Oriented majority-vote model in social dynamics

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Mass events ruled by collective behaviour are present in our society every day. Some of these events require a large number of people interacting in contact networks or lattices. Could the spatial orientation of the agents involved have an effect in promoting or hindering the emergence of consensus states? To address this issue, we propose a simple model for a group of agents interacting in a lattice where an attention-driving action can take place, and possibly spread globally. Computer simulations are performed in order to describe the dynamics of the model and its implications in social systems. The model yields a polarization between global activity or inactivity when orientation dynamics are present, whereas mixed-states arise when they are not.

I. INTRODUCTION

The idea of using laws -in the physical sense of the term- in order to describe human societies goes way back to the early eighteenth century. Nonetheless, actual attempts to describe social phenomena by means of a physical approach had to wait until the mid 1900s, when similarities between statistical regularities and collective behaviour of social and physical systems were pointed out [1]. In the past few decades, the interest in descriptions of social systems within the framework of statistical physics have gained momentum. The availability of large databases and computing power are some of the reasons for it.

It appears natural in this scheme that Ising-like models must be present to some extent. Two excellent examples are the so-called *voter* and *majority-vote* models [2], which try to simulate opinion dynamics working on a group of agents that interact with each other. Both these models can be embedded and simulated on a regular lattice. On the former model, each agent copies the opinion of a neighbour [3], while in the latter the agent takes the opinion of the majority of its neighbours [4]. These are both simple models that poorly reproduce real social dynamics: social networks do not live in the geometry of lattices or simplified graphs, and their interactions are far more complex than the ones considered in these models. However, their simplicity allow for a better understanding of their complexity and both models can be solved analytically.

The problem that will be addressed in the present work will be a close relative of the majority vote-model, and will be defined with the aim of describing the dynamics that can be observed when a crowded gathering of people takes place (such as a concert or a demonstration) and a group of people spontaneously undertake an action that drives the attention of other people. This action could be holding up a lighter, lighting with a mobile phone or chanting a slogan. The focus will be set at the spatial orientation of the agents and the attention-driving property of the action: their field of view will determine the group of agents with whom they interact, setting the state to reproduce with a majority rule at the same time. A dynamical rule will favour the active agents to be looked at.

Results will be computed for this dynamical orientation model, and a null model (i.e. without the orientation dynamics considered). Their statistics in the stationary state are to be compared.

II. THE MODEL

A. Dynamics

The model considers a two dimensional lattice with N agents embedded in its nodes, with a certain geometry to be specified. A state s_i, that is either 1 (active) or -1 (inactive), is assigned to each agent i. One of the nearest-neighbours of every agent characterizes its orientation in space, as if the latter was looking directly to the former. This node will be designated by ω_i. The group of agents that happens to be 'in the field of view' of a given node will be called its *potential neighbourhood.* This group includes the agent that is being looked at, ω_i, and the common neighbours of both ω_i and i (see figure 1). The potential neighbourhood of a node i will be notated by Ω_i.

The time is taken to be discrete. The state of s_i of node i at time $t+1$ is redefined according to the most common state in its potential neighbourhood Ω_i at time t:

$$s_i(t+1) = \begin{cases} +1 \;\; if \;\; \sum_{j\in\Omega_i(t)} s_j(t) > 0 \\ -1 \;\; if \;\; \sum_{j\in\Omega_i(t)} s_j(t) < 0 \end{cases} \tag{1}$$

Afterwards, its orientation w_i may also change. The rule that is implemented is not symmetrical between the two states: those agents that are in the active state in a time step t are more likely to be looked by its neighbours at time $t+1$. In more precise words:

- If $s_{\omega_i}(t) = -1$, and there is one or more active nodes in $\Omega_i(t)$, one of these (j) is picked at random

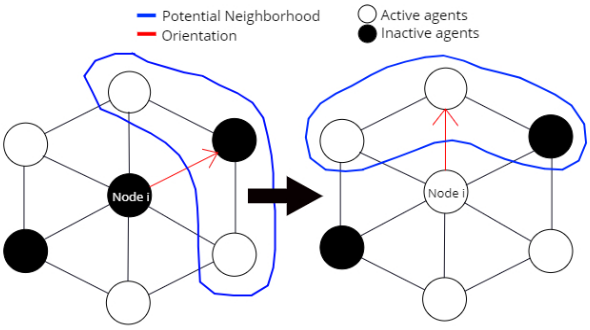


FIG. 1: A sample node i of the lattice considered in the model, represented with its orientation w_i and the states of the nodes that are shown.

and $w_i(t+1) = j$. In other words, if the agent i was looking at an inactive node at time t, but had any active nodes in its field of view, one of these j catches its attention and i rotates to look at j.

