Brane-world localized gravity and the AdS/CFT correspondence

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Abstract: In this report, corrections to the gravitational potential of a massive point particle are computed for observers located in a brane-world scenario with intrinsic de Sitter (dS_4) geometry embedded in an Anti de Sitter (AdS_5) bulk. The results are compared, using the AdS/CFT correspondence, to a description in terms of four dimensional general relativity coupled to a conformal field theory.

I. INTRODUCTION

In 1999, Randall and Sundrum proposed a geometric mechanism to solve the hierarchy problem [1, 2]. In their scenario, matter fields are confined in four dimensional branes, embedded in a higher-dimensional Anti de Sitter spacetime. By making the brane separation arbitrarily large, they realized that even in the case of single brane embedded in AdS_5, something close to four dimensional gravity can be recovered [2, 3], despite the presence of an arbitrarily large extra dimension. Here we focus on this particular scenario.

Our background geometry of interest consists of two identical balls of AdS_5 (B_1 and B_2) glued alongside their dS_4 boundaries. A useful tool to analyze the dynamics of the spacetime in that setting is the gauge/gravity duality, between supergravity in AdS_5 (or in our case of interest, the classical limit of it) and a conformal field theory (CFT) living on its boundary. As stated in [4], the duality can be also formulated for finite regions of the AdS spacetime, which amounts to a UV cut-off in the dual theory. The correspondence states that the partition functions for both theories are related as (g^4 is the induced metric on the boundary)

$$Z[g^4] = \int d[g]e^{-S_{grav}[g]} = \int d[\phi]e^{-S_{CFT}[\phi;g^4]} = e^{-W_{CFT}[g^4]}.$$
(1)

The bulk path integral is taken over all the metrics g that induce a conformal equivalence class of metrics g_4 over the boundary, and ϕ are the degrees of freedom of the CFT. The classical gravitational action for AdS_5 is divergent, but counterterms which depend on the geometry of the boundary can be introduced in order to render it finite [5]. Using them, the gravitational action reads

$$S_{grav} = S_{EH} + S_{GH} + S_1 + S_2 + S_3,$$
(2)

where S_{EH} is the usual Einstein-Hilbert term with negative cosmological constant $\Lambda = -6/l^2$, S_{GH} is the

Gibbons-Hawking boundary term, and the others are given by (in Euclidean signature)

$$S_1 = \frac{3}{8\pi G_{(5)}l} \int d^4x\sqrt{g^4}, \quad S_2 = \frac{l}{32\pi G_{(5)}} \int d^4x\sqrt{g^4}R,$$

$$S_3 = \frac{l^3 \ln(R/\rho)}{64\pi G_{(5)}} \int d^4x\sqrt{g^4}\left[R_{\mu\nu}R^{\mu\nu} - \frac{R^2}{3}\right].$$
(3)

Here, $G_{(5)}$ is Newton's constant in five dimensions. The four dimensional integrals are to be taken over the boundary, and so, the curvature tensors which appear are the intrinsic ones. The R appearing in the logarithm of the third term measures the radius of the boundary, and ρ is a finite renormalization length scale.

Keeping this in mind, the action for the Randall-Sundrum scenario is given by

$$S_{RS} = S_{EH} + S_{GH} + 2S_1 + S_M = S_G + 2S_1 + S_M,$$
(4)

where $2S_1$ accounts for the action of a brane, located at the boundary of the two balls B_1 and B_2, with tension $\sigma_0 = 3/(4\pi G_{(5)}l)$, and S_M is the action for the matter on the brane (later, we will take it to be the action for a point particle plus a vacuum energy contribution in order to sustain inflation of the brane). The partition function is then given by:

$$Z_{RS}[g^4] = \int_{B_1\cup B_2} d[g]e^{-S_{RS}} = e^{-2S_1-S_M}\left(\int_{B_1} d[g]e^{-S_{EH}-S_{GH}}\right)^2.$$
(5)

The second integral can be related via the AdS/CFT correspondence to the generating functional of the dual theory plus two of the counterterms. Thus, we obtain a dual description in terms of the 4D action

$$S_{4D} = -2S_2 - 2S_3 + S_M + 2W_{CFT}.$$
(6)

The term S_2 has exactly the form of Einstein-Hilbert action for the 4D brane geometry, and S_3 renormalizes the generating functional W_{CFT}. Thus, we can see that studying corrections to the gravitational potential on the brane in a 5D setting is analogous to a 4D analysis of the gravitational potential in dS_4 coupled to a CFT. In what follows we do the bulk computation and compare it to the boundary result which was found in [6].

