# Micro-rheometer with a modular pump: theoretical approach

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In this work we will propose a theoretical approach for a microfluidic device which can be used as a micro and portable rheometer. We will propose a theoretical model based on the dynamics of the fluid interface in the microchannel in a regime of constant pressure. In this experimental device we use a pump module of polydimethylsiloxane (PDMS) that absorbs the air in the microfluidic system. Furthermore, we will be able to predict velocities and viscosities of the different Newtonian fluids by means of the properties of the fluid and the geometrical parameters of the experimental set-up. We will also study the diffusion process of the modular pump in order to understand the pressure generated for driving fluid. Finally, we demonstrated the utility of this model by comparing our predictions with the experimental data.

## I. INTRODUCTION

Nowadays having a lab-on-a-chip has become one of the best ways to take advantage of the modern technology[1]. Specifically, microfluidics has shown great advantages in various areas such as biotechnological and chemical analysis and, in addition, a good understanding of the dynamics of a liquid front in a micro-device can be used to understand and classify different fluids ( viscosity measuring, contaminated and non-contaminated fluids, healthy and infected blood, etc.)[2]. This kind of devices have some common features as low fluid volume consumption, faster response times to short diffusion distances, compactness and portability. These assets can be really interesting when we aim to solve a biological problem where using an small quantity of substances can simplify and agilise the procedure of any activity[3]. However, in spite of these benefits, microfluidic devices with real applications are not being introduced with relatively speed into the society.

Looking closer to this emergent technology we realize that when working with microfluidic devices we typically need a bulky system. At the present time, traditional pumps including syringe and peristaltic pumps, or even pressure source regulators have portability limitations due to their bulky hardware. However, when the goal is to improve the portability of the device, several researchers have developed and proposed plenty of new options for pumping mechanisms and integrated micropumps on-chip such as gravity-driven flow, evaporation process, valves, diaphragms and more. Short while ago, a proposed pumping method for portable microfluidic systems was a pump module made of PDMS (polydimethylsiloxane) [4] . This is a pumping mechanism that takes advantage of its inherent porosity and air solubility in order to absorb the air in the microfluidic system and then create a negative pressure for driving fluid. The key advantage of this pump is that its power-free and easy-to-use as we just need to place the degassed PDMS slab on the outlet port of microfluidic devices.

Using a modular pump to start and control the fluid flow through a micro-channel increases the flexibility and reduces the complexity and cost. This independent pumping mechanism is a degassed PDMS micro-device that when it is brought back to the atmosphere a re-dissolution process of air into the PDMS takes place. This pumping activity is as short as the air distribution inside and outside the PDMS reaches the equilibrium, which is related to the geometry of the pump and the surface through which the diffusion process is happening.

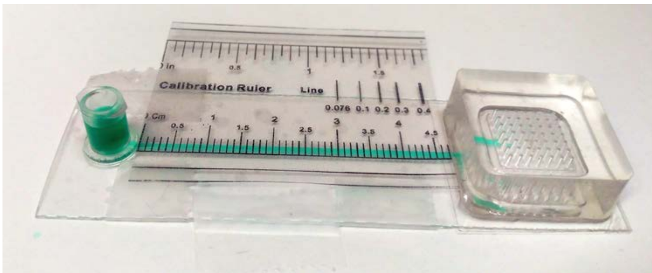


FIG. 1: Photograph showing the actual experimental setup. On the left we can see our reservoir that is full of fluid meanwhile on the opposite site, connected with it through the microchannel, we have our PDMS pump. *Courtesy of J. Etxebarria*

Here we study a microfluidic device designed to study fluid flow driven by pressure inside a capillary micro-channel. In this device a fluid container is placed at the beginning of the experimental setup and the fluid goes from the container through a tube to the micro-channel where on the other side a PDMS pump sets the pressure difference. The tube acts as a flux controller and by measuring the velocity of the fluid-air

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interface inside the micro-channel we will be able to deduce rheological properties as the viscosity of the fluid.

Our aim in this study is to propose a theoretical approach to the dynamics inside of the microfluidic device where a fluid is pushed through a micro-channel against a second fluid, gas, that is getting into the PDMS pump due to the gradient of concentration. When a PDMS pump is connected to the micro channel a negative pressure is generated by the particles leaving the channel to get into the pump. This diffusion process leads to an hydrodynamic process in which the fluid front starts moving forward in order to compensate the variation of pressure looking for being in equilibrium.