This procedure runs through the entire lattice, redefining both states and orientations of every agent at each time step.

B. Choosing a geometry for the lattice

The most immediate geometry one can think of when it comes to a 2D-lattice is a square lattice. However, the choice made in the computation at the present work has been a triangular lattice. The subsequent lines are an attempt to justify this choice.

In the first place, the connectivity in a square lattice equals four. With the description given for the model, the number of agents in a potential neighbourhood would be three. That means, essentially, that the ratio between the number of neighbours in a certain potential neighbourhood and the total is 75%. Most part of the neighbourhood is taken into account, and the role of orientation would seem to be less relevant than with a higher connectivity.

With a triangular lattice, however, the connectivity is six, and the analogous ratio is 50%. On the one hand, it seems pleasing to allow for more possible orientations, being that the one thing is being explored; on the other hand, this ratio seems more reasonable if one thinks of a real field-of-view for humans (we have a frontal field of view).

The model could be as well implemented in other geometries, but it has not been the interest in presenting the results on the present work.

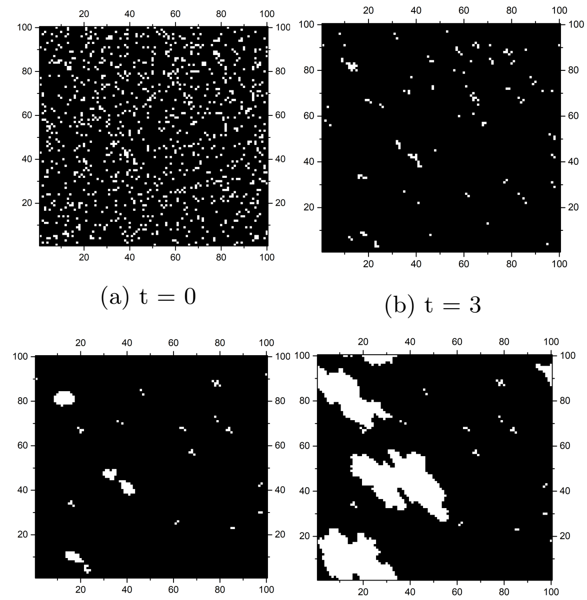
III. RESULTS

The model have been implemented on a lattice as described above of $N = 10^4$ nodes with periodic boundary conditions (PBC). Different initial conditions have been used. The initial orientations have been set totally *ad random* in every case. The initial states for the lattice have been fixed using two different rules, which are specified in their corresponding subsections.

A. Random initial states

In the lines that follow, the initial condition of the states is that an agent is active with probability p, and otherwise inactive with probability $1-p$. This could be representative of certain real situations, in which people attending an event have information that an action will take place (in a concert, for example, holding up a lighter when the artists start to play a certain song). The more people are aware of the action, the greater p would be.

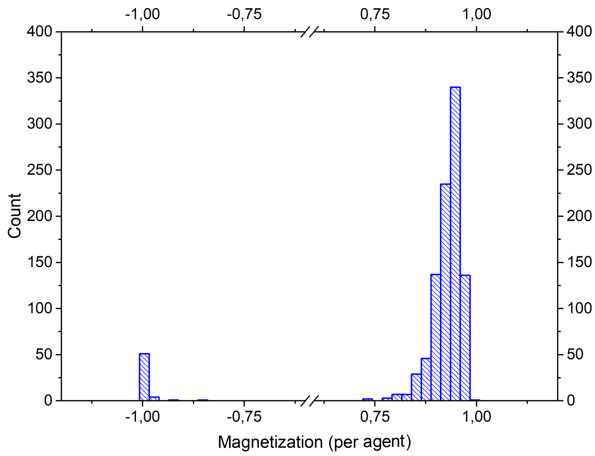
1. Single realizations and time evolution



