Cooperative dynamics of active heterogeneous systems

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Abstract: Collective motion is fundamental in flocking or schooling dynamics and represent one of the most fascinating sights in nature. In this paper, to describe collective motion, we use one of the most representative models called the Vicsek Model. The main features of this model is a noise-driven phase transition between ordered and disordered phase and its mathematical and computational simplicity. We reproduce its basic results and continue our work adding a leader to study how leadership affects its dynamics, showing that this new ingredient breaks the phase transition.

I. INTRODUCTION

In nature, collective motion plays an important role in life being behavior. Examples of that fact are seen worldwide and are well-known since it can be seen in our everyday life. Representative examples, which highlight that fact, are schools of fish providing protection or outmaneuvering, in a matter of seconds, a predator, a flock of birds moving around or migrating in formation or a sheep herd.

Although those are the most intuitive examples, there is a wide range of other systems where collective motion takes place. From an overcrowded street to cells or bacteria, even non-living systems show that kind of collective phenomena [1], like micrometer-sized silver chloride (AgCl) particles acting as an autonomous micromotor in an special environmental conditions [2].

This wide range of scales and systems got the attention of researchers from several fields. Biologists were the first who started studying that phenomena [3], followed by computer graphics researchers, like C.W. Reynolds, who created realistic animation of birds flocking by setting basic rules like collision avoidance and velocity matching, for instance [4]. However, it was not until T.Vicsek proposed his model that the physics community took interest in that field [5].

T.Vicsek presented a model where each individual is constantly updating its velocity direction by averaging with its closest neighbors, being the system fully determined by the position and velocity direction of each particle [5]. From a mathematical and computational point of view, Vicsek model is quite simple and doesn't require big computational efforts and from a physical view, this model shows a phase transition between ordered and disordered phase, which makes this model really interesting for a physicist. For those reasons, we decided to use Vicsek Model to study collective motion.

The structure of our work is the following: in Section II, we explain how Vicsek Model works, the way we implemented it and finally we present its basic results. After that, in Sec. III we introduce a new role (leader) in our system and explore how our system evolves and if the phase transition still takes place in function of the fraction of followers. Finally, in Sec. IV we end our study adding a second leader and tanking into consideration the effects of uninformed individuals.

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II. STANDAR VICSEK MODEL

The Vicsek Model is based in N self-propelled particles which are able to move in a d-dimensional box of sides L with constant speed v_0. Each particle interacts with the other ones through averaging its velocity direction with other particles inside a range of interaction defined in terms of an euclidean distance of radius R_0. Since perfect interaction is not realistic, a noise term η is introduced in order to simulate difficulty in gathering or processing information, etc.

In a 2-dimensional box, every particle is characterized by two variables, its position $\mathbf{r}_i$(t) and its velocity direction, which can be represented with only one angle θ(t) arbitrarily chosen between $[-\pi,\pi]$. By knowing how both parameters evolve over time, we can fully determine our system's evolution. Then, given that each particle dynamics depend just on its velocity, its future position can be determined following the Eq.(1):

$$\mathbf{r_i}(t+\Delta t) = \mathbf{r_i}(t) + \mathbf{v_i}(t)\Delta t, \tag{1}$$

where Δt represents our integration time step and $\mathbf{v_i}(t)$ the velocity at time t, which can be decomposed in terms of θ(t):

$$\mathbf{v_i}(t) = v_0(cos(\theta_i(t)), sin(\theta_i(t))) \tag{2}$$

It must be remarked that the only time dependence is found in θ_i(t) so it is the parameter which rules the motion. The conditions that defines θ_i(t)'s time dependence are two, the way interaction between particles is summarized and the impossibility of perfect alignment. As said before, each particle averages its direction with all individuals inside its radius of interaction with a noise term added to fulfill the second condition. Those conditions lead to:

$$\theta_i(t+\Delta t) = arg(\sum_{j\in R_0} \mathbf{v}_j(t)) + \eta\xi_i \tag{3}$$

where η is a parameter which allows us to control the noise intensity and ranges from 0 to 1, ξ is an uniformly distributed random number set between [-π,π] and the argument arg(...)=arctan($\frac{v_y}{v_x}$) is the velocity direction.

At this point, we need to define a suitable order parameter so as to capture the statistical information of the collective motion and to analyze the transition between an ordered and a disordered phase. A possibility to define it is as shown in Eq.(4):

$$|\phi(t)| = \frac{1}{Nv_0}|\sum_{i=1}^{N} \mathbf{v}_i(t)| \tag{4}$$

This order parameter gives us information about the disorder of our system in function of η.

