Large N QCD from rotating branes

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We study large N SU(N) Yang-Mills theory in three and four dimensions using a one-parameter family of supergravity models which originate from non-extremal rotating D-branes. We show explicitly that varying this ''angular momentum'' parameter decouples the Kaluza-Klein modes associated with the compact D-brane coordinate, while the mass ratios for ordinary glueballs are quite stable against this variation, and are in good agreement with the latest lattice results. We also compute the topological susceptibility and the gluon condensate as a function of the ''angular momentum'' parameter. [S0556-2821(99)00906-6]

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I. INTRODUCTION

Generalizing the conjectured duality [1] between large N superconformal field theories and superstring or M theory on anti–de Sitter (AdS) backgrounds, Witten proposed an approach to studying large N non-supersymmetric theories such as pure QCD using a dual supergravity (string theory) description [2]. The basic idea is to start with $d+1$ dimensional superconformal field theories at finite temperature—thus breaking the superconformal invariance—and obtain a d dimensional non-supersymmetric gauge theory at zero temperature by dimensional reduction in the Euclidean time direction. The AdS space is then replaced by a certain limit of the Schwarzschild geometry describing a black hole in an AdS space.

When the curvature of the space is small compared to the string scale (or, in the case of M theory, Planck scale), supergravity provides an adequate effective description that exhibits a qualitative agreement with pure QCD in three and four dimensions [2,3]. The supergravity limit of string theory (i.e. infinite string tension, $\alpha' \to 0$ limit) corresponds to the strong coupling limit of the gauge theory ($\lambda = g_{YM}^2 N \gg 1$), with $1/\lambda$ playing the role of α'. In the approach of [2], the gauge theory has an ultraviolet cutoff proportional to the temperature T; the supergravity approximation should describe the large N gauge theory in the strong coupling regime with a finite ultraviolet cutoff. This is analogous to a strong coupling lattice gauge theory with lattice spacing $\sim 1/T$ [3]. In the limit that the ultraviolet cutoff is sent to infinity, one has to study the theory at small λ, and the supergravity description breaks down. To calculate the spectrum in this regime, a better understanding of string theory with Ramond-Ramond (R-R) background fields is required.

Glueball masses in the supergravity approximation have been computed in [4,5]. The Witten model [2] contains in addition to the glueballs certain Kaluza-Klein (KK) particles with masses of the order of the glueball masses. These KK modes do not correspond to any states in the Yang-Mills theory, and therefore they should decouple in the ''continuum'' limit. The KK states do not decouple with the inclusion of the leading $1/\lambda$ corrections [6]. Although such states can decouple in a full string theory treatment, there may be generalizations of the Witten model that have a more direct connection with the continuum gauge theory already at the supergravity level (at least in the sector of states with spin ≤ 2 that can appear in a supergravity description). A similar situation arises in lattice gauge theory. It is well known that the action one starts with has a significant effect on the speed at which one gets to the continuum limit. One can add to the lattice action deformations which are irrelevant in the continuum and arrive at an appropriate effective description of the continuum theory while having a larger lattice spacing (such a deformed action is called an ''improved'' lattice action). A similar strategy in the dual supergravity picture would correspond to a suitable modification of the background metric, so as to have an appropriate effective description of the gauge theory while still having a finite ultraviolet cutoff. An important test of the proposal is to check that the KK modes in the supergravity description that do not correspond to gauge degrees of freedom are heavy and decouple, and at the same time the infrared physics is not significantly altered. In this paper we make the first step in this direction by examining a generalization of the Witten model that has an additional parameter.

A more general approach to the conjectured correspondence between gauge theories and M-theory requires the investigation of supergravity compactifications which asymptotically approach anti–de Sitter backgrounds, e.g. $AdS_5 \times S^5$ or $AdS_7 \times S^4$ (see, e.g., [7,8]). There exist a few super-

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gravity backgrounds that generalize the Witten model and which are regular everywhere. These are essentially obtained by starting with the rotating version of the non-extremal D4 brane background (or rotating D3 brane background, in the case of QCD_3) and taking a field theory limit as in [1]. These models were investigated in [9]. The deformation of the background proposed in [2] is parametrized by an ''angular momentum'' parameter (the supergravity background is actually static, with the Euclidean time playing the role of an internal angle). In this paper we determine numerically the scalar and pseudoscalar spectrum of these models as a function of the angular momentum parameter and compare the results to those obtained by lattice calculations. We also compute the gluon condensate and the topological susceptibility

The paper is organized as follows: Sec. II is devoted to the study of supergravity models for pure QCD in $3+1$ dimensions. The models are described in Sec. II A. In Sec. II B we compute the scalar glueball mass spectrum and analyze its dependence on the angular momentum. In Sec. II C we calculate the mass spectrum of some KK modes. It will be shown that the KK modes associated with the compact D-brane coordinate decouple as the angular momentum parameter is increased. This, however, is not the case for the SO(3) non-singlet KK modes with vanishing U(1) charge in the compact D-brane coordinate. In Sec. II D we compute the gluon condensate from the free energy associated with the supergravity background. In Sec. II E we compute the topological susceptibility and its dependence on the angular momentum parameter. Section III contains an analogous study for the case of QCD in $2+1$ dimensions. The conclusions are similar in both cases, and they are summarized in Sec. IV.

II. QCD IN 3+1 DIMENSIONS

A. Supergravity models for QCD_4

One way to construct non-supersymmetric models of QCD based on supergravity is to start from the non-extremal D4 brane metric, and view the Euclidean time coordinate as an internal coordinate compactified on a circle of radius $(2\pi T_H)^{-1}$ [2]. Possible generalizations of this proposal are constrained by the no-hair theorem, which implies that the most general regular manifold with only D4 brane charges (and an isometry group containing $\mathbf{R}^4$) is given by the rotating version of the non-extremal D4 brane, which has two additional parameters representing angular momenta in two different planes. The Euclidean version of this metric (related to the rotating M5 brane metric by dimensional reduction) was used in [9] to construct models for QCD with extra parameters. Here we investigate in detail the case when there is one non-vanishing angular momentum, parametrized by a. The field theory limit of the Euclidean rotating M5 brane with angular momentum component in one plane is given by the metric [9]

$$ds_{11}^2=\Delta^{1/3}\left\{\frac{U^2}{(\pi N)^{1/3}}\left[\sum_{i=1}^{5}dx_i^2+\left(1-\frac{U_0^6}{U^6\Delta}\right)d\tau^2\right]\right.$$
$$+(\pi N)^{2/3}\left(d\theta^2+\frac{\tilde{\Delta}}{\Delta}\sin^2\theta d\varphi^2+\frac{1}{\Delta}\cos^2\theta d\Omega_2^2\right.$$
$$\left.\left.-\frac{2a^2U_0^3}{U^4\Delta(\pi N)^{1/2}}\sin^2\theta d\tau d\varphi\right)+\frac{4(\pi N)^{2/3}dU^2}{U^2\left(1-\frac{a^4}{U^4}-\frac{U_0^6}{U^6}\right)}\right\},\tag{2.1}$$

where $x_1,\ldots,x_5$ are the coordinates along the M5 brane where the gauge theory lives, U is the ''radial'' coordinate of the AdS space, while the remaining four coordinates parametrize the angular variables of S^4, and where we have introduced

$$\Delta=1-\frac{a^4\cos^2\theta}{U^4},\quad \tilde{\Delta}=1-\frac{a^4}{U^4}.\tag{2.2}$$

Dimensional reduction along x_5 (which will play the role of the ''eleventh'' dimension) gives N rotating non-extremal D4 branes, which in the low energy regime should be described by SU(N) Yang-Mills theory at finite temperature T_H, perturbed by some operator associated with the rotation. The $3+1$ dimensional SU(N) Yang-Mills theory at zero-temperature can be described by making $x_4\to -ix_0$, and viewing τ as parametrizing a space-like circle with radius $R_0=(2\pi T_H)^{-1}$, where fermions obey anti-periodic boundary conditions. At energies much lower than $1/R_0$, the theory is effectively $3+1$ dimensional. Because of the boundary conditions, fermions and scalar particles get masses proportional to the inverse radius, so that, as $R_0\to 0$, they should decouple from the low-energy physics, leaving pure Yang-Mills theory as low-energy theory.

