

Decay of Υ mesons to $B\bar{B}$ meson pairs

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Abstract: Starting from a simple interaction model and previous numerical data, we study the decay and interaction between the Υ meson and $B\bar{B}$ meson pairs. We find an expression for the Υ binding potential, the mixing potential and we compute the decay widths for 3 states.

I. INTRODUCTION

The Υ mesons are a high mass family of states composed of a bottom quark and its antiparticle relative. These states decay quickly into other particles, the composition of which presents a big variation with respect to the energy state of the original meson. When it passes a certain mass threshold, there is enough energy available to induce a process of string breaking, in which a pair of $B\bar{B}$ mesons is generated (where a B meson is composed by a heavy \bar{b} quark and a very light d quark). This also generates an increase in the decay width of around 3 orders of magnitude when comparing states below and above the threshold. In this article, we will attempt to compute the decay widths associated to this specific process of string breaking (above the mass threshold). As the particles considered in this decay are very massive, we will use a non-relativistic approach. To obtain our results, we use the lattice QCD numerical results obtained by Bali et al. [1] on the transition from the quark-antiquark state to the meson-antimeson system. The raw numerical data is shown in the appendix. We will find the decay width of various states in the Υ spectrum using Fermi's golden rule.

Throughout this article we use natural units, so $\hbar = c = 1$.

II. DEVELOPMENT

We propose an interaction Hamiltonian of the form shown in eq. (1). V_Υ is the binding potential of the Υ mesons, which we will use later to determine its wavefunctions. V_{mix} is the interaction potential between the two states. From the numerical energy data we will reconstruct it and find

a function to describe it. Finally, we consider the $B\bar{B}$ system to be unbound, and we will describe it as a plane wave.

$$H = \begin{pmatrix} V_\Upsilon & V_{mix} \\ V_{mix} & 0 \end{pmatrix} \quad (1)$$

Knowing the eigenvalues of this matrix (energies provided by the numerical data), we reconstruct the original potentials:

$$V_\Upsilon = E_1 + E_2 \quad (2)$$

$$V_{mix} = \sqrt{-E_1 E_2} \quad (3)$$

A. Υ mesons wavefunctions

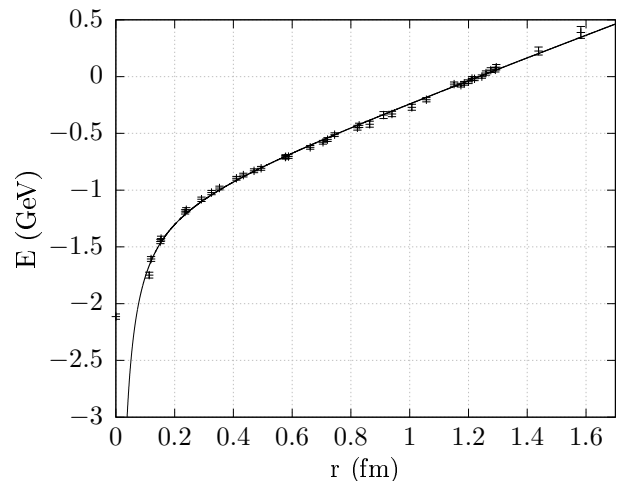


FIG. 1: Binding potential of the Υ mesons, with the Cornell potential fit overlaid

In fig. 1 we show the result of the Υ binding potential. We describe this as a Cornell potential

(eq. 4). The first term represents the Coulomb-like attraction at short distances and the second represents the linear increase of the gluon binding force.

$$V_{\Upsilon}(r) = -\frac{a}{r} + \sigma r + c \quad (4)$$

The numerical fit values are

$$\sigma = 0.190(3) \text{ GeV}^2 \quad (5)$$

$$a = 0.369(15) \quad (6)$$

$$c = -1.13(2) \text{ GeV} \quad (7)$$

In this model, the constant in the potential c is a free parameter, and we will have to adjust it manually to obtain correct results when finding the meson spectrum. We have found the best results using $c \approx 9.39 \text{ GeV}$. Using this potential we now solve the Schrödinger equation using a numerical method [2]. This will give us the Υ spectrum, including the energies and the wavefunctions. We show this in table I, comparing with the bibliographical data of the PDG [3]. To compare with these values we had to use energy spin-averages of all the particles listed in the PDG, as our calculation does not specify the spin of the particles, only the N and L quantum numbers.