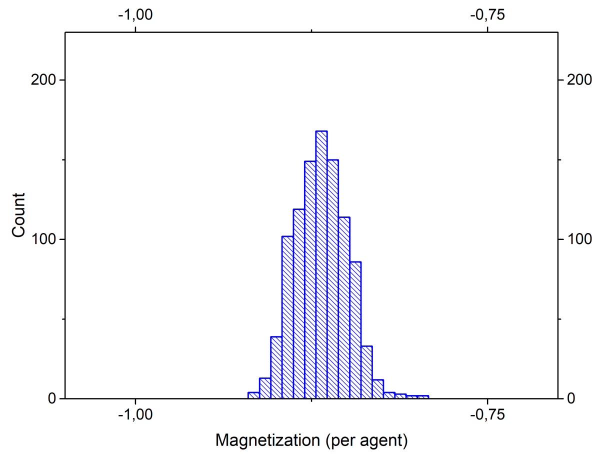
(a) t = 0 (b) t = 3 (c) t = 10 (d) t = 50

FIG. 2: The configuration of the active(white)/inactive(black) states are plotted in different time steps in a single realization, with initial probability of activation p = 0.1 for the whole lattice. In (a), the initial configuration shows a homogeneous distribution. From (a) to (b), the most common state grows in number, and the formation of small clusters of the active state can be seen from (b) to (c). Some of these clusters grow as time goes forward (d), an will lead to a global cluster of actives agents of the size of the entire lattice, at the stationary state.

In Figure 2, the lattice state configuration of a single realization for the model with the orientation dynamics is pictured at different time steps. Initially, each agent has been set to have a probability $p = 0.1$ of being active. The initial time evolution ($t = 3$ is shown in subfigure (b)) makes the homogeneity disappear, making the most common state (in this case, the inactive state) even more present. Small clusters of the minority state prevail in this case. As time goes on, these clusters can evolve in two different fashions: they either start to grow in size or remain small. The latter case corresponds to local cycles, in which the orientations and states of a vicinity of nodes causes their states to indefinitely alternate.



(a) Dynamical model



(b) Null model

FIG. 3: Histogram depicting the distribution of the stationary magnetization per agent at $p = 0.1$ for (a) Dynamical model and $p = 0.3$ for (b) Null model, for 1000 realizations. The magnetization is computed in analogy with ferromagnetic systems, summing the states over the complete lattice. There is a break in the horizontal axis in figure (a), as the stationary states are polarized between global activity or inactivity, and therefore the middle part of the histogram is empty. Comparatively, the stationary magnetization in figure (b) is distributed around a sole value.

The interesting case is the former, which leads to global activation of the entire lattice, as the only way a cluster could stop growing would be that all the immediately surrounding agents be oriented in such a way that none of the cluster's nodes are in their potential neighbourhood. This is increasingly unlikely as the cluster grows, as the orientation dynamic favours the agents to turn towards active nodes in the lattice. To establish some physical correspondence between this dynamic and the social system that models, it could be said that the action will spread globally if there is, initially, some group of people that are active and looking to each other, so that they keep being active even if their state is in clear minority in global terms. Enough mutual support makes the action spread.

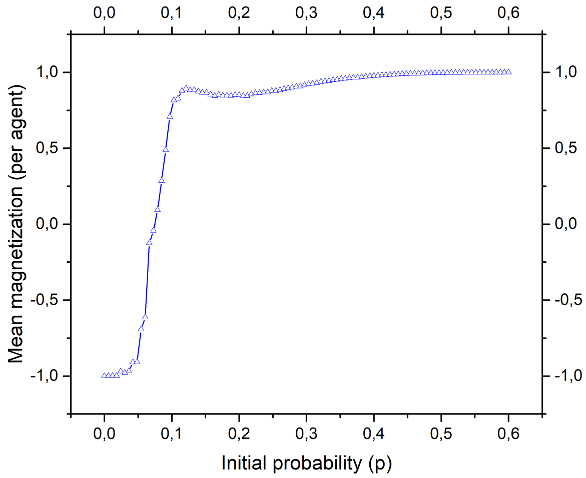
Thus, a reasonable question would be: how probable is that at least one such active nuclei forms and spreads over the entire lattice? It seems logical that this must depend, firstly, on the initial probability of being active p. For greater p, the initial density of active agents is bigger, so it is more likely that such a cluster will form.

