Some aspects of the $N=2$ superstring

Joaquim Gomis*
Department of Physics, Indiana University, Bloomington, Indiana 47405
(Received 3 October 1988)

The string model with $N=2$ world-sheet supersymmetry is approached via ghosts, Becchi-Rouet-Stora-Tyutin cohomology, and bosonization. Some amplitudes involving massless scalars and vectors are computed at the tree level. The constraints of locality on the spectrum are analyzed. An attempt is made to "decompactify" the model into a four-dimensional theory.

I. INTRODUCTION

The $N=2$ superstring, introduced by Ademollo *et al.*[1,2] has not attracted too much interest because the critical dimension is 2 (Ref. 1). Recent studies have focused on the representations of the $N=2$ superconformal algebra,[3–13] the applications to Calabi-Yau compactification,[14,15] the relation between world-sheet and space-time supersymmetry,[16–18] and various other topics.[19–21]

Our purpose in this paper is to carry out the ghost-extended quantization of the open $N=2$ model and discuss related topics. Section II reviews the model and the Becchi-Rouet-Stora-Tyutin (BRST) quantization. Vertex operators for the massless scalar and the massless vector are constructed in Sec. III. In Sec. IV we calculate Yukawa couplings and four-point amplitudes for these states. When massless vectors appear in $D=2$ particle theories, they may be eliminated, at the cost of introducing nonlocal interactions. Although they contain no degrees of freedom, it is more natural not to eliminate them. In string theory, the same is likely to be true.

Section V discusses the twisted sectors.[1,22] They contain $D=2$ fermions. The requirement of locality imposes severe Gliozzi-Scherk-Olive- (GSO-)type constraints on the physical spectrum.

In Sec. VI we try to "decompactify" the model by regrouping the two scalar fields in the $N=2$ theory, $X^\mu(z)$ and $Y^\mu(z)$ ($\mu=1,2$), into a four-component object. One motivation in studying the $N=2$ system was to try to construct a four-dimensional string theory. We come quite close to succeeding but encounter an obstruction (see Sec. V).

Our notation is that of Ref. 23; $z=\exp(\sigma+i\tau)$, $\alpha'=\frac{1}{2}$, $g^{\mu\nu}=\eta^{\mu\nu}=\left(\begin{smallmatrix}-1&0\\0&1\end{smallmatrix}\right)$ in Minkowski space and $\eta^{\mu\nu}=\left(\begin{smallmatrix}1&0\\0&0\end{smallmatrix}\right)$ in Euclidean space. We work in the latter and eventually Wick rotate results. The $N=2$ superspace variable Z stands for (z,θ_-,θ_+). The Appendix presents the decomposition of the BRST operator into various ghost charges and the two picture-changing operators.

II. COVARIANT QUANTIZATION OF THE $N=2$ STRING

The $N=2$ superstring is covariantly quantized by introducing ghosts and obtaining the BRST charge.[24–28] In $N=2$ superspace, there are two Grassmann variables θ_α^+ and θ_α^-, two supersymmetry generators Q_α^+ and Q_α^-, and two covariant derivatives D_α^+ and D_α^- (Ref. 1). Here, α is a world-sheet spinor index. Switching from the index α to complex notation, one uses θ^+, $\bar\theta^+$, θ^-, $\bar\theta^-$, and D^+, $\bar D^+$, D^-, and $\bar D^-$, etc., where

$$D^+=\partial_{\theta^-}+\theta^+\partial_z,\quad D^-=\partial_{\theta^+}+\theta^-\partial_z\ ,$$
$$\bar D^+=\partial_{\theta^-}+\bar\theta^+\partial_{\bar z},\quad \bar D^-=\partial_{\bar\theta^+}+\bar\theta^-\partial_{\bar z}\ . \tag{1}$$

The nonzero anticommutators are

$$\{D^+,D^-\}=2\partial_z,\quad \{\bar D^+,\bar D^-\}=2\partial_{\bar z}\ . \tag{2}$$

The action can be written in terms of two fields $S(z,\bar z,\theta^+,\bar\theta^+,\theta^-,\bar\theta^-)$ and $S^*(z,\bar z,\theta^+,\bar\theta^+\theta^-,\bar\theta^-)$ satisfying the constraints $D^-S=\bar D^-S=0$ and $D^+S^*=\bar D^+S^*=0$: action$=\int dz\,d\bar z\int d\theta^+d\bar\theta^+d\theta^-\bar d\theta^-S^*S$. The field S^* is related to S by interchanging $+$ and $-$ indices and complex conjugating the field but not z (see $*$ involution below). The solution of the equation of motion $D^+\bar D^+S=0$ is $S=S_1+S_2$, where $\bar D^+S_1=D^-S_1=\bar D^-S_1=0$ and $D^+S_2=D^-S_2=0$ (Ref. 1). A real superfield $\mathbf{X}^\mu$ is constructed via

$$\mathbf{X}^\mu(z,\theta^+,\theta^-)=S_1(z+\theta^-\theta^+,\theta^-)+S_1^*(z+\theta^+\theta^-,\theta^+)\ .$$

A U(1) operator acts on tensors carrying $+$ and $-$ indices 1. The number of plusses minus the number minuses is the U(1) charge. It is convenient to switch to an SO(2) formulation, in which indices become 1 and 2 instead of $+$ and $-$

$$A_\pm=\frac{A_1\pm iA_2}{\sqrt2}\ . \tag{3}$$

The components of $\mathbf{X}^\mu(Z)$ are

$$\mathbf{X}^\mu(Z)=X^\mu(z)+\theta_+\psi_+^\mu(z)+\theta_-\psi_+^\mu(z)+i\theta_-\theta_+\partial Y^\mu(z)\ ,$$

where $X^\mu(z)$ and $Y^\nu(z)$ are free bosons and $\psi_i^\mu(z)$ are free fermions. Note that $\partial Y^\mu(z)$ and not $Y^\mu(z)$ appears in $\mathbf{X}^\mu(Z)$, this means that the zero mode, iy_0^μ in $Y^\mu(z)$ does not appear.

