# Signatures of $CP$ violation in the presence of multiple $b$-pair production at hadron colliders

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We calculate the production of two $b$-quark pairs in hadron collisions. Sources of multiple pairs are multiple interactions and higher order perturbative QCD mechanisms. We subsequently investigate the competing effects of multiple $b$-pair production on measurements of $CP$ violation: (i) the increase in event rate with multiple $b$-pair cross sections which may reach values of the order of 1 b in the presence of multiple interactions and (ii) the dilution of $b$ versus $\bar{b}$ tagging efficiency because of the presence of events with four $B$ mesons. The impact of multiple $B$-meson production is small unless the cross section for producing a single pair exceeds 1 mb. We show that even for larger values of the cross section the competing effects (i) and (ii) roughly compensate so that there is no loss in the precision with which $CP$-violating CKM angles can be determined.

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## I. INTRODUCTION

$CP$ violation can be accommodated in the standard model with three families in terms of a phase angle in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. There are several initiatives to study $CP$ violation in the $B$ sector of the CKM matrix. Failure to observe the standard model predictions implies physics beyond the standard model. It is customary to describe the $B$ sector of the CKM matrix in terms of the three angles of the unitary triangle $\alpha$, $\beta$, and $\gamma$ [1]. It is generally expected that a complete determination of the three angles will require experiments at both $B$ factories and hadron colliders.

In this paper we study the measurement of $CP$-violation angles at the CERN Large Hadron Collider (LHC) as a function of its luminosity. The advantages and disadvantages of the collider have been discussed in various publications [2]. The LHC is projected to reach a peak luminosity of $10^{34}$ cm$^{-2}$ s$^{-1}$. This combined with a large $b$-quark production cross section at high energy guarantees a number of events of the order of $10^{10}$ per year. Even after taking into account the branching ratios and detector efficiencies one is still left with a high number of events, of the order of $10^4$. Therefore the LHC is a $B$ factory.

The highest luminosities are, however, reached at a price: the possible presence of multiple interactions per beam crossing which could lead to the production of multiple $b$-quark pairs in the detector. The multiple $b$-pair cross section increases as the number of interactions per beam crossing and may reach 1 b at the highest luminosity where it dominates the single pair cross section. In extreme cases this effect may increase the number of $b$ pairs produced by two orders of magnitude. Unfortunately, double pair production will also introduce an additional source of fake asymmetry. In a double pair event the gold-plated tagging, e.g., $B^0B^- \to J/\psi K_S l^- X$ as well as the abnormal pairing, e.g., $B^0B^+ \to J/\psi K_S l^+ X$ are possible. Therefore the number of mistagged $b$'s increases since there is no possibility of tagging all four $b$ hadrons in the event.

We will in the end conclude that the impact of multiple $B$-meson production is small unless the cross section for producing a single pair exceeds 1 mb. Even in this case, the competing effects of (i) a gain in event rates from the additional production of multiple $b$-pair events and (ii) the dilution of the $CP$ signature by the introduction of fake asymmetry associated with wrong pairing of $b$'s, are likely to cancel. If one efficiently controls the systematics of the mistagging, the sensitivity of the experiments is actually improved. We will present results quantifying these statements.

On the theoretical side, we compute the production of four $b$ quarks in a single $pp$ interaction to leading order in QCD. Its cross section is comparable to the cross section for producing two pairs of $b$ quarks by multiple partons when no overlapping events occur.

