Variational analysis of the Gross-Neveu model in an S^1 space

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The Gross-Neveu model in an S^1 space is analyzed by means of a variational technique: the Gaussian effective potential. By making the proper connection with previous exact results at finite temperature, we show that this technique is able to describe the phase transition occurring in this model. We also make some remarks about the appropriate treatment of Grassmann variables in variational approaches.

The main aim of this paper is to point out that the variational approach known as the Gaussian effective potential[1] (GEP) is able to correctly describe phase transitions, at least in a particular well-studied theory: the Gross-Neveu (GN) model.[2] The GN model has already been analyzed in $\mathbb{R}$ space by means of the GEP in Ref. 3 where, apart from correctly reproducing the well-known features of the model in the large-N limit (chiral-symmetry breaking and mass generation), close analogies to the so-called precarious phase in the $(\lambda\phi^4)_4$ scalar theory[4] were found. Since the behavior of precarious $(\lambda\phi^4)_4$ theory has been recently studied in $\mathbb{R}^2\times S^1$ space[5] and a phase transition to a trivial theory has been found for small enough S^1 radius, we feel it is worthwhile having a check of the GEP reliability in describing phase transitions in this model which is soluble in the large-N limit. This will be achieved by making the proper connection with the exact results at finite temperature.[6] We also make some remarks about the validity of different Gaussian *Ansätze* and the appropriate treatment of Grassmann variables in the GEP approach.

The Gross-Neveu Hamiltonian reads

$$H=\int_{S^1}dx\{-\pi^a(x)\gamma^0\gamma^1\partial_1\psi^a(x)-\tfrac{1}{2}g_B^2[\pi^a(x)\gamma^0\psi^a(x)]^2\}\ ,$$
$$\{\pi_i^a(x),\psi_j^b(y)\}=i\delta^{ab}\delta_{ij}\delta(x-y),\quad a,b=1,\ldots,N\ . \tag{1}$$

We shall use as a variational *Ansatz* the wave functional in the Schrödinger picture:

$$W[\psi;\psi_0,\pi_0,\Omega]=\text{const}\times\exp\left[i\pi_{0i}^a\int_{S^1}dx\,\psi_i^a(x)\right]\times\prod_{x,a,i}[\psi_{+i}^a(x)-\psi_{0+i}^a]\ , \tag{2}$$

where $\psi_i^a(x)$, ψ_{0i}^a, $\pi_i^a(x)$, π_{0i}^a are considered the generators of a Grassmann algebra with involution $[\psi_1^{a*}(x)=-i\pi_i^a(x),\ \psi_{0i}^{a*}=-i\pi_{0i}^a]$ (Ref. 7). The + (−) means projection onto the positive- (negative-) energy subspaces of the operator $-i\gamma^0\gamma^1\partial_1+\Omega\gamma^0$, $\Omega=\Omega(\psi_0,\pi_0)$ being a variational "mass" parameter, and $\psi_{01}^a=\langle W\mid\psi_i^a(x)\mid W\rangle$, $\pi_{0i}^a=\langle W\mid\pi_i^a(x)\mid W\rangle$. On arbitrary functionals $\tilde{W}[\psi;\psi_0,\pi_0]$ a scalar product that maps

$$(\tilde{W}[\psi;\psi_0,\pi_0],\tilde{W}'[\psi;\psi_0,\pi_0])\rightarrow R_{\tilde{W}\tilde{W}'}(\psi_0,\pi_0)\sim\int\mathcal{D}\psi\,\tilde{W}^\dagger[\psi;\psi_0,\pi_0]\times\tilde{W}'[\psi;\psi_0,\pi_0]$$

can be defined;[8,9] in our cases [(2) and later (4)] $R_{WW}(\psi_0,\pi_0)\in\mathbb{R}^+$ as can be seen by changing $\psi\rightarrow\psi+\psi_0$ in the functional integral. In a way completely analogous to Ref. 3 creation and annihilation operators can be defined and the field and momentum operators can be expressed in terms of these:[10]