We have the following operator-product expansions (OPE's):

$$X^\mu(z)X^\nu(w)\sim\eta^{\mu\nu}\ln(z-w)\sim Y^\mu(z)Y^\nu(w)\ ,$$
$$\psi_i^\mu(z)\psi_j^\nu(w)\sim\frac{\delta_{ij}\eta^{\mu\nu}}{(z-w)}\ .$$

These OPE's are all encoded in $\mathbf{X}^\mu(Z_a)\mathbf{X}^\nu(Z_b)$ $\sim\eta^{\mu\nu}\ln(Z_{ab})$ where $Z_{ab}=z_a-z_b-(\theta_a^+\theta_b^-+\theta_a^-\theta_b^+)$. The derivatives of Z_{ab} are $\partial_{z_a}Z_{ab}=1$, $\partial_{z_b}Z_{ab}=-1$, $D_a^\pm(Z_{ab})=D_b^\pm(Z_{ab})=\theta_{ab}^\pm$ where $\theta_{ab}^i=\theta_a^i-\theta_b^i$. In addition, $D_{ab}^\pm\theta_{ab}^\pm=D_b^\pm\theta_{ab}^\pm=0$, $D_a^\mp\theta_{ab}^\pm=1$, and $D_b^\mp\theta_{ab}^\pm=-1$.

The $N=2$ superspace Taylor-series expansion is

$$f(Z_a)=\sum_{n=0}^{\infty}\frac{1}{n!}(Z_{ab})\partial_{z_b}^n[1+\theta_{ab}^+D_b^-+\theta_{ab}^-D_b^++\tfrac{1}{2}(\theta_{ab}^+\theta_{ab}^-D_b^+D_b^-+\theta_{ab}^-\theta_{ab}^+D_b^-D_b^+)]f(Z_b)\ . \tag{4}$$

Defining $\int d\theta^+\int d\theta^-\theta^-\theta^+\equiv 1$ and $\oint DZ_a\equiv\oint(dz_a/2\pi i)\int d\theta_a^+\int d\theta_a^-$, one can derive the integration formulas

$$\begin{aligned}
&\oint DZ_a\frac{\theta_{ab}^-\theta_{ab}^+}{Z_{ab}^{n+1}}f(Z_a)=\frac{1}{n!}\partial_{z_b}^n f(Z_b)\ ,\\
&\oint DZ_a\frac{\theta_{ab}^+}{Z_{ab}^{n+1}}f(Z_a)=\frac{1}{n!}\partial_{z_b}^n D_b^+f(Z_b)\ ,\\
&\oint DZ_a\frac{\theta_{ab}^-}{Z_{ab}^{n+1}}f(Z_a)=-\frac{1}{n!}\partial_{z_b}^n D_b^-f(Z_b)\ ,\\
&\oint DZ_a\frac{f(Z_a)}{Z_{ab}^{n+1}}=\frac{1}{2n!}\partial_{z_b}^n(D_b^-D_b^+-D_b^+D_b^-)f(Z_b)\ .
\end{aligned}\tag{5}$$

Other $N=2$ formulas are similar to the $N=1$ ones given in Sec. III of Ref. 25.

Primary $N=2$ conformal fields $\Psi_q^h(Z)$ are characterized by a weight h and a charge q. They have the following OPE with the stress-energy tensor $T(Z)$:

$$\begin{aligned}
T(Z_a)\Psi_q^h(Z_b)\sim{}&h\frac{\theta_{ab}^-\theta_{ab}^+}{Z_{ab}^2}\Psi_q^h(Z_b)-\frac{q}{2Z_{ab}}\Psi_q^h(Z_b)\\
&+\frac{1}{2Z_{ab}}(\theta_{ab}^-D_b^+-\theta_{ab}^+D_b^-)\Psi_q^h(Z_b)\\
&+\frac{\theta_{ab}^-\theta_{ab}^+}{Z_{ab}}\partial_{z_b}\Psi_q^h(Z_b)\ .
\end{aligned}\tag{6}$$

The contribution to the stress-energy tensor from $\mathbf{X}^\mu$ is $T^X(Z)=\frac{1}{2}D_-\mathbf{X}^\mu D_+\mathbf{X}^\mu(Z)$. The theta-independent components of $T^X(Z)$ are the U(1) current-algebra fields T_n, the $\theta_+\theta_-$ component contains the Virasoro generators L_n, and the theta components are the superpartners of the L_n: G_n^+ and G_n^- [see Eq. (2.7) of Ref. 1].

The superfields $\mathbf{X}^\mu(Z)$, $D_+\mathbf{X}^\mu(Z)$, $D_-\mathbf{X}^\mu(Z)$, and $\exp(k\cdot\mathbf{X})(Z)$ satisfy Eq. (6) with, respectively $(h,q)=(0,0)$, $(h,q)=(\frac{1}{2},+1)$, $(h,q)=(\frac{1}{2},-1)$, and $(h,q)=(\frac{1}{2}k^2,0)$. The OPE of $T^X(Z)$ with itself is

$$\begin{aligned}
T^X(Z_a)T^X(Z_b)\sim{}&\frac{D}{4Z_{ab}^2}+\frac{\theta_{ab}^-\theta_{ab}^+}{Z_{ab}^2}T^X(Z_b)\\
&+\frac{1}{2Z_{ab}}(\theta_{ab}^-D_b^+-\theta_{ab}^+D_b^-)T^X(Z_b)\\
&+\frac{\theta_{ab}^-\theta_{ab}^+}{Z_{ab}}\partial_{z_b}T^X(Z_b)\ ,
\end{aligned}\tag{7}$$

where $D=\eta_\mu^\mu$ is the dimension of space-time. Using the integration formulas in Eq. (4), Eq. (5) can be shown to embody the $N=2$ Virasoro algebra[1] with a central term equal to D.