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II.   ACTION AND EQUATIONS OF MOTION

The gravitational action with a negative cosmological constant reads

$$S_G = \frac{1}{16\pi G_{(5)}} \left[\int d^5x \sqrt{-g} \left({}^{(5)}R + \frac{12}{l^2} \right) + 2 \int d^4x \sqrt{-g} K \right],$$
(7)

where K is the extrinsic curvature scalar and the integral of the Gibbons-Hawking boundary term is to performed over the brane. For simplicity, we work in Gaussian normal coordinates (GNC) with respect to the brane,

$$ds^2 = g_{MN} dx^M dx^N = dy^2 + g^4_{\mu\nu} dx^\mu dx^\nu.$$
(8)

In what follows, we drop the 4 superscript on the induced metric, as it is the only one appearing in our equations. It is useful to express the five-dimensional curvature scalar in terms of its four-dimensional counterpart and extra terms related with the extrinsic curvature. In our sign convention, the contraction of the Gauss-Codazzi relations reads:

$${}^{(5)}R = R - K^2 - K_{\mu\nu}K^{\mu\nu} - 2\mathcal{L}_n K.$$
(9)

Here n is the normal vector to the brane. Using this slicing and carefully integrating by parts, we obtain

$$S_G \sim \int d^5x \sqrt{-g} \Big(R + \frac{1}{4} (g^{\mu\nu} \partial_y g_{\mu\nu})^2 + \frac{1}{4} (\partial_y g_{\mu\nu})(\partial_y g^{\mu\nu}) + \frac{12}{l^2} \Big).$$
(10)

The action for the matter source on the brane is

$$2S_1 + S_M = \int d^5x \delta(y) \left(-\sigma\sqrt{-g} + \frac{1}{2} g_{\mu\nu} T^{\mu\nu} \right),$$
(11)

where σ is the brane tension. The explicit form of $T^{\mu\nu}$ for a point particle will be discussed later. Imposing that the variations of the total action vanish, we find the equation of motion

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - \frac{6}{l^2} g^{\mu\nu} + \frac{1}{4} \partial_y^2 g^{\mu\nu} + \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \partial_y^2 g_{\alpha\beta} +$$

$$- \frac{1}{4} g^{\mu\alpha} g^{\nu\beta} \partial_y^2 g_{\alpha\beta} + \frac{1}{4} g^{\alpha\beta} (\partial_y g_{\alpha\beta})(\partial_y g^{\mu\nu}) + \frac{3}{8} g^{\mu\nu} (\partial_y g^{\alpha\beta})^2 +$$

$$+ \frac{1}{8} (g^{\alpha\beta} \partial_y g_{\alpha\beta})^2 g^{\mu\nu} = \delta(y) \left[\frac{1}{2} \left(\partial_y g^{\mu\nu} + g^{\mu\nu} g^{\alpha\beta} \partial_y g_{\alpha\beta} \right) \right.$$

$$\left. - 8\pi G_{(5)} \left(-\sigma g^{\mu\nu} + T^{\mu\nu} \right) \right].$$
(12)

Integrating the equation over a pill-box around $y = 0$ gives a condition on the normal derivative of the metric at the brane.

