Anomaly cancellation at finite cutoff

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We describe a perturbative framework in which finite cutoff (Λ) effects can be taken into account. Essentially it consists of keeping terms of $O(\Lambda^{-2})$ in the usual perturbation theory once the complete set of dimension-six operators have been included in the Lagrangian with coupling constants proportional to Λ^{-2}. This is motivated by Wilson renormalization-group arguments. The occurrence of local gauge anomalies is analyzed within this framework. It is proven that no genuine contribution to the anomaly arises at $O(\Lambda^{-2})$. The discussion is completely general though special attention is paid to the standard model.

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I. INTRODUCTION

The understanding of local gauge anomalies [1] in chiral gauge theories on the one hand (see [2] for a review), and of the triviality problem [3] in scalar (nonasymptotically free) theories on the other (see [4] for a review) have been quoted as two major events in theoretical physics during the last decade [5]. Both issues, although unrelated at first sight,[1] turn out to be relevant for the standard model (SM). The local gauge anomaly vanishes in the SM because the sum over hypercharges of the leptons and quarks equals zero for each generation, avoiding the inconsistencies that otherwise would appear [7]. On the other hand, the scalar-gauge-boson sector, which is crucial for providing masses to the gauge bosons, suffers from the triviality problem; i.e., it becomes noninteracting if the regulating cutoff (Λ) is exactly (nonperturbatively) removed [8]. In nature interactions are expected to exist. If this is so the SM must be considered as an effective "low-energy" theory with a finite built-in cutoff. This fact has physical consequences. For instance the Higgs-boson mass can be related to the built-in cutoff by using nonperturbative renormalization group techniques so that an upper bound for the former arises [9]. Since standard perturbation theory (PT) is a fixed cutoff formalism in itself where the cutoff is removed up to $O(\Lambda^{-2})$, the triviality problem does not alter the usual perturbative calculations. Yet when probing the SM at high enough energies effects due to the built-in cutoff are likely to appear and therefore the standard perturbation theory will have to be modified to accommodate these effects.

In this paper we outline a perturbative framework which incorporates the leading effects of a finite built-in cutoff in a regularization-independent way. Although it is motivated by Wilson renormalization-group arguments, the practical implementation is entirely based upon standard PT. We then analyze the occurrence of gauge anomalies at $O(\Lambda^{-2})$. Anomaly cancellation at $O(\Lambda^{-2})$ might in principle put constraints on the couplings of the dimension-six operators, and hence on the physics beyond the SM. We shall prove that this is not the case: no genuine contributions to the anomaly arise at $O(\Lambda^{-2})$.

In Sec. II we describe the above-mentioned framework which essentially consists of adding to the Lagrangian dimension-six ("irrelevant") operators with couplings proportional to Λ^{-2} and consistently keeping terms $O(\Lambda^{-2})$ in a standard perturbative calculation. In Sec. III we explain why local gauge anomalies must also cancel in this framework and show, under certain assumptions, that no genuine anomalies arise at $O(\Lambda^{-2})$. In Sec. IV the formalism is pinned down for the fermionic sector of a $R_L(G)\times R_R(G)$ theory where $R_{L,R}$ combine into an anomaly-free [at $O(\Lambda^0)$] unitary representation of the gauge group G, so that the assumptions made in Sec. III can be proven. In Sec. V we show that the fermionic sector of an N-generation SM can be accommodated to the form employed in Sec. IV, and hence the results obtained there also hold for the SM. We close with a brief discussion in Sec. VI. For completeness we display the results of a calculation of the covariant anomaly at $O(\Lambda^{-2})$ in the Appendix.

II. CUTOFF-DEPENDENT PERTURBATION THEORY

Perturbation theory consists of a well-defined set of rules which allow us to calculate physical amplitudes in a local quantum field theory. It requires the introduction of a regularization, which possesses a characteristic momentum scale Λ (the cutoff), in order to smooth out the short-distance singularities. Any physical amplitude is expanded until $O(\Lambda^0)$. A nontrivial statement of perturbative renormalization of local quantum field theories is that all the divergent pieces (when $\Lambda\rightarrow\infty$) of all physi-

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[1]The only discussions we know of on the interrelation between anomalies and triviality are due to Bég [6].

cal amplitudes can be absorbed by adding local counterterms to the original Lagrangian [10]. If the theory is power-counting renormalizable this amounts to redefinitions of the (bare) parameters existing in the original Lagrangian only. This implies that the formally unitary theory is actually unitary after renormalization. The validity of perturbative renormalization is constrained by conditions of the type

$$\lambda_R \ln\left[\frac{\Lambda}{\mu}\right] \ll 1, \quad \frac{\mu}{\Lambda} \ll 1 , \tag{2.1}$$

where λ_R is the renormalized coupling constant, say in a scalar $\lambda\Phi^4$ theory, and μ the subtraction point typically chosen at the scale where the experiments are carried out. The second inequality reflects the fact that one systematically neglects terms $O(\Lambda^{-2})$ and implies that Λ must be large compared to physical scales μ. The first inequality is needed when perturbatively expanding the bare parameters in terms of the renormalized ones and implies that Λ cannot be too large. The fact that the first inequality has a logarithmic dependence on Λ and the second a quadratic one allows in many cases a wide range of Λ's in which perturbative renormalization is valid.