The gauge coupling g_4^2 in the $3+1$ dimensional Yang-Mills theory is given by the ratio between the periods of the 11-dimensional coordinates x_5 and τ times 2π. It is convenient to introduce ordinary angular coordinates θ_1, and θ_2 which are 2π-periodic by

$$\tau=R_0\theta_2,\quad x_5=\frac{g_4^2}{2\pi}R_0\theta_1=\frac{\lambda}{N}R_0\theta_1,\tag{2.3}$$

$$R_0=(2\pi T_H)^{-1}=\frac{A}{3u_0},\quad A\equiv\frac{u_0^4}{u_H^4-\frac{1}{3}a^4},\tag{2.4}$$

where u_H is the location of the horizon, and we have introduced the 't Hooft coupling

$$\lambda\equiv\frac{g_4^2N}{2\pi},\tag{2.5}$$

the coordinate u by $U=2(\pi N)^{1/2}u$, and rescaled $a \to 2(\pi N)^{1/2}a$. By dimensional reduction in θ_1, one obtains the metric

$$ds^2_{\mathrm{IIA}}=\frac{2\pi\lambda A}{3u_0}u\Delta^{1/2}\Bigg[4u^2(-dx_0^2+dx_1^2+dx_2^2+dx_3^2)+\frac{4A^2}{9u_0^2}u^2\left(1-\frac{u_0^6}{u^6\Delta}\right)d\theta_2^2+\frac{4du^2}{u^2\left(1-\frac{a^4}{u^4}-\frac{u_0^6}{u^6}\right)}+d\theta^2+\frac{\tilde{\Delta}}{\Delta}\sin^2\theta d\varphi^2+\frac{1}{\Delta}\cos^2\theta d\Omega_2^2-\frac{4a^2Au_0^2}{3u^4\Delta}\sin^2\theta d\theta_2 d\varphi\Bigg], \tag{2.6}$$

with a dilaton background

$$e^{2\phi}=\frac{8\pi}{27}\frac{A^3\lambda^3u^3\Delta^{1/2}}{u_0^3}\frac{1}{N^2}. \tag{2.7}$$

With this normalization, the metric reduces to Eq. (4.8) of Ref. [2] after setting $a=0$. The string coupling e^{ϕ} is of order $1/N$, and the metric has become independent of N, which is consistent with the expectation that in the large N limit the string spectrum should be independent of N. The metric is regular, and the location of the horizon is given by

$$u_H^6-a^4u_H^2-u_0^6=0, \tag{2.8}$$

i.e.,

$$u_H^2=\frac{a^4}{\gamma u_0^2}+\frac{1}{3}\gamma u_0^2, \quad \gamma=3\left[\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{4}{27}\left(\frac{a}{u_0}\right)^{12}}\right]^{1/3}. \tag{2.9}$$

Note that for large a, one has the approximate expressions

$$u_H^2\approx a^2+\frac{u_0^6}{2a^4}, \quad A\approx\frac{3u_0^4}{2a^4} \tag{2.10}$$

(u_H^2 is always real). This shows, in particular, that the radius $R_0=A/(3u_0)$ can be made very small by increasing a/u_0. This is essentially the mechanism that will make the corresponding KK modes decouple at large a/u_0. At small a, such KK states have masses of the same order as the masses of the lightest glueball states.

The string tension is given by [9]

$$\sigma=\frac{4}{3}\lambda A u_0^2=4\lambda\frac{u_0^6}{3u_H^4-a^4}. \tag{2.11}$$

String excitations should have masses of order $\sigma^{1/2}$. The spin $\leqslant 2$ glueballs that remain in the supergravity approximation—whose masses are determined from the Laplace equation—have masses which are independent of λ. The supergravity approximation is valid for $\lambda A \gg 1$ so that all curvature invariants are small [9]. In this limit the spin >2 glueballs corresponding to string excitations will be much heavier than the supergravity glueballs.

B. Spectrum of glueball masses

The glueball masses are obtained by computing correlation functions of gauge invariant local operators or the Wilson loops, and looking for particle poles. Following [7,8], correlation functions of local operators $\mathcal{O}$ are related at large N and large g_{YM}^2N to tree level amplitudes of supergravity. The generating functional for the correlation function of $\mathcal{O}$ is the string partition function evaluated with specified boundary values φ_0 of the string fields. When the supergravity description is applicable we have

$$\langle e^{-\int d^4x\varphi_0(x)\mathcal{O}(x)}\rangle=e^{-I_{SG}(\varphi_0)}, \tag{2.12}$$

where I_{SG} is the supergravity action.

The spectrum of the scalar glueball[1] 0^{++} is obtained by finding the normalizable solution to the supergravity equation for the dilaton mode Φ that couples to $\mathrm{Tr}\,F^2$, which is the lowest dimension operator with 0^{++} quantum numbers.

The equation for Φ reads

$$\partial_\mu[\sqrt{g}e^{-2\phi}g^{\mu\nu}\partial_\nu\Phi]=0, \tag{2.13}$$

where $g_{\mu\nu}$ is the string frame metric.

We look for θ-independent solutions of the form $\Phi=\chi(u)e^{ik\cdot x}$. One obtains the equation

$$\frac{1}{u^3}\partial_u[u(u^6-a^4u^2-u_0^6)\chi'(u)]=-M^2\chi(u), \quad M^2=-k^2, \tag{2.14}$$

where the eigenvalues M are the glueball masses. The solution of this (ordinary) differential equation has to be normalizable and regular both at $u\to u_H$ and $u\to\infty$. The eigenvalues of this equation can be easily obtained numerically [4] by using the "shooting" method. One first finds the asymptotic behavior of $\chi(u)$ for large u, and then numerically integrates this solution back to the horizon. The solutions regular at the horizon will have a finite derivative at u_H. This condition will determine the possible values of the glueball masses M. The results of this numerical procedure are presented in Table I and in Fig. 1. One can see from Fig. 1 that the ratios of the masses of the excited glueball states compared to the ground state are very stable with respect to the variation in the parameter a, even though both quantities

[1] In the following we will use the notation J^{PC} for the glueballs, where J is the glueball spin, and P, C refer to the parity and charge conjugation quantum numbers, respectively.

TABLE I. Masses of the first few 0^{++} glueballs in QCD_4, in GeV, from supergravity compared to the available lattice results. The first column gives the lattice result [10,11], the second gives the supergravity result for $a=0$ while the third gives the supergravity result in the $a\to\infty$ limit. The authors of Ref. [11] do not quote an error on the preliminary lattice result for 0^{++*}. Note that the change from $a=0$ to $a=\infty$ in the supergravity predictions is tiny.

State	Lattice, $N=3$	Supergravity $a=0$	Supergravity $a\to\infty$
0^{++}	1.61 ± 0.15	1.61(input)	1.61(input)
0^{++*}	2.8	2.55	2.56
0^{++**}		3.46	3.48
0^{++***}		4.36	4.40

themselves grow like $M^2\propto a^2$ for large a. The asymptotic value of the mass ratios is taken on very quickly, $a/u_0\approx 2$ is sufficiently large to be in the asymptotic region. *A priori* one could have expected that these mass ratios may change significantly when a is varied. This leads one to suspect that there is a dynamical reason for the stability of the ratios of masses.