The $N=2$ superstring action is invariant under several local transformations. Gauge fixing generates a Faddeev-Popov determinant expressible as an action term using an $N=2$ superfield ghost $C(Z)$ and an antighost $B(Z)$:

$$\begin{aligned}
&C(Z)\equiv c(z)+i\theta_+\gamma_-(z)-i\theta_-\gamma_+(z)+i\theta_-\theta_+\xi(z)\ ,\\
&B(Z)\equiv -i\eta(z)-i\theta_+\beta_-(z)-i\theta_-\beta_+(z)+\theta_-\theta_+b(z)\ .
\end{aligned}\tag{8}$$

The ghosts $c(z)$ and $b(z)$ are for the τ-σ reparametrization invariances; $\gamma_i(z)$ and $\beta_i(z)$ are the spinor ghosts for the two local supersymmetry transformations and $\xi(z)$ and $\eta(z)$ are the ghosts associated with the local U(1) symmetry. Their Lagrangians are first-order systems[25] with background charges Q and statistics ϵ of $(Q,\epsilon)=(-3,+)$, $(2,-)$, and $(-1,+)$, respectively. The ghost OPE is $C(Z_a)B(Z_b)\sim\theta_{ab}^-\theta_{ab}^+/Z_{ab}\sim B(Z_a)C(Z_b)$. The ghost stress-energy tensor $T^{\rm gh}(Z)$ is $T^{\rm gh}(Z)=\partial_z(CB)(Z)-\frac{1}{2}D_+CD_-B(Z)-\frac{1}{2}D_-CD_+B(Z)$. $B(Z)$ and $C(Z)$ are $q=0$ conformal fields with $h=+1$ and $h=-1$. The OPE of $T^{\rm gh}(Z)$ with itself is Eq. (7) with $X\to{\rm gh}$ and $D\to -2$. Defining $T(Z)\equiv T^X(Z)+T^{\rm gh}(Z)$, the central term in the $N=2$ algebra vanishes if D is two. Throughout this paper we take $D=2$ so that $T(Z)$ itself is a $q=0$, $h=1$, $N=2$ conformal field.

The $N=2$ BRST charge $Q_{\rm BRST}$ (Refs. 26–28) is $Q_{\rm BRST}\equiv\oint dZ\,j_{\rm BRST}(Z)$ where $j_{\rm BRST}(Z)=C(T^X+\frac{1}{2}T^{\rm gh})(Z)+\frac{1}{4}[D_-(CBD_+C)-D_+(CBD_-C)]$. The term in [ ] does not contribute to $Q_{\rm BRST}$ but is needed so that $j_{\rm BRST}$ is an $h=0$, $q=0$ conformal field. Only for $D=2$ is $Q_{\rm BRST}^2=0$ (Refs. 26–28). The expression for $j_{\rm BRST}$ in terms of the field components is written in the Appendix.

The factors of i in the field expansions, e.g., in Eq. (8), have been chosen so that superfields have definite transformation properties under an involution operator * defined as

$$\begin{aligned}
&z^*=z,\quad (a+bi)^*=(a-bi)\quad (a\text{ and }b\text{ real})\ ,\\
&(A_+)^*=A_-,\quad (A_-)^*=A_+\ ,\\
&(AB)^*=A^*B^*,\quad (A+B)^*=A^*+B^*,\quad A_i^*=A_i\ .
\end{aligned}\tag{9}$$

The involution operation acts on linear combinations of superfields and is similar to complex conjugation except that it does not complex conjugate z. For any field, $(A^*)^*=A$. With our conventions

$$\mathbf{X}^{\mu *}=\mathbf{X}^{\mu}, \quad T^*=-T, \quad (D_+\mathbf{X}^{\mu})^*=D_-\mathbf{X}^{\mu} , \qquad C^*=C, \quad B^*=-B, \quad j^*_{\mathrm{BRST}}=-j_{\mathrm{BRST}} . \tag{10}$$

III. MASSLESS STATES AND VERTEX OPERATORS

Physical states correspond to BRST-invariant vertex operators. In this section we construct two such operators: one for a massless scalar and one for a massless vector. The scalar is the same particle for which amplitudes were computed in Ref. 1.

There are four sectors, NS-NS, R-NS, NS-R, and R-R, where NS stands for Neveu and Schwarz and R for Ramond, corresponding to the different boundary conditions for ψ_1 and ψ_2. The scalar is in the NS-NS sector and the vector is in the R-R sector. In constructing the vertex vertex operator, ghosts play an important role. The twisted sectors R-NS and NS-R are discussed in Sec. V.

As in the $N=1$ case,[25] bosonize β_i and γ_i: $\beta_i=\partial\xi_i\exp(-\varphi_i)$, $\gamma_i=\exp(\varphi_i)\eta_i$ as well as the $\psi_i^{\mu}(z)$: $[\psi_i^1(z)\mp i\psi_i^2(z)]/\sqrt{2}=\exp(\pm\phi_i)$. Define a generalized bosonized field by

$$\psi^{p_1p_2}_{q_1q_2}(z)\equiv e^{p_1\phi_1}e^{p_2\phi_2}e^{q_1\varphi_1}e^{q_2\varphi_2}c^{p_1p_2}_{q_1q_2} , \tag{11}$$

where $c^{p_1p_2}_{q_1q_2}$ is a cocycle operator. Using methods in Ref. 23 we define

$$c^{p_1p_2}_{q_1q_2}=\exp\{i\pi[p_2N_{\phi_1}+q_1(N_{\phi_1}+N_{\phi_2}) +q_2(-N_{\phi_1}+N_{\phi_2}-N_{\varphi_1})]\} , \tag{12}$$

where $N_{\phi_i}=[\partial\phi_i]_0$ and $N_{\varphi_i}=-[\partial\varphi_i]_0$ are the number operators for the ψ_i and $\gamma_i-\beta_i$ systems: $[N_{\phi_i}, \exp(q\phi_j)(z)]=q\delta_{ij}\exp(q\phi_j)(z)$. $[N_{\varphi_i},\exp(q\varphi_j)(z)]=q\delta_{ij}\exp(q\varphi_j)(z)$. The Ramond sector for the ith fermion occurs when p_i and q_i are half-integer. An alternative approach would be to bosonize in the $\pm$ basis.