State	E (GeV) ± 0.01	E_{PDG} (GeV)	Relative Error (%)
1S	9.49	9.445	0.42
1P	9.87	9.900	0.34
1D	10.10	10.164	0.65
2S	9.98	10.020	0.34
2P	10.22	10.260	0.42
3S	10.32	10.355	0.32
3D	10.66	-	-
4S	10.60	10.579	0.24
5S	10.86	10.890	0.31
6S	11.09	10.99	0.90

TABLE I: Spectrum of the Υ mesons obtained by numerically solving the Schrödinger equation.

The energies are compared with bibliographical values. To make these simulations we used as the mass of the b quark $m_b = 4.88 \text{ GeV}$, a value used previously by Oncala & Soto [4] to compute meson spectrums.

We computed all the identified $b\bar{b}$ states found in the PDG listings, and found compatibilities between the $\Upsilon(10860)$ and the 5S state, and between

the $\Upsilon(11020)$ and the 6S state. These are the values included as E_{PDG} for those states. We can see all the resulting energies are close to these bibliographical values, with most staying at less than a 0.5% relative error.

B. Mixing potential

In figure 2 we show the mixing potential obtained. The V_{mix} points are directly obtained from applying equation (3). The uncertainty in these points is big because the values reach very close to zero, especially in the E_2 case (as seen in the appendix). For the last two points, the values even go to positive (making the square root complex) while the uncertainties cross over to negative values.

Apart from these, we also plot the transition rate g obtained by Bali et al. applying their mixing angles model. As their model includes higher order corrections, the values are more accurate than the simple effective model we presented above.

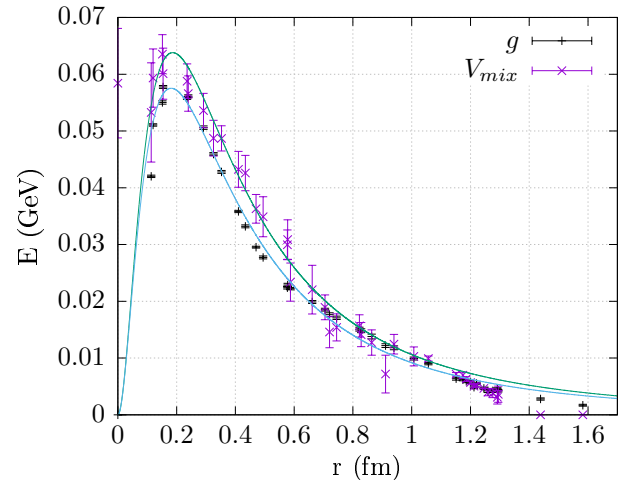


FIG. 2: Mixing potential between the Υ and the $B\bar{B}$ states. V_{mix} is the result of our effective model, and g is the transition rate obtained by Bali et al. [1]. A fit for both models is shown.

To describe the potential behaviour we have found the expression:

$$V_{mix}(r) = \frac{\alpha r^2 + \beta r^3}{(r + k)^6} \quad (8)$$

with numerical parameters (for the g fit)

$$\alpha = 6.3(8) \text{ GeV}^{-3} \quad (9)$$

$$\beta = 21(2) \text{ GeV}^{-2} \quad (10)$$

$$k = 1.10(1) \text{ GeV}^{-1} \quad (11)$$

We can see how the relative uncertainties in this fit are much bigger than in the binding potential. In figure 3 we also show the behaviour of the mixing potential for big values of r , to characterize its decreasing behaviour. To determine the dependency, we left the exponent as a free parameter in the fit, and we checked the best description is a decreasing cubic exponential.

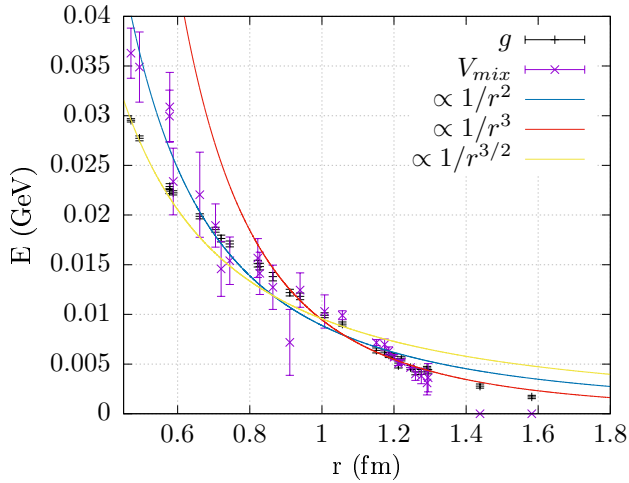


FIG. 3: Mixing potential behaviour for big r values. Multiple possible exponents are fitted, we can see how the best model is a decreasing cubic exponential.