III.   BACKGROUND GEOMETRY

As we have stated in the introduction, we are interested in a geometry consisting of two balls of AdS_5 glued alongside its dS_4 boundaries. For this purpose, consider an ansatz of the form:

$$g_{\mu\nu} = e^{2w} \gamma_{\mu\nu}.$$
(13)

The justification of this form is perhaps clearer in Euclidean signature. The Euclidean version of dS_4 is simply a 4-sphere, and so, $\gamma_{\mu\nu}$ would simply be the its metric. In GNC, the fifth coordinate y is simply the physical distance between the spheres and the factor $e^{w(y)}$ accounts for the change in the volume of the spheres. The Ricci tensor for dS_4 is proportional to the metric, with a factor related to the dS_4 length, or alternatively, to its Hubble parameter,

$$R_{\mu\nu} = 3H^2 \gamma_{\mu\nu}.$$
(14)

Using this ansatz, the differential equation for the warp factor $w(y)$ is:

$$w'' + 2(w')^2 - \frac{2}{l^2} - H^2 e^{-2w} = \delta(y) \left(w' - \frac{8}{3}\pi G_{(5)}\sigma \right).$$
(15)

The solution is given by (after imposing the boundary conditions on the brane)

$$w(y) = \ln \left[Hl \sinh \left(\frac{y_0 - |y|}{l} \right) \right],$$
(16)

and the different parameters are related via

$$\sigma = \frac{3}{4\pi G_{(5)} l} \sqrt{1 + H^2 l^2}, \quad H = \left[l \sinh \left(\frac{y_0}{l} \right) \right]^{-1}.$$
(17)

The flat brane results [2] are recovered with $H \to 0$, and the normal coordinate ranges from $y \in (-y_0, y_0)$.

IV.   PERTURBATIONS

Studying general perturbations in this background becomes rather involved, as the brane would be moved from $y = 0$ to somewhere else. For this matter, we introduce the perturbations so that we are still working in GNC with respect to the brane

$$ds^2 = e^{2w}(\gamma_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + dy^2.$$
(18)

In general, a linearised change of coordinates will transform the metric as

$$h_{MN} \to h_{MN} + e^{-2w} \mathcal{L}_{e^{2w}\xi}(e^{2w}\gamma_{MN})$$

$$= h_{MN} + e^{2w} \left[2\nabla_{(M}\xi_{N)} + 2w'(y) \left(\gamma_{MN}\zeta^y + 2\delta^y_{(N}\xi_{M)} \right) \right],$$
(19)

with $\gamma_{yy} = e^{-2w}$, $\gamma_{y\mu} = 0$ and ∇_N the covariant derivative related to the metric γ_{MN}. We are interested in studying the residual gauge freedom, keeping in mind

that we want to restrict our study to normal coordinates respect to the wall. The conditions that we obtain after imposing the components h_{yM} to be zero in every gauge are

$$\partial_y \xi_y = -3w' \xi_y, \quad \partial_y \xi_\mu = -\partial_\mu \xi_y - 2w' \xi_\mu. \quad (20)$$

Keeping the brane at $y = 0$, amounts to $\xi_y = 0$. Then, the solution for the second condition is

$$\xi_\mu = e^{-2w} \xi_\mu^{(0)}(x^\nu), \quad (21)$$

and we are left with the usual (four dimensional) gauge freedom

$$h_{\mu\nu} \to h_{\mu\nu} + 2\nabla_{(\mu} \xi_{\nu)}^{(0)}. \quad (22)$$

Once we have justified the particular form of our perturbations, we can study its linearized equations of motion. For that matter, it is useful to introduce a new coordinate orthogonal to the brane, $\zeta = e^{|y|/l}$. Under this change, we are imposing symmetry under $y \to -y$, as we cannot study any other case with it. To first order in $h_{\mu\nu}$, the equations read

$$\nabla^2 h_{\mu\nu} - 2\nabla^\rho \nabla_{(\mu} h_{\nu)\rho} + \nabla_\mu \nabla_\nu h + 6H^2 h_{\mu\nu} \quad (23)$$

$$+ D_\zeta \left(h_{\mu\nu} + \frac{1}{2}\gamma_{\mu\nu} h \right) = -\delta(\zeta - 1)\frac{8\pi G_{(5)}}{l}\left(2T_{\mu\nu} - \gamma_{\mu\nu} T \right),$$

where ∇_μ is the covariant derivative associated with the induced metric in the brane $\gamma_{\mu\nu}$ (traces are also with regard to this metric). We have also introduced a derivative operator in the bulk coordinate, given by

$$D_\zeta = \frac{e^{2w}}{l^2}\left[\zeta^2 \partial_\zeta^2 + (\zeta + 4\zeta^2 \partial_\zeta w)\partial_\zeta + 2\delta(\zeta - 1)\partial_\zeta \right]. \quad (24)$$