## II. SINGLE VERSUS DOUBLE PAIR PRODUCTION

At high energies events with more than one heavy quark pair become abundant [3, 4]. In a hadron collider event there are three sources of events with two $b$-quark pairs: (i) higher order QCD diagrams, (ii) double parton interaction in a single $pp$ collision, and (iii) multiple $pp$ interactions in the crossing of the two bunches. For high luminosity LHC running mechanisms (ii) and (iii) dominate and the cross section for the production of two $b$-quark pairs can be written as

$$\sigma_{4b} = \sigma_{2b}^2 \left\{ \frac{1}{2\pi R^2} + 2\sqrt{\frac{1}{2\pi R^2}\frac{(N-1)}{\sigma_{\text{inel}}}} + \frac{(N-1)}{\sigma_{\text{inel}}} \right\}. \tag{1}$$

The three terms in Eq. (1) correspond, respectively, to the three diagrams in Fig. 1. The first term is the cross

 

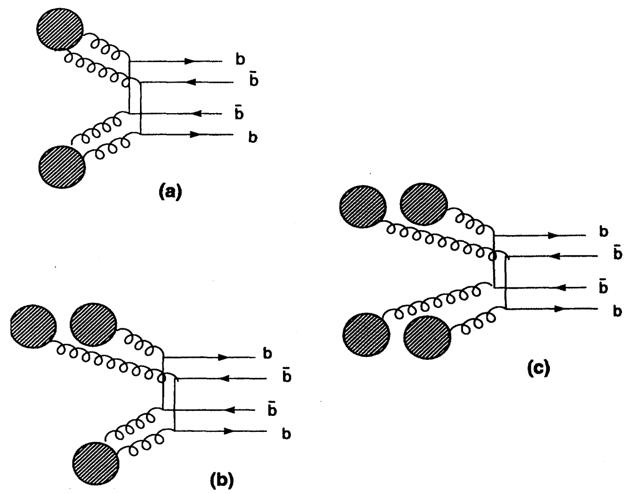


FIG. 1. Diagrams for double pair production by multiple parton interaction in high energy $pp$ collisions. The three contributions correspond to the three terms in Eq. (1): (a) $\sigma_{2b}^2 \frac{1}{2\pi R^2}$, (b) $\sigma_{2b}^2 \sqrt{\frac{1}{2\pi R^2} \frac{(N-1)}{\sigma_{\rm inel}}}$, and (c) $\sigma_{2b}^2 \frac{(N-1)}{\sigma_{\rm inel}}$.

section for double parton interaction as estimated in [5]. $\sigma_{2b}$ is the single $b$-pair production cross section. In our calculation we include $\sigma_{2b}$ from perturbative QCD to order $O(\alpha_s^3)$ [6]. $R$ is the radius of the proton occupied by the partons, mostly gluons, producing the $b$ pairs (in our calculation we use $R = 1$ fm). $N$ is the average number of interactions per bunch crossing:

$$N = \mathcal{L}\Delta t \sigma_{\rm inel}, \tag{2}$$

where $\mathcal{L}$ is the luminosity and $\Delta t$ the time between collisions.

The last term in Eq. (1) is the standard form for the cross section for multiple $pp$ interaction in the bunch crossing with $\sigma_{\rm inel}$ being the proton-proton inelastic cross section. As for the second term in Eq. (1), this is the cross section for the interaction of two partons from one proton in one bunch with two protons from the other bunch. It can be understood as the interference of the two previous effects. From Eq. (1) we see that $\sigma_{4b}$ increases linearly with the luminosity. We present our results on cross sections next.

In Fig. 2(a) the single and double $b\bar{b}$ pair production cross sections in $pp$ collisions are shown as a function of $\sqrt{s}$. As seen in the figure, at high energy, production by multiple parton interaction dominates.

For the sake of comparison we also show the cross section for the the higher-order QCD process $pp \to b\bar{b}b\bar{b}$. Its calculation is involved since it receives contributions from 72 Feynman diagrams. Our results are in good agreement with the leading-logarithm calculation of Ref. [4]. Figure 3 shows several characteristics of these events. The invariant energy of the interaction is around 100 GeV which corresponds to a Bjorken $x$ of about 0.07. Since gluon distributions are well tested at this energy, we expect our results to be fairly insensitive to the choice of parton distributions. The rapidity, transverse energy, and $\eta$-$\phi$ separation show that the $b$ pairs are experimentally accessible. However the cross section is two orders of magnitude smaller than the single $b$-pair production cross section.