On the one hand, when η is equal to 0, we reach the maximum order possible because all particles are aligned exactly to the same direction. Therefore, there is no cancellation in the summation, so our order parameter reaches its maximum value ϕ=1. However, if we increase slightly the noise intensity η, that perfect alignment is broken, although all particles move approximately at the same direction, as seen in Fig.(1). Because of that, there are some cancellations in the sum and our order parameter decreases its value.

On the other hand, when we have a high noise intensity η the term $\eta\xi_i(t)$ introduces a huge distortion to the alignment and produces a random distribution of each particle's direction. That produce the cancellation of the sum, which makes our order parameter vanish, reaching ϕ=0 when η=1.

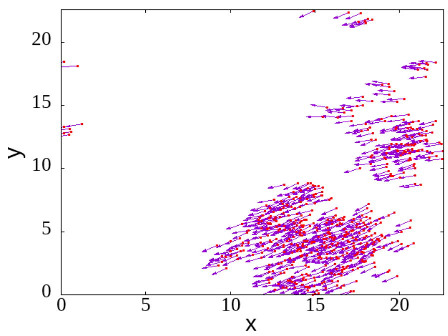

FIG. 1: Final ordered configuration when η=0.05.

Finally, one can estimate the error of the order parameter by computing its variance as follows:

$$Var = <\phi^2> - <\phi>^2 \tag{5}$$

Once we have already talked about Vicsek Model from a dynamical and statistical point of view we must concern about the computational strategy to follow:

First of all we must initialize our system by setting N particles moving with velocity v_0=0.3 inside a 2-dimentional box of side L, with density ρ=$\frac{N}{L^2}$. Given that there is no privileged position nor direction, we must set them uniformly distributed ranging all the possible values [0,L] for the positions and [-π,π] for the velocity direction.

Secondly, because of the fact that we are working with a finite sized system, we need to reduce finite size effects. To do so, we work with periodic boundary conditions (PBC) where a particle that exits from one side, enter from the opposite one, fulfilling the conservation of the number of particles.

Finally, when considering the distance between particles, in order to know if a particle is inside the range of interaction, one must take into account PBC since particles that are at a distance greater than $\frac{L}{2}$ are, in fact, at a shorter distance when considering PBC. For instance, consider two particles in the opposite frontier of the box, its distance is greater than $\frac{L}{2}$ but they could interact because of the PBC.

At this point, we begin our simulations by studying the order parameter in function of the noise intensity η for different sizes but with constant density ρ=1.

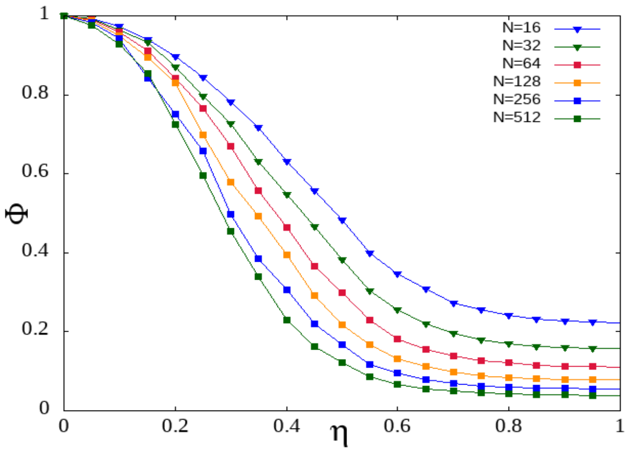

FIG. 2: Order parameter in function of the noise intensity for different sizes, with ρ=1.

From Fig.(2) it can be seen how the order parameter evolves in function of η for different sizes. When the noise intensity is low, for all sizes the order parameter is close to 1. While we increase the intensity, the order parameter gets lower in a different way depending on the size. Although for all sizes the order parameter decreases, for bigger systems it decreases faster. It is due to finite size effects. Furthermore, it must be remarked that $\phi(\eta)$ should tend to zero when η tends to 1, however this value is never reached, and not only that, for small sizes, ϕ is not even close to 0. The reason is because when the noise intensity tends to 1, our system's configuration is basically a set of N random vectors which, from a mathematical point of view, its order parameter goes as $\phi \sim \frac{1}{\sqrt{N}}$ [6].