At this point, it is necessary to introduce some coordinates on the brane. For simplicity, we can study the dS_4 geometry of the domain wall in terms of conformally flat coordinates. These coordinates cover only half of the hyperboloid [7], but are suitable for our study as we are interested in a local property, such as the gravitational potential, and these coordinates can be constructed around every point of dS_4. They take the form of a flat FLRW geometry, with scale factor $a(\eta) = -1/(H\eta)$,

$$ds^2 = \gamma_{\mu\nu} dx^\mu dx^\nu = \left(\frac{-1}{H\eta} \right)^2 \eta_{\mu\nu} dx^\mu dx^\nu. \quad (25)$$

A. Decomposition

As dS_4 is maximally symmetric, we can classify the perturbations according to their behaviour under spatial rotations of the spacetime. The different components of the perturbations can transform either as a scalar, vector or tensor. The metric decomposed in terms of the scalar perturbations reads

$$h_{\mu\nu} = a^2 \left[2\delta_\mu^0 \delta_\nu^0 (\Phi_A + \Phi_H) + 2\eta_{\mu\nu}\Phi_H + 2\nabla_{(\mu} X_{\nu)} \right], \quad (26)$$

where we have redefined $a^{-2}\zeta_\nu^{(0)} = X_\nu$, as the generator of gauge transformations. At linear order, the equations for the different modes are decoupled, and so we need not worry about the tensor and vector modes. Then, our four variables of interest are the two Bardeen potentials Φ_A, Φ_H and the two gauge variables X, X_0 (X is the potential of the curl-free part of the gauge vector $X_i = X_{,i} + S_i$ with $S_{i,i} = 0$).

B. Stress energy tensor

A point particle located at the origin has energy momentum tensor

$$T_{\mu\nu} = ma^{-1}\delta_\mu^0 \delta_\nu^0 \delta^{(3)}(x) = -\frac{1}{4\pi}ma^{-1}\delta_\mu^0 \delta_\nu^0 \Delta\left(\frac{1}{r} \right), \quad (27)$$

which has indeed the perfect fluid form with zero solenoidal velocity. This justifies our omission of the vector modes. The energy momentum tensor satisfies the appropriate conservation law.

V. SOLUTION FOR THE SCALAR PART

After decomposing the linearized version of Einstein's equation, four differential equations for the scalar modes are found. As we are studying the gravitational potential of a point mass, we would like to search for a static solution. The existence of one is clearer if we introduce cosmological time and physical distances via

$$\hat{x}^i(t) = a(t)x^i = e^{Ht}x^i, \quad P = -\delta^{kl}\frac{\partial^2}{\partial \hat{x}^k \partial \hat{x}^l}. \quad (28)$$

Then, the following equations are obtained:

$$2P\Phi_H - \frac{1}{2}D_\zeta\left(6\Phi_H - 2P\Xi - 6H^2\Xi + 3H\dot{\Xi} - 3H\Xi_0 \right)$$
$$= \delta(\zeta - 1)\frac{2G_{(5)}m}{l}P\frac{1}{\hat{r}},$$

$$2P\Phi_A - \frac{1}{4}D_\zeta\left(6\Phi_A - 3\ddot{\Xi} + 6H^2\Xi + 3H\dot{\Xi} + 3\ddot{\Xi}_0 - 3H\Xi_0 + 4P\Xi \right)$$
$$= \delta(\zeta - 1)\frac{2G_{(5)}m}{l}P\frac{1}{\hat{r}},$$

$$D_\zeta\left(6\dot{\Phi}_H + 6H\Phi_H + 6H\Phi_A - P\dot{\Xi} + P\Xi_0 - 6H^2\Xi_0 \right) = 0,$$

$$D_\zeta\left(4\Phi_H - 2\Phi_A + \ddot{\Xi} - 6H^2\Xi + H\dot{\Xi} - \ddot{\Xi}_0 - H\Xi_0 \right) = 0. \quad (29)$$

