -\{\varepsilon_\mu, \pi_\nu\} = -\{\pi_\nu, \varepsilon_\mu\} = g_{\mu\nu}, \\ 1 &= \{e, \pi_e\} = -\{\pi_e, e\} = -\{\varepsilon_5, \pi_5\} = -\{\pi_5, \varepsilon_5\} = -\{\chi, \pi_\chi\} = -\{\pi_\chi, \chi\}. \end{aligned} \tag{27}$$

Notice that the form of the constraint Φ_1 is different from the usual Dirac constraint.[10] Using the Dirac brackets

$$\{\varepsilon_\mu, \varepsilon_\nu\}^* = ig_{\mu\nu}, \quad \{\varepsilon_5, \varepsilon_5\}^* = -i, \tag{28}$$

we can eliminate π_μ and π_5. The canonical action is given by

$$S = \int_{\tau_a}^{\tau_b} d\tau \left[-\dot{x}p + \frac{i}{2}(\dot{\varepsilon}^\mu\varepsilon_\mu - \dot{\varepsilon}_5\varepsilon_5) + \dot{\chi}\pi_\chi + \dot{e}\pi_e - \chi\Phi_1 - e\Phi_2\right]. \tag{29}$$

To construct the BRST effective action, let us introduce the ghosts

$$\eta^a = (P_1, -iP_2, C^1, C^2), \quad P^a = (-\bar{C}^1, i\bar{C}^2, \bar{P}_1, \bar{P}_2), \quad \{\bar{P}_\alpha, C^\beta\} = \{P_\alpha, \bar{C}^\beta\} = -\delta_\alpha^\beta. \tag{30}$$

The BRST charge is

$$\Omega = P_1\pi_\chi - iP_2\pi_e + C^1\Phi_1 + C^2\Phi_2 - \frac{i}{4}(C^1)^2\bar{P}_2. \tag{31}$$

$\bar{P}_1$, P_1, $\bar{C}^1$, C^1 are even, while $\bar{P}_2$, P_2, $\bar{C}^2$, C^2 are odd. The gauge-fixing fermion

$$\Psi = \bar{P}_1\chi + \bar{P}_2 e \tag{32}$$

leads to the relativistic gauge $\dot{\chi} = \dot{e} = 0$. The transition amplitude will be computed with the BRST-invariant boundary conditions

$$\pi_\chi = \pi_e = C^\alpha = \bar{C}^\alpha = 0 \quad \text{at } \tau = \tau_a, \tau_b \tag{33}$$

and

$$p^\mu(\tau_a) = p_a^\mu, \quad \varepsilon^\mu(\tau_a) = \varepsilon_a^\mu, \quad \varepsilon_5(\tau_a) = \varepsilon_{5a}, \quad p^\mu(\tau_b) = p_b^\mu, \quad \varepsilon^\mu(\tau_b) = \varepsilon_b^\mu, \quad \varepsilon_5(\tau_b) = \varepsilon_{5b}. \tag{34}$$

The effective action may be written as

$$S_{\text{eff}} = \int_{\tau_a}^{\tau_b} d\tau \left[x\dot{p} + \frac{i}{2}(\dot{\varepsilon}^\mu\varepsilon_\mu - \dot{\varepsilon}_5\varepsilon_5) + \dot{\chi}\pi_\chi + \dot{e}\pi_e - \chi\Phi_1 - e\Phi_2 - \dot{\bar{C}}^\alpha P_\alpha + \dot{C}^\alpha\bar{P}_\alpha - \bar{P}_1 P_1 - i\bar{P}_2 P_2 + \frac{i}{2}\chi C^1\bar{P}_2\right]. \tag{35}$$

The corresponding amplitude will be

$$\langle p_b^\mu \varepsilon_b^\mu \varepsilon_{5b}, \tau_b | p_a^\mu \varepsilon_a^\mu \varepsilon_{5a}, \tau_a \rangle = \int [dp][dx][d\varepsilon][d\varepsilon_5][d\pi_e][de][d\pi_\chi][d\chi][d\bar{P}_\alpha][dP_\alpha][d\bar{C}^\alpha][dC^\alpha]\exp(iS_{\text{eff}}). \tag{36}$$

Several remarks are in order. (a) This amplitude is not the physical amplitude from which we shall compute the Dirac propagator; however, it will be useful as an intermediate step. (b) The states $|p_a^\mu \varepsilon_a^\mu \varepsilon_{5a}, \tau_a\rangle$ are not eigenstates of the operators $\hat{\varepsilon}_a^\mu, \hat{\varepsilon}_{5a}$ (Ref. 12, p. 256); therefore, the amplitude does not generate contradictions with the principles of quantum mechanics.