Let us now consider the 0^{-+} glueballs. The lowest dimension operator with 0^{-+} quantum numbers is $\mathrm{Tr}\,F\tilde{F}$. On the D4 brane worldvolume, the field that couples to this operator is the R-R 1-form A_μ. In order to find the 0^{-+} glueball masses we have to solve its equation of motion

$$\partial_\nu[\sqrt{g}g^{\mu\rho}g^{\nu\sigma}(\partial_\rho A_\sigma-\partial_\sigma A_\rho)]=0, \quad \mu,\nu=1,\ldots,10. \tag{2.15}$$

Consider solutions of the form

$$A_{\theta_2}=\chi_{\theta_2}(u)e^{ik\cdot x}, \quad A_\mu=0 \ \text{ if } \ \mu\neq\theta_2. \tag{2.16}$$

Plugging this into Eq. (2.16), we obtain

$$\partial_\nu[\sqrt{g}g^{\theta_2\theta_2}g^{\nu\sigma}\partial_\sigma A_{\theta_2}]=0, \tag{2.17}$$

which reads

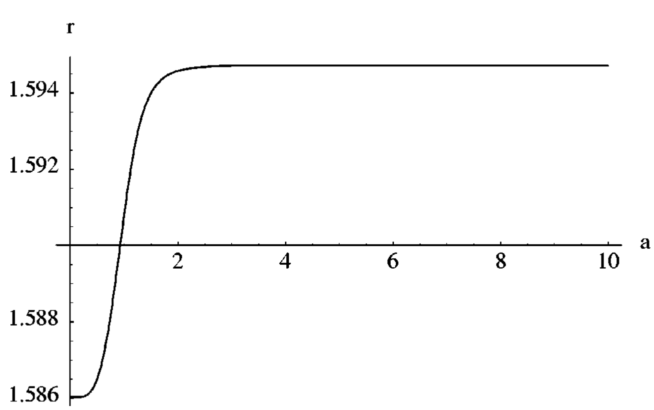

FIG. 1. The dependence of the ratio $r=M_{0^{++*}}/M_{0^{++}}$ of the masses of the first excited (0^{++*}) glueball state to the lowest 0^{++} glueball state on the parameter a (in units where $u_0=1$). The ratio changes very little and takes on its asymptotic value quickly.

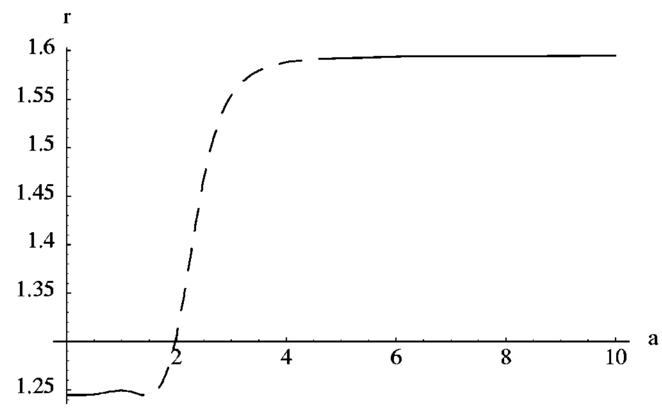

FIG. 2. The dependence of the ratio $r=M_{0^{-+}}/M_{0^{++}}$ on the parameter a (in units where $u_0=1$). The change in the ratio is stable against the variation of a, however it increases by about 25% while going to $a=\infty$. The change is in agreement with lattice simulations. As explained in the text, this figure is reliable only for the regions $a\ll u_0$ or $a\gg u_0$ which are shown in the plot with a solid line, while for the intermediate region denoted by a dashed line there are corrections due to the non-vanishing off-diagonal component of the metric $g_{\theta_2\varphi}$.

$$\frac{1}{u^5}(u^6-a^4u^2-u_0^6)\partial_u[u^3(u^4-a^4)\chi'_{\theta_2}(u)] = -M^2(u^4-a^4)\chi_{\theta_2}(u). \tag{2.18}$$

For $a=0$, it yields Eq. (2.9) of [12], as required. When $a\neq0$ there are no solutions of the form (2.16). The reason is that the $g_{\theta_2\varphi}$ component of the metric (2.6) is non-vanishing for $a\neq0$ and, as a result, the φ component of the Maxwell equation is not satisfied automatically (solutions contain a non-vanishing component A_φ).

We will work in the approximation $a/u_0\gg1$. In this approximation the non-diagonal $g_{\theta_2\varphi}$ part of the metric can be neglected, and there are solutions of the form (2.16). Effectively, we can solve Eq. (2.18) in the limit $a\gg u_0$. We must however keep in mind that we need $u_0\neq0$ to regularize the horizon, and the actual limit that is taken is a/u_0 large at fixed u_0 (so that curvature invariants are bounded from above and they are small for sufficiently large 't Hooft coupling λ).

The mass spectrum from Eq. (2.18) can be obtained using a similar numerical method as for the 0^{++} glueballs. The dependence of the lightest 0^{-+} glueball mass on a is presented in Fig. 2, whereas the 0^{-+} glueball mass spectrum in Table II. Note that while masses ratios are fairly stable against the variation of a (they again grow like $M^2\propto a^2$), the actual values of the mass ratios compared to 0^{++} increase by a sizeable ($\sim$25%) value. The change is in the right direction as suggested by recent improved lattice simulations [13]. The mass of the second 0^{-+} state also increases and is in agreement with the new lattice results [13].

We can directly compare the ratio of masses of the lowest glueball states 0^{-+} and 0^{++} with lattice results [10,11,13]. Since one of the largest errors in the lattice calculation of

TABLE II. Masses of the first few 0^{-+} glueballs in QCD_4, in GeV, from supergravity compared to the available lattice results. The first column gives the lattice result, the second is the supergravity result for $a=0$ while the third is the supergravity result in the $a\to\infty$ limit. Note that the change from $a=0$ to $a=\infty$ in the supergravity predictions is sizeable, of the order $\sim 25\%$.

State	Lattice, $N=3$	Supergravity $a=0$	Supergravity $a\to\infty$
0^{-+}	2.59 ± 0.13	2.00	2.56
0^{-+*}	3.64 ± 0.18	2.98	3.49
0^{-+**}		3.91	4.40
0^{-+***}		4.83	5.30

glueball masses comes from setting the overall scale[2] the ratios of masses are even more accurately known from the lattice than the masses themselves. Using the lattice results [11,13,14] in the more accurate ''lattice units'' r_0:

$$r_0 M_{0^{++}} = 4.33\pm0.05, \quad r_0 M_{0^{-+}} = 6.33\pm0.07,$$

$$r_0 M_{0^{-+*}} = 8.9\pm0.1, \tag{2.19}$$

we find

$$\left(\frac{M_{0^{-+}}}{M_{0^{++}}}\right)^{a=0}_{\text{supergravity}} = 1.24,$$

$$\left(\frac{M_{0^{-+}}}{M_{0^{++}}}\right)^{a\to\infty}_{\text{supergravity}} = 1.59,$$

$$\left(\frac{M_{0^{-+}}}{M_{0^{++}}}\right)_{\text{lattice}} = 1.46\pm0.03, \tag{2.20}$$

$$\left(\frac{M_{0^{-+*}}}{M_{0^{++}}}\right)^{a=0}_{\text{supergravity}} = 1.85,$$

$$\left(\frac{M_{0^{-+*}}}{M_{0^{++}}}\right)^{a\to\infty}_{\text{supergravity}} = 2.17,$$

$$\left(\frac{M_{0^{-+*}}}{M_{0^{++}}}\right)_{\text{lattice}} = 2.06\pm0.05. \tag{2.21}$$

One can see that taking the $a\to\infty$ improves the agreement between the supergravity and lattice predictions significantly. One should however keep in mind that the supergravity results presented here are for the limit $N\to\infty$ and $\lambda\to\infty$, while the lattice results are for $N=3$ and λ small.[3] Direct lattice calculations for the large N limit have just started to become available [15], however no reliable direct estimate for the mass of the 0^{-+} is known yet.

[2]We thank M. Peardon for emphasizing this point to us.

[3]For example, in Ref. [11] the results are extrapolated to $\lambda=0$ from calculations in the region $7.5<g^2N<10$.

TABLE III. Masses of the KK modes which wrap around the θ_2 circle and have no corresponding states in QCD_4, in GeV. The first column gives the masses for $a=0$ while the second the masses for $a=3$. Note that even for $a=3$ these states are heavier by a factor of 10 than the 0^{++} glueball mass and are effectively decoupled from the spectrum even in the supergravity limit.