The computation of scattering amplitudes requires picture-changing operators[25] $\{\xi_i,Q_{\mathrm{BRST}}\}$ [see Eqs. (A4) and (A5)] so that integral and c-type (indicated by c and $\int$ superscripts) vertex operators can be generated in various pictures.[25] Let S and V stand for scalar and vector. If $k^2=0$ and $k^{\mu}\epsilon_{\mu}=0$ then the following vertex operators are BRST invariant:

$$S^{\int}_{(-1,-1)}(k)=\int dz\, \psi^{0\;\;0}_{-1\,-1}\exp(k\cdot X)(z) , \tag{13}$$

$$S^{c}_{(-1,-1)}(k,z)=c\Psi^{0\;\;0}_{-1\,-1}\exp(k\cdot X)(z) , \tag{14}$$

$$S^{\int}_{(0,0)}(k)=\int dz[k\cdot\partial Y+(k\cdot\psi_1)(k\cdot\psi_2)]\exp(k\cdot X)(z) , \tag{15}$$

$$S^{c}_{(0,0)}(k,z)=\{c[k\cdot\partial Y+(k\cdot\psi_1)(k\cdot\psi_2)] -\tfrac{1}{2}(k\cdot\psi_1\gamma_1+k\cdot\psi_2\gamma_2)\}\exp(k\cdot X)(z) , \tag{16}$$

$$V^{\int}_{(-1/2,1/2)}(k,\epsilon)=\int dz\, \epsilon_{\mu}V^{\mu}_{-1/2-1/2}\exp(k\cdot X)(z) , \tag{17}$$

$$V^{c}_{(-1/2,-1/2)}(k,\epsilon,z)=c\epsilon_{\mu}V^{\mu}_{-1/2-1/2}\exp(k\cdot X)(z) . \tag{18}$$

In Eqs. (13)–(18), k^{μ} and ϵ^{μ} are momentum and polarization vectors, the (q_1,q_2) subscripts denote the various (φ_1,φ_2) ghost pictures,[25] and the vector field $V^{\mu}_{-1/2-1/2}$ is related to a R-R spin field by $V^{\mu}_{-1/2-1/2}\equiv(1/\sqrt{2})(C\gamma^{\mu})_{\alpha\beta}S^{\;\;\alpha\;\;\;\beta}_{-1/2\,-1/2}$, $S^{\;\;\alpha\;\;\;\beta}_{-1/2\,-1/2}\equiv\Psi^{\alpha/2\;\;\beta/2}_{-1/2\,-1/2}$. The γ^{μ} are $D=2$ gamma matrices: $(\gamma^{\mu})^{\alpha\beta}=(\sigma^{\mu})^{\alpha\beta}$ and $C_{\alpha\beta}=(\sigma^2)_{\alpha\beta}$ is the charge-conjugation matrix. The $D=2$ Lorentz decomposition of $S^{\;\;\alpha\;\;\;\beta}_{-1/2\,-1/2}$ is as follows. There are two scalars [when $(\alpha,\beta)=(+,-)$ and $(-,+)$] and one vector [when $(\alpha,\beta)=(+,+)$ and $(-,-)$]. Because $(C\gamma^{\mu})$ is diagonal, only terms for which $\alpha=\beta$ enter. Note that in the expressions of the vertex operator for the scalar and massless vectors the zero mode iy^0 of Y does not appear.

IV. SOME YUKAWA COUPLINGS AND SCATTERING AMPLITUDES AT THE TREE LEVEL

We use Eqs. (13)–(18) and the techniques in Ref. 23 to compute scattering amplitudes. Three c-type vertices must be used and the total $\gamma_1-\beta_1$ and $\gamma_2-\beta_2$ ghost numbers must be -2 in a correlation function in order to cancel the background ghosts charges.

The three-scalar Yukawa coupling is zero for n odd.[1] Three-vector and vector–two-scalar couplings vanish because correlation functions with an odd number of vector fields are zero. The latter is a bit unusual since in particle theories vector-scalar-scalar interactions are usually nonzero. The vector-vector-scalar coupling is

$$\langle V^{c}_{(-1/2,-1/2)}(k^1,\epsilon^1,z_1)V^{c}_{(-1/2-1/2)}(k^2,\epsilon^2,z_2)S^{c}_{(-1,-1)}(k^3,z_3)\xi(z_3)\rangle=i\epsilon^{\mu\nu}\epsilon^1_{\mu}\epsilon^2_{\nu} , \tag{19}$$

where $\epsilon^{12}=1=-\epsilon^{21}$ but vanishes for kinematic reasons. When momentum conservation is combined with $\epsilon^1\cdot k^1=0$ and $\epsilon^2\cdot k^2=0$, one finds $\epsilon^{\mu\nu}\epsilon^1_{\mu}\epsilon^2_{\nu}=0$.

The four-scalar amplitude is

$$\langle S^{c}_{(-1,-1)}(k^1,z_1)S^{\int}_{(-1,-1)}(k^2)S^{c}_{(0,0)}(k^3,z_3)S^{c}_{(0,0)}(k^4,z_4)\xi(z_4)\rangle=\frac{st}{4}\int_0^1 dx\, x^{s/2-1}(1-x)^{t/2-1} , \tag{20}$$

where $s=(k^2+k^3)^2=k^2\cdot k^3$ and $t=(k^1+k^2)^2=k^1\cdot k^2$. Equation (20) agrees with Ref. 1.

The two-scalar–two-vector amplitude is

$$\langle S^{c}_{(-1,-1)}(k^1,z_1)S^{\int}_{(0,0)}(k^2)V^{c}_{(-1/2,-1/2)}(k^3,\epsilon^3,z_3)V^{c}_{(-1/2,-1/2)}(k^4,\epsilon^4,z_4)\xi(z_4)\rangle =(\epsilon^3\cdot k^3)\epsilon^{\mu\nu}\epsilon^4_{\mu}k^3_{\nu}\int_0^1 dx\, x^{3/2}(1-x)^{t/2-1} . \tag{21}$$

The four-vector amplitude is

$$\langle V^{c}_{(-1/2,-1/2)}(k^1,\epsilon^1,z_1)V\!\!\int_{-1/2,-1/2}(k^2,\epsilon^2)V^{c}_{(-1/2,-1/2)}(k^3,\epsilon^3,z_3)V^{c}_{(-1/2-1/2)}(k^4,\epsilon^4,z_4)\xi(z_4)\rangle$$
$$= -\int_0^1 dx\, x^{s/2}(1-x)^{t/2}\left[\frac{\epsilon^1\cdot\epsilon^2\epsilon^3\cdot\epsilon^4}{x}-\frac{\epsilon^1\cdot\epsilon^4\epsilon^2\cdot\epsilon^3}{(1-x)}\right] . \quad (22)$$

There are poles in the s channel at $s=0$ corresponding to the massless scalar state. This is expected since the OPE of two vector fields contains the scalar field:

$$V^{\mu}_{-1/2-1/2}(z)V^{\nu}_{-1/2-1/2}(w)\sim -i\frac{\epsilon^{\mu\nu}\Psi^{0}_{-1}{}^{0}_{-1}(w)}{(z-w)} .$$

As should be the case from SL(2,R) invariance, Eqs. (20)–(22) do not depend on z_1,z_3,z_4. The insertions of $\xi(z_i)$ are needed because of the nonzero background ghost charge, $Q=-1$ for the η-ξ system. These insertions do not ruin the BRST invariance of the scalar and vector vertex operators.