We are going to be concerned with the case where Λ is relatively small so that the first inequality holds with no problem but corrections $O(\Lambda^{-2})$ must be taken into account. In order to do so we introduce two modifications to the standard PT: (i) we keep terms until $O(\Lambda^{-2})$ (included) in all our expressions, and (ii) we enlarge our original Lagrangian by adding the complete set of dimension-six ("irrelevant") operators with (bare) coupling constants proportional to Λ^{-2} but otherwise arbitrary. The latter point is crucial for unitarity since otherwise our results would strongly depend on the chosen regularization spoiling the formal unitarity of the theory. The arbitrariness of the (renormalized) coupling constants of the dimension-six operators reflects the fact that we need more parameters to probe our theory beyond the regime it was originally designed for. It is common to well-established low-energy approaches (see, for instance, [11]).

Although the rules above have been given in a purely perturbative framework, they are best motivated by a Wilson renormalization-group point of view (see [12] for a review), which in addition clarifies the regularization independence of the procedure. Suppose that our physical amplitudes at the scales μ are ultimately calculable from a fundamental well-defined theory which admits a path-integral representation. Suppose next that the fundamental degrees of freedom can be separated according to momentum scales so that one can integrate out the high energy degrees of freedom until an intermediate scale Λ. For physical amplitudes at the scale $\mu \ll \Lambda$ it is reasonable to assume that the details of the fundamental theory at scales beyond Λ do not matter. Then a local expansion of the (highly nonlocal) effective action up to $O(\Lambda^0)$ (i.e., keeping only power-counting renormalizable terms) should be enough to describe the physics occurring at the scale μ. The same should be true if we change Λ by $\Lambda+\delta\Lambda$ and hence small changes in the scale Λ (or the regulator) must be compensated for by the corresponding change of the bare parameters, up to $O(\Lambda^{-2})$ terms which have already been neglected in the local expansion of the effective action. This corresponds to nothing but to the hypothesis of universality [13] which was proved within PT in [14]. In our case we still want to make experiments at the scale μ but we also want to take into account that an intermediate scale Λ exists. The natural way to do so is by just including the leading so-called irrelevant operators in the local expansion of the effective action [i.e., we stop at $O(\Lambda^{-2})$ instead of at $O(\Lambda^0)$]. These would be dimension-six operators with couplings proportional to Λ^{-2} but otherwise arbitrary. By naive generalization of the argument above to $O(\Lambda^{-2})$, any small change of the intermediate scale $\Lambda \to \Lambda+\delta\Lambda$ must be compensated for by the corresponding change of the bare parameters and the coupling constants of the dimension-six operators, up to $O(\Lambda^{-4})$.

In order to describe the main features of this framework concerning Feynman diagrams let us take the simplest $\lambda\Phi^4$ theory as example. There are three dimension-six operators to be added to the Lagrangian:

$$\frac{1}{\Lambda^2} Z_0 \partial_\mu \partial^\mu \Phi \partial_\nu \partial^\nu \Phi, \quad \frac{1}{\Lambda^2} Z_1 \partial_\mu \Phi \partial^\mu \Phi \Phi^2 , \quad \frac{1}{\Lambda^2} Z_2 \Phi^6 . \tag{2.2}$$

$Z_i = Z_i(\Lambda/\mu)$ are bare dimensionless coupling constants which are assumed to admit an expansion in powers of $\hbar$:

$$Z_i(\Lambda/\mu) = Z_{i0} + \hbar Z_{i1}(\Lambda/\mu) + \cdots . \tag{2.3}$$

Let us restrict our discussion to one loop and demonstrate that the $Z_{i1}(\Lambda/\mu)$ are enough to compensate the regularization dependence at $O(\Lambda^{-2})$ in any diagram. Let us further take $Z_{00}=0$ which will be justified later on. Consider first the two-point function. The tadpole diagram gives rise to just mass renormalizations even at $O(\Lambda^{-2})$. (One may consider a sharp cutoff regularization for simplicity.) From the dimension-six operators only the second in (2.2) contributes but it only gives rise to mass and wave-function renormalizations (there are no nonlocal contributions). Consider next the four-point function. The contribution with no dimension-six operator vertex at $O(\Lambda^{-2})$ is regularization dependent but local and amounts to a Z_1 renormalization. The local contributions to the diagram with one vertex containing the second operator in (2.2) correspond to wave function and Z_1 renormalizations. The nonlocal contribution is regularization independent because the only cutoff dependence is a global factor Λ^{-2} coming from the coupling of the dimension-six operator. The diagram with one vertex containing the third operator in (2.2) gives rise to a coupling-constant renormalization only. Consider finally the six-point function. The contribution with no dimension-six operator at $O(\Lambda^{-2})$ is regularization dependent but local and amounts to a Z_2 renormalization. The local contributions to diagrams with one vertex containing the second or third operators in (2.2) correspond to Z_2 renormalizations whereas the nonlocal contributions are regularization independent for the same reasons as in the four-point function. For the n-point function, $n \geq 8$, the contribution with no dimension-six operator at $O(\Lambda^{-2})$ is zero whereas nonlocal regularization-independent contributions arise due to the dimension-six operators similarly to the $n=6$ case. Diagrams with more than one vertex containing dimension-six operators amount to just renormalizations [the nonlocal parts are $O(\Lambda^{-4})$] and can be consistently neglected. Notice that the fact that the nonlocal contributions of the dimension-six operators are $O(\Lambda^{-2})$ is consistent with the "irrelevance" of these operators in usual ($O(\Lambda^0)$) PT.

Although the first operator in (2.2) has a leading ultraviolet behavior, it should not be included in the propagator, since otherwise the latter would contain unphysical poles. This is overcome by choosing $Z_{00}=0$ as a renormalization condition. (At one loop one may also choose $Z_{01}=0$ since no diagram requires Z_0 renormalizations.) Then this operator is treated as a perturbation $O(\hbar)$. The choice $Z_{00}=0$ may be justified as follows. If we have an intrinsic scale Λ we can construct more than one field with a given dimension and quantum numbers, for instance Φ and $(1/\Lambda^2)\partial_\mu\partial^\mu\Phi$. A natural way to single out our basic field is by requiring that it must satisfy the Klein-Gordon equation when the interactions are switched off. It is not difficult to convince oneself that such a field can always be obtained perturbatively in Λ^{-2} by a formal change of variables in the path integral.

If we were interested in higher energies (still small in comparison with the cutoff) we could extend this procedure to higher powers of the inverse cutoff. However, there is a big lose of predictivity each time we need to take into account an extra power of the inverse cutoff. In fact if we wanted to take into account all inverse cutoff powers we would need and infinite number of (renormalized) coupling constants, the theory would become nonlocal, and the physics described by that theory (if it existed at all) would probably be unrelated to the one of the power-counting Lagrangian. This reflects nothing but the fact that in order to describe physics at the cutoff scale we need a new more fundamental theory. Then the procedure above must be understood as an improvement of PT for energies closer to the cutoff but still small in comparison with it. We would like to emphasize that we only remove the regularization dependence of our theory. The cutoff (scale) dependence is still there. In this way, even if we have a built-in cutoff we can preserve unitarity at a given order of inverse cutoff power. The price we pay is the introduction of more (renormalized) coupling constants.

The framework above is somewhat similar to considering a nonrenormalizable theory, i.e., adding dimension-six operators with dimensionful arbitrary coupling constants. This is so, for instance, as far as the number of free parameters is concerned. However, in a nonrenormalizable framework loop corrections require the introduction of higher and higher dimensional operators. In order to retain predictive power a range of momenta must be found such that higher orders are smaller and smaller (see [11]). In our framework the diagrams that would require the introduction of higher-order counterterms (operators) are suppressed by powers of Λ^{-2}. As discussed above, the divergences introduced can be accounted for by the usual (dimension ≤ 4) bare parameters, whereas local terms containing $\ln\Lambda/\Lambda^2$ or similar can be disposed of by renormalizations of the couplings of the dimension-six operators. We believe that the framework above is essentially equivalent to a suitable nonrenormalizable setting. It just explicitly displays the "small" expansion parameter which is identified with Λ^{-2}. Since the nonlocal contributions to a given diagram involving a dimension-six operator are proportional to Λ^{-2}, they must be proportional to the square of the external momenta as well, which reinforces the view of this framework as a "low-energy" expansion.