State	Value for $a=0$	Value for $a=3$
KK	2.24	20.25
KK*	3.12	20.37
KK**	4.01	20.52
KK***	4.89	20.72

C. Masses of Kaluza-Klein states

In the supergravity approximation the $a=0$ model contains additional light KK modes in the spectrum whose masses are of the same order as those of the glueball states [6]. In this section we investigate whether the additional parameter of the model considered here can be tuned to decouple the KK modes already at the supergravity tree-level. In the following, it will be shown that this is indeed the case for the KK modes wrapped around the θ_2 direction, which become very heavy for $a/u_0 \gg 1$. We thus look for solutions of the dilaton equation (2.14) of the form

$$\Phi = \chi(u) e^{ik\cdot x} e^{in\theta_2}. \tag{2.22}$$

One finds the following equation:

$$\frac{1}{u^3}\partial_u[u(u^6-a^4u^2-u_0^6)\chi'(u)] = \left(-M^2+\frac{n^2}{R_0^2}\frac{u^4-a^4}{u^4-a^4-\frac{u_0^6}{u^2}}\right)\chi(u), \tag{2.23}$$

where R_0 is given in Eq. (2.4). This generalizes Eq. (2.14) to the case $n\neq0$. We want to compare $M_0 \equiv M(n=0)$ with $M_{KK} \equiv M(n=1)$. The question is how M_0/M_{KK} behaves as a function of a (we can set $u_0=1$). The extra term proportional to $1/R_0^2$ gives a positive contribution to the mass, so that M_{KK} should increase as $M_{KK}^2 \propto a^8$ as a is increased (the KK radius $R_0=A/3u_0$ shrinks to zero as $a\to\infty$). Thus one expects that $M_0/M_{KK} \propto 1/a^3 \to 0$ as a increases. The numerical values of the masses of these KK modes are displayed in Table III and Fig. 3. Note that the numerical evaluation of the masses of these KK modes becomes more and more difficult as a increases. This is because the term with $1/R_0^2$ causes an overall shift of the masses, while the splittings between the excited KK modes still remain of the same order as for the ordinary glueballs. As a result, the solutions become more and more quickly oscillating as a increases, making numerical treatments increasingly difficult. For this reason we display only values up to $a/u_0=3$.

Above we have demonstrated that the KK modes which correspond to states that wrap the θ_2 direction are effectively decoupled from the spectrum even in the supergravity ap-

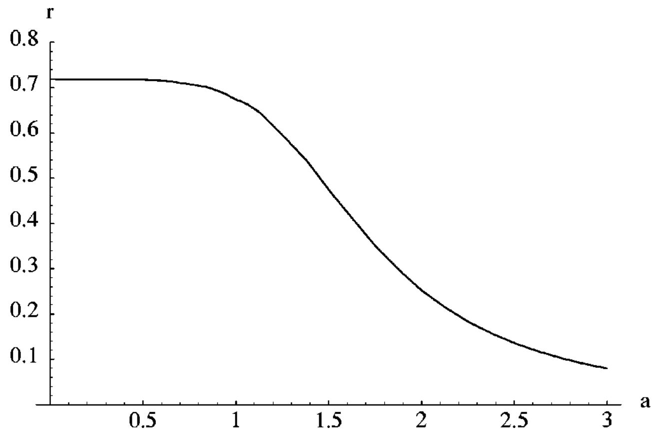

FIG. 3. The dependence of the ratio $r=M_{0^{++}}/M_{KK}$ of the lowest 0^{++} glueball state compared to the KK mode wrapping the θ_2 circle on the parameter a in units where $u_0=1$. This KK mode decouples very quickly from the spectrum even in the supergravity approximation.

proximation. However, there are other KK modes in this theory, and one would like to know whether these are decoupled as well. The reason for the decoupling of the modes on θ_2 is clear: the radius of this direction shrinks to zero when $a\to\infty$. However, the radii of the other compact directions do not behave similarly. Therefore it is reasonable to expect that these states will not decouple at the level of supergravity from the spectrum (but they could decouple once string theory corrections are incorporated). We now demonstrate by explicit calculation of the corresponding mass spectrum that this is indeed the case.

Consider non-singlet modes which are independent of θ_2, of the form $f(u)e^{ik\cdot x}\cos\varphi\sin\theta$. This corresponds to spherical harmonics on S^4 with angular momentum $l=1$. Plugging this ansatz into the dilaton equation (2.13) we find that $f(u)$ satisfies the equation $(u_0=1)$

$$\frac{1}{u^3}\,\partial_u[(u^7-a^4u^3-u)f'(u)] - f(u)\left(k^2+\frac{4u^2(4+3a^4u^2-4u^6)}{1+a^4u^2-u^6}\right)=0. \tag{2.24}$$

The results of the numerical analysis of the eigenvalues are presented in Table IV and Fig. 4. One can see that these

TABLE IV. Masses of the KK modes corresponding to $l=1$ angular momentum on the S^4, in GeV. The first column gives the masses for $a=0$ while the second the masses in the $a\to\infty$ limit. Note that the change from $a=0$ to $a=\infty$ in the supergravity predictions is not sufficiently large in order to decouple these particular states from the spectrum in the supergravity limit.

State	Value for $a=0$	Value for $a\to\infty$
KK	2.30	2.84
KK*	3.29	3.80
KK**	4.23	4.74
KK***	5.15	5.65

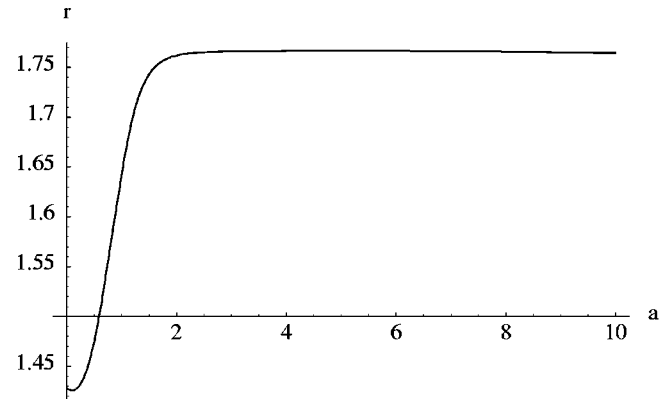

FIG. 4. The dependence of the ratio $r=M_{KK}/M_{0^{++}}$ of KK modes (corresponding to spherical harmonics with $l=1$ on S^4) compared to the lowest 0^{++} glueball state on the parameter a in units where $u_0=1$. This KK mode does not decouple from the spectrum in the supergravity approximation even in the $a\to\infty$ limit.

states do not decouple from the spectrum at the supergravity level, instead their masses remain comparable to the ordinary glueball masses.

D. Free energy and gluon condensation

The standard relation between the thermal partition function and free energy $Z(T)=\exp(-F/T)$ relates the free energy associated with the supergravity background to the expectation value of the operator $\mathrm{Tr}\,F^2_{\mu\nu}$. This relation was exploited in [12] to obtain a prediction for the gluon condensate in the Witten $(a=0)$ supergravity model. Let us now derive the corresponding supergravity result for general a. From the rotating M5 brane metric [given in Eq. (3.1) of [9]], one can obtain the following formulas for the Arnowitt-Deser-Misner (ADM) mass, entropy and angular momentum (see also [16,17]):

$$M_{\mathrm{ADM}}=\frac{V_5V(\Omega_4)}{4\pi G_N}2m\left(1+\frac{3}{4}\sinh^2\alpha\right),\quad V(\Omega_4)=\frac{8\pi^2}{3}, \tag{2.25}$$

$$S=\frac{V_5V(\Omega_4)}{4G_N}2mr_H\cosh\alpha, \tag{2.26}$$

$$J_H=\frac{V_5V(\Omega_4)}{4\pi G_N}ml\cosh\alpha, \tag{2.27}$$

$$G_N=\frac{\kappa_{11}^2}{8\pi}=2^4\pi^7l_P^9, \tag{2.28}$$

where G_N is Newton's constant in 11 dimensions, and l_P is the 11 dimensional Planck length. The (magnetic) charge N is related to α and m by

$$2m\cosh\alpha\sinh\alpha=\pi Nl_P^3. \tag{2.29}$$

The Hawking temperature and angular velocity are given by

$$T_H=\frac{3r_H^2+l^2}{8\pi m\cosh\alpha},\quad \Omega_H=\frac{lr_H}{2m\cosh\alpha}. \tag{2.30}$$