To determine the existence of a true phase transition and when it takes place we must take a close look to the parameter of order and its evolution over time. An evidence of a phase transition can be found in the behavior of the order parameter. Fluctuations on it become more and more important while it approaches to the phase transition, being infinite when the transition takes place.

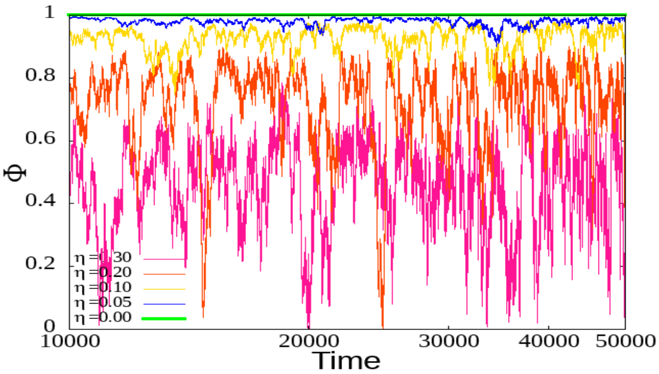

FIG. 3: Order parameter over time with log scale in the x-axis once the system reached an stationary state.

Analyzing Fig.(3) we clearly see how $\phi(t)$ fluctuates when we increase η. When the noise intensity is far from its critical value, it doesn't fluctuates as much as it does while we approach its critical value. When we get closer, its fluctuations start being considerable until it reaches its maximum value at $\eta = \eta_c$. In order to determine η_c with precision we must quantize the magnitude of the fluctuations. To do so, with analogy with ferro-magnetism, we define the susceptibility as in Eq.(6):

$$\chi = NVar \tag{6}$$

Recalling Eq.(5) we get:

$$\chi = N(<\phi^2> - <\phi>^2) \tag{7}$$

Plotting χ versus η allows us to get some information about the phase transition. Once again, taking the analogy with ferro-magnetism, the presence of a peak is a clear evidence of a second-order phase transition between ordered and disordered phase which takes place where the peak is found.

A peak is found in the susceptibility versus the noise intensity (Fig.4), being a clear evidence of the existence of a phase transition in the Vicsek Model.

Once we have already reproduced Vicsek Model's results we can move an step forward by introducing new features in the model. Our first move is to introduce a new role in the system, a leader.

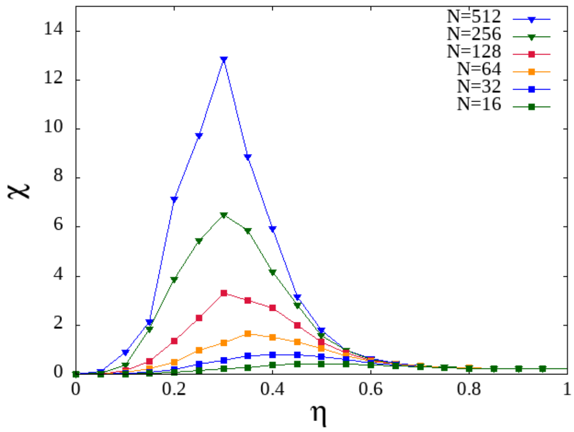

FIG. 4: Susceptibility in function of η for different sizes.

III. LEADERSHIP

When considering social interaction, leadership is an important feature to take into consideration when studying collective motion. Several species of animals follow a hierarchical structure where some individuals are followed by the rest of the community, playing the role of a leader. This structure, in some cases, can be quite complex, leading to a situation where two or more individuals compete to be the dominant leader. This competition brings us to a situation where some individuals follow one leader and others follow a second one.

In this part, we are interested in the effect produced by a single leader in the collective motion and, particularly, how the introduction of that new feature affects the behavior of the Standard Vicsek Model from a statistical point of view.

We introduce a non-local leader, in such a way that its followers (informed individuals) interact with it no matter at what distance is found and considering it to have an immutable velocity $\mathbf{v}_{leader}$. In spite of having this privilege, if an individual is a follower, it interacts with the leader by taking into consideration its direction with the same weight as a normal individual when averaging with its surrounding neighbors, as shown in Eq.(8).

$$\theta_i(t + \Delta t) = arg(\sum_{j \in R_0} \mathbf{v}_j(t) + \epsilon_i \mathbf{v}_{leader}) + \eta \xi_i \tag{8}$$

where ϵ_i is a coefficient equal to 1 if the i-th particle is a follower and 0 otherwise.