As for the multiple parton interaction events, one should expect them to have similar distributions as the

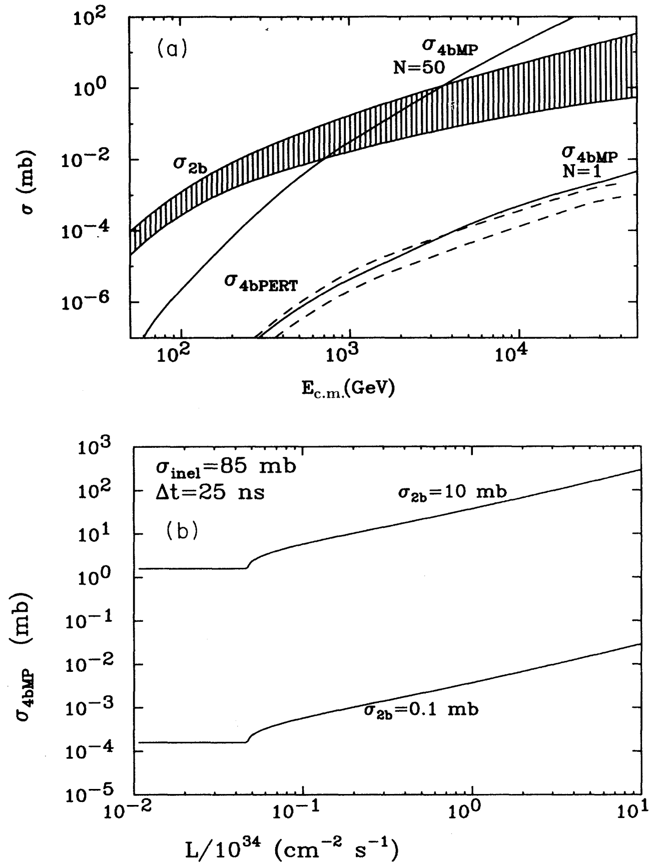


FIG. 2. (a) Single and double $b\bar{b}$ pair production cross sections in $pp$ collisions as a function of $\sqrt{s}$. The shaded band represents the prediction for the single pair production cross section $\sigma_{2b}$ from perturbative QCD to order $O(\alpha_s^3)$ obtained for several parametrizations of proton structure functions and different values of the scale $\mu$ and the quark mass $m_b$. The upper (lower) edge of the band corresponds to the prediction for the parametrization in Ref. [12] [Eichten-Hinchlifferlane-Quigg (EHLQ) [13]] for the structure functions with $m_b$ = 4.5 (5.0) GeV and $\mu = mb/2$ ($2m_b$). $\sigma_{4b\rm PERT}$ (dashed lines) is the cross section from higher-order QCD processes $pp \to b\bar{b}b\bar{b}$ for the EHLQ structure functions with $m_b$ = 5 GeV. The upper (lower) curve corresponds to $\mu = 4m_b$ ($\sqrt{\hat{s}}$). $\sigma_{4b\rm MP}$ (full lines) is the double pair production cross section from multiple parton processes [Eq. 1]. The lower (upper) curve corresponds to $N = 1$ (50) and the lower (upper) values of $\sigma_{2b}$. We have used the prediction of $\sigma_{\rm inel}$ from [14]. Here $N$ is the number of interactions per bunch crossing. (b) Double $b$-pair production cross section from multiple parton processes [Eq. 1] at LHC ($\sqrt{s}$ = 16 TeV) as a function of luminosity for $\sigma_{\rm inel}$($\sqrt{s}$ = 16 TeV)=85 mb [14] and $\Delta t$ = 25 ns. The upper (lower) curve corresponds to $\sigma_{2b}$ = 10 (0.1) mb.