## II. EXPERIMENTAL RESULTS

We have studied fluid flow inside a rectangular microchannel using a pressure-driven flow where the pressure difference is set and the velocity of the fluid front interface, $\dot{l}(t)$, inside the microchannel is measured by tracking the mean position as a function of time, what we will call $l(t)$, between several contiguous images and averaging its values through the channel length [5]. The microsystem consists of different microchannels of length, $l_c = 4cm$ and heights from $b = 175\mu m$ to $b = 612\mu m$ and width of $w = 1mm$, molded using PDMS. We also have a fluid container which we will consider large enough to consider a constant height of the fluid in the deposit which will go to the microchannel through a tube of length $l_t = 1cm$ and radius $r_t = 256\mu m$.

Once the experiment has started, the mean front velocity remains approximately constant since the position of the interface in the channel passes the 20mm [5]. In table [I] we show the mean front velocity for each height of the micro-channel for water and olive oil for.

| **Olive Oil** | **Water(I)** | **Height(I)** | **Water(II)** | **Height(II)** |
|---|---|---|---|---|
| $\dot{l}(mm/s)$ | $\dot{l}(mm/s)$ | $b(\mu m)$ | $\dot{l}(mm/s)$ | $b(\mu m)$ |
| 0,33 | 0,40 | 188 | 0,5 | 175 |
| 0,20 | 0,22 | 336 | 0,39 | 250 |
| 0,14 | 0,17 | 424 | 0,24 | 380 |
| 0,10 | 0,10 | 612 | 0,19 | 500 |

TABLE I: Table showing experimental measurements of the mean values of the front velocities averaged over the length of the channel as a function of its height. Experiments were carried out with a liquid column of height $H = 8,3mm$. Olive oil and Water I are the experimental values of the heights I meanwhile Water II are the experimental results for heights II; they are two different experiences at the laboratory.

## III. THEORETICAL MODEL

In order to give a mathematical model for our device we will split our problem in two parts. The first one is about the dynamics of the gas getting into the pump and the second one is the hydrodynamics of the fluid front. We will use the subindex 2 for the closed gas system and the subindex 1 for the pump system. Notice that, as time progresses and the position of the fluid front, $l(t)$, increases, the volume of 2 will be reduced, $V_2 = V_2(t)$, meanwhile the volume of the system 1 will maintain constant all the time, $V_1 \neq V_1(t)$.We can say then that we have $N$ particles and we can write: $N = N_1(t)+N_2(t)$

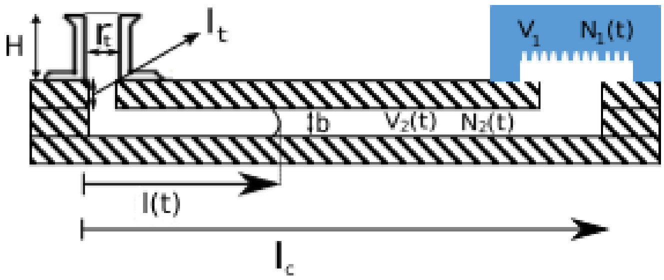


FIG. 2: Schematic representation of the experimental set up. The box on the left represents our reservoir that will be full of fluid meanwhile on the opposite site, connected with it through the microchannel, we have a representation of our PDMS pump of volume $V_1$.

Since we are able to know the quantity of particles that we have in the micro-channel at the initial time because we know that is at atmospheric pressure, $N_2(t = 0) = N = \frac{p_{atm} V_2(t=0)}{k_B T}$, we will be able to determine how it varies through time due to the diffusion process along the surface, $A$, connecting the closed gas system (2) and the pump system (1)

$$\dot{N_2}^A(t) = -AD \left.\frac{\partial u_1(x,t)}{\partial x}\right|_{x=0} \tag{1}$$

where $u_1(x,t)$ is defined as the linear concentration of the system 1 and $D$ is the diffusion coefficient. Moreover, from mass conservation we know that the amount of particles that get into the pump are the same that escape from the micro-channel through the contact surface, $\dot{N}_1(t) = -\dot{N}_2(t)$. So the number of particles in the pump at a given time t is the integral from $t = 0$ to $t$ of (1). In addition, the position of the interface water-air is related to the volume of enclosed air $V_2(t)$

$$V_2(t) = V_2(t = 0) - wbl(t) \tag{2}$$

where $l(t)$ represents the position of the front in the micro-channel. Pressure of each system, 1 and 2, can be deduced from ideal gas law by using the respective variables