Overdots denote derivatives with regard to cosmological time, and we have redefined the gauge variables as $\Xi = a^2 X$ and $\Xi_0 = aX_0$. As there is no explicit dependence on time, both in the equation and in the source, we can search for a static solution. After some algebra, we can write the equations as

$$D_\zeta \Sigma = (P - 6H^2)\left[\Sigma - \delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r}\right], \quad (30)$$

$$D_\zeta \Delta = 2H^2(\Sigma - \Delta) - 3H^2\delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r},$$

$$D_\zeta \Xi = \frac{1}{3}(\Sigma + 2\Delta), \quad D_\zeta \Xi_0 = -6H\Sigma + 6H\delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r},$$

where $\Sigma = \Phi_A + \Phi_H$ and $\Delta = \Phi_A - 2\Phi_H$ are simply combinations of the Bardeen potentials.

A. Expansion in H^2

Despite the simple form of the equation for the scalar modes, and the fact that we will need only the first three to solve for the Bardeen potentials, obtaining an exact solution is a challenging task.

One possible alternative is to expand the solution in powers of H^2, and try to find a perturbative result. At first order, we would expect to recover the flat brane results [8], and the second order would introduce the first corrections due to the intrinsic geometry of the domain wall. This expansion will enable us to solve the differential equations, but will unfortunately complicate the task of imposing the appropriate boundary conditions. At order 0, the equations read

$$D_\zeta^{(0)}\Sigma^{(0)} = P\left[\Sigma^{(0)} - \delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r}\right], \quad D_\zeta^{(0)}\Delta^{(0)} = 0,$$

$$D_\zeta^{(0)}\Xi^{(0)} = \frac{1}{3}(\Sigma^{(0)} + 2\Delta^{(0)}), \quad D_\zeta^{(0)}\Xi_0^{(0)} = 0, \quad (31)$$

with the differential operator $D_\zeta^{(0)}$ given by

$$D_\zeta^{(0)} = \frac{1}{l^2\zeta^2}\left[\zeta^2\partial_\zeta^2 - 3\zeta\partial_\zeta + 2\delta(\zeta-1)\partial_\zeta\right]. \quad (32)$$

The delta terms enforce discontinuity of the derivative of the metric (in this particular case, of a combination of the scalar modes appearing in the metric). This amounts to one of the boundary conditions needed to solve the linear second order differential equation. The second one can be obtained by imposing that the horizons remain regular. At order zero ($H = 0$) the horizons are located at infinite coordinate distance $\zeta \to \infty$, and so, the second condition amounts to cancelling all the divergent terms (terms that diverge including the exponential of the warp factor e^{2w} appearing on the metric) as $\zeta \to \infty$. The solution is then (with K_n a modified Bessel function of the second kind)

$$\Sigma^{(0)} = \frac{\sqrt P G_{(5)}m\zeta^2 K_2(\sqrt P l\zeta)}{K_1(l\sqrt P)\hat r}, \quad \Delta^{(0)} = -\frac{G_{(5)}m}{l\hat r}. \quad (33)$$

To solve the equation for $\Delta^{(0)}$ we need to solve the equation at order zero for the gauge function $\Xi^{(0)}$. This is due to the fact that the second boundary condition

(regularity at the horizon) was not applicable, as it was already regular. The third equation in (31) does depend in $\Delta^{(0)}$, and both boundary conditions apply to $\Xi^{(0)}$. Luckily, imposing the boundary conditions for the latter case, determines the form of $\Delta^{(0)}$. Having cleared the first order, we can move on to the second order.