These quantities satisfy the first law of black hole thermodynamics:

$$dM_{\mathrm{ADM}}=T_H dS+\Omega_H dJ_H. \tag{2.31}$$

We are interested in the field theory limit $l_P\to 0$, obtained by rescaling variables as follows:

$$r=u^2 l_P^3(4\pi N),\quad m=\frac{1}{2}u_0^6 l_P^9(4\pi N)^3,\quad l=ia^2 l_P^3(4\pi N). \tag{2.32}$$

We get

$$E\equiv M_{\mathrm{ADM}}-M_{\mathrm{extremal}}=\frac{5}{3\pi^3}V_5N^3u_0^6, \tag{2.33}$$

$$T_H=\frac{3u_0}{2\pi A},\quad S=\frac{4}{3\pi^2}V_5N^3u_H^2u_0^3, \tag{2.34}$$

$$\Omega_H=i\frac{2a^2u_H^2}{u_0^3},\quad J_H=i\frac{2}{3\pi^3}V_5N^3a^2u_0^3, \tag{2.35}$$

with

$$A=\frac{3u_0^4}{3u_H^4-a^4},\quad u_H^4-a^4=\frac{u_0^6}{u_H^2}. \tag{2.36}$$

The free energy is given by

$$F=E-T_HS-\Omega_HJ_H=-\frac{V_5}{3\pi^3}N^3u_0^6. \tag{2.37}$$

For $a=0$ this reproduces the result of [18] ($u_0=2\pi AT_H/3$). The M5 brane coordinate x_5 is compactified on a circle with radius $R_0\lambda/N$, given by Eq. (2.4), so that

$$V_5=\frac{V_4\lambda}{T_HN}. \tag{2.38}$$

The gluon condensate is then given by

$$\left\langle\frac{1}{4g_{\mathrm{YM}}^2}\mathrm{Tr}\,F_{\mu\nu}^2(0)\right\rangle=-\frac{F}{V_4T_H}=\frac{4}{27\pi}\lambda N^2u_0^4A^2. \tag{2.39}$$

For $a=0$ this reduces to the corresponding result in [12] (setting $A=1$ and $u_0=2\pi T_H/3$). Expressing u_0 in terms of the string tension (2.11) we obtain

$$\left\langle\frac{1}{4g_{\mathrm{YM}}^2}\mathrm{Tr}\,F_{\mu\nu}^2(0)\right\rangle=\frac{1}{12\pi}\frac{N^2}{\lambda}\sigma^2. \tag{2.40}$$

Note that this relation is independent of a (in particular, it applies to the $a=0$ case as well). It has the expected dependence on N, and a simple dependence on λ.

E. Topological susceptibility

The topological susceptibility χ_t is defined by

$$\chi_t=\frac{1}{(16\pi^2)^2}\int d^4x\langle\mathrm{Tr}\,F\tilde{F}(x)\mathrm{Tr}\,F\tilde{F}(0)\rangle. \tag{2.41}$$

The topological susceptibility measures the fluctuations of the topological charge of the vacuum. At large N the Witten-Veneziano formula [19,20] relates the mass $m_{\eta'}$ in SU(N) with N_f quarks to the topological susceptibility of SU(N) without quarks:

$$m_{\eta'}^2=\frac{4N_f}{f_\pi^2}\chi_t. \tag{2.42}$$

The effective low-energy four dimensional brane theory contains the coupling

$$\frac{1}{16\pi^2}\int d^4x\,d\theta_2\,A_{\theta_2}\mathrm{Tr}\,F_{\mu\nu}F_{\lambda\sigma}\epsilon^{\mu\nu\lambda\sigma}, \tag{2.43}$$

where A_{θ_2} is the component along the coordinate θ_2 of the R-R 1-form A_μ. We will consider zero mode ($M^2=0$) configurations where A_{θ_2} is independent of the world-volume coordinates. Comparing to the standard Yang-Mills coupling,

$$\frac{1}{16\pi^2}\int d^4x\,\hat{\theta}\,\mathrm{Tr}\,F\tilde{F}, \tag{2.44}$$

one obtains the relation

$$\hat{\theta}=\int_0^{2\pi}d\theta_2A_{\theta_2}=2\pi A_{\theta_2}. \tag{2.45}$$

The action of the R-R 1-form is given by

$$I=\frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{g}\frac{1}{4}(\partial_\mu A_\nu-\partial_\nu A_\mu)\times(\partial_{\mu'}A_{\nu'}-\partial_{\nu'}A_{\mu'})g^{\mu\mu'}g^{\nu\nu'}. \tag{2.46}$$

As discussed in Sec. II B, in the approximation that a/u_0 is either very large or very small, the metric is diagonal and there are zero mode solutions of the form $A_{\theta_2}=A_{\theta_2}(u), A_\mu=0$, $\mu\neq\theta_2$. The action reduces to

$$I=\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{g}\left(\frac{dA_{\theta_2}(u)}{du}\right)^2g^{uu}g^{\theta_2\theta_2}. \tag{2.47}$$

Using Eq. (2.6) and integrating over the angular coordinates, this becomes

$$I=\frac{2^7\pi^6A^2\lambda^3}{27u_0^2\kappa_{10}^2}V_4\int_{u_H}^\infty du\,u^3(u^4-a^4)\left(\frac{dA_{\theta_2}(u)}{du}\right)^2. \tag{2.48}$$

The equation of motion is then given by

$$\partial_u[u^3(u^4-a^4)\partial_u A_{\theta_2}]=0. \tag{2.49}$$

Therefore,

$$\partial_u A_{\theta_2}=6A_{\theta_2}^{\infty}C(a)\frac{u_0^6}{u^3(u^4-a^4)}, \tag{2.50}$$

$$A_{\theta_2}=A_{\theta_2}^{\infty}\left[1+6C(a)\left(\frac{u_0^6}{2a^4u^2}+\frac{u_0^6}{4a^6}\log\frac{u^2-a^2}{u^2+a^2}\right)\right]. \tag{2.51}$$

The integration constant $C(a)$ will be fixed by assuming that $A_{\theta_2}(u)$ vanishes at the horizon [21]. This gives

$$\frac{1}{C(a)}=-\frac{3u_0^6}{a^4u_H^2}+\frac{3u_0^6}{2a^6}\log\frac{u_H^2+a^2}{u_H^2-a^2}. \tag{2.52}$$

The other integration constant $A_{\theta_2}^{\infty}$ is related to the $\hat{\theta}$-parameter by Eq. (2.45), $2\pi A_{\theta_2}^{\infty}=\hat{\theta}$. Note that in the limit $a=0$ one gets $C(0)=1$ and $A_{\theta_2}(u)=A_{\theta_2}^{\infty}(1-u_0^6/u^6)$.

Using $\kappa_{10}^2=\kappa_{11}^2/2\pi=2^6\pi^7$ and $u_0=2\pi A T_H/3$, we obtain

$$I=\hat{\theta}^2V_4\frac{16\pi}{729}A^6C(a)\lambda^3T_H^4. \tag{2.53}$$

The topological susceptibility (2.41) can then be obtained by differentiating twice with respect to $\hat{\theta}$:

$$\chi_t=\frac{32\pi}{729}A^6C(a)\lambda^3T_H^4, \tag{2.54}$$

or, in terms of the string tension (2.11),

$$\chi_t=\frac{C(a)}{8\pi^3}\lambda\sigma^2. \tag{2.55}$$

The $\hat{\theta}$-dependence of the vacuum energy of the form $\hat{\theta}^2$ is the result anticipated in [21] for the $a=0$ model, and Eq. (2.53) shows that it holds for large a too. In the large N limit, this must be the case for consistency [21]. For $a=0$, one has $A=1=C(a)$, and Eq. (2.54) reproduces the result obtained in [12]. In the large a limit we have [see Eq. (2.10)]

$$C(a)\approx\frac{a^6}{9u_0^6\log\dfrac{a}{u_0}}, \tag{2.56}$$

so that

$$\chi_t\approx\frac{1}{18\pi^3}\lambda^3\frac{u_0^6}{a^2\log\dfrac{a}{u_0}}=\frac{1}{72\pi^3}\frac{a^6}{u_0^6\log\dfrac{a}{u_0}}\lambda\sigma^2. \tag{2.57}$$

This decreases if we increase a/u_0 at fixed λ and u_0.