C. $B\bar{B}$ system wavefunction

We consider the $B\bar{B}$ system to be unbound (free particles), and so we describe it as a plane wave. To include it in our calculations we use the plane wave expansion (eq. 12)

$$e^{i\vec{k}\cdot\vec{r}} = 4\pi \sum_{l=0}^{\infty} i^l j_l(kr) \sum_{m=-l}^l Y_m^{l*}(\hat{k}) Y_m^l(\hat{r}) \quad (12)$$

In our case, using non-relativistic expressions, we find the expression $k = \sqrt{mE}$, where m is the

reduced mass of the $B\bar{B}$ system ($m = \frac{m_B}{2} = 2.640$ GeV [3]) and E is its non-relativistic energy, $E = \frac{\vec{p}_B^2}{m_B}$.

When the plane wave is integrated, we will treat the radial and angular parts separately. Here, the l number will come determined by the Υ meson and we will have to sum over its polarizations.

D. Decay widths

As said earlier, we will use Fermi's golden rule (eq. 13) to find the decay widths of some Υ states. Of course, only states that are above the $m_{B\bar{B}}$ threshold can be considered physical.

$$\Gamma = 2\pi \left| \left\langle \Psi_{\Upsilon} \left| V_{mix} \frac{r^i}{r} \right| \Psi_{B\bar{B}} \right\rangle \right|^2 \rho \quad (13)$$

In this expression we have added the $\frac{r^i}{r}$ term, which represents the symmetrical coupling between the spins of the quarks in the Υ mesons ([1], sec. II-A). This is a necessary term as these mesons have a $J^{PC} = 1^{--}$, while the B mesons have $J^P = 0^-$. To conserve angular momentum, there needs to be a relative spin of 1 between the two mesons of the $B\bar{B}$ system. This also implies C-parity conservation (as $C = (-1)^L = -1$).

The last piece we need is the state density ρ . The expression is the following:

$$\rho = k^2 \frac{dk}{dE} = \frac{1}{2} \sqrt{m^3 E} \quad (14)$$

Firstly, we will compute the angular parts, constant across all Υ states. Expressing the spherical coordinates $\frac{r^i}{r}$ in function of spherical harmonics, we have:

$$\frac{x}{r} = \sqrt{\frac{2\pi}{3}} (Y_{-1}^1 + Y_1^1) \quad (15)$$

$$\frac{y}{r} = i\sqrt{\frac{2\pi}{3}} (Y_{-1}^1 - Y_1^1) \quad (16)$$

$$\frac{z}{r} = \sqrt{\frac{4\pi}{3}} Y_0^1 \quad (17)$$

These expressions were used while making the calculation implicit in eq. (18). Retrieving the sum from the free wave expansion (eq. 12), the whole angular part will be:

$$\sum_{m=-1}^1 \left| \left\langle Y_0^0 \left| \frac{r^i}{r} \right| Y_m^1 \right\rangle \right|^2 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \quad (18)$$

From this result, we conclude the radial parts play the relevant role in the decay widths. These parts will include the wavefunction from the Schrödinger equation, the mixing potential V_{mix} and the spherical Bessel function from the free wave expansion.

For the $\Upsilon(4S)$ state we find a decay width of 15 ± 2 MeV. Comparing this to the PDG value¹ of 20.0 ± 2.5 MeV, we see that it lies within the same order of magnitude, with the uncertainty bounds being close to overlap. For the $\Upsilon(5S)$ state we find a value of 40 ± 5 MeV. In this case, the bare $B\bar{B}$ decay width is listed in the PDG as 2.8 ± 0.4 MeV (for the $\Upsilon(10860)$). However, there are numerous other decays involving B mesons that may be the cause of the bigger value divergence. Finally, for the $\Upsilon(6S)$ we find a value of 67 ± 7 MeV, further showing the increasing tendency. The PDG total width for the $\Upsilon(11020)$ state, which we tentatively identified as 6S, is of 49_{-15}^{+9} MeV. This is a state for which the widths of specific channels have not yet been determined experimentally, but we can see the general value remains within the order of magnitude of our prediction.