III. LOCAL GAUGE ANOMALIES AT FINITE CUTOFF

The fact that local gauge anomalies [1] may spoil the consistency of theories with chiral couplings was first discussed within the framework of perturbatively renormalizable quantum field theories (QFT's) [7]. It was realized by 't Hooft that the possible existence of local gauge anomalies has implications beyond perturbation theory [15]. His "anomaly matching" conditions [15] (see also [16]) have played a major role in theoretical physics since, the most celebrated example in the last years being the cancellation of local gauge anomalies for certain groups in the 10-dimensional "low"-energy theory of superstrings [17]. In view of this, it seems to need little justification to require anomaly cancellation in the framework outlined in the previous section. Yet we would like to briefly comment on it. Having local anomalies means that the number of degrees of freedom changes due to quantum corrections [18]. This is clearly unacceptable, at least within perturbation theory: one loop effects are supposed to be small corrections to tree-level calculations, but in case of having a local anomaly they show up new degrees of freedom. (Clearly a new degree of freedom cannot be regarded as a small correction.) The same philosophy applies for finite cutoff corrections. Suppose that we have a theory which has no local anomalies in the usual ($O(\Lambda^0)$) PT, i.e., if one neglects $O(\Lambda^{-2})$ terms. Suppose next that we calculate $O(\Lambda^{-2})$ corrections according to the framework of Sec. II. If the local anomaly did not cancel, we would find that when probing the theory at scales closer to Λ it suddenly describes extra degrees of freedom, which is not acceptable for the same reasons as before. As a consequence anomaly cancellation at $O(\Lambda^{-2})$ is a physical requirement which may put constraints on the couplings of the dimension-six operators.

Chiral anomalies are known to be given entirely by one loop contributions in the usual ($O(\Lambda^0)$) PT [19]. They remain a purely one-loop effect even at $O(\Lambda^{-2})$ as we argue next. Chiral anomalies are better discussed in the framework of the effective action obtained after integrating out the fermions. Usually the fermionic fields appear as bilinears and the effective action can be obtained exactly. This is not the case at $O(\Lambda^{-2})$ since among the dimension-six operators there are four-fermion interactions. Nevertheless, we can always transform the four-fermion operators into bilinears by introducing auxiliary fields (see next section). The propagators can always be regularized in a gauge-covariant way by using ϵ regularization in a Schwinger proper-time representation, for instance, even for the fermionic fields [20]. This is not so for the determinants of Dirac operators containing chiral couplings, the essential reason being that they are not Hermitian in Euclidean space [21]. Therefore the anomaly is going to remain a one-loop effect caused by the determinants of Dirac operators only, at least at $O(\Lambda^{-2})$.

As mentioned above the propagator of a Dirac operator D, which generically transforms as $\delta_u D = uD - D\hat{u}$ under the action of the gauge group, can always be regularized in a gauge-covariant way. In order to regularize the determinant on the same footing we can regularize its formal variation

$$\delta\,\mathrm{tr}\ln D = \mathrm{tr}\delta D D^{-1} \to L_\epsilon(\delta D, D) = \mathrm{tr}\delta D D_{\mathrm{reg}}^{-1} ,$$
$$D_{\mathrm{reg}}^{-1} = \int_\epsilon^\infty d\lambda\, D^\dagger \exp(-\lambda D D^\dagger) \tag{3.1}$$

($\epsilon = \Lambda^{-2}$) and integrate afterwards. The right-hand side (RHS) of (3.1) is a functional of the external fields and their variations. If it can be written as a total variation of some functional, i.e., if it is integrable, the effective action can be identified with this functional, which by construction respects all the symmetries of the classical Lagrangian (except for scale invariance). If the RHS of (3.1) is not integrable, it signals a possible obstruction to implementing all the classical symmetries in the effective action. In the last case, a local integrating functional, which is unique up to total variations (local counterterms), can always be added to (3.1), and hence an effective action defined [20]. The integrating functional takes the form

$$\begin{aligned} l_c(\delta D, D) &= -\int_0^1 dt\, C_\epsilon(\delta D(t), \dot{D}(t), D(t)) ,\\ C_\epsilon(\delta_1 D, \delta_2 D, D) &= \delta_2 L_\epsilon(\delta_1, D, D) - \delta_1 L_\epsilon(\delta_2 D, D) \\ &= \int_0^\epsilon d\lambda\, \mathrm{tr}[\delta_2 D \exp(-\lambda D^\dagger D)\delta_1 D^\dagger \\ &\qquad \times \exp(-(\epsilon-\lambda) D D^\dagger) \\ &\qquad -(1\leftrightarrow 2)] . \end{aligned} \tag{3.2}$$

$l_\epsilon(\delta, D, D)$ was shown in [20] to be proportional to the $\epsilon^{\mu\nu\rho\sigma}$ at $O(\Lambda^0)$. This is also so at $O(\Lambda^{-2})$ as we shall see in next section. Then the effective action can always be written as [22]

$$\begin{aligned} \Gamma_{\mathrm{eff}} &= \int_0^1 dt\, L_\epsilon(\dot{D}, D) + l_\epsilon(\dot{D}, D) + \text{local counterterms} \\ &= \eta_{\mathrm{inv}}^\epsilon + Q_5^\epsilon ,\\ \eta_{\mathrm{inv}}^\epsilon &= \int_0^1 dt\, L_\epsilon(d_t D(t), D(t)) ,\\ Q_5^\epsilon &= \int_0^1 dt [A_{\mathrm{cov}}^\epsilon(A_t, D) + l_\epsilon(\dot{D}, D)] ,\\ A_{\mathrm{cov}}^\epsilon(u, D) &= \mathrm{tr}\delta_u D D_{\mathrm{reg}}^{-1} \\ &= \mathrm{tr}[u \exp(-\epsilon D D^\dagger) - \hat{u} \exp(-\epsilon D^\dagger D)] , \end{aligned} \tag{3.3}$$