Note that in the computations of the scattering amplitudes at the tree level the zero modes of Y do not enter; however, they are essential for loop amplitudes; for a discussion see Refs. 21 and 28.

V. TWISTED STATES

Usually world-sheet symmetries translate into space-time symmetries. Hence one would expect $N=2$ supersymmetry in the model. Such states appear in the NS-R and R-NS sectors.[1,22] The boundary conditions are as follows: η, ξ, and $\partial Y^{\mu}(z)$ are antiperiodic in σ; ψ_1^{μ}, β_2, and γ_2 are periodic for the R-NS case and $\psi_2^{\mu}(z)$, β_1, and γ_1 are periodic for the NS-R case. The conditions of the The NS-R and R-NS sectors are called twisted:[1] the behavior of $\partial Y^{\mu}(z)$ is similar to a Z_2 orbifold.[29]

Let t (for "twist") represent an operator which implements the boundary conditions on $Y^{\mu}(z)$:

$$\partial Y^{\mu}(z)t(w)\sim\frac{O(w)}{\sqrt{z-w}} \quad (23)$$

for some operator $O(w)$. Bosonizing ξ and η via $\xi=\exp(\chi)$, $h=\exp(-\chi)$, the following are BRST invariant:

$$\begin{aligned} &cu_{\alpha}\Psi^{1/2}_{-1}{}^{0}_{-1/2}e^{\chi/2}t\exp(k\cdot X)(z) \quad (\text{R-NS}) ,\\ &cu_{\alpha}\Psi^{0}_{-1/2}{}^{\alpha/2}_{-1}e^{\chi/2}t\exp(k\cdot X)(z) \quad (\text{NS-R}) , \end{aligned} \quad (24)$$

if $\frac{1}{2}k^2=0$ and the spinor wave function u_{α} satisfies the massless Dirac equation $u\not k=0$. One component of u_{α} must be zero. When $u_{\alpha}\neq 0$ for $\alpha=+$, the Dirac equation condition becomes $k^1-ik^2=0$; when $u_{\alpha}\neq 0$ for $\alpha=-, k^1+ik^2=0$. In Minkowski space, there is a fermion moving to the right or to the left at the speed of light.

The requirement of locality imposes constraints on the spectrum. Locality is equivalent to the GSO projection and dictates that the algebra of operator products not involve root singularities. Since

$$\Psi^{0}_{-1/2}{}^{\alpha/2}_{-1}(z)\Psi^{0}_{-1}{}^{0}_{-1}(w)\sim -\Psi^{0}_{-3/2}{}^{\alpha/2}_{-1}(w)/(z-w)^{3/2} ,$$

the operators in Eq. (24) are not local with respect to the scalar vertex operator $\Psi^{0}_{-1}{}^{0}_{-1}$. Either (a) the fermions cannot be retained in the theory, (b) the scalar must be removed, or (c) the scalar vertex operator must be modified by inserting operators with nontrivial operator products with t or χ. As an example of (c) one could insert a factor of $\xi(z)=e^{\chi}$ in the scalar and vertex vertex operators.

Unfortunately, one does not know much about the twist operator t. If one assumes that $t(z)t(w)\sim 1/(z-w)^{1/4}$, then

$$\Psi^{+1/2}_{-1}{}^{0}_{-1/2}e^{\chi/2}t(z)\Psi^{+1/2}_{-1}{}^{0}_{-1/2}e^{\chi/2}t(w)$$
$$\sim \text{phase}\times\Psi-2^{+1}{}^{0}_{-1}e^{\chi}/(z-w)$$

but since $\Psi^{+1/2}_{-1}{}^{0}_{-1/2}e^{\chi/2}t(z)$ and $\Psi^{+1}_{-2}{}^{0}_{-1}\chi(x)$ are not mutually local, the fermionic state cannot be included in the theory. Hence, if $t(z)t(w)\sim 1/(z-w)^{1/4}$, the GSO projection would remove all fermions. To better understand the constraints of locality we have created some models for t.

The derivative of a boson is expressible as a product of two fermions[20]

$$\partial Y^1(z)=if^1f^3(z), \quad \partial Y^2(z)=if^2f^4(z) . \quad (25)$$

As long as $f^i(z)f^j(w)\sim\delta^{ij}/(z-w)$ all correlation functions of $\partial Y^{\mu}(z)$ are reproduced by the representation in Eq. (25). Usually, $f^i=(1/\sqrt{2})(e^{H^i}+e^{-H^i})$ and $f^{i+2}=(i/\sqrt{2})(e^{H^i}-e^{-H^i})$ for $i=1,2$, but one can bosonize the f^i differently:

$$\begin{aligned} &f^1=\frac{1}{\sqrt{2}}(e^{H'}+e^{-H'}), \quad f^2=\frac{i}{\sqrt{2}}(e^{H'}-e^{-H'}) ,\\ &f^3=\frac{1}{\sqrt{2}}(e^{H}+e^{-H}), \quad f^4=\frac{i}{\sqrt{2}}(e^{H}-e^{-H}) , \end{aligned} \quad (26)$$

where H and H' are free bosons. The advantage of Eq. (26) is that t can be represented as a spin field for the f^3-f^4 system. Define

$$t^{+}=\frac{1}{\sqrt{2}}(e^{H/2}+e^{-H/2}), \quad t^{-}=\frac{i}{\sqrt{2}}(e^{H/2}-e^{-H/2}) . \quad (27)$$

The operators t^{+} and t^{-} satisfy $t^{\pm}(z)t^{\pm}(w)\sim 1/(z-w)^{1/4}$. The conformal dimension of t^{+} and t^{-} is $\frac{1}{8}$, as should be the case for a Z_2 twist field.[30–33] In what follows we take $t=t^+$. Since $t(z)t(w) \sim 1/(z-w)^{1/4}$, the fermionic operators lead to a nonlocal theory as demonstrated above. The GSO projection for this representation of t throws out the fermions. This is understandable because the operators in Eq. (24) have fractional statistics. The statistics of an exponential of bosons, $\exp(v\cdot H)$, is $(-1)^{\langle v\cdot v\rangle}$ where $\langle\cdot\rangle$ denotes the Lorentz inner product: $+$ signature for $\epsilon=+$ bosons and $-$ signature for $\epsilon=-$ bosons. For the fermionic vertex operators, $\langle v\cdot v\rangle=-\frac{1}{2}$.