If $H \neq 0$, the horizon is at finite coordinate distance. In terms of the old coordinate, we must now impose regularity at y_0. Using Eq. (17), we can see that in terms of the new coordinate, it is located at

$$\zeta_0 = \frac{1}{Hl}\left[1 + \sqrt{1 + H^2l^2}\right]. \quad (34)$$

Problems arise right here. In our expansion in H^2, $\Sigma = \Sigma^{(0)} + H^2\Sigma^{(1)} + ...$, the $\Sigma^{(i)}$ at every order do not depend in H. Trying to impose regularity (at every order) at a point whose location depends on H, would be inconsistent with the expansion. But, it is interesting to recall that we are considering the first order where $H \neq 0$ but it is arbitrarily small. Thus, we should impose regularity at a point at finite distance, but arbitrarily far away. Keeping this in mind, the relevant second order in H^2 differential equations are

$$\left(D_\zeta^{(0)} - P\right)\Sigma^{(1)} = -D_\zeta^{(1)}\Sigma^{(0)} - 6\Sigma^{(0)} + 6\delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r},$$

$$D_\zeta^{(0)}\Delta^{(1)} = -D_\zeta^{(1)}\Delta^{(0)} - 2\Delta^{(0)} + 2\Sigma^{(0)} - 3\delta(\zeta-1)\frac{2G_{(5)}m}{l\hat r},$$

$$D_\zeta^{(0)}\Xi^{(1)} = -D_\zeta^{(1)}\Xi^{(0)} + \frac{1}{3}\left(\Sigma^{(1)} + 2\Delta^{(1)}\right), \quad (35)$$

where the RHS are strictly source terms. The derivative operator $D_\zeta^{(1)}$ can be obtained by expanding in H^2 Eq.(24) (as we have done for one of the combinations of the Bardeen potentials, we keep the terms $D_\zeta^{(1)}$ independent of H in the expansion).

The solution to the first of the three equations is a linear combination of two modified Bessel functions (both resemble exponentials, one decaying and the other growing) plus some terms which are solution of the inhomogeneous part, which decay as we move away from the brane. Taking a combination of the two solutions so that regularity is achieved at the horizon (which for our purposes, can be taken as far away as we desire), would give exponentially suppressed corrections to the result in the brane. For that reason, we take the coefficient of the growing mode to be zero and impose the brane condition on the second one. Using again the equation for one of the gauge potentials, as the equation for $\Delta^{(1)}$ is unconstrained by fall-off conditions, we obtain the second order solution. In order to analyze the behaviour on the brane, we take $\zeta = 1$:

$$\Sigma^{(0)} = \frac{\sqrt P G_{(5)}m}{K_1\left(l\sqrt P\right)\hat r}K_2\left(\sqrt P l\right), \quad \Delta^{(0)} = -\frac{G_{(5)}m}{l\hat r},$$

$$\Sigma^{(1)} = \left[l^2 P + 15\right] \frac{l G_{(5)} m}{6\hat{r}}$$

$$-\frac{G_{(5)} lm}{6\hat{r}} \left[(12 + l^2 P) K_2/K_1 - (24 + 3l^2 P) \frac{1}{l\sqrt{P}}\right] K_2/K_1,$$

$$\Delta^{(1)} = 2 \frac{G_{(5)} m}{\sqrt{P} \hat{r}} K_2/K_1 - \frac{(4 + l^2 P)}{P} \frac{G_{(5)} m}{l\hat{r}}, \qquad (36)$$

where K_2/K_1 is the ratio of Bessel functions evaluated at the same point as in the order 0 solution. In order to obtain the solution, we still need to eliminate the Laplacians appearing in the expression and recombine Σ and Δ into the Bardeen potentials. Expanding for small $\sqrt{P}l$

$$\Sigma^{(0)} = \frac{G_{(5)} m}{l\hat{r}} \left[1 - Pl^2 \left[\gamma + \ln\left(\frac{l\sqrt{P}}{2}\right)\right]\right], \quad \Delta^{(0)} = -\frac{G_{(5)} m}{l\hat{r}},$$