where δ_u means gauge variation, d_t is a covariant derivative,

$$\begin{aligned} d_t \cdot &= \frac{d}{dt} \cdot + A_t(t)\cdot - \cdot \hat{A}_t(t) ,\\ A_t(t) &\to A_t^g(t) = g(t) A_t(t) g(t)^\dagger + g(t)\partial_t g(t)^\dagger , \end{aligned} \tag{3.4}$$

and $A_t(t)$ has been added and subtracted for convenience. We have also introduced an interpolation $A_\mu(t)$, $g(t)$ such that $A_\mu(0)=0$, $g(0)=1$ and $A_\mu(1) = A_\mu$, $g(1)=g$ [22]. From (3.3) we see that the effective action can be decomposed into a manifestly gauge-invariant nonlocal piece ($\eta_{\mathrm{inv}}^\epsilon$) and a local piece in (4+1) dimensions (Q_5^ϵ) (when $\epsilon \to 0$). Notice, furthermore, that $\delta_u Q_5^\epsilon$ must be a local functional in 4 dimensions since the anomaly is local and $\eta_{\mathrm{inv}}^\epsilon$ is gauge invariant. At $O(\Lambda^0)$, Q_5^ϵ consists of 5-dimensional terms (we associate dimension one to ∂_t and A_t), which are Euclidean invariant in 4 dimensions and proportional to $\epsilon^{\mu\nu\rho\sigma}$. It is not difficult to see that no gauge-invariant term satisfying such conditions exist. Therefore at $O(\Lambda^0)$, Q_5^ϵ either vanishes for group-theoretical reasons or gauge invariance is violated. Moreover, there is only a nontrivial 5-dimensional form $Q_5(A)$ such that $\delta_u Q_5(A) = d A_{\mathrm{con}}(u, A)$ and it can actually be seen that Q_5^ϵ reduces to $Q_5(A)$ at this order [22]. At $O(\Lambda^{-2})$, it may occur that neither $A_{\mathrm{cov}}^\epsilon(u, D)$ nor $l_\epsilon(\dot{D}, D)$ vanish but nevertheless the $O(\Lambda^{-2})$ part of Q_5^ϵ is gauge invariant. This is due to the fact that dimension-7, Lorentz-covariant (in 4 dimensions) terms proportional to the ϵ tensor which are gauge invariant do exist. For a simple gauge group they read

$$\begin{aligned} &\mathrm{tr}(FFK) ,\\ &\mathrm{tr}(F_{t\rho} D_\rho F F), \quad \mathrm{tr}(F_{t\rho} F D_\rho F) ,\\ &\mathrm{tr}(D_\rho F_\rho F_t F), \quad \mathrm{tr}(F_\rho D_\rho F_t F) ,\\ &\mathrm{tr}(D_\rho F_\rho F F_t), \quad \mathrm{tr}(F_\rho F D_\rho F_t) ,\\ &\mathrm{tr}(D_t F_\rho F_\rho F), \quad \mathrm{tr}(F_\rho D_t F_\rho F) ,\\ &F_r = [D_r, D], \quad D_r = \partial_r + A_r, \quad r = \rho, t , \end{aligned} \tag{3.5}$$

where form notation is to be understood for the omitted indices. K is a gauge-covariant dimension-three one-form which accounts for the four-fermion interactions as we shall see in next section. Furthermore, the only way we know of to obtain a local anomaly from terms of that type is that there exists a Lorentz-invariant nontrivial five-form of dimension-7 such that its gauge variation is a total derivative. If we assume that all these forms can be derived from six forms of dimension-8 in the manner of descendent equations [23], the answer is negative. It is not difficult to check that all six forms of dimension-8 which are closed are globally exact (exterior derivatives of gauge-invariant forms). More explicitly, there are only two gauge-invariant closed forms which can be written as

$$\begin{aligned} &\mathrm{tr}(FFF_\rho F_\rho + \tfrac{1}{2} F D_\rho F D_\rho F) = d\,\mathrm{tr}(D_\rho F F_\rho) ,\\ &\mathrm{tr}(FFDK) = d\,\mathrm{tr}(FFK) . \end{aligned} \tag{3.6}$$

Thus we are led to the conclusion that the $O(\Lambda^{-2})$ contributions induce no genuine anomalies.