$$\psi_i^a(x)=\psi_{0i}^a+\frac{1}{L}\sum_{n=-\infty}^{\infty}\frac{1}{2\omega(k,\Omega)}[u_i(k,\Omega)a^a(k,\Omega)e^{ik\cdot x}+v_i(k,\Omega)b^{\dagger a}(k,\Omega)e^{-ik\cdot x}]\ ,$$
$$\pi_i^a(x)=\pi_{0i}^a+\frac{i}{L}\sum_{n=-\infty}^{\infty}\frac{1}{2\omega(k,\Omega)}[v_i^*(k,\Omega)b^a(k,\Omega)e^{ik\cdot x}+u_i^*(k,\Omega)a^{\dagger a}(k,\Omega)e^{-ik\cdot x}]\ ,$$
$$\{a^a(k,\Omega),a^{\dagger b}(k',\Omega)\}=\{b^a(k,\Omega),b^{\dagger b}(k',\Omega)\}=2\omega(k,\Omega)\delta_{nn'}L\ , \tag{3}$$
$$a^a(k,\Omega)\mid W\rangle=b^a(k,\Omega)\mid W\rangle=0,\quad W[\psi;\psi_0,\pi_0,\Omega]\equiv\langle\psi\mid W\rangle\ ,$$
$$\omega(k,\Omega)=(k^2+\Omega^2)^{1/2},\quad k=(2n+1)\pi/L\ ,$$

where $u_i(k,\Omega),v_i(k,\Omega)$ are the usual Dirac spinors; $k=(2n+1)\pi/L$ implements the physical antiperiodic boundary conditions for fermions and L is the circumference length.

To justify *Ansatz* (2) we have proceeded as follows. We started by considering the most general translational-invariant Gaussian wave functional satisfying $\langle W\mid\psi\mid W\rangle=\psi_0$, $\langle W\mid\pi\mid W\rangle=\pi_0$ which reads

$$W[\psi;\psi_0,\pi_0,\Delta]=\text{const}\times\exp\left[i\int_{S^1}dx\,\pi_{0i}^a\psi_i^a(x)+\tfrac{1}{2}\int_{S^1\times S^1}dx\,dy[\psi_i^a(x)-\psi_{0i}^a]\Delta_{ij}^{ab}(x-y)[\psi_j^b(y)-\psi_{0j}^b]\right]\ ,$$
$$\Delta_{ij}^{ab}(x-y)=-\Delta_{ji}^{ba}(y-x)\ . \tag{4}$$

We computed next the expectation value of H and observed that in the large-N limit it reduces to

$$\langle H\rangle = N\,\mathrm{tr}\left[i\gamma^0\gamma^1\partial_1\frac{1}{1+\Delta^\dagger\Delta}\right] + \frac{g_B^2}{2}N^2\left\{2\alpha\,\mathrm{tr}\left[\gamma^0\frac{1}{1+\Delta^\dagger\Delta}\right] + \frac{1}{L}\left[\mathrm{tr}\left[\gamma^0\frac{1}{1+\Delta^\dagger\Delta}\right]\right]^2\right\}, \tag{5}$$

where the Hamiltonian U(N) symmetry was exploited to trivialize the color dependence; $\alpha\equiv i(\pi_0^a\gamma^0\psi_0^a)/N$; and the trace is taken over space and Dirac indices. Minimization of (5) is rather subtle, the reason being its Dirac structure similar to that of a free fermionic theory of mass

$$g_B^2N\left[\alpha+\frac{1}{2L}\mathrm{tr}\left[\gamma^0\frac{1}{1+\Delta^\dagger\Delta}\right]\right],$$

where we know that the minimization of $\langle H\rangle$ leads Δ to two different end points in the two different positive- and negative-energy subspaces.[8,9] This suggests computing the traces of (5) in suitable variational positive- and negative-energy subspaces with respect to the operator $-\gamma^0\gamma^1k^1+\tilde{m}(k)\gamma^0$ in momentum space [$\tilde{m}(k)$ is a variational function] while considering Δ proportional to the identity in each subspace. If we neglect the Grassmann character of α and treat it as a real number, we find after functional minimization that $\Delta_+\to\infty$, $\Delta_-\to 0$, $\tilde{m}(k)=\Omega$ (k independent), the final result being identical to that found by starting from (2): namely,