Width ratios	V_{mix} model	Constant model
Γ_{4S}/Γ_{5S}	0.38	0.09
Γ_{5S}/Γ_{6S}	0.59	0.46
Γ_{4S}/Γ_{6S}	0.23	0.04

TABLE II: Comparison of decay width ratios from our model and a hypothetical model where the interaction is a constant

Lastly, we will compare the results obtained from this analysis with the case in which the interaction potential V_{mix} is a constant, to see the effect

of our mixing model in the calculations. Without knowing the value of this hypothetical constant, we can only compare the ratios between two decay widths (as the constant term will cancel). This is shown in table II. We can see how the constant model provides much different predictions, with relations far from the bibliographical data.

III. CONCLUSIONS

- We have built a simple model to describe the interaction between the Υ mesons and the $B\bar{B}$ system, using a non-relativistic approach based on numerical lattice QCD data.
- From this data, we found a binding potential that reproduces bibliographical results in the energy levels of the Υ . We have also determined an empirical expression for the mixing potential between both systems.
- We have determined 3 decay widths of the studied process, numerically close values to the bibliographical data.
- Our biggest numerical uncertainty has come from the parameters of the mixing potential, where these depend on the accuracy of the numerical data we used. Our model is a simple approach to reality, and would need other corrections to describe fully this interaction.

Acknowledgments

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¹ The PDG data for this particle gives the total decay width and a lower bound of 96% for the $B\bar{B}$ case. Here we show the value corresponding to this bound.

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- [2] W. Lucha and Franz F. Schöberl, "Solving the Schrödinger equation for bound states with Mathematica 3.0", Int. J. Mod. Phys. C **10**, 607 (1999).
- [3] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update.
- [4] R. Oncala and J. Soto, "Heavy Quarkonium Hybrids: Spectrum, Decay and Mixing", Phys. Rev. D **96**, 014004 (2017).

APPENDIX: NUMERICAL DATA

r (fm)	E_1 (GeV)	E_2 (GeV)	g (GeV)
0	-2.164(7)	0.050(17)	0
0.113	-1.799(9)	0.050(17)	0.236(12)
0.120	-1.673(9)	0.066(12)	0.287(12)
0.152	-1.536(9)	0.083(9)	0.309(17)
0.154	-1.503(9)	0.076(12)	0.324(14)
0.235	-1.277(9)	0.085(9)	0.314(14)
0.240	-1.254(9)	0.081(9)	0.315(14)
0.292	-1.159(9)	0.078(9)	0.284(17)
0.326	-1.090(9)	0.069(9)	0.258(14)
0.353	-1.052(9)	0.071(7)	0.240(14)
0.410	-0.957(7)	0.062(9)	0.201(12)
0.434	-0.929(7)	0.062(9)	0.187(14)
0.470	-0.877(9)	0.047(7)	0.166(9)
0.494	-0.853(7)	0.045(9)	0.156(14)
0.577	-0.747(9)	0.038(7)	0.126(14)
0.578	-0.747(7)	0.040(9)	0.128(17)
0.588	-0.728(9)	0.024(7)	0.125(12)
0.661	-0.647(7)	0.024(9)	0.112(14)
0.705	-0.597(12)	0.019(5)	0.104(14)
0.720	-0.566(12)	0.012(5)	0.099(19)
0.745	-0.526(12)	0.014(5)	0.096(17)
0.822	-0.467(17)	0.017(5)	0.085(19)
0.828	-0.443(14)	0.014(5)	0.083(19)
0.864	-0.431(21)	0.012(5)	0.077(24)
0.911	-0.344(26)	0.005(5)	0.068(19)
0.939	-0.344(19)	0.014(5)	0.066(19)
1.007	-0.282(21)	0.012(5)	0.056(14)
1.057	-0.218(17)	0.014(2)	0.051(12)
1.151	-0.085(14)	0.019(5)	0.036(14)
1.174	-0.092(12)	0.017(5)	0.035(12)
1.186	-0.076(12)	0.017(5)	0.033(14)
1.200	-0.062(12)	0.017(5)	0.032(14)
1.212	-0.038(9)	0.021(7)	0.027(14)
1.220	-0.040(9)	0.021(7)	0.031(12)
1.246	-0.026(7)	0.026(7)	0.026(12)
1.260	-0.012(7)	0.043(12)	0.024(17)
1.276	-0.007(5)	0.064(17)	0.024(14)
1.292	-0.005(5)	0.064(14)	0.026(9)
1.295	-0.005(5)	0.088(19)	0.024(12)
1.438	0.002(5)	0.225(31)	0.016(12)
1.582	0.007(5)	0.389(47)	0.009(9)

TABLE III: Raw data used for the calculations. Original from [1], converted into physical units from lattice results