An alternative way of looking at the problem is the following. If the effective action is gauge invariant, the expectation value of a certain current must be covariantly conserved [1,2]. If a gauge-covariant regularization is introduced, the current is to be normalized by gauge-covariant counterterms only:

$$\langle \mathrm{tr}(uD_\mu(j_\mu+\text{gauge-covariant counterterms}))\rangle = A^{\epsilon}_{\mathrm{cov}}(u,D)=\mathrm{tr}\delta_u D D^{-1}_{\mathrm{reg}} \quad (3.7)$$

(angular brackets denote an expectation value over the fermionic fields). From this point of view, the problem of whether or not a genuine anomaly exists reduces to whether gauge-covariant counterterms can be found such that they account for all the contributions to $A^{\epsilon}_{\mathrm{cov}}(u,D)$ in (3.7). The possible counterterms give rise to the contributions to the LHS of (3.7) listed below:

$$\begin{gathered} \mathrm{tr}(uD(FK)),\quad \mathrm{tr}(uD(KF)) , \\ \mathrm{tr}(uD_\rho(FD_\rho F)),\quad \mathrm{tr}(uD_\rho(D_\rho FF)) , \\ \mathrm{tr}(uD(F_\rho D_\rho F)),\quad \mathrm{tr}(uD(D_\rho F_\rho F)) , \\ \mathrm{tr}(uD(FD_\rho F_\rho)),\quad \mathrm{tr}(uD(D_\rho FF_\rho)) . \end{gathered} \quad (3.8)$$

On the other hand, the possible contributions to $A^{\epsilon}_{\mathrm{cov}}(u,D)$ follow:

$$\begin{gathered} \mathrm{tr}(uDKF),\quad \mathrm{tr}(uFDK) , \\ \mathrm{tr}(u\{D_\rho F,D_\rho F\}),\quad \mathrm{tr}(u\{D_\rho D_\rho F,F\}) , \\ \mathrm{tr}(u\{F\{F_\rho,F_\rho\}\}),\quad \mathrm{tr}(u[D_\rho D_\rho F,F]) . \end{gathered} \quad (3.9)$$

It is a matter of linear algebra to see that a suitable combination of the counterterms (3.8) can always account for any combination of the terms (3.9). Thus we reach the same conclusion as before, i.e., there is no genuine anomaly at $O(\Lambda^{-2})$.

IV. THE $R_L(G)\times R_R(G)$ CASE

In order to make the arguments in the previous section substantial we have to prove that $A^{\mathrm{cov}}(u,D)$ and $l_\epsilon(\delta D,D)$ are proportional to the ϵ tensor, and that only one extra (dimension-3) one-form (K) in addition to gauge fields appears in these objects at $O(\Lambda^{-2})$.

Consider a gauge theory in Euclidean space with left (right) fermions belonging to the unitary representation $R_L(G)$ [$R_R(G)$] of a Lie group G, as it is customary in general discussions on anomaly cancellation [2]. Its fermionic part is given by the Euclidean Lagrangian

$$L=\bar\Psi D\Psi,\quad D=\gamma^\mu D_\mu,\quad D_\mu=\partial_\mu+A_\mu , \quad (4.1)$$

$$A_\mu=A_{L\mu}p_L+A_{R\mu}p_R,\quad A_{H\mu}=T^a_H A^a_\mu,\quad H=L,R ,$$

where $p_{L,R}=(1\pm\gamma_5)/2$ and $T^{a\dagger}_H=-T^a_H$ are the generators of two representations of the Lie algebra of G. L is invariant under the local gauge transformations

$$\begin{gathered} \Psi\to g\Psi,\quad \bar\Psi\to\bar\Psi\hat g , \\ A_\mu\to gA_\mu g^{-1}+g\partial_\mu g^{-1} , \\ g=g_Lp_L+g_Rp_R , \end{gathered} \quad (4.2)$$

where g_H belong to the corresponding representations of the gauge group G. Carets mean interchanging p_L and p_R here and in the rest of the paper. If G is simple the complete set of dimension-six operators to be added to (4.1) reads ($H,H'=L,R$)

$$\bar\Psi\gamma^\mu c_1(D_\rho F_{\mu\rho})\Psi , \quad (4.3a)$$

$$i\bar\Psi\gamma^\mu c_2\{D_\rho,F_{\mu\rho}\}\Psi , \quad (4.3b)$$

$$\bar\Psi\epsilon^{\mu\nu\rho\sigma}\gamma^\mu c_3F_{\nu\rho}D_\sigma\Psi , \quad (4.3c)$$

$$f_S^{HH'}\bar\Psi\gamma^\mu p_H\Psi\bar\Psi\gamma^\mu p_{H'}\Psi , \quad (4.3d)$$

$$f_V^{HH'}\bar\Psi\gamma^\mu T^a_H p_H\Psi\bar\Psi\gamma^\mu T^a_{H'}p_{H'}\Psi , \quad (4.3e)$$

$$h\bar\Psi p_L\Psi\bar\Psi p_R\Psi , \quad (4.3f)$$

where

$$\begin{gathered} F_{\mu\nu}=[D_\mu,D_\nu],\quad (D_\rho\cdot)=\partial_\rho\cdot+[A_\mu,\cdot] , \\ c_n=c_{nL}p_L+c_{nR}p_R,\quad n=1,2,3 . \end{gathered} \quad (4.4)$$