$$\langle H\rangle/N = 2[-I_1^L(\Omega)+\Omega^2 I_0^L(\Omega)] + \frac{\lambda_B}{2}[\alpha^2-4\Omega I_0^L(\Omega)\alpha+4\Omega^2 I_0^{L\,2}(\Omega)],$$
$$\lambda_B\equiv g_B^2N,\quad I_n^L(\Omega)\equiv\frac{1}{L}\sum_{n=-\infty}^{\infty}\frac{1}{2\omega(k,\Omega)}[\omega^2(k,\Omega)]^n. \tag{6}$$

Taking into account the Grassmann character of $\tilde{m}$ and Δ (both even functions of ψ_0,π_0 to be found by variational analysis) brings not only technical problems but also conceptual ones. One way to proceed would be expanding $\langle H\rangle$, $\tilde{m}$, and Δ in terms of the relevant subset of the Grassmann algebra $\{\alpha^n\}$, $n=0,\ldots,2N$ in (5), i.e.,

$$\langle H\rangle=\sum_{n=0}^{2N}\langle H\rangle_n\alpha^n,\quad \tilde{m}=\sum_{n=0}^{2N}\tilde{m}_n\alpha^n,$$

and

$$\Delta=\sum_{n=0}^{2N}\Delta_n\alpha^n,$$

and minimizing each term $\langle H\rangle_n$ with respect all m_n and Δ_n. However, since α^n do not have a definite sign for $n\geq 1$ due to their Grassmann nature, minimization is only meaningful for $\langle H\rangle_0$. For $\langle H\rangle_n$, $n\geq 1$ we will follow instead the stationarity principle, i.e., $\delta\langle H\rangle_n/\delta\tilde{m}_k=\delta\langle H\rangle_n/\delta\Delta_k=0$, $n=1,\ldots,2N$, $k=0,\ldots,2N$ (in contrast with $\min_{\{\tilde{m}_k,\Delta_k\}}\langle H\rangle_n$). In the general case stationarity of $\langle H\rangle$ under the restrictions $\langle\psi\rangle=\psi_0$, $\langle\pi\rangle=\pi_0$ amounts to saying that we are looking for approximate eigenfunctionals of the Hamiltonian in the presence of external sources,[11] while, in the non-Grassmann case, minimization selects the eigenfunctional that best approaches the ground state in the presence of those sources. Here we expect minimization of $\langle H\rangle_0$ to accomplish just this. We have not attempted to fully derive (6) from (5) following the above guidelines, but have instead limited ourselves to verify that they lead to (6) up to order α^2 which is actually the only thing that we will need below.

One should be aware, however, that for finite N the expression corresponding to (5) is much more involved so the simpler *Ansatz* (2) is not expected to give sensible results. In fact, we have checked that for a fixed-cutoff theory (2) reproduces the Callan-Symanzik β function at one-loop level only if $N\to\infty$, while perfect agreement for all N is found if one uses the general Gaussian *Ansatz* (4).

From now on let us focus on (6). The divergences appearing there are regularized by changing

$$\sum_{n=-\infty}^{\infty}\to\sum_{n=-\infty}^{\infty}e^{-k^2/\Lambda^2},$$

where Λ is an ultraviolet cutoff. Each $I_n^L(\Omega)$ can be separated in an L-independent part [$I_n(\Omega)$] that contains all the divergences and an L-dependent finite part [$\delta_n^L(\Omega)$], $I_n^L(\Omega)=I_n(\Omega)+\delta_n^L(\Omega)$ (Ref. 5). The effective potential itself can also be separated in an L-independent part which is that found in Ref. 3 and a finite (when Λ is removed) L-dependent part