A representation of $t=\exp(b/2)$ by-passes the problem of fractional statistics if b is an $\epsilon=-$ boson. If t is to have conformal weight $\frac{1}{8}$ then the background charge Q associated with b is -1. Bosonize f^3 and f^4 in the following manner:

$$f^3=\frac{1}{\sqrt{2}}(e^He^Be^b+e^{-H}e^{-B}e^{-b}) ,$$
$$f^4=\frac{i}{\sqrt{2}}(e^He^Be^b-e^{-H}e^{-B}e^{-b}) , \tag{28}$$

where H has $\epsilon=+$ and $Q=0$, B has $\epsilon=+$ and $Q=-1$ and b has $\epsilon=-$ and $Q=-1$. The operator $t=\exp(b/2)$ satisfies $t(z)t(w)\sim\exp(b)/(z-w)^{1/4}$. The statistics of the fermionic vertex operators is anticommunting. The operators,

$$\Psi^{+1/2}_{-1}{}^{0}_{-1/2}e^{\chi/2}t(z) \text{ and } \Psi_{-1/2}^{0}{}^{+1/2}_{-1}e^{\chi/2}t(z) ,$$

form a local set. They still are not local with respect to the scalar and the vector. We thus have two GSO projections: retain two left-moving fermions or retain the scalar and the vector. The former has a trivial S matrix since (a) a system in which all massless particles move to the left cannot interact and (b) it is impossible to construct vertex operators which cancel the background χ charge.

In short, it appears difficult to obtain $N=2$ supersymmetric multiplets. Although we cannot make a definitive statement because there might exist some representation of the twisting of $\partial Y^\mu(z)$ which avoids the problem of locality, it seems unlikely that scalars, vectors, and fermions can all be retained.

Some general arguments suggest that $N=2$ supersymmetry cannot be obtained. The $N=2$, $D=2$ supersymmetry algebra is $\{Q_i^\alpha,Q_j^\beta\}=2\delta_{ij}(\gamma_\mu C)^{\alpha\beta}p^\mu$ where C is the charge-conjugation matrix. More explicitly, $\{Q_i^+,Q_j^+\}=2\delta_{ij}(p^2+ip^1)$ and $\{Q_i^-,Q_j^-\}=2\delta_{ij}(p^2-ip^1)$ so that in the massless case either the Q_i^- are null operators (for $p^2=ip^1$) or the Q_i^+ are null operators (for $p^2=-ip^1$). One massless multiplet consists of a scalar, two fermions, and one component of a vector all moving to the left. A string theory based on this multiplet has a trivial S matrix. To obtain nontrivial scattering a right-moving multiplet must be added so that fermions of both chiralities enter. In 10-dimensional spinning strings, there are massless fermions of both chiralities. BRST-invariant vertex operators can be constructed for both but both operators cannot be retained if locality is to be satisfied. The GSO projection removes one of the states. Likewise in $D=2$, one expects that locality requires the removal of fermions of one chirality. A nontrivial $N=2$ supersymmetric theory is thus not possible. This is the result that we have uncovered above.

VI. A $d=4$ THEORY?

Limiting oneself to the vector and scalar, one can try to extend the model to four dimensions. As mentioned above, the zero mode, iy_0^μ, of $Y^\mu(z)$, does not appear in $\mathbf{X}^\mu$ and in vertex operators. Nevertheless, X^μ and Y^μ appear symmetrically in $Q_{\rm BRST}$. Can one append y_0^μ to the theory to create a $d=4$ theory? Reference 34 attempted such a "decompactification"; however, the space-time symmetry group was O(2,2).

Let us construct O(4) Lorentz generators (we are working with the Euclidean metric $\eta^{\mu\nu}$ for the two-dimensional objects) $M^{\mu\nu}$, and BRST-invariant Lorentz-covariant vertex operators. The strategy is to group X^μ and Y^μ [as well as $\psi_1^\mu(z)$ and ψ_2^μ] into one four-component vector. Set $X^1(z)\equiv X^1(z)$, $X^2(z)\equiv X^2(z)$, $X^3(z)\equiv Y^1(z)$, $X^4(z)\equiv Y^2(z)$, and from ψ_i^μ, set $\psi^1\equiv\psi_1^1$, $\psi^2\equiv\psi_1^2$, $\psi^3\equiv\psi_2^1$, $\psi^4\equiv\psi_2^2$. Using $X^\mu(z)$ and $\psi^\mu(z)$, $\mu=1,2,3,4$, define in the standard fashion[35] the Lorentz generators $M^{\mu\nu}\equiv\int dx\,[\frac{1}{2}(X^\mu\partial X^\nu-X^\mu\partial X^\mu)+\psi^\mu\psi^\nu]$.