III. QCD IN 2+1 DIMENSIONS

A. Supergravity Models for QCD_3

Analogous models for QCD_3 can be obtained by starting with the Euclidean rotating D3 brane [Eq. (3.16) in [9] with $x_0\to -i\tau, l\to il$] and taking $\alpha'\to 0$ by rescaling variables as follows:

$$r=U\alpha',\quad 2m=U_0^4\alpha'^4,\quad l=a\alpha'. \tag{3.1}$$

In the limit $\alpha'\to 0$ at fixed U,a,U_0 we obtain

$$\begin{aligned}ds_{\mathrm{IIB}}^2=\alpha'\Delta_0^{1/2}\Bigg[&\frac{U^2}{\sqrt{4\pi g_sN}}(h_0d\tau^2+dx_1^2+dx_2^2+dx_3^2)\\&+\frac{\sqrt{4\pi g_sN}dU^2}{U^2\left(1-\dfrac{a^2}{U^2}-\dfrac{U_0^4}{U^4}\right)}+\sqrt{4\pi g_sN}\\&\times\left(d\theta^2+\frac{\tilde{\Delta}_0}{\Delta_0}\sin^2\theta d\varphi^2+\frac{\cos^2\theta}{\Delta_0}d\Omega_3^2\right)\\&-\frac{2aU_0^2}{U^2\Delta_0}\sin^2\theta d\tau d\varphi\Bigg],\end{aligned} \tag{3.2}$$

where

$$h_0=1-\frac{U_0^4}{U^4\Delta_0},\quad \Delta_0=1-\frac{a^2\cos^2\theta}{U^2},\quad \tilde{\Delta}_0=1-\frac{a^2}{U^2}, \tag{3.3}$$

$$d\Omega_3^2=d\psi_1^2+\sin^2\psi_1d\psi_2^2+\cos^2\psi_1d\psi_3^2. \tag{3.4}$$

The theory describes fermions with anti-periodic boundary conditions on the circle parametrized by τ, which has radius $(2\pi T_H)^{-1}$ with

$$T_H=\frac{u_0}{\pi B},\quad B\equiv\frac{2u_0^3}{u_H(2u_H^2-a^2)}, \tag{3.5}$$

$$u_H^2=\frac{1}{2}(a^2+\sqrt{a^4+4u_0^4}). \tag{3.6}$$

For convenience, we have rescaled variables by $U=(4\pi g_sN)^{1/2}u, a\to(4\pi g_sN)^{1/2}a$. At energies much lower than T_H the theory should be effectively $2+1$ dimensional (with $x_0=ix_3$ playing the role of time). The gauge coupling of the $2+1$ dimensional field theory is given by

$$g_{\mathrm{YM}_3}^2=g_{\mathrm{YM}_4}^2T_H,\quad g_{\mathrm{YM}_4}^2=2\pi g_s, \tag{3.7}$$

$$\lambda\equiv\frac{g_{\mathrm{YM}_3}^2N}{2\pi}=g_sN\frac{u_0}{\pi B}. \tag{3.8}$$

TABLE V. Masses of the 0^{++} glueballs and their excited states in QCD_3. The first column gives the lattice results extrapolated to $N\to\infty$, the second column the supergravity results for $a=0$ and the third column the supergravity limit $a\to\infty$. The lattice results are in the units of the square root of the string tension. The error given is statistical and does not include the systematic error.

State	Lattice $N\to\infty$	Value for $a=0$	Value for $a\to\infty$
0^{++}	4.065 ± 0.055	4.07 (input)	4.07 (input)
0^{++*}	6.18 ± 0.13	7.03	7.05
0^{++**}	7.99 ± 0.22	9.93	9.97
0^{++***}	–	12.82	12.87

In this model Wilson loops exhibit an area-law behavior with string tension

$$\sigma=\frac{1}{2\pi}\sqrt{4\pi g_s N}u_0^2=\sqrt{\lambda B}u_0^{3/2}. \tag{3.9}$$

This can be obtained by minimizing the Nambu-Goto action [22], and it is essentially given by the coefficient of $\Sigma_{i=1}^3 dx_i^2$ at the horizon times $1/(2\pi)$ [2,23,24]. In the limit of large 't Hooft coupling, the light physical states are the supergravity modes, whose masses can be determined from the equations of motion of the string theory effective action. In the next sections we calculate the mass spectrum of the light physical states and of KK modes.

B. Spectrum of glueball masses

In order to find the spectrum of the 0^{++} glueball states one has to consider the supergravity equation for the dilaton mode Φ that couples to the operator $\mathrm{Tr}\,F^2$

$$\partial_\mu[\sqrt{g}g^{\mu\nu}\partial_\nu\Phi]=0, \tag{3.10}$$

evaluated in the above background. For functions of the form $\Phi=\chi(U)e^{ik\cdot x}$, we obtain

$$\partial_u[(u^4-u_0^4-a^2u^2)u\chi'(u)]+M^2u\chi(u)=0,\quad M^2=-k^2. \tag{3.11}$$

The eigenvalues of this equation can again be determined numerically. The results are presented in Table V and Fig. 5. Figure 5 gives the dependence on a of the mass ratio of the first excited 0^{++} glueball state compared to the ground state 0^{++}. One obtains a very similar behavior to the case of QCD_4, that is the mass ratio changes very little, and takes on its asymptotic value quickly. The comparison to the available lattice results [25] are given in Table V.

C. Masses of KK states

Just like in the case of QCD_4, we would like to analyze the behavior of the masses of the different KK modes. We will find very similar results: the KK modes wrapping the coordinate τ are decoupling (even though a bit slower than in

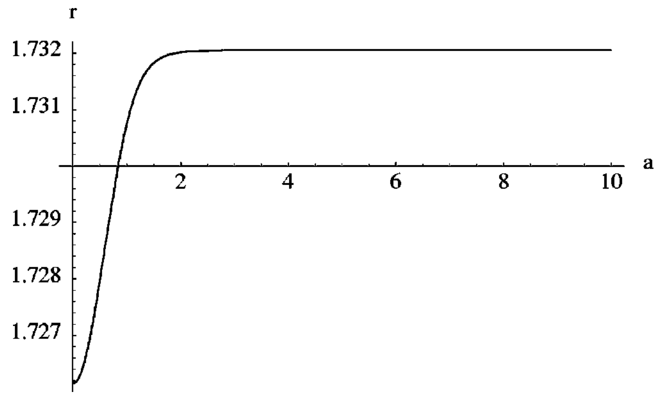

FIG. 5. The dependence on a of the ratio $r=M_{0^{++*}}/M_{0^{++}}$ in QCD_3. One can see that the ratio is very stable to changes in a, and reaches its asymptotic value quickly. a is given in units of u_0.

QCD_4), while the other KK modes corresponding to states with angular momentum on the S^5 are not decoupling in the supergravity limit.

First we consider the KK modes wrapping the compact τ direction. Let us consider solutions to the Laplace equation $\nabla^2\Phi=0$ of the form

$$\Phi=\chi(u)e^{ik\cdot x}e^{i\beta\tau}. \tag{3.12}$$

The coordinate τ is periodic with period T_H^{-1}, where T_H is given in Eq. (3.5). Therefore

$$\beta=2\pi T_H n, \tag{3.13}$$

where n is an integer. Using the metric (3.2) we find

$$\partial_u[(u^4-u_0^4-a^2u^2)u\chi'(u)]=u\left(-M^2+n^2 4\pi^2T_H^2\frac{u^2-a^2}{u^2-a^2-\dfrac{u_0^4}{u^2}}\right)\chi(u). \tag{3.14}$$

One can see that just like in the case of QCD_4 there is an additional positive contribution to the masses, which grows like a^4, therefore the masses of these KK states should grow as $M_{KK}^2\approx a^4$. Thus these KK modes decouple from the spectrum, but slower than the corresponding KK modes in QCD_4. The results of the numerical analysis are summarized in Table VI and Fig. 6.