## IV. VELOCITY OF THE FLUID FRONT INSIDE THE MICROCHANNEL

In order to explain the motion of the interface between liquid and gas we will assume Darcy's law inside of a rectangular micro-channel of height $b$. Being $l(t)$ the position of the front at time $t$, $\eta$ is the viscosity and $\Delta P(t)$ is the total pressure drop we have:

$$\dot{l}(t) = \frac{b^2}{12\eta}\frac{\Delta P(t)}{l(t)} \quad (3)$$

The total pressure drop along through the liquid until just after the interfase is:

$$\Delta P = \rho g H + p_{atm} + p_c - p_2(t) - \frac{8 v_t \eta l_t}{r^2}$$

where $p_2(t)$ represents the pressure difference generated by the pump. The last term, which is the one related to the microtube connecting the fluid container to the microchannel (deduced from Poiseulle's law), has variables as $v_t$ that is the mean value of the velocity in the microtube and $l_t$ which is the length of the microtube feeding the capillary channel. In addition, due to mass conservation we can also write the last term in terms of the geometry of the device instead of $v_t$ as we have $\dot{l}bw = v_t \pi r^2$. Then, by reorganizing and combining each relation, we arrive at a general differential equation describing the temporal evolution of the fluid front inside the channel

$$\dot{l}(t) = \frac{b^2}{12\eta}\frac{\rho g H + p_{atm} + p_c - p_2(t)}{l(t) + \frac{8bwl_t}{\pi r^4}\frac{b^2}{12}} \quad (4)$$

Notice that, as $p_2(t)$ is the pressure drop generated by the pump, as initially it was a gas at $p = p_{atm}$ and what happens through time is that gas particles get inside the pump due to the diffusion process, we can write:

$$p_2(t) = p_{atm} - p_1(t) \quad (5)$$

where $p_1(t)$ is the pressure corresponding to the particles that got inside the pump structure. Using this last relation, we can give a more clearfull relation between $\dot{l}(t)$ and the pressure drop generated by the pump $p_1(t)$

$$\dot{l}(t) = \frac{b^2}{12\eta}\frac{\rho g H + p_c + p_1(t)}{l(t) + \frac{8bwl_t}{\pi r_t^4}\frac{b^2}{12}} \quad (6)$$

In addition, for initial times and basically during all the experiment we have that $l(t) << \frac{8bwl_t}{\pi r^4}\frac{b^2}{12} \simeq 20cm$[6] resulting as a consequence a regime of time where $\dot{l}(t)$ and $p_1(t)$ are no longer time dependent as a consecuence of the time scale solution for the diffusion process which is presented in section V.

$$\dot{l} = \frac{\rho g H + p_c + p_1}{\frac{8\eta bwl_t}{\pi r^4}} \quad (7)$$

Moreover, by taking natural logarithms to each side of Eq (7)

$$ln(\dot{l}(b)) = ln\left[\frac{(\rho g H + p_c + p_1)}{\eta bwl_t}\pi r^4\right] - ln(b) \quad (8)$$

## V. DIFFUSION PROCESS

The working mechanism of the PDMS pump is based on air absorption through a difussion process. The pressure generated by the modular pump depends on the amount of air that is getting into it. When the air in the closed system is diffused in the pump, we expect that the pressure will fall in response to the change in the amount of air. However, from our experiences in the laboratory [5] and other experiences from literature [4] we can see how at a limited time, the pumping pressure generated by the modular PDMS pump arrives to a plateau which means that it remains approximately constant.

According to the pumping performace we expect to arrive in the diffusion process to a solution that remains approximately constant for a certain regime of time. This means that the diffusion proces arrives to a situation where $N_1$ doesn't change significatively.

### A. The diffusion equation

The diffusion equation is a partial differential equation which describes density fluctuations in a material undergoing diffusion. The equation for our system 1 can be written as:

$$\frac{\partial u_1(\mathbf{r},t)}{\partial t} = D\nabla^2 u_1(\mathbf{r},t) \quad (9)$$

where $u_1(\mathbf{r},t)$ is the density of the diffusing fluid at location $\mathbf{r} = (x,y,t)$ and time t. $D$ is the diffusion coefficient and we supposed that does not depend on the position or time, i.e., D is constant.