[A few more operators may exist in particular cases, e.g., $G\supset SU(2)$, see the next section.] All the dimensionless couplings $c_{nH},f_{S,V}^{HH'}$, and h are real so that each term in (4.3) is Hermitian in Minkowski space. The global factor Λ^{-2} has been omitted in each term of (4.3). If G is semisimple two basic modifications have to be introduced. First, there are as many terms (4.3a)–(4.3c) as normal subgroups of G since one can define a curvature $F^{(i)}_{\mu\nu}$ for each ideal of the Lie algebra of G corresponding to a normal subgroup i. Then one may introduce $F_{n\mu\nu}=\sum_i c_n^{(i)}F^{(i)}_{\mu\nu}$, $n=1,2,3$, which takes values in the Lie algebra of G. Second, there are as many terms (4.3d)–(4.3e) as direct products can be formed between the ideals of the Lie algebra of G by taking in each one either the given representation or the identity. If in addition some normal subgroups have the same representation for the left and right fermions; i.e., there are vectorlike normal subgroups, there are as many terms (4.3f) as direct products of the representations of the Lie algebra of the vectorlike normal subgroups and the identity can be formed as before.

The four-fermion interactions (4.3d) and (4.3e) can be written as

$$\frac{1}{\Lambda^2}[-\tfrac14 S_{H\mu}(f_S^{-1})^{HH'}S_{H'\mu}-\tfrac14 V^a_{H\mu}(f_V^{-1})^{HH'}V^a_{H'\mu} + \bar\Psi\gamma^\mu S_\mu\Psi+\bar\Psi\gamma^\mu V_\mu\Psi] ,$$

$$S_\mu=S_{L\mu}p_L+S_{R\mu}p_R,\quad V_\mu=V^a_{L\mu}T^a_Lp_L+V^a_{R\mu}T^a_Rp_R , \quad (4.5)$$

by introducing the auxiliary fields $S_{H\mu}$ and $V^a_{H\mu}, H=L,R$.

Notice that $V^a_{H\mu}$ can be encoded in a simple redefinition of the gauge fields $A_{H\mu}$ whereas $S_{H\mu}$ can also be traced by thinking of $A_{H\mu}$ as taking values in the Lie algebra of G plus the identity. If G is semisimple one has as many auxiliary fields as direct products can be formed according to the rule given after (4.4). The same connections $A_{H\mu}$ are enough to keep track of the auxiliary fields in this case as well but they must be thought of as taking values in the direct sum of all the above-mentioned direct products.

The four-fermion interaction (4.3f) can be written as

$$\frac{1}{\Lambda^2}\left[\overline{\Psi}p_L\varphi\Psi-\overline{\Psi}p_R\varphi^\dagger\Psi+\frac{1}{h}\varphi^\dagger\varphi\right] \tag{4.6}$$

by introducing the auxiliary scalar field φ. If G is semisimple and has vectorlike normal subgroups there are as many φ's as direct products can be formed according to the rules given above.

The introduction of auxiliary fields is not unique. Any choice, however, is formally equivalent to (4.3d)–(4.3f), since the functional integral over the auxiliary fields is Gaussian and can be done exactly. Upon regularization the equivalence is not so clear but the usual universality arguments suggest that it still holds. In our case there is an extra subtlety since the distribution of powers of the cutoff between the quadratic and linear term in the auxiliary fields [in (4.5) and (4.6)] does matter in the end of the calculations. This is due to the fact that the cutoff acts as a counting parameter so, as such, it must be kept explicit after the introduction of the auxiliary fields. This forces the auxiliary fields to be of dimension-3 as explicitly displayed in (4.5) and (4.6). (Of course, counterterms depending on the auxiliary fields must now be allowed.) Let us finally stress that the introduction of the auxiliary fields is due to technical reasons only. An alternative way to proceed would be by using an expansion about background fermionic fields $\Psi_0(\Psi=\Psi_0+\Psi_q)$ and calculating an effective action for A_μ and $\Psi_0,\overline{\Psi}_0$. This is feasible, though lengthy and more involved. After some (not completely trivial) calculations one may convince oneself that both procedures are equivalent.