The next step is to define four-dimensional BRST-invariant vertex operators. This is done by letting the $k\cdot X$ dot product in $S^{\int}_{(-1,-1)}(k)$ and $S^c_{(-1,-1)}(k,z)$ be four dimensional. Replace $\epsilon_\mu V^\mu_{(-1/2,1/2)}$ by $\epsilon_A S^{A}_{-1/2-1/2}$ in $V^{\int}_{(-1,-1)}(k)$ and $V^c_{(-1,-1)}(k,z)$ where $A=\alpha\beta=(++$ or $--)$ and allow ϵ_A to be complex. $V^{\int}_{(-1,-1)}(k)$ and $V^c_{(-1,-1)}(k,z)$ become $d=4$ vertex operators for a Weyl fermion of positive chirality as long as ϵ_A satisfies the massless Dirac equation, $\epsilon_A(k_\mu\gamma^\mu)^A_B=0$, where $\gamma^1=(\sigma^1\times 1)$, $\gamma^2=(\sigma^2\times 1)$, $\gamma^3=(\sigma^3\times\sigma^2)$, and $\gamma^4=-(\sigma^3\times\sigma^1)$ (Ref. 23). As is well known (see, for example, Ref. 23), a two-spinor label $A=\alpha\beta$ transforms as a fermion:

$$\psi^\mu\psi^\nu(z)S^{A}_{-1/2-1/2}(w) \sim -\frac{1}{(z-w)}\frac{1}{2}\gamma^{\mu\nu A}_{B}S^{B}_{-1/2-1/2}(w) , \tag{29}$$

where $\gamma^{\mu\nu}=\frac{1}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$. In checking the BRST invariance, $\mathcal{Q}_0^0\sim\int dz j_0^0(z)$ [see Eq. (A1)] generates the chirality condition (a) $\epsilon_A\gamma^{5A}_B=\epsilon_A$ where $\gamma^5=(\sigma^3\times\sigma^3)$; the $T^M_{\theta 1}$ [see Eq. (A3)] piece of $\mathcal{Q}_1^0$ requires ϵ_A to satisfy the Dirac equation (b) $\epsilon_A(k_\mu\gamma^\mu)^A_B=0$, and the $T^M_{\theta 2}$ piece of $\mathcal{Q}_1^0$ gives a second Dirac equation, (c) $\epsilon_A(k_1\gamma^3+k_2\gamma^4-k_3\gamma^1-k_4\gamma^2)^A_B=0$. Miraculously, (c) is a consequence of (a) and (b) and hence is not a new constraint.

Operators products are $d=4$ Lorentz covariant because the vertex operators are Lorentz covariant. For example, the OPE of $S^{A}_{-1/2-1/2}$ with itself is

$$S^{A}_{-1/2-1/2}(z)S^{B}_{-1/2-1/2}(w) \sim\frac{C^{AB}}{(z-w)}\Psi^{0}_{-1}{}^{0}_{-1}(w)+\frac{1}{4}(\gamma^{\mu\nu}C)^{AB}\psi_\mu\psi_\nu\Psi^{0}_{-1}{}^{0}_{-1}(w) -\frac{1}{2}C^{AB}(\partial\phi_1+\partial\phi_2)\Psi^{0}_{-1}{}^{0}_{-1}(w)+O(z-w) , \tag{30}$$

where $C=i(\sigma^2\times\sigma^1)$ is the four-dimensional charge-conjugation matrix. The cocycles in Eq. (12) and bosonized ghosts are crucial in generating the correct gamma matrix structure.

It seems as if one has all the necessary ingredients to extend the $N=2$ theory to four dimensions; however, a problem arises. The picture-changing operator $\{\xi_1, Q_{\mathrm{BRST}}\}$ is not Lorentz invariant due to $T^M_{\theta 1}$. When $S^{\int}_{(00)}(k)$ and $S^c_{(00)}(k,z)$ are constructed by picture-changing $S^{\int}_{(-1-1)}(k)$ and $S^c_{(-1-1)}(k,z)$, they are not Lorentz invariant. Although states and most vertex operators can be extended to four dimensions, we are unable to obtain Lorentz-invariant amplitudes.

VII. CONCLUSION

In this paper we constructed vertex operators for massless scalar and vector states for the $N=2$ superstring and we computed Yukawa couplings and scattering amplitudes. Several interesting questions remain. (A) We have come quite close to constructing a $d=4$, $N=2$ string theory. Is there some way to get around the difficulty that we have encountered? (B) How can one compute amplitudes involving the twisted states of Sec. V? Is there a viable concrete representation of twist operators and is it unique? (C) What is the BRST-cohomology modulo picture-changing operations? In other words, what are the BRST-invariant but BRST-nontrivial vertex operators? What are the possible GSO projections, that is, which are the possible spectrums of the theory satisfying locality? We hope these questions will stimulate further thinking about the $N=2$ string model.

ACKNOWLEDGMENTS

This work was supported in part by a NATO Collaborative Research Grant No. (0763/87) and by the United State Department of Energy under Contract No. DE-AC02-84ER40125. I am indebted to Stuart Samuel whose participation played a crucial role in this paper. I thank Alan Kostelecký for assistance with Sec. V.

APPENDIX: THE PICTURE-CHANGING OPERATORS

In checking BRST invariance, it is useful to decompose j_{BRST} into definite ghost numbers: $j_{\mathrm{BRST}}=j_0^1+j_0^0+j_1^0+j_2^{-1}+j_2^0$, where the superscript denotes the $c-b$ ghost number and the subscript denotes the $\gamma_1-\beta_1+\gamma_2-\beta_2$ ghost number. The $\xi-\eta$ enter in such a way as to make the overall ghost number of Q 1:

$$\begin{aligned} j_0^1(z)&=c(\tilde{T}+\partial cb)(z)\ ,\\ j_0^0(z)&=\tfrac{1}{2}\xi(\psi_2\cdot\psi_1+\beta_2\gamma_1-\beta_1\gamma_2)(z)\ ,\\ j_1^0(z)&=-(\gamma_1T^M_{\theta 1}+\gamma_2T^M_{\theta 2})(z)\ ,\\ j_2^{-1}(z)&=-\tfrac{1}{4}b(\gamma_1^2+\gamma_2^2)(z)\ ,\\ j_2^0(z)&=\tfrac{1}{2}\eta(\gamma_2\partial\gamma_1-\gamma_1\partial\gamma_2)(z)\ , \end{aligned} \tag{A1}$$

where

$$\tilde{T}=\tfrac{1}{2}\partial X\partial X+\tfrac{1}{2}\partial Y\partial Y+\tfrac{1}{2}\partial\psi_i\psi_i-\tfrac{3}{2}\partial\gamma_i\beta_i-\tfrac{1}{2}\partial\beta_i\gamma_i+\partial\xi\eta\ , \tag{A2}$$

$$T^M_{\theta 1}=\tfrac{1}{2}(\psi_1\cdot\partial Y-\psi_2\cdot\partial X),\quad T^M_{\theta 2}=\tfrac{1}{2}(\psi_1\cdot\partial X+\psi_2\cdot\partial Y)\ . \tag{A3}$$