The integrations over x^μ, π_e, and π_χ lead to delta functions of the derivatives of their conjugate variables, so the functional integrations over p^μ, e, and χ reduce to $\delta(p_b - p_a)$ and ordinary integrations over e_0 and χ_0, respectively.

The integration over the odd and even ghosts amounts to just a constant:

$$\int [d\bar{P}_\alpha][dP_\alpha][d\bar{C}^\alpha][dC^\alpha]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau \left[P_2\dot{\bar{C}}^2 + \dot{C}^2\bar{P}_2 - i\bar{P}_2P_2 + \frac{i}{2}\chi_0 C^1\bar{P}_2 - P_1\dot{\bar{C}}^1 + \dot{C}^1\bar{P}_1 - \bar{P}_1P_1\right]\right]$$
$$= \int [d\bar{C}^\alpha][dC^\alpha]\exp\left\{i\int_{\tau_a}^{\tau_b} d\tau\left[-i\dot{\bar{C}}^2\left[\dot{C}^2 - \frac{i}{2}\chi_0 C^1\right] - \dot{\bar{C}}^1\dot{C}^1\right]\right\} = -1\ . \quad (37)$$

In spite of this trivial contribution, the amplitude will be τ independent.

The remaining integrations are

$$\langle p_b^\mu \varepsilon_b^\mu \varepsilon_{5b}, \tau_b | p_a^\mu \varepsilon_a^\mu \varepsilon_{5a}, \tau_a\rangle = -\delta(p_b - p_a)\int_0^\infty de_0 \int d\chi_0 \exp\left[-\frac{ie_0}{2}(\tau_b - \tau_a)(p^2 - m^2)\right]$$
$$\times \int [d\varepsilon_\mu][d\varepsilon_5]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau\left[\frac{i}{2}(\dot{\varepsilon}^\mu\varepsilon_\mu - \dot{\varepsilon}_5\varepsilon_5) + \frac{i\chi_0}{2}(p\varepsilon + m\varepsilon_5)\right]\right]. \quad (38)$$

In order to proceed it will be useful to compute the functional integration:

$$\langle \varepsilon_b^M, \tau_b | \varepsilon_a^M, \tau_a\rangle = \int [d\varepsilon_M]\exp\left[i\int_{\tau_a}^{\tau_b} d\tau\left[\frac{i}{2}\varepsilon^N\dot{\varepsilon}_N + i\eta^N\varepsilon_N\right]\right], \quad (39)$$

where η^M is, in general, a τ-dependent external source. Note also that the states $|\varepsilon_a^M, \tau_a\rangle$ are not eigenstates of the operator $\hat{\varepsilon}_a^M$ (Ref. 12).

Performing a discretization of the τ interval in N equal parts it may be seen that we obtain different results depending on whether N is odd[12] or even:[13]

$$N \text{ even:}\quad \langle \varepsilon_b, \tau_b | \varepsilon_a, \tau_a\rangle = \prod_{M=0}^{D} \delta\left[\varepsilon_a^M - \varepsilon_b^M + \int_{\tau_a}^{\tau_b} d\tau\, \eta^M(\tau)\right]\exp\left[-\tfrac{1}{2}\varepsilon_b^N\varepsilon_{aN} - \tfrac{1}{2}\int_{\tau_a}^{\tau_b} d\tau\,\eta^N(\tau)\int_{\tau_a}^{\tau} d\tau'\,\eta_N(\tau')\right], \quad (40a)$$

$$N \text{ odd:}\quad \langle \varepsilon_b, \tau_b | \varepsilon_a, \tau_a\rangle = \exp\left[\tfrac{1}{2}\varepsilon_b^M\varepsilon_{aM} + \tfrac{1}{2}(\varepsilon_b + \varepsilon_a)^M\int_{\tau_a}^{\tau_b} d\tau\,\eta_M(\tau) - \tfrac{1}{2}\int_{\tau_a}^{\tau_b} d\tau\,\eta^M(\tau)\int_{\tau_a}^{\tau} d\tau'\,\eta_M(\tau')\right]. \quad (40b)$$