$$V_G(\alpha,\Omega)\equiv\langle H\rangle/N=V_G^\infty(\alpha,\Omega)+V_G^L(\alpha,\Omega),$$
$$V_G^\infty(\alpha,\Omega)=\Omega\alpha+\frac{\Omega^2}{4\pi}\left[\ln\frac{\Omega^2}{m^2}-1\right], \tag{7}$$
$$V_G^L(\alpha,\Omega)=-2\delta_1^L(\Omega),$$

where renormalization has been carried out in V_G^{∞}[3]; and m^2 is the dynamically generated fermion mass at $L=\infty$. Formulas for $\delta_n^L(\Omega)$ can be found in Appendix A. By expanding Ω in terms of the subset $\{\alpha^n\}$ ($\Omega=\sum_{n=0}^{2N}\Omega_n\alpha^n$), minimizing the α-independent part of V_G, and taking the stationary value of the α-dependent part, one can see that there is a unique nontrivial ($V_G\neq$const) solution which leads to[12]

$$V_G(\alpha)\equiv V_G(\alpha,\Omega(\alpha))=E_C+m_L\alpha+\tfrac{1}{2}\lambda_L\alpha^2+\cdots, \tag{8}$$

where

$$E_C=\frac{m^2}{4\pi}+\frac{m_L^2}{4\pi}\left[\ln\frac{m_L^2}{m^2}-1\right]-2\delta_1^L(m_L),$$
$$\lambda_L=-1/[1/\pi+2m_L^2\delta_{-1}^L(m_l)], \tag{9}$$

and m_L given by the solution of

$$\frac{1}{4\pi}\ln\frac{m_L^2}{m^2}-\delta_0^L(m_L)=0, \tag{10}$$

$$\frac{1}{4\pi}\frac{1}{m_L^2}+\tfrac{1}{2}\delta_{-1}^L(m_L)>0. \tag{11}$$

$V_G(\alpha)$ has been normalized in such a way that E_C is the

Casimir energy, i.e., subtracting $V_G^\infty(0)$. Notice that λ_L is a negative increasing function of L [from (A4)], m_L^2 is also an increasing function of L [from (11) and (A3)], and E_C is negative and lower than that corresponding to a free fermion of mass m [making a Taylor expansion of E_C to first order and using (A3), $m_L^2 \le m^2$]. Thus, when the circumference shrinks the particles become less massive and the interaction stronger. Equation (10), however, cannot always be satisfied. When $L < L_C = m^{-1}\pi e^{-\gamma}$ (γ=Euler number), (10) is always positive so the minimum of E_C is given by the end point $m_L^2=0$, meaning that the system undergoes a phase transition at $L=L_C$. This can easily be seen from the equivalent expression for (10) [from (A5) and (A6)]:

$$\frac{1}{4\pi}\ln(1/m^2L^2)-\gamma/2\pi+\frac{1}{2\pi}\ln\pi+\sum_{n=0}^{\infty}\left[\frac{1}{\{(m_LL)^2+[(2n+1)\pi]^2\}^{1/2}}-\frac{1}{(2n+1)\pi}\right]=0\,. \tag{12}$$

Since for $L=L_C$, $m_L^2=0$ is a solution of (12) the phase transition is continuous (second order). Furthermore, starting from (7) again for $L \le L_C$ one finds that V_G reduces to a constant. We are then left with a theory of free massless fermions.

Let us return to the case $L > L_C$ and ask ourselves which $\{\alpha\}$ configuration in (8) corresponds to the vacuum. To answer it we apply the stationary principle equating to zero the variation of (8) with respect to ψ_{0i}^a, π_{0i}^a. Since

$$\left\{\frac{\delta}{\delta\psi_{0i}^a}\alpha^n\right\}\cup\left\{\frac{\delta}{\delta\pi_{0i}^a}\alpha^m\right\},\quad n,m=1,\ldots,2N$$

form a subset of the Grassmann algebra, the solution cannot be anything but $\psi_{0i}^a=\pi_{0i}^a=0$. (In Appendix B we show from a more fundamental point of view that ψ_{0i}^a and π_{0i}^a must vanish in the vacuum if no external sources are present.) In the $L=\infty$ case this means that, in our approach, formal Lorentz invariance (absence of mean fields) is essentially recovered because of the Grassmann character of the fermionic fields. We have also explicitly checked that our vacuum functional (2) is annihilated by the generators of the "hidden" O(2N) symmetry of (1) (Ref. 13) as well as by those of the explicit U(N) symmetry including the normal-ordered fermionic number charge, which does not commute with all the O(2N) generators.[13]