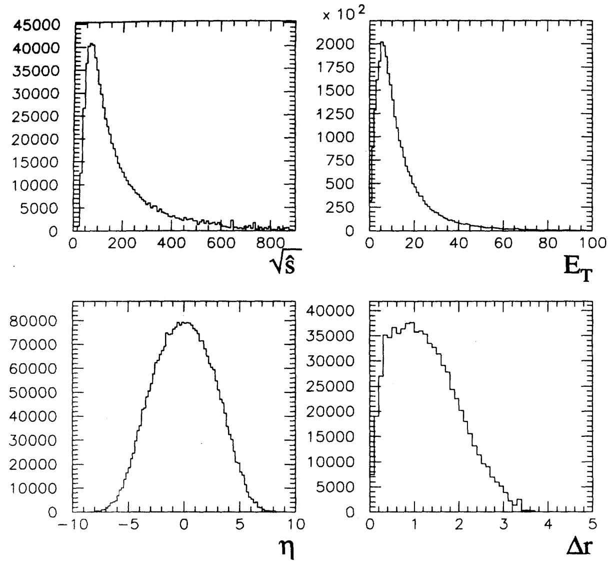


FIG. 3. Invariant mass ($\sqrt{\hat{s}}$), pseudorapidity ($\eta$), transverse energy, and $\eta$-$\phi$ separation ($\Delta r$) distributions for the 4-$b$'s events from higher-order QCD diagrams. The figures are shown for EHLQ structure functions with $\mu = 4m_b$. As discussed in the text the distributions are not very sensitive to this choice.

ones from single events since they are not more than an overlapping of those.

Figure 2(b) shows the double pair production cross section by multiple parton interactions as a function of the LHC luminosity for two values of the single pair cross section. The values bracket the expectations which strongly depend on the gluon structure function. Multiple parton processes may dominate single pair production for $\mathcal{L} >$2–20$\times 10^{33}$ cm$^{-2}$ s$^{-1}$ for $\sigma_{2b} > 1 - 10$ mb.

It should be pointed out that one cannot expect to make a reliable estimate of $\sigma_{2b}$ at the LHC from leading-order perturbation theory. The leading order perturbative calculations, which require inclusion of $O(\alpha_s^2)$ and $O(\alpha_s^3)$ diagrams, are unreliable because the results are sensitive to the assumed quark mass and the renormalization scale. They seem to underestimate already the experimentally observed cross section at the Fermilab Tevatron [7]. Because of its intermediate mass the calculation of the bottom quark cross section is believed not to be well understood. A perturbative calculation lies beyond the scope of existing QCD technology because it requires resummation of large logarithms of $1/x$ with $x \simeq m_b/\sqrt{s}$ [8]. The range of values, considered in this paper, is conservative, but cannot be guaranteed.

## III. *CP* VIOLATION MEASUREMENT: RESULTS

We will illustrate the implications of multiple $b$-pair production for the study of $CP$ violation using the gold-plated channel $\bar{B}_d^0 \to J/\psi K_S$ followed by $K_S \to \pi^+\pi^-$ and $J/\psi \to l^+l^-$, which yields information on the angle $\beta$. The $b$ or $\bar{b}$ nature of the second $B$ is established by the sign of the lepton produced in its semileptonic decay. $CP$ violation results in a nonvanishing asymmetry,

$$A = \frac{N_+ - N_-}{N_+ + N_-}, \tag{3}$$

where $N_{+,-}$ represent the number of events of the type $B \to J/\psi K_S, \bar{B} \to J/\psi K_S$, respectively. The integrated asymmetry is given by

$$A = \frac{x_d}{1 + x_d^2}\sin(2\beta), \tag{4}$$

where $x_d = \Delta m/\Gamma \sim 0.69$, $\Delta m$ is the mass difference between the two states, and $\Gamma$ is their width. The factor $\frac{x_d}{1+x_d^2}$ accounts for the possibility of $B_d^0$ oscillation.

For proton-proton interactions the initial state is not a $CP$ eigenstate, so one expects to have a fake contribution to the asymmetry, $F$, which modifies the asymmetry to

$$A \simeq a[\sin(2\beta) + F], \tag{5}$$

with $a = D\frac{x_d}{1+x_d^2}$. $D$ is related to the so-called "dilution" factor. The equation is valid for small $\beta$. $F$ has been estimated to be of order of a few percent of the signal [9].