This feature shows that the path integration (39) gives rise to two different scalar products:[13]

$$N \text{ even:}\quad \langle \varepsilon_b | \varepsilon_a\rangle = \prod_{M=0}^{D} \delta[\varepsilon_a^M - \varepsilon_b^M]\ , \quad (41a)$$

$$N \text{ odd:}\quad \langle \varepsilon_b | \varepsilon_a\rangle = \exp(\tfrac{1}{2}\varepsilon_b^M\varepsilon_{aM})\ . \quad (41b)$$

From (39) we see that the matrix elements of $\hat{\varepsilon}_M$ can be obtained through functional derivatives of the transition amplitude with respect to the external source η^M:

$$N \text{ even:}\quad \langle \varepsilon_b | \hat{\varepsilon}_M | \varepsilon_a\rangle = (-1)^D \lim_{\tau_b \to \tau_a} \frac{\delta}{\delta\eta^M(\tau)}\langle \varepsilon_b, \tau_b | \varepsilon_a, \tau_a\rangle$$
$$= (-1)^{D+1-M}\prod_{\substack{N=0\\ N\neq M}}^{D}(\varepsilon_{aN} - \varepsilon_{bN})\exp(-\tfrac{1}{2}\varepsilon_b^S\varepsilon_{aS})\ , \quad (42a)$$

$$N \text{ odd:}\quad \langle \varepsilon_b | \hat{\varepsilon}_M | \varepsilon_a\rangle = -\lim_{\tau_b \to \tau_a} \frac{\delta}{\delta\eta^M(\tau)}\langle \varepsilon_b, \tau_b | \varepsilon_a, \tau_a\rangle$$
$$= \tfrac{1}{2}(\varepsilon_b^M + \varepsilon_a^M)\langle \varepsilon_b | \varepsilon_a\rangle\ . \quad (42b)$$

From these expressions we can see that $|\varepsilon_a\rangle$ are not eigenstates of the operator $\hat{\varepsilon}_M$.

The transition amplitude can be written for both cases, N odd and even, as

$$\langle \varepsilon_b, \tau_b | \varepsilon_a, \tau_a\rangle = \left\langle \varepsilon_b \left| \exp\left[\hat{\varepsilon}^M\int_{\tau_a}^{\tau_b} d\tau\,\eta_M(\tau)\right]\right| \varepsilon_a\right\rangle \exp\left[-\tfrac{1}{2}\int_{\tau_a}^{\tau_b} d\tau\,\eta^M(\tau)\int_{\tau_a}^{\tau} d\tau'\,\eta_M(\tau')\right]. \quad (43)$$

From this expression, deriving functionally twice with respect to the external sources one can see that the expected anticommutation relations are satisfied:

$$\langle \varepsilon_b | [\hat{\varepsilon}_M, \hat{\varepsilon}_N]_+ | \varepsilon_a \rangle = \langle \varepsilon_b | \varepsilon_a \rangle g_{MN} \ . \tag{44}$$

We want to use $|\varepsilon\rangle$ as a basis to build the Hilbert space of physical states $\mathcal{H}$. In order to do so we need to have a resolution of the identity for the states $|\varepsilon\rangle$. However, it turns out that this is not possible using the odd scalar product only, so we will restrict ourselves to the even scalar product. In such a case we have

$$\langle \varepsilon_b | \varepsilon_a \rangle = \int \langle \varepsilon_b | \varepsilon \rangle d^D\varepsilon \langle \varepsilon | \varepsilon_a \rangle \ . \tag{45}$$

The wave function of a physical state $|\psi\rangle$, $\psi(\varepsilon) = \langle \varepsilon | \psi \rangle$ can be written as an expansion in terms of ε_M with complex coefficients.