TABLE VI. Masses of the KK modes wrapping the circle τ in QCD_3, using the same normalization as in Table V. The first column gives the masses for $a=0$ while the second the masses for $a=4$. Note that these states decouple quickly from the spectrum even in the supergravity approximation.

State	Value for $a=0$	Value for $a=4$
KK	5.79	23.77
KK*	8.64	24.63
KK**	11.50	25.78
KK***	14.36	27.19

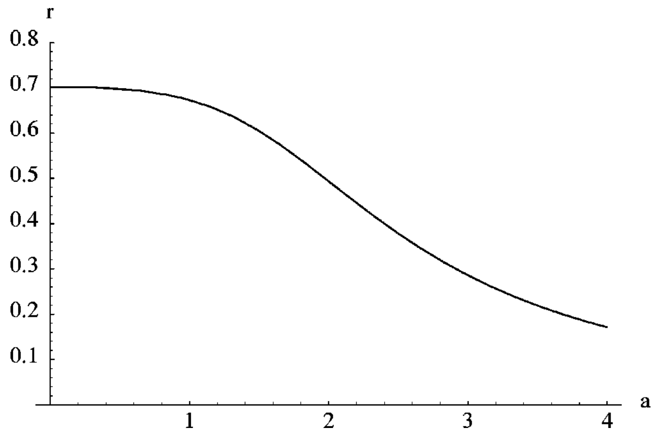

FIG. 6. The dependence on a of the ratio $r=M_{0^{++}}/M_{KK}$ of the lowest 0^{++} glueball state compared to the KK states wrapping the τ circle in units where $u_0=1$. These KK modes decouple from the spectrum in the supergravity approximation very quickly.

Next we analyze the KK modes which correspond to states with angular momentum $l=1$ on S^5 in the $a=0$ case. For $a=0$ these states have been examined in [6], and found to be non-decoupling in the supergravity limit and including the lowest order α' corrections. Here we repeat this analysis and find (just like in the case of QCD_4) them to be non-decoupling even in the $a\to\infty$ case, in the supergravity limit.

In order to do the analysis of these KK modes one needs to find the explicit form of the spherical harmonics. The spherical harmonics on S^d can be constructed in the following way. One takes S^d embedded in R^{d+1}, and expresses the Cartesian coordinates y_i in terms of the angles. Then the spherical harmonics are just the functions $C^{i_1,\ldots,i_k}y_{i_1}\ldots y_{i_k}$, where C is a symmetric traceless tensor [26]. This way, the simplest non-trivial spherical harmonic is just the coordinate y_i itself. In the case of our QCD_3 theory, we actually have to use the ''spheroidal coordinates'' y_i given in [9] on p. 9. Thus one looks for solutions of the dilaton equation of the form $f(u)e^{ik\cdot x}y_i$, $i=1,2,3,4,5,6$. For $a=0$ the isometry group of S^5 is SO(6), and the $l=1$ KK mode is in the representation **6** of SO(6). Introducing the angular momentum a breaks SO(6) to SO(4)×U(1)×U(1), and the **6** decomposes into **4**+**1**+**1**. These states satisfy different eigenvalue equations. For $i=1,2$ (the two singlets are degenerate) the equation one gets is

$$\partial_u[u(u^4-a^2u^2-1)f'(u)] -f(u)\left(k^2u+\frac{5u^3+4a^2u^5-5u^7}{1+a^2u^2-u^4}\right)=0. \tag{3.15}$$

Note that for $a=0$ this indeed reduces to the equation given in [6] for $l=1$. We have numerically solved this equation, and find that the mass of these KK modes is growing slightly, until it becomes degenerate with the first excited state of the 0^{++} glueball. Thus it does not decouple from the spectrum in the supergravity limit. The results are summarized in Table VII and Fig. 7.

TABLE VII. Masses of the KK modes corresponding to the two degenerate singlet pieces of the $l=1$ sextet of the original SO(6) isometry in QCD_3, using the same normalization as in Table V. The first column gives the masses for $a=0$ while the second the masses in the $a\to\infty$ limit. Note that these states do not decouple from the spectrum in the supergravity approximation.

State	Value for $a=0$	Value for $a\to\infty$
KK	5.27	7.05
KK*	8.29	9.97
KK**	11.23	12.87
KK***	14.14	15.76

For the other 4 KK states which are in the **4** of SO(4) one finds the equation

$$\partial_u[u(u^4-a^2u^2-1)f'(u)]-f(u)(k^2u+5u^3-3a^2u)=0, \tag{3.16}$$

which for $a=0$ reproduces the equation in [6] with $l=1$. The additional mass term is now negative, which actually makes these KK modes lighter in the $a\to\infty$ limit than for $a=0$. However, they are still of the same order (and slightly heavier) than the 0^{++} glueballs. The numerical results for this state are summarized in Table VIII and Fig. 8.

D. Free energy and gluon condensation

From the rotating D3 brane metric [see Eq. (3.16) in [9]], one can find the following formulas for the thermodynamic variables:

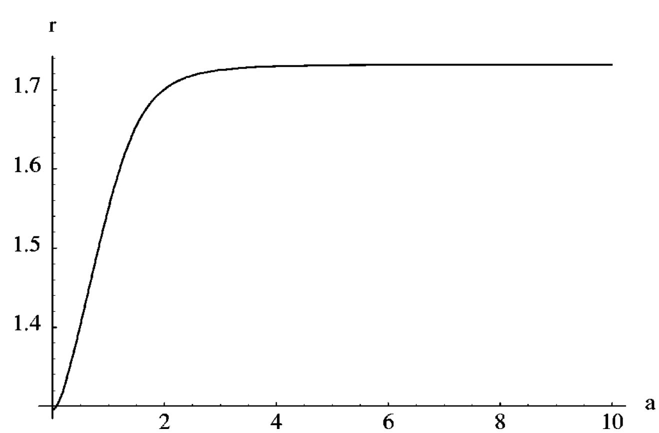

FIG. 7. The dependence of the ratio $r=M_{KK}/M_{0^{++}}$ of the KK modes corresponding to the two singlet $l=1$ states compared to the lowest 0^{++} glueball state on the parameter a in units where $u_0=1$. This KK mode does not decouple from the spectrum in the supergravity approximation even in the $a\to\infty$ limit, even though it increases slightly and becomes exactly degenerate with the first excited glueball state 0^{++*}.

TABLE VIII. Masses of the KK modes corresponding to the quartet piece of the $l=1$ sextet of the original SO(6) in QCD_3, using the same normalization as in Table V. The first column gives the masses for $a=0$ while the second the masses in the $a\to\infty$ limit. Note that these states actually get lighter from $a=0$ to $a=\infty$ in the supergravity approximation.

State	Value for $a=0$	Value for $a\to\infty$
KK	5.27	4.98
KK*	8.29	8.14
KK**	11.23	11.15
KK***	14.14	14.10

$$M_{\rm ADM}=\frac{V_3V(\Omega_5)}{4\pi G_N}\frac{5}{4}2m\left(1+\frac{4}{5}\sinh^2\alpha\right),\quad V(\Omega_5)=\pi^3, \tag{3.17}$$

$$S=\frac{V_3V(\Omega_5)}{4G_N}2mr_H\cosh\alpha, \tag{3.18}$$

$$J_H=\frac{V_3V(\Omega_5)}{4\pi G_N}ml\cosh\alpha, \tag{3.19}$$

$$T_H=\frac{r_H(2r_H^2+l^2)}{4\pi m\cosh\alpha},\quad \Omega_H=\frac{lr_H^2}{2m\cosh\alpha}, \tag{3.20}$$

$$G_N=\frac{\kappa_{10}^2}{8\pi}=8g_s^2\pi^6(\alpha')^4, \tag{3.21}$$

where

$$2m\cosh\alpha\sin\alpha=4\pi g_sN\alpha'^2. \tag{3.22}$$

One can check that they satisfy the first law of black hole thermodynamics Eq. (2.31). In the limit $\alpha'\to 0$ [rescaling variables as in Eq. (3.1)], we get