The two picture-changing operators are

$$\begin{aligned}\{\xi_1(z),Q\}&=c\partial\xi_1(z)-T^M_{\theta 1}e^{\varphi_1}(z)+\tfrac{1}{4}[b\partial\eta_1e^{2\varphi_1}+\partial(b\eta_1e^{2\varphi_1})](z)-\tfrac{1}{2}\xi\partial\xi_2e^{\varphi_1}e^{-\varphi_2}(z)\\ &\quad-(\eta\partial\eta_1+\tfrac{1}{2}\partial\eta\eta_2+\eta\eta_2\partial\varphi_2)e^{\varphi_1}e^{\varphi_2}(z)\ ,\end{aligned} \tag{A4}$$

$$\begin{aligned}\{\xi_2(z),Q\}&=c\partial\xi_2(z)-T^M_{\theta 2}e^{\varphi_2}(z)+\tfrac{1}{4}[b\partial\eta_2e^{2\varphi_2}+\partial(b\eta_2e^{2\varphi_2})](z)-\tfrac{1}{2}\xi\partial\xi_1e^{-\varphi_1}e^{\varphi_2}(z)\\ &\quad-(\eta\partial\eta_1+\tfrac{1}{2}\partial\eta\eta_1+\eta\eta_1\partial\varphi_1)e^{\varphi_1}e^{\varphi_2}(z)\ .\end{aligned} \tag{A5}$$

---

*Permanent address: Department d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Diagonal, 647, Barcelona 08028, Spain. Electronic address: D3EMJGT0 @ EB0UB011 BITNET.

[1]M. Ademollo, L. Brink, A. D'Adda, R. D'Auria, E. Napolitano, S. Sciuto, E. Del Guidice, P. Di Vecchia, S. Ferrara, F. Gliozzi, R. Musto, and R. Pettorino, Phys. Lett. **62B**, 105 (1976); M. Ademollo, L. Brink, A. D'Adda, R. D'Auria, E. Napolitano, S. Sciuto, E. Del Guidice, P. Di Vecchia, S. Ferrara, F. Gliozzi, R. Musto, R. Pettorino, and J. H. Schwarz, Nucl. Phys. **B111**, 77 (1976).

[2]L. Brink and J. H. Schwarz, Nucl. Phys. **B121**, 285 (1977).

[3]P. Di Vecchia, J. L. Petersen, and H. B. Zheng, Phys. Lett. **162B**, 327 (1985).

[4]W. Boucher, D. Friedan, and A. Kent, Phys. Lett. B **172**, 316 (1986).

[5]J. L. Petersen, in *Proceedings of the 19th International Symposium: Special Topics in Gauge Field Theories,* Ahrenshoop, Germany, 1985 (Akademie der Wissenchatten der DDR, Berlin, 1985).

[6]P. Di Vecchia, J. L. Petersen, and M. Yu, Phys. Lett. B **172**, 211 (1986).

[7]S. Nam, Phys. Lett. B **172**, 323 (1986).

[8]P. Di Vecchia, J. L. Petersen, M. Yu, and H. B. Zheng, Phys. Lett. B **174**, 280 (1986).

[9]M. Yu and H. B. Zheng, Nucl. Phys. **B288**, 275 (1987).

[10]A. Schwimmer and N. Seiberg, Phys. Lett. B **184**, 191 (1987).

[11]T. Eguchi and A. Taormina, Phys. Lett. B **210**, 125 (1988).

[12]K. Li and N. P. Warner, Phys. Lett. B **211**, 101 (1988).
[13]Y. Kazama and H. Suzuki, Phys. Lett. B **216**, 112 (1989).
[14]D. Gepner, Phys. Lett. B **199**, 380 (1987).
[15]D. Gepner, Nucl. Phys. **B296**, 757 (1988).
[16]T. Banks, L. Dixon, D. Friedman, and E. Martinec, Nucl. Phys. **B299**, 613 (1988).
[17]A. Sen, Nucl. Phys. **B278**, 289 (1986); **B287**, 423 (1987).
[18]E. Martinec and G. M. Sotkov, Phys. Lett. B **208**, 249 (1988).
[19]J. D. Cohn, Nucl. Phys. **B284**, 349 (1987).
[20]M. Green, Nucl. Phys. **B293**, 593 (1987).
[21]M. Corvi, V. A. Kostelecký, and P. Moxhay, Phys. Rev. D **39**, 1611 (1989).
[22]S. D. Mathur and S. Mukhi, Nucl. Phys. **B302**, 130 (1988).
[23]V. A. Kostelecký, O. Lechtenfeld, W. Lerche, S. Samuel, and S. Watamura, Nucl. Phys. **B288**, 173 (1987).
[24]M. Kato and K. Ogawa, Nucl. Phys. **B212**, 443 (1983).
[25]D. Friedan, E. Martinec, and S. Shenker, Phys. Lett. **106B**, 55 (1985); Nucl. Phys. **B271**, 93 (1986).
[26]A. Bilal, Phys. Lett. B **180**, 255 (1986).
[27]A. R. Bogojevic and Z. Hlousek, Phys. Lett. B **179**, 69 (1986).
[28]S. D. Mathur and S. Mukhi, Phys. Rev. D **36**, 465 (1987).
[29]L. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. **B261**, 678 (1985); **B274**, 285 (1986).
[30]S. Hamidi and C. Vafa, Nucl. Phys. **B279**, 465 (1987).
[31]L. Dixon, D. Friedan, E. Martinec, and S. Shenker, Nucl. Phys. **B282**, 13 (1987).
[32]A. B. Zamolodchikov, Nucl. Phys. **B285** [FS19], 481 (1987); S. A. Apikyan and A. B. Zamolodchikov, Zh. Eksp. Teor. Fiz. **92**, 34 (1987) [Sov. Phys. JETP **65**, 19 (1987)].
[33]G. Mussardo, G. Sotkov, and M. Stanishkov, Int. J. Mod. Phys. A **4**, 1135 (1989).
[34]A. D'Adda and F. Lizzi, Report No. RAL 87-009, 1987 (unpublished).
[35]J. Scherk, Rev. Mod. Phys. **47**, 123 (1975).