The connection between the results in the S^1 space and the finite-temperature ones is not direct in the Schrödinger picture. However, since the effective potential computed in the manner of Symanzik turns out to be equivalent to the usual one computed through the path integral method[11,14] and, considering that in this last formulation computing at finite temperature is completely equivalent to computing with one spatial dimension compactified (in the Euclidean space), our approach in the S^1 space must be equivalent to some approximation for finite temperature in the $\mathbb{R}$ space by just identifying T (temperature) and L^{-1}. In order to make thorough contact with the existing results, we first introduce the composite operator effective potential in terms of $\sigma \equiv -i\lambda_B\langle:\pi\gamma^0\psi:\rangle=\Omega$ (Ref. 15) by taking the stationary value of (7) with respect to ψ_{0i}^a,π_{0i}^a keeping σ (Ω_n actually) fixed. This amounts to dropping α in (7) and considering $\Omega=\sigma$ as a real number. Since V_G^∞ in (7) has already been proved to coincide with the exact result[15] we only have to prove that V_G^L also does. This is easily achieved by performing the variable change $-L(x^2+\Omega^2)=-\Omega^2t-L^2/4t$ in (A2) which leads to

$$V_G^L=-\frac{2}{\pi L}\int_0^\infty dx\,\ln(1+e^{-L(x^2+\Omega^2)})\qquad(\Omega=\sigma,\ L=T^{-1})\,. \tag{13}$$

in accordance with Ref. 6. This shows that the GEP approach is able to properly describe phase transitions, at least in this soluble model.

Finally, let us comment on the analogy between this model and the precarious phase in $\lambda\phi_4^4$ theory first noticed in Ref. 3. Precarious $\lambda\phi_4^4$ theory in $\mathbb{R}^2\times S^1$ space has been studied in Ref. 5. We find here the same qualitative behavior for λ_L, m_L, and E_C in L, and a phase transition to a noninteracting theory that also exists there, though the order of the transition differs. The presented results give us further confidence that the triviality-restoring phase transition in precarious $\lambda\phi_4^4$ really exists.

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APPENDIX A

In order to find the expressions for $\delta_n^L(\Omega)$ we use

$$\sum_{n=-\infty}^{\infty} f(2n+1)=\sum_{n=-\infty}^{\infty} f(n)-\sum_{n=-\infty}^{\infty} f(2n)$$

in the regulated $I_n^L(\Omega)$ (6) and follow the steps of Ref. 5 for each sum. We obtain

$$\delta_1^L(\Omega)=-\frac{1}{4\pi}\int_0^\infty dt\,t^{-2}e^{-\Omega^2 t}\times\sum_{n=1}^{\infty}(2e^{-L^2n^2/t}-e^{-L^2n^2/4t})\,,$$
$$\delta_n^L(\Omega)=\frac{1}{n+\frac{1}{2}}\frac{d}{d\Omega^2}\delta_{n+1}^L(\Omega),\quad n<1\,. \tag{A1}$$