Considering the diffusion equation in one dimension on the interval $x \in [0, x_L]$ we have:

$$\frac{\partial u_1(x,t)}{\partial t} = D\frac{\partial^2 u_1(x,t)}{\partial x^2} \quad (10)$$

Using separation of variables we find a solution for the concentration of our system one which is not identically zero satisfying the following boundary conditions:

$$u_1(x,t=0) = 0 \qquad \forall x \neq 0 \quad (11a)$$

$$u_1(0,t) = u_0 \qquad \forall t \quad (11b)$$

$$\left.\frac{\partial u_1}{\partial x}\right|_{x=x_L} = 0 \qquad \forall t \quad (11c)$$

The resulting solution for the non-homogeneous boundary conditions can be written as:

$$u_1(x,t) = (1 - \frac{x}{x_L})u_0 + \frac{x}{x_L}u_L + \sum_{m=1}^{\infty} A_n \sin\left(\frac{(m-\frac{1}{2})\pi x}{x_L}\right) e^{-D\frac{\pi^2}{x_L^2}(m-\frac{1}{2})^2 t} \quad (12)$$

$$A_m = \frac{2}{x_L}\int_0^{x_L} f(\zeta)\sin\left(\frac{(m-1/2)\pi\zeta}{x_L}\right)d\zeta \quad (13)$$

Where in our case $u_0$ is related to the density of the gas in the micro-channel. It is possible to find the pressure generated by the gas inside the pump, $p_1(t)$, by integrating $\dot{N}_1(t)$ and dividing by the volume $V_1$.

$$N_1(t) = \int_0^t dt'\dot{N}_1^A(t') = -\int_0^t dt'\dot{N}_2^A(t') \quad (14)$$

where the expansion of $m$ in Eq.(12) must be truncate for a "$m$" where the exponential is small enough to not being considered. The pressure for driving fluid generated by the pump can be calculated for a given time, $t$, as:

$$p_1(t) = \frac{N_1(t)}{V_1}K_B T \quad (15)$$

From previous studies [5] [7] we know that there is a plateau on the fluid flow that is proportional with our $\dot{l}(t)$. We know that this stationary time is the order of magnitude of minuts, so we will take $t_{st} \simeq 1min$

## VI. RESULTS AND DISCUSSION

As we can see in Fig.3 that all our experimental results follows a linear relation where represented in natural logarithms as predicted in Eq.(8) within the time, pressure and height ranges that our experiments and model take place. The theoretical model that we present allows us to relate the parameters of this linear relation to physical properties of the fluid and geometrical parameters of our experimental setup as can be seen in Fig.4 where we plot the average velocity of the front as a function of the channel height.

The predicted value of the viscosity by means of the shear rate in Fig.5 in the case of the water gives $0.87mPas$ which perfectly concur with the results available in previous studies [6]

About the diffusion process, we did an analysis of the time regimes that we need to work with in order to be able to choose the last term of our expansion in $m$ taking into account that our experiment takes approximately 4 minuts to finish. By introducing the parameters of the experiment, $V_1 = 350mm^3$, $A = 1cm^2$, $D = 3,4\cdot10^{-9}\frac{m^2}{s}$ and $x_L = 5mm$, into the time dependant part of the

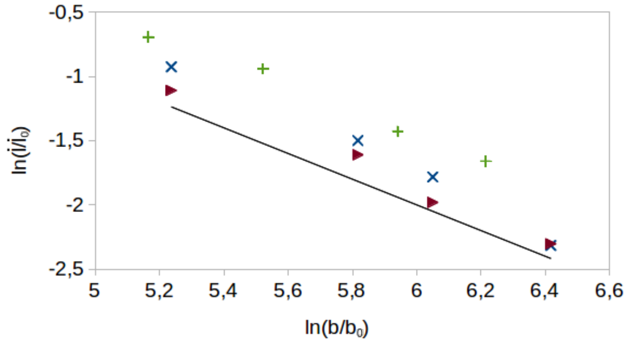


FIG. 3: Plot showing the natural logarithm of the mean front velocity as a function of the natural logarithm of the channel height where the shown values are dimensionless variables from the original values divided by the unity. X marks and green crosses are the representation for water (I) and (II) meanwhile red triangle shape marks correspond to the olive oil experimental values. This plot shows a solid dark line of gradient minus one, a value predicted in our theoretical model Eq (7)