The first question that arises is what happens with the order parameter when we introduce a fraction of informed individuals. We observe from Fig.(5) that when a portion of informed individuals is introduced, ϕ is greater compared with the situation where there are no informed individuals. Besides, the greater the fraction, the greater the order of our system.

Its interesting to observe in Fig.(6) that, when a fraction ω of informed individuals is introduced, even for a

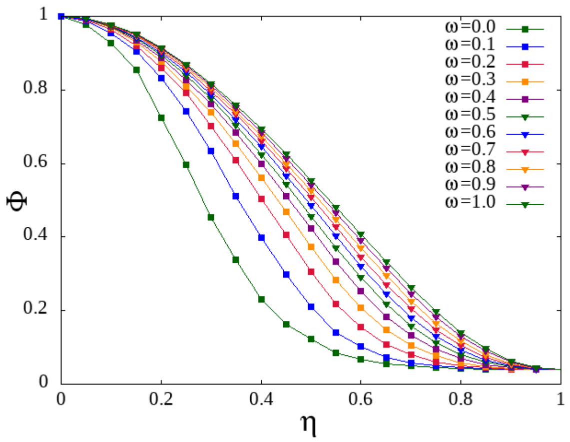

FIG. 5: Order parameter as a function of η for different fractions of informed individuals, with N=512 and ρ=1.

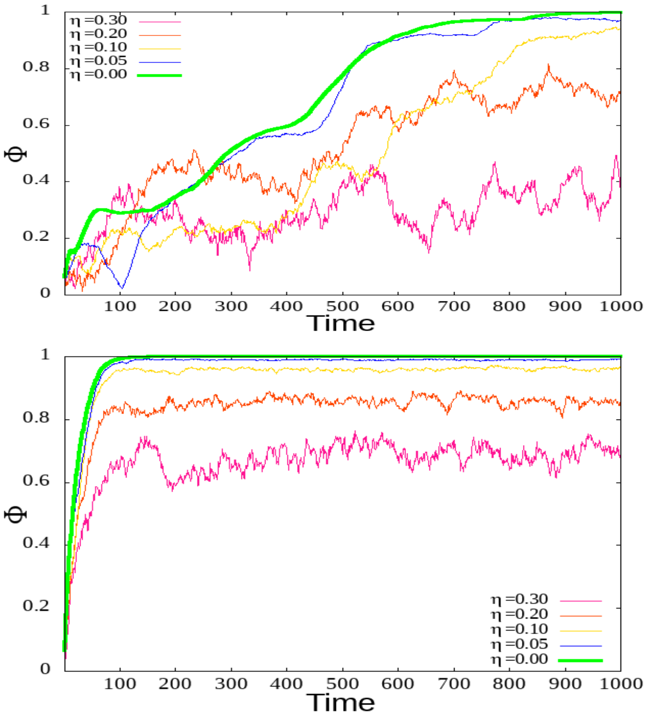

FIG. 6: Detailed evolution of the order parameter over time with ω=0.0 (top), and ω=0.2 (bottom).

small ω, the time required to achieve an ordered stationary state drops drastically and the fluctuations are reduced. That arises the question whether the phase transition still takes place or not when there are informed individuals.

To answer the previous question, once again we must look at the susceptibility, starting with the extreme case where ω=1. Analogously with ferro-magnetism, one would expect to break the phase transition when all individuals follow the leader, since it acts like a magnetic field in the ferro-magnetic case. Remember that in the ferro-magnetism we have a phase transition at T_c between polarized and unpolarized phase. When we introduce a magnetic field, all spins align to its direction and the phase transition is broken, so we expect the leader to have the same qualitative effect at η_c.

Comparing Fig.(6) with Fig.(3) we clearly see huge differences in the order's parameter fluctuations. When there is no leader, there is a phase transition between ordered and disordered phase that we clearly see in its fluctuation's behavior. However, when all the N individuals are informed, the fluctuations drop drastically and our system align itself to the leader's direction, breaking the phase transition.

An other remarkable fact is that when ω=1, apart from reducing the fluctuations, the order parameter takes a greater value than when there is no leader. That means that the presence of a leader makes the order prevail even when high noise intensities are applied, only when η=1 a disordered phase is observed.

Once we studied the opposite cases (ω=0 and ω=1) one is interested in the intermediate case. Being proved that there is no phase transition when ω=1 we will look how the fluctuations behave and scale with N for different fractions.