Since

$$\langle \psi' | \psi \rangle = \int \langle \psi' | \varepsilon \rangle d^D\varepsilon \langle \varepsilon | \psi \rangle \tag{46}$$

we define $\langle \psi | \varepsilon \rangle$ such that we get a Hilbert space with positive norm

$$\langle \psi' | \psi \rangle = \langle \psi | \psi' \rangle^*, \quad \langle \psi | \psi \rangle \geq 0 \ . \tag{47}$$

The dimension of $\mathcal{H}$ is 2^{D+1}; the number of independent coefficients in a generic expansion of $\psi(\varepsilon)$. Our aim is to get a representation of the operators $\hat{\varepsilon}_\mu$ and $\hat{\varepsilon}_5$ in this space. With this purpose we define a new basis $|\varphi_k\rangle$ $(k=1,\ldots,2^{D+1})$ of states belonging to $\mathcal{H}$ and satisfying

$$\sum_k \langle \varepsilon | \varphi_k \rangle \langle \varphi_k | \varepsilon' \rangle = \langle \varepsilon | \varepsilon' \rangle \ . \tag{48}$$

It is easy to see that the matrix $(\hat{\varepsilon}_M)_{kl} = \langle \varphi_k | \hat{\varepsilon}_M | \varphi_l \rangle$ provides a representation of the algebra (44). This means that we have a reducible representation of the Clifford algebra of dimension 2^{D+1}.

Now we can apply the previous discussion and results to the computation of the amplitude (38). Using (43) we obtain

$$\begin{aligned} \langle p_b^\mu \varepsilon_b^\mu \varepsilon_{5b}, \tau_b | p_a^\mu \varepsilon_a^\mu \varepsilon_{5a}, \tau_a \rangle &= \frac{\delta(p_b - p_a)}{\frac{i}{2}(\tau_b - \tau_a)(p_a^2 - m^2 - i\epsilon)} \int d\chi_0 \left[\langle \varepsilon_b | \varepsilon_a \rangle + \langle \varepsilon_b | (p_a^\mu \hat{\varepsilon}_\mu + m\hat{\varepsilon}_5) | \varepsilon_a \rangle \frac{\chi_0}{2}(\tau_b - \tau_a) \right] \\ &= -\frac{i\delta(p_b - p_a)}{p_a^2 - m^2 - i\epsilon} \langle \varepsilon_b | (p_a^\mu \hat{\varepsilon}_\mu + m\hat{\varepsilon}_5) | \varepsilon_a \rangle \ . \end{aligned} \tag{49}$$

Finally, writing $\hat{\varepsilon}_\mu = (1/\sqrt{2})\gamma_5\gamma_\mu$, $\hat{\varepsilon}_5 = (1/\sqrt{2})\gamma_5$ the physical transition will be

$$\langle p_b \psi', \tau_b | p_a \psi, \tau_a \rangle = -\frac{i}{\sqrt{2}} \frac{\delta(p_b - p_a)}{p_a^2 - m^2 - i\epsilon} \langle \psi' | \gamma_5(\not{p}_a + m) | \psi \rangle \ . \tag{50}$$

This is not the usual form of the Dirac propagator. This is due to the first-class structure of Dirac constraint Φ_1 in $\mathcal{H}$, i.e.,

$$[\hat{\Phi}_1, \hat{\Phi}_1]_+ = \tfrac{1}{2}\hat{\Phi}_2 \ . \tag{51}$$

This standard form is obtained multiplying by γ_5.

IV. CONCLUSIONS

We have considered the BRST-invariant path-integral method for spinless and spinning relativistic free particles to get the Klein-Gordon and Dirac propagators, respectively. The contribution of the ghost part in the spinless case is essential in order to find a kernel which does not depend on the unphysical parameter $(\tau_b - \tau_a)$. In the spinning case, the contributions of the fermionic and bosonic ghosts cancel each other and produce just a constant. However, the amplitude is again independent of $(\tau_b - \tau_a)$. Both in the spinless and spinning cases, the BRST-invariant boundary conditions play a crucial role.

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