The summation in (A1) can be carried out giving

$$\delta_1^L(\Omega)=\frac{1}{4\pi}\int_0^\infty dt\, t^{-2}\ln(1+e^{-\Omega^2 t-L^2/4t}) , \tag{A2}$$

$$\delta_0^L(\Omega)=-\frac{1}{2\pi}\int_0^\infty dt\, t^{-1}(e^{\Omega^2 t+L^2/4t}+1)^{-1} , \tag{A3}$$

$$\delta_{-1}^L(\Omega)=-\frac{1}{4\pi\Omega^2}\int_0^\infty dt \cosh^{-2}(t/2+L^2\Omega^2/8t) , \tag{A4}$$

where the required properties are explicit. We need a further formula to prove that the phase transition is continuous. From (A1) we have

$$\delta_0^L(\Omega)=2\pi[2\delta_{-1}(\Omega,2L)-\delta_{-1}(\Omega,L)] , \tag{A5}$$

where $\delta_{-1}(\Omega,L)$ is defined in the Appendix of Ref. 5 and shown to admit the representation[16]

$$\delta_{-1}(\Omega,L)=\frac{1}{2\pi^2}\left[\frac{\gamma}{2}+\frac{1}{2}\ln\frac{\Omega L}{4\pi}+\frac{\pi}{2\Omega L}+\pi\sum_{n=1}^{\infty}\{[(\Omega L)^2+(2\pi n)^2]^{1/2}-(2\pi n)^{-1}\}\right] \tag{A6}$$

($\gamma=$Euler number).

APPENDIX B

In this appendix we formally prove that the expectation values of the field and momentum (ψ_0 and π_0) vanish in any stationary state of a given fermionic Hamiltonian in the absence of external sources. The question we address might seem like nonsense since ψ_0 and π_0 are usually taken to be zero by invoking Lorentz invariance or fermionic number conservation. However, in our case, Lorentz invariance is lost for $L\neq\infty$ due to the S^1 space topology, and the fermion number charge does not commute with some generators of the O(2N) symmetry so we cannot assure that it belongs to the complete set of compatible observables [actually the fermionic charge is given up in favor of the O(2N) generators in Ref. 13]. Since continuous symmetries cannot be spontaneously broken in two dimensions[17] one could use the O(2N) symmetry of the vacuum to set ψ_0 and π_0 equal to zero; however, it seems unnatural to us requiring such a heavy external result to achieve this goal. Let us proceed then with our proof. We borrow from Ref. 11 that the problem of stationarizing $\langle W|H|W\rangle$ with fixed $\psi_0=\langle W|\psi|W\rangle$, $\pi_0=\langle W|\pi|W\rangle$ is equivalent to solving the Schrödinger equation for the Hamiltonian with external sources:

$$(H-\eta^*\psi-\pi\eta)|W\rangle=E(\eta^*,\eta)|W\rangle . \tag{B1}$$

From here one easily deduces[11]

$$\psi_0=-\frac{\delta E}{\delta\eta^*},\quad \pi_0=\frac{\delta E}{\delta\eta} . \tag{B2}$$

Now from (B1) we see that E must be an even function of the Grassmann sources η^*,η. Then the right-hand sides of (B2) are odd Grassmann functions of η^*,η, so they vanish when η^*,η vanish. If we switch off the sources in (B1), $|W\rangle$ becomes a stationary state of the original Hamiltonian which from the previous argument satisfies $\langle W|\psi|W\rangle=\langle W|\pi|W\rangle=0$. Notice that this formal proof is based entirely on the Grassmann nature of the sources and not on any symmetry of the theory.

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[8]See T. Barnes and G. I. Ghandour, Nucl. Phys. **B146**, 483 (1978) and Ref. 1 for the $\psi_{0i}^a=\pi_{0i}^a=0$ case.

[9]Writing $W^\dagger[\psi;\psi_0,\pi_0]=\int\mathcal{D}\pi' e^{-i\pi'\psi}(W[\psi;\psi_0,\pi_0])^*$ the scalar product takes the standard form of Ref. 7 for the $\psi_{0i}^a=\pi_{0i}^a=0$ case.

[10]In Ref. 3 the possible phase occurring in (2) was overlooked. Then, in the formula corresponding to (3) only the positive-energy projections of ψ_{0i}^a and π_{0i}^a appeared.

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[12]Here we take into account the full Grassmann structure of V_G, in contradistinction to Ref. 3 where α was approximated by a real positive number. Though the philosophy is quite different there, the results concerning the usual GN phase remain the same.

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