We are now ready to analyze the effect of multiple parton interactions on the measurement of $CP$ violation at LHC. When including double pair production we must account for all possible pairings. We will assume that the trigger is designed to tag a single $B$ hadron. In the presence of multiple pair production the "other" $B$ hadron may, or may not, be assigned the "correct" charge. This will increase the fake contribution to the asymmetry.

Let us define $\alpha$ as the ratio of double to single pair production:

$$\alpha = p\frac{\sigma_{4b}}{\sigma_{2b}} = p\left\{\frac{\sigma_{2b}}{2\pi R^2} + (N-1)\frac{\sigma_{2b}}{\sigma_{\rm inel}} + 2\sigma_{2b}\sqrt{\frac{1}{2\pi R^2}\frac{(N-1)}{\sigma_{\rm inel}}}\right\}, \tag{6}$$

where $p \sim 0.64$ is the probability that two or more $B$'s do not decay semileptonically since we are assuming only one prompt lepton in the event. Then the asymmetry can be written as

$$A = \frac{N_+^1 + \alpha N_+^2 - N_-^1 - \alpha N_-^2}{N_+^1 + \alpha N_+^2 + N_-^1 + \alpha N_-^2}, \tag{7}$$

where $N_{+,-}^{1,2}$ represent the number of events produced with + or − signature and coming from a double (2) or single (1) $b$-pair production. We have that

$$
\begin{aligned}
N_+^{i=1,2} = N(J/\psi\, K_s\, l^+ X)^{1,2} \propto & \{\tilde{n}^+ + R_d\tilde{n}^0 + R_s\tilde{n}^s\}\left\{1 + \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\{W_d\tilde{n}^0 + W_{\bar{s}}\tilde{n}^{\bar{s}}\}\left\{1 - \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\delta_{i,2}\{n^+ + R_d n^0 + R_s n^s\}\left\{1 - \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\delta_{i,2}\{W_d n^0 + W_s n^s\}\left\{1 + \frac{x_d}{1+x_d^2}\sin 2\beta\right\}, \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
N_-^{i=1,2} = N(J/\psi\, K_s\, l^- X) \propto & \{\tilde{n}^- + R_d\tilde{n}^0 + R_s\tilde{n}^{\bar{s}}\}\left\{1 - \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\{W_d\tilde{n}^0 + W_s\tilde{n}^{\bar{s}}\}\left\{1 + \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\delta_{i,2}\{n^- + R_d n^0 + R_s n^{\bar{s}}\}\left\{1 + \frac{x_d}{1+x_d^2}\sin 2\beta\right\} \\
& +\delta_{i,2}\{W_d n^0 + W_s n^{\bar{s}}\}\left\{1 - \frac{x_d}{1+x_d^2}\sin 2\beta\right\}. \qquad (9)
\end{aligned}
$$

The effect of neutral $B$ oscillation in the tagging is parametrized in terms of the $W_i$ and $R_i = 1 - W_i$ coefficients. $W_i$ is the probability of a $B_i^0$ meson to decay as a $\bar{B}_i^0$:

$$W_i = \frac{x_i^2}{2+2x_i^2}. \qquad (10)$$

We have used $x_d = 0.69$ and $x_s = 9$ [9].

The terms proportional to $\delta_{i,2}$ account for the possibility of abnormal pairing in the double pair processes. We used the same notation as in Ref. [9]:

$$
\begin{aligned}
\tilde{n}^+ &= 2C[N(B^+\bar{B}^0)\mathcal{B}(B^+ \to l^+) + N(\bar{\Lambda}_b\bar{B}^0)\mathcal{B}(\bar{\Lambda}_b \to l^+)], \\
\tilde{n}^- &= 2C[N(B^-\bar{B}^0)\mathcal{B}(B^- \to l^-) + N(\Lambda_b\bar{B}^0)\mathcal{B}(\Lambda_b \to l^-)], \\
\tilde{n}^0 &= CN(B^0\bar{B}^0)[\mathcal{B}(B^0 \to l^+) + \mathcal{B}(\bar{B}^0 \to l^-)], \\
\tilde{n}^s &= 2CN(B_s^0\bar{B}^0)\mathcal{B}(B_s^0 \to l^+), \\
\tilde{n}^{\bar{s}} &= 2CN(B_{\bar{s}}^0\bar{B}^0)B(B_{\bar{s}}^0 \to l^-), \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
n^+ &= C[N(B^+B^0)\mathcal{B}(B^+ \to l^+) + N(\bar{\Lambda}_b B^0)\mathcal{B}(\bar{\Lambda}_b \to l^+)], \\
n^- &= C[N(B^-\bar{B}^0)\mathcal{B}(B^- \to l^-) + N(\Lambda_b\bar{B}^0)\mathcal{B}(\Lambda_b \to l^-)], \\
n^0 &= C[N(B^0B^0)\mathcal{B}(B^0 \to l^+) + N(\bar{B}^0\bar{B}^0)\mathcal{B}(\bar{B}^0 \to l^-)], \\
n^s &= CN(B_s^0B^0)\mathcal{B}(B_s^0 \to l^+), \\
n^{\bar{s}} &= CN(\bar{B}_s^0\bar{B}^0)\mathcal{B}(\bar{B}_s^0 \to l^-). \qquad (12)
\end{aligned}
$$

Here $\tilde{n}^\alpha$ and $n^\alpha$, $\alpha = +,-,0,s,\bar{s}$ correspond to "favorable" and "unfavorable" assignments, respectively. $C$ is a constant. Even after including all possibilities the asymmetry can still be cast in the form of Eq. (5) for small $\sin(2\beta)$. $F$ and $a$ must, however, include the contributions from multiple pair production.

Because the initial state is not a $CP$ eigenstate the $N(B_i\bar{B}_j)$ are not all equal since protons are made of quarks rather than antiquarks, and therefore a $\bar{b}$ is more likely to pair with a valence quark than a $b$. This difference is parametrized [9] in terms of baryon and meson fragmentation probabilities $l_b, l_m$ which are implicitly defined by the equations

$$N(B_i\bar{B}_j) \propto P(B_i)P(\bar{B}_j) \qquad (13)$$

with

$$
\begin{aligned}
&P(B^+) = p_d(1-l_m) + 2l_m/3, \quad P(B^-) = p_d(1-l_b), \\
&P(B_d^0) = p_d(1-l_m) + l_m/3, \quad P(\bar{B}_d^0) = p_d(1-l_b), \\
&P(B_s^0) = p_s(1-l_m), \quad P(\bar{B}_s^0) = p_s(1-l_b), \\
&P(\bar{\Lambda}_b) = p_\Lambda(1-l_m), \quad P(\Lambda_b) = p_\Lambda(1-l_b) + l_b. \qquad (14)
\end{aligned}
$$

We will assume that $p_d = 0.38$, $p_s = 0.14$, and $p_\lambda = 0.1$ and take the semileptonic branching ratios for $\Lambda_b$ and $B$ to be equal [9]. Clearly for $l_m = l_b = 0$ it follows that $N(B_i\bar{B}_j) = N(B_j\bar{B}_i)$ and the fake asymmetry vanishes. The results are not very sensitive to $l_m$ but depend, in contrast, very strongly on the value of $l_b$.

In Figs. 4, 5, and 6 we have displayed the value of $A(0)$, the coefficient $a$, and the fake contribution to the asymmetry, $F$, as a function of the LHC luminosity for different values of $\sigma_{2b}$, $\sigma_{\rm inel}$ and the parameters $l_b$ and $l_m$. As expected the values of $A(0)$ and $F$ depend strongly on the parameter $l_b$ while $a$, which measures the "true" final state asymmetry, is rather insensitive to it. As for the effect of multiple pair production, it increases the value of the fake contribution to the asymmetry while decreasing the value of the $a$. As expected, it increases the ratio of "fake" to "true" asymmetry. The value of the luminosity at which this effect becomes important depends on the value of the $\sigma_{2b}$. For $\sigma_{2b} < 1$ mb the effect is not very important for any possible value of the luminosity.