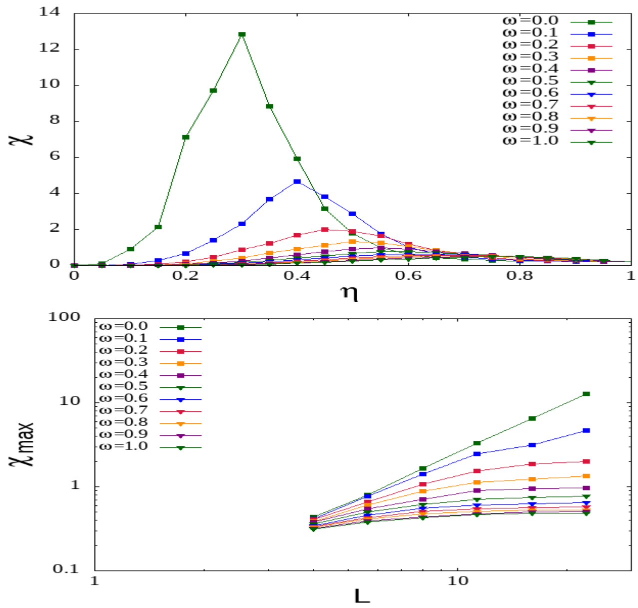

FIG. 7: Susceptibility versus η for different ω with N=512 (top) and susceptibility peak height in function of L with log scale for different ω (bottom).

From Fig.(7) we can see how the fluctuations behave in the Standar Vicsek Model (ω=0) and how they evolve when informed individuals are introduced and its proportion raises. There is a drastic drop in the susceptibility compatible with the fact that a leader strengthen the order when it has some followers. Apart from that, looking at how susceptibility's peak scales in function on the size L, one sees how practically between ω=0.4 and ω=0.5

those fractions of informed individuals behave approximately like ω=1, so we expect no phase transition between those fractions and absolute leadership when ω=1.

Finally, for lower fractions we would expect a Vicsek-like behavior where the peak scales in function of the size with a power law which could have different but close exponents for different fractions. However, comparing our results when ω=0 (Standard Vicsek Model) with the ones obtained in Ref.[7], makes us suspect that we had no enough statistics since we worked with small-sized systems and with not enough steps and seeds. So we can not ensure its behavior.

IV. MULTIPLE ROLES

To end our work, we are going to study if with our model is possible to reproduce some kind of rich behavior as in I.D.Couzin experiment[8]. Couzin managed to build a multiple-leader system with different number of followers and strength. He found that the introduction of uninformed individuals played a central role in his system's behavior, being able to modify the dominant collective motion in a non-trivial way. For instance, in some cases, depending on the number of uninformed individuals, the stronger minority were the dominant and determined the collective motion and in other cases the uninformed individuals promoted a democratic consensus and made the weaker majority to be the dominant one.

To begin with, we consider a situation where all the individuals are informed and there is a leader with the majority of followers. We have seen that in this situation all the system align itself to a preferred direction closer to the leader with the majority. Furthermore, comparing this case with the ideal situation where there is no interaction and each individual just follow its leader, we see that interaction actually favor the order.

Finally, we introduce uninformed individuals. In spite of not having the strength parameter in our model, we wonder if in our case such a rich behavior arises when uninformed individuals are in presence of two different leaders with its respective followers. Since each leader has the same strength in our model, the relevant parameter is the number of followers each leader has. Exploring different situations such as low fraction of uninformed, majority respect of the total informed, approximately the same fraction between informed and uninformed etc. no relevant role has been seen for the uninformed individuals, so in our future studies we must implement the strength parameter to reproduce this characteristic features.

V. CONCLUSIONS

During this work we studied collective motion using a basic model of N self-propelled particles with a really basic interaction rules called Vicsek Model. We reproduced its basics results, showing a noise-driven phase transition between ordered and disordered phase. Then, a new role (leader) was introduced, we studied how this new feature affected Vicsek Model's behavior. We saw how its behavior changed quite drastically when a relatively low fraction of follower individuals were introduced. We saw how followers represent a big contribution to order and how those reduce the required time to achieve the alignment to a collective direction. Although we have seen that the phase transition disappeared for a high fraction of a non-local leader followers ($\omega \sim 0.5$), we have not been able to determine with precision at which fraction of followers occurs. Finally, we tried to reproduce, with that modified model, Couzin's experimental results. Even though he took into consideration a new parameter (strength of leadership) we studied if with our model the same effects could be achieved, were a leader with a minority of followers resulted to be the dominant. With our model, we have not been able to see such a behavior although we continued by introducing this new feature. For a matter of time, those results could not be reflected in that work.

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