The histogram in Figure 3a shows the distribution of the stationary magnetization per agent m of 1000 realizations at $p = 0.1$. As it turns out, the distribution is polarized between global activity or inactivity. Approximately 5% of the realizations ended with global inactivity ($\bar{m}$ close to -1) in a sharp fashion. This would mean that the probability of one or more clusters forming and growing for this conditions is around 0.95. These remaining realizations are distributed close to $m = +1$. The fact that the distribution at the globally active realizations is not sharp is due to the formation of stable inactive clusters when the system is close to the stationary state.

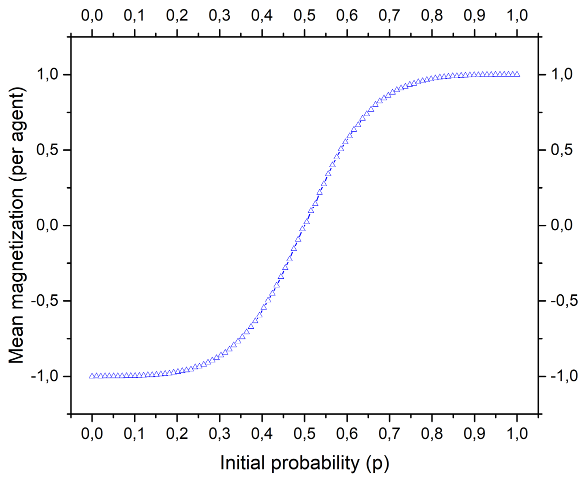
There is a huge contrast between this results and the ones obtained for the null version of the same model (see Fig.3b). If orientations are initially set at random, and do not change throughout the simulation, the system evolves so that clusters of the same state are formed. However, unlike in the oriented-dynamical case, this clusters never get to grow afterwards. The system rapidly tends to a stationary state of mixed active/inactive agents, being the initially most frequent state even more frequent at the stationary state than before. This fact can be spotted in Figure 3b, as the values are distributed around a sole value ($m = 0.88$ approximately in the case $p = 0.3$).

2. *Averaging over realizations*

In this subsection, the shape of the mean magnetization per agent as a function of the initial probability is computed using a Monte-Carlo simulation. The plot that is obtained is that of Figure 4a.



(a) Dynamical model



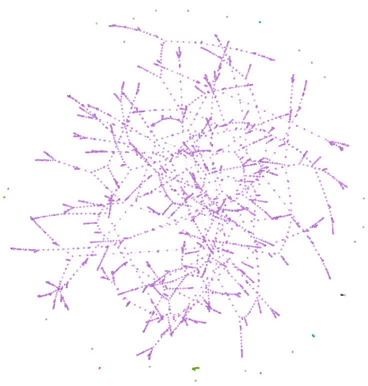
(b) Null model

FIG. 4: Mean magnetization per particle as a function of the initial probability of activation p. Each point is the mean value of 200 realizations and there are 100 points for each plot. Only values for $p \in [0, 0.6]$ were computed for plot (a) Dynamical model, as for larger values the curve is approximately a constant. Note that plot (b) belongs to the null model, in which states +1 and -1 are symmetrical.

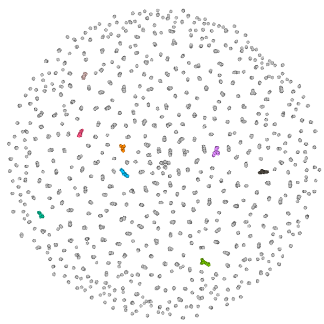
With $m = \pm 1$ being absorbing states, the cases $p = 0$ and $p = 1$ are trivial. For values in the interval (0,0.5), approximately, there is a slightly increase in the mean magnetization, mainly due to the increasing probability of an active cluster being formed in the initial time steps. There is a rapid increase in magnetization as p shifts between $p = 0.05$ and $p = 0.1$. The probability of a cluster that takes on the whole lattice becomes eventually 1 for some value slightly above $p = 0.1$.

Being this probability essentially unity, the behaviour of the system is driven mainly by the amount and size of the inactive clusters in the stationary state. The possible explanation for the relative maximum present around $p = 0.1$ is that the two mechanisms -probability that the system shifts to global activation and size of the inactive clusters in the stationary state- are both important around that range of p values.