$$\Sigma^{(1)} = \frac{G_{(5)} ml}{\hat{r}} \left[\frac{17}{6} + 4\gamma + 4\ln\left(\frac{l\sqrt{P}}{2}\right)\right],$$

$$\Delta^{(1)} = -\frac{G_{(5)} ml}{\hat{r}} \left[1 + 2\gamma + 2\ln\left(\frac{l\sqrt{P}}{2}\right)\right], \qquad (37)$$

where γ is Euler's constant, which appears in the expansion of the ratio of Bessel functions for small arguments. The result contains no negative powers of the Laplacian, as they would introduce non-local terms in our analysis. In order to get rid of the Laplacians it is useful to recall two results:

$$\ln\sqrt{P}\frac{1}{\hat{r}} = -\frac{\ln\hat{r} + \gamma}{\hat{r}}, \quad P\frac{\ln\hat{r}}{\hat{r}} = \frac{1}{\hat{r}^3}. \qquad (38)$$

Which leads to the final expression

$$\Phi_A = \frac{G_{(5)} m}{l\hat{r}} \left[1 + \frac{2l^2}{3\hat{r}^2} + H^2 l^2 \left[\frac{14}{9} + 2\ln\left(\frac{l}{2\hat{r}}\right)\right]\right],$$

$$\Phi_H = \frac{G_{(5)} m}{l\hat{r}} \left[1 + \frac{l^2}{3\hat{r}^2} + H^2 l^2 \left[\frac{23}{18} + 2\ln\left(\frac{l}{2\hat{r}}\right)\right]\right]. \qquad (39)$$

VI.   DISCUSSION

The order zero in H^2 results agree from those in [8]. This is what we expected, as $H = 0$ is simply the flat brane case studied by Garriga and Tanaka. The second order result can checked with those of [6] if we take into account the relation between the constants appearing in the two cases:

$$G_{(5)} = l G_{(4)}, \quad l^2 = 64\pi b l_{PL}^2. \qquad (40)$$

$G_{(4)}$ is the four dimensional gravitational constant (with Planck length $l_{PL} = \sqrt{\hbar G_{(4)}/c^3}$) and b is the central charge of the CFT, related to the number of free fields as

$$b = \frac{N_0 + 6N_{1/2} + 12N_1}{1920\pi^2}, \qquad (41)$$

where N_0 is the number of conformally coupled scalar fields, $N_{1/2}$ massless spinor fields and N_1 vectors. Their analysis was based in computing first the gravitational potential of a test mass in dS_4, then studying the back reaction to it from the non-zero expected value of the energy momentum tensor of conformal fields in the new geometry.

Our first order results coincide with [6], but there is some discrepancy involving the second order solution. From our point of view, there are two possible reasons for this difference, either the term S_3 in the four dimensional action plays a role that we have not considered, or our assumptions over the second order boundary conditions are wrong. This is left for further research.

[1] Randall, L. and Sundrum, R.. "A Large mass hierarchy from a small extra dimension". Phys. Rev. Lett. **83**: 3370-3373 (1999).

[2] Randall, L. and Sundrum, R.. "An Alternative to compactification". Phys. Rev. Lett. **83**: 4690-4693 (1999).

[3] Hawking, S. W. and Hertog, T. and Reall, H. S.. "Brane New World". Phys. Rev. D. **62**: 043501 (2000).

[4] Gubser, S. S.. "AdS / CFT and gravity". Phys. Rev. D. **63**: 084017 (2001).

[5] Emparan, R. and Johnson, C. V. and Myers, R.C.. "Surface terms as counterterms in the AdS / CFT correspondence". Phys. Rev. D. **60**: 104001 (1999).

[6] Fröb, M. B. and Verdaguer, E.. "Quantum corrections to the gravitational potentials of a point source due to conformal fields in de Sitter". JCAP **1603**: 015 (2016).

[7] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time*, (Cambridge University Press 2011).

[8] Garriga, J. and Tanaka, T.. "Gravity in the brane world". Phys. Rev. Lett. **84**: 2778-2781 (2000).