To quantify the impact of double pair production on the measurement of $\sin(2\beta)$ we have computed the statistical and systematic error on $\sin(2\beta)$, with

$$\sigma_{\rm stat} = \frac{\sqrt{1-A^2}}{\sqrt{2N_{\rm events}}}, \qquad (15)$$

$$\sigma_{\rm syst} = \sqrt{\sin^2(2\beta)\sigma_a^2/a^2 + \sigma_F^2}, \qquad (16)$$

where $N_{\rm events}$ is the total number of signal events given by

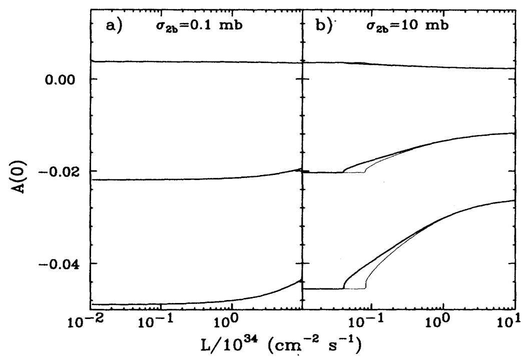


FIG. 4. Asymmetry for $\sin(2\beta) = 0$, $A(0)$, at LHC as a function of the collider luminosity for two values of $\sigma_{2b}$. The upper curves correspond to $l_b = 0$ and $l_m = 0.05$. The central curves correspond to $l_b = l_m = 0.05$, and the lower curves correspond to $l_b = 0.1$ and $l_m = 0.05$. In all cases the dark (light) lines correspond to $\sigma_{\rm inel} = 100$ (50) mb.

$$\begin{aligned} N_{\rm events} &= N(J/\psi K_s l^+) + N(J/\psi K_s l^-) \\ &= 2\times N(b\bar{b}) \times P(B_d) \times \mathcal{B}(B\to lX) \times \mathcal{B}(B_d \to J/\psi K_s)\mathcal{B}(K_s \to \pi^+\pi^-) \times \mathcal{B}(J/\psi \to l^+l^-) \times \epsilon \\ &= 6.76\times 10^6 \times (\mathcal{L}/10^{34}) \times (\sigma_{2b}/\text{mb}) \times (1+\alpha) \ \ \text{events/yr.} \end{aligned} \qquad (17)$$

The $\epsilon$ factor is the product of the triggering, tagging, and reconstruction efficiencies combined with the geometrical acceptance. We use $\epsilon = 7.7\times 10^{-3}$ [10]. The systematic errors on $F$ and $a$ have been estimated to be of the order of a percent [11].

In summary, the presence of multiple pair production increases the total number of events and therefore improves the statistical error. It also increases the fake contribution to the asymmetry, $F$, and, as a result of this, it increases the systematic error. The minimum value of $\sin(2\beta)$ that can be measured with a given statistical significance $N_\sigma$ is

$$\sin(2\beta)_{\rm min} = N_\sigma \frac{\sqrt{\sigma^2_{\rm stat} + \sigma_F^2}}{\sqrt{1 - N_\sigma^2 \dfrac{\sigma_a^2}{a^2}}}. \qquad (18)$$

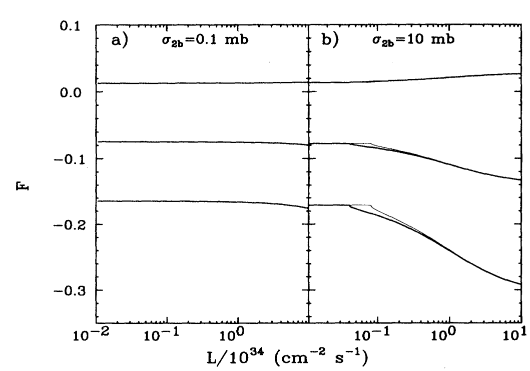


FIG. 5. Same as previous figure for the fake contribution to the asymmetry, $F$.