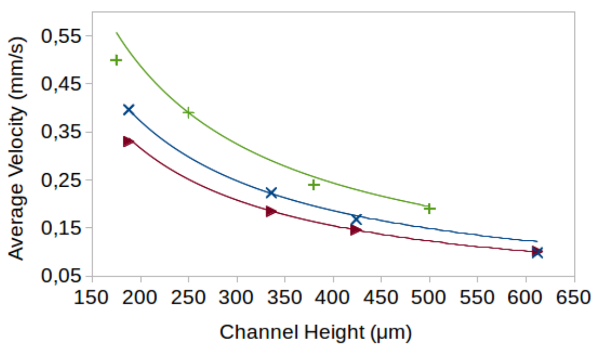


FIG. 4: Plot showing the mean front velocity as a function of the channel height. Experimental data is plotted by using its respective marks meanwhile the predicted values from our theoretical model Eq.(7) are showed in solid lines of the same color of its marks.

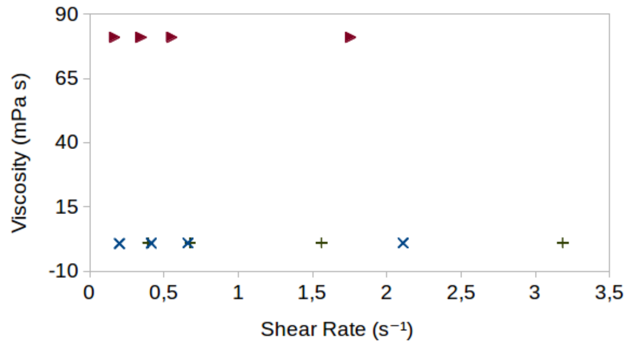


FIG. 5: This plot shows the estimte viscosity, using our model (7), of the fluids (water and olive oil) . We observe, as it should be expected for a Newtonian fluid, that for the values of shear rate the viscosity remains constant.

Eq. (12) we can see the regimes of each order of the expansion.

The corresponding values of the $p_1$ as a function of the order of the expansion $m$ and time $t$ are:

| t(s) | p1(m=4) | p1(m=3) | p1(m=2) | p1(m=1) |
|---|---|---|---|---|
| 0,5 | 78,59 | 58,99 | 39,35 | 19,68 |
| 1,5 | 234,93 | 176,61 | 117,94 | 59,03 |
| 2 | 312,70 | 235,25 | 157,19 | 78,70 |
| 10 | 1521,23 | 1158,27 | 780,70 | 392,96 |
| 15 | 2244,26 | 1721,01 | 1166,21 | 588,95 |
| 30 | 4280,03 | 3347,89 | 2303,90 | 1174,94 |
| 60 | 7855,25 | 6353,93 | 4498,24 | 2338,12 |

TABLE II: Table showing our predicted values of the pressure generated by the pump. Pressure values are expressed in $Pa$ and time in seconds.

Where from the experiments we know that for times of $1mins$ the pressure arrives to a plateau that our diffusion equation does not contain but it is included in the fluid front model. The value of this constant pressure can be deduced from our model for $t = 2s$ and for larger times it must be constant.

## VII. CONCLUSIONS AND FUTURE WORK

We have proven our theoretical model of velocity for fluid interfaces in micro-channels with a newtonian fluids. We obtained a good prediction model which for a given micro-channel height, pressure drop from the pump and fluid, gives the mean value of the fluid front velocity for a regime where it remains approximately constant. We can see from our equations that when increasing the channel height the velocity of the fluid drops inversely proportional to this variable. Going further, when doing numbers in our equations, we can see how important is the value of the channel-height due to the slightly difference between the pressure balance. The negative pressure generated by the pump is subtly above of the capillary and hydrostatic pressure, fact that takes importance when dividing by the order of magnitude of the micro-channel, $1\mu m$, gives a notorious speed of the interface. Fact that also explains why the pressure of the pump leads the dynamics of this device. Our model also predicts a viscosity that is not shear-rate dependent as should be for a newtonian fluid. In the future, is possible to develop a theoretical model useful for a non-newtonian fluids as it could be used for substances such as blood. It is believed that this microdevice might be useful for point-of-care diagnostic tests. When using our two models together we predict the appropiate experimental behaviour for our micro-rheometer.

**Acknowledgments**

I would like to thank A. Hernandez Machado, J. Etxebarria, Mauricio M. Moreno and L. Basabe to help me in the whole process of this work.

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