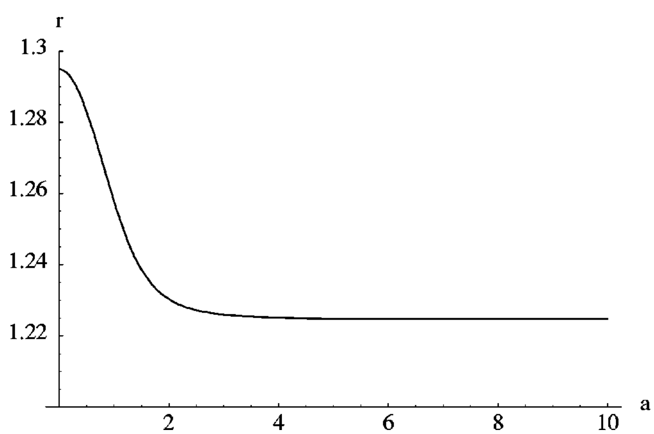

FIG. 8. The dependence of the ratio $r=M_{KK}/M_{0^{++}}$ of the KK states compared to the lowest 0^{++} glueball state on the parameter a in units where $u_0=1$. This KK mode does not decouple from the spectrum in the supergravity approximation even in the $a\to\infty$ limit, they instead get even slightly lighter than for $a=0$.

$$E\equiv M_{\rm ADM}-M_{\rm ext}=\frac{3}{8\pi^2}V_3N^2u_0^4, \tag{3.23}$$

$$T_H=\frac{u_0}{\pi B},\quad S=\frac{1}{2\pi}V_3N^2u_Hu_0^2, \tag{3.24}$$

$$\Omega_H=i\frac{au_H^2}{u_0^2},\quad J_H=i\frac{1}{4\pi^2}V_3N^2au_0^2, \tag{3.25}$$

$$u_H^2-a^2=\frac{u_0^4}{u_H^4},\quad B=\frac{2u_0^3}{u_H(2u_H^2-a^2)}. \tag{3.26}$$

The free energy is then given by

$$F=E-T_HS-\Omega_HJ_H=-V_3\frac{N^2u_0^4}{8\pi^2}. \tag{3.27}$$

This gives for the gluon condensate the following expression:

$$\left\langle\frac{1}{4g_{\rm YM}^2}{\rm Tr}\,F_{\mu\nu}^2(0)\right\rangle=-\frac{F}{V_3T_H}=\frac{1}{8\pi}N^2Bu_0^3. \tag{3.28}$$

In terms of the Yang-Mills string tension, this is

$$\left\langle\frac{1}{4g_{\rm YM}^2}{\rm Tr}\,F_{\mu\nu}^2(0)\right\rangle=\frac{1}{8\pi}\frac{N^2}{\lambda}\sigma^2. \tag{3.29}$$

We find again that supergravity predicts that the gluon condensate is proportional to N^2/λ times the string tension squared. The result expressed in terms of the string tension is thus independent of a.

IV. CONCLUSIONS

In this paper we investigated quantitative aspects of large N SU(N) Yang-Mills theory in three and four dimensions using a one-parameter family of supergravity models related to non-extremal rotating D-branes. The new feature of this class of models is the decoupling of the KK modes associated with the compact D-brane coordinate as the angular momentum parameter is increased. The mass ratios for ordinary glueballs were found to be very stable against this variation. While the mass ratios of the 0^{++} glueballs change only slightly compared to the case with zero angular momentum, there is a substantial change in the mass ratios of 0^{-+}, 0^{-+*} given in Eqs. (2.20),(2.21), which for large a are in better agreement with the lattice values than for $a=0$.

It is worth emphasizing that the ratio a/u_0 should be large enough to have $M_{KK}\gg M_{\rm glueball}$, but not infinite, since there are also string states winding around the compact D-brane coordinate with masses of order σR_0 that should decouple, i.e., $M_{\rm wind}\gg M_{\rm glueball}$. This requires $\lambda u_0^8/a^8\gg 1$, which is consistent with the condition that curvature invariants are small compared to the string scale [9]. In general, for any given ratio a/u_0 which is large enough to decouple KK states from the low-energy physics, it is possible to choose λ sufficiently large so that string winding states also decouple.

We have found that the [SO(3) or SO(4)] non-singlet KK modes with vanishing U(1) charge in the compact D-brane coordinate do not decouple in this class of models. One can hope that those KK modes may decouple in a model with more angular momenta (since there is room to take other limits). In this case the isometry group of the internal space is smaller. For example, in QCD_3, for $a=0$ it is given by $SO(6)\times U(1)$, whereas for $a\neq 0$ it is $SO(4)\times U(1)\times U(1)$. The isometry group of the model with the maximum number of angular momenta only contains U(1) factors. This is consistent with the fact that in pure QCD there can only be singlets of the original R-symmetry.

We have also found some features which seem to be universal, i.e., which do not depend on the extra supergravity parameter. In particular, both in QCD_3 and QCD_4 supergravity gives a gluon condensate of the form $(N^2/\lambda)\sigma^2$, with a coefficient which is the same for all models parametrized by a. Another feature that seems to be common to all supergravity models is a topological susceptibility of the form $\lambda\sigma^2$, with a coefficient which is independent of N but depends on a/u_0. This result suggests that in the regime $\lambda \gg 1$ the η' particle of QCD_4 with $N=3$ is much heavier than other mesons (whose masses are proportional to the string tension).

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[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[3] D. J. Gross and H. Ooguri, Phys. Rev. D **58**, 106002 (1998).
[4] C. Csáki, H. Ooguri, Y. Oz, and J. Terning, ''Glueball mass spectrum from supergravity,'' hep-th/9806021.
[5] R. de Mello Koch, A. Jevicki, M. Mihailescu, and J. Nunes, Phys. Rev. D **58**, 105009 (1998); M. Zyskin, Phys. Lett. B **439**, 373 (1998).
[6] H. Ooguri, H. Robins, and J. Tannenhauser, Phys. Lett. B **437**, 77 (1998).
[7] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998).
[8] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
[9] J. G. Russo, ''New compactifications of supergravities and large N QCD,'' hep-th/9808117.
[10] M. J. Teper, ''Physics from the lattice: Glueballs in QCD: Topology: SU(N) for all N,'' hep-lat/9711011.
[11] C. Morningstar and M. Peardon, Phys. Rev. D **56**, 4043 (1997).
[12] A. Hashimoto and Y. Oz, ''Aspects of QCD dynamics from string theory,'' hep-th/9809106.
[13] M. Peardon, Nucl. Phys. B (Proc. Suppl.) **63**, 22 (1998); C. Morningstar and M. Peardon, hep-lat/9901004.
[14] C. Michael, ''Hadronic spectroscopy from the lattice: Glueballs and Hybrid Mesons,'' hep-ph/9810415.
[15] S. Dalley and B. van de Sande, ''Glueball calculations in large N_c gauge theory,'' hep-ph/9810236.
[16] G. Horowitz and A. Sen, Phys. Rev. D **53**, 808 (1996).
[17] M. Cvetič and D. Youm, Nucl. Phys. **B499**, 253 (1997).
[18] I. Klebanov and A. A Tseytlin, Nucl. Phys. **B475**, 164 (1996).
[19] E. Witten, Nucl. Phys. **B156**, 269 (1979).
[20] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).
[21] E. Witten, Phys. Rev. Lett. **81**, 2862 (1998).
[22] J. M. Maldacena, Phys. Rev. Lett. **80**, 4859 (1998).
[23] A. Brandhuber, N. Itzhaki, J. Sonnenschein, and S. Yankielowicz, J. High Energy Phys. **6**, 001 (1998).
[24] S. Rey and J. Yee, ''Macroscopic Strings as Heavy Quarks in Large N Gauge Theory and Anti-de Sitter Supergravity,'' hep-th/9803001; S. Rey, S. Theisen, and J. Yee, Nucl. Phys. **B527**, 171 (1998).
[25] M. J. Teper, Phys. Rev. D **59**, 014512 (1998).
[26] S. Lee, S. Minwalla, M. Rangamani, and N. Seiberg, ''Three point functions of chiral operators in $D=4$, $N=4$ SYM at large N,'' hep-th/9806074.