In Fig. 7 we plot $\sin(2\beta)_{\rm min}$ for $\sigma_{2b} = 10$ mb as a function of the collider luminosity for different values of the systematic errors on $F$ and $a$ for a $3\sigma$ effect. For comparison we show the corresponding value of $\sin(2\beta)_{\rm min}$ for $\sigma_{2b} = 0.1$ mb. As expected the value of $\beta_{\rm min}$ and its dependence on luminosity depend critically on the systematic error on the fake asymmetry and are less sensitive to other parameters such as the systematic error on $a$. This is easy to understand. Unless the systematic error on $F$ is extremely small, its systematic error represents the limiting factor on $\sin(2\beta)_{\rm min}$ in Eq. (18.) For large values of $\sigma_{2b}$, the presence of multiple pair production increases $F$ at higher luminosities and therefore the value of the $\sin(2\beta)_{\rm min}$. If, on the other hand, one can determine $F$ with good precision, the limiting factor in Eq. (18) becomes the statistical error and the increased statistics as a result of multiple $b$-pair production improves the measurement.

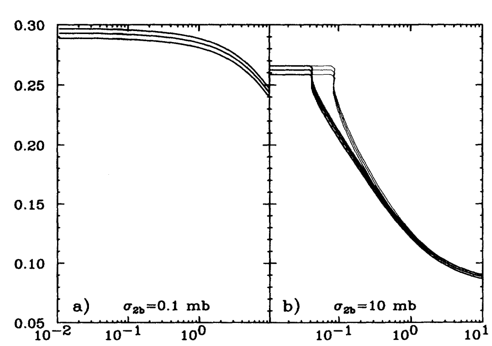


FIG. 6. Same as previous figure for the coefficient $a$.

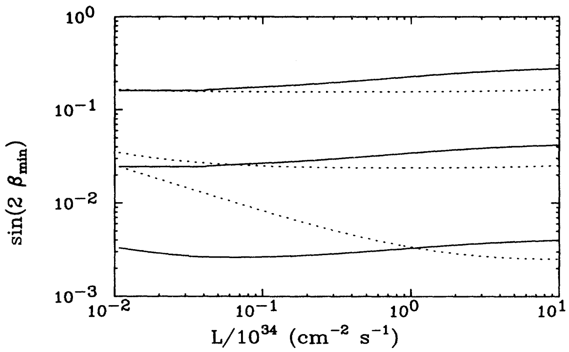


FIG. 7. Minimum value of $\sin(2\beta)$ measurable at the LHC with $3\sigma$ effect as a function of the collider luminosity for $\sigma_{2b} =$ 10 mb (solid lines) and $\sigma_{2b} = 0.1$ mb (dotted lines), $\sigma_{\rm inel} =$ 100 mb, and $l_m = l_b = 0.05$. From upper to lower the curves correspond to $\sigma_a = \sigma_F = 30,\ 10,\ 1\,\%$.

## IV. CONCLUSIONS

In this paper we have studied $CP$-violation measurements at hadron colliders. We have investigated the effect of multiple $b$ production as a function of collider luminosity. We conclude that multiple $b$-pair production dominates over single pair production for $\mathcal{L} > 2\text{–}20\times10^{33}$ cm$^{-2}$ s$^{-1}$ for $\sigma_{2b} > 1\text{–}10$ mb.

The presence of multiple pair production produces two competing effects in the measurement of the $CP$-violating asymmetry. It increases the total number of events and therefore it improves the statistics. It also increases the number of mistagged events introducing an additional source of fake asymmetry and therefore worsens the systematics.

In the end the impact of multiple $B$ meson production is small unless the cross section for producing a single pair exceeds 1 mb. In this case the dominant factor weighing the competing effects is the error on the determination of the fake contribution to the asymmetry. Lower luminosities are advantageous when one cannot measure the fake contribution to better than a few percent. However, if a higher precision is achieved, it is still advantageous to perform the measurement at a higher luminosity.

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