An easy way to picture the typical outcomes for different values of p is to build the directed graph of the system (i.e. a directed edge is considered from each node to the node it is looking at). The inactive regions are composed by small connected components while the active ones are distributed between the number of initial active clusters that made the action spread (if the outcome is globally active). For relatively small values of p, such as $p = 0.1$, typically 1 or 2 giant connected components form that sum up to most of the lattice (see Fig. 5a). For greater values, many small connected components are present (Fig. 5b). A possible explanation for



(a) p=0.1



(b) p=0.6

FIG. 5: The graphs for the stationary states for two realizations, (a) p=0.1 and (b) p=0.6. In the former, there is a single giant connected component of the 98% of nodes, while in the latter, the graph appears sparse, with many small connected components. In both cases, more than 98% of the nodes are active.

the shape of figure 4a is as follows: the probability of an active nuclei forming eventually gets to be almost unity. However, the number of these nuclei keeps growing as p increases, and as a consequence, the more sparse the oriented graph is. The ratio between the inactive and active connected components changes as there are more connected components, and more initially active agents as p gets bigger.

B. Initial nucleation conditions

If a small group of people is organized to start an action, how likely is that this action will spread globally? This issue is addressed in this section by setting the following initial conditions: a small nuclei of 7 nodes -a central one and its nearest neighbours- is active initially, and all the others are inactive. This is, somehow, a way to force the nucleation process that was observed in the previous section with random initial conditions.

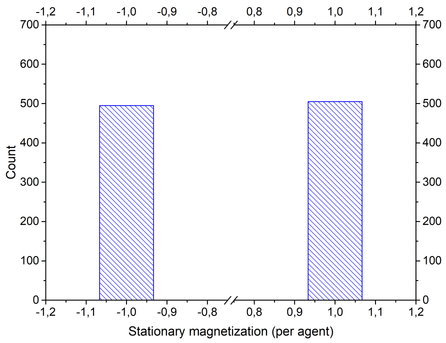


FIG. 6: Histogram depicting the stationary magnetization for the initial nucleation conditions, for a total of 1000 realizations.

Figure 6 shows the outcomes at the stationary state for 1000 realizations in these conditions. It turns out that the system reaches consensus, in either activity or inactivity with an almost one to one ratio. This points out the importance of the nucleating process, since it is to be noted that 7 agents can change the opinion state of the whole lattice (of $N = 10^4$ agents) with a probability around 50%.

IV. CONCLUSIONS

We have given some visualization tools in order to explore some of the key features of this oriented majority-vote model. The computer simulations run with random initial conditions point out the formation of active nuclei in the lattice as a key feature to yield global activity in the lattice. The probability of such active clusters being formed shapes the global statistics of the system: the number that are formed in the very first time steps determine the stationary-state magnetization.

By introducing this simple orientation dynamics, the outcomes are polarized between global activity or inactivity, while the null model allows for mixed active-inactive states. With the introduction of a small initial nucleation (7 neighbour agents active out of 10^4), the system is already driven to global activation by the orientation dynamics in a considerable number of cases ($\sim 50\%$ of the cases). Thus, spatial orientation is a possible mechanism for the emergence of global behaviour in mass events.

At this point, many further work can be visualized. From the one hand, an analytical description of this or a similar model would help to elucidate the characteristics of such a model, with the aim of describing orientation dynamics as a pathway for collective behaviour. On the other hand, the model allows for a huge diversity of modifications to explore a variety of phenomena. Spontaneously allowing for long-range interactions between agents or introducing some random noise in the states or orientations are some of the possibilities.

Acknowledgments

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[1] D. Stauffer, "A Biased Review of Sociophysics". J. Stat. Phys. **151**, 9 (2013)
[2] C. Castellano, S. Fortunato, V. Loretto, "Statistical physics of social dynamics". Reviews of Modern Physics **81**, 591-646 (2009)
[3] E. Ben-Naim, P. L. Krapivsky, S. Redner, *Fundamental Kinetic Processes*, Ch 7 "Spin Dynamics" Boston University 2008
[4] T.M. Liggett, *Interacting Particle Systems*, Springer, New York, 1985.