After all the steps above have been carried out, the fermionic part of the Lagrangian can be accommodated to the form $L=\overline{\Psi}\mathrm{D}\Psi$ where

$$\begin{aligned}
\mathrm{D}&=\gamma^\mu\left[\partial_\mu+\mathrm{A}_\mu+\frac{i}{\Lambda^2}\{D_\rho,F_{2\mu\rho}\}+\frac{1}{\Lambda^2}\epsilon^{\mu\nu\rho\sigma}F_{3\nu\rho}D_\sigma\right]+\frac{1}{\Lambda^2}\Phi\ ,\\
\mathrm{A}_\mu&=A_\mu+\frac{1}{\Lambda^2}c_1(D_\rho F_{\mu\rho})+\frac{1}{\Lambda^2}K_\mu\ ,\\
F_{n\mu\rho}&=c_nF_{\mu\rho},\quad n=1,2,3\ ,\\
K_\mu&=S_\mu+V_\mu,\quad \Phi=\varphi p_L-\varphi^\dagger p_R\ .
\end{aligned} \tag{4.7}$$

D is the basic operator in our discussion, which we are going to develop for $\Lambda^\epsilon_{\rm cov}(u,\mathrm{D})$ only. A totally parallel argument leads to the same conclusions for $C_\epsilon(\delta_1\mathrm{D},\delta_2\mathrm{D},\mathrm{D})$ in (3.2) and hence for $l_\epsilon(\delta\mathrm{D},\mathrm{D})$. The operator $\mathrm{D}^\dagger$ reads

$$\mathrm{D}^\dagger=\gamma^\mu\left[\partial_\mu+\hat{\mathrm{A}}_\mu+\frac{i}{\Lambda^2}\{\hat{D}_\rho,\hat{F}_{2\mu\rho}\}-\frac{1}{\Lambda^2}\epsilon^{\mu\nu\rho\sigma}\hat{F}_{3\nu\rho}\hat{D}_\sigma\right]-\frac{1}{\Lambda^2}\hat{\Phi} \tag{4.8}$$

and hence it can be obtained from D by making $\gamma_5\to-\gamma_5$, $F_{3\mu\nu}\to-F_{3\mu\nu}$, and $\Phi\to-\Phi$. Since gauge covariance forces Φ to appear in pairs, this implies that the contribution to $A^\epsilon_{\rm cov}(u,\mathrm{D})$ in (3.3) from terms with an even number of $F_{3\mu\nu}$'s is proportional to γ_5 whereas that from an odd number of $F_{3\mu\nu}$'s is proportional to the identity. Since each $F_{3\mu\nu}$ is accompanied by an ϵ tensor, only terms proportional to $\epsilon^{\mu\nu\rho\sigma}$ arise.

A direct consequence of the covariant anomaly being proportional to $\epsilon^{\mu\nu\rho\sigma}$ is that it receives no contributions from the operator (4.3f). This operator may only contribute through the scalar field Φ which has dimension-three. Since the terms contributing to the covariant anomaly at $O(\Lambda^{-2})$ must have dimension-six, be gauge and Lorentz invariant, and proportional to $\epsilon^{\mu\nu\rho\sigma}$, they cannot accommodate two scalars of dimension three. This means that no restriction arises from anomaly cancellation for the four-fermion operator (4.3f). We drop the scalar field in the following.

Once the scalar field has been disposed of the contributions to the covariant anomaly factorize for the left and right sectors. The calculation can be carried out by using standard techniques (see [24] for a recent review), the only peculiarity being that the cutoff $\Lambda^{-2}=\epsilon$ appears both in the regularization (3.2), (3.3) and in the operators D (4.7) and $\mathrm{D}^\dagger$ (4.8), and that we keep all the contributions up to $O(\Lambda^{-4})$. The precise form of the result was not necessary for the discussion in the previous section. We present it for completeness in the Appendix. Notice finally that if G is not simple the list of possible contributions to $A^\epsilon_{\rm cov}(u,\mathrm{D})$ in (3.9) increases as it does the list of possible counterterms (3.8). A general proof that there exist enough counterterms to remove all the contributions to $A^\epsilon_{\rm cov}(u,\mathrm{D})$ at $O(\Lambda^{-2})$ is given at the end of the next section.

V. THE *N*-GENERATION STANDARD MODEL

In this section we rewrite the fermion-gauge-boson sector of N-generation SM at $O(\Lambda^{-2})$ in such a way that the results of the previous section can be applied. The relevant part of the power-counting renormalizable Lagrangian in Euclidean space reads

$$L=\sum_{p=1}^{N}\sum_{i=q,l}\overline{\Psi}{}^p_iD_i\Psi^p_i\ , \tag{5.1}$$

where

$$\Psi^p_q=p_Hq^p_H,\quad \Psi^p_l=p_Hl^p_H,\quad H=L,R \tag{5.2}$$

(the summation is implicit in repeated indices), q stands for quark and l for lepton and $p_{L,R}=(1\pm\gamma_5)/2$. Both left and right quarks and leptons are allocated in doublets. The left doublets correspond to the fundamental representation of SU(2), whereas the right doublets are taken as such for technical reasons [25]. The quark fields belong in addition to the fundamental representation of SU(3). The differential operator D_i is defined

$$D_i=\gamma^\mu D_{\mu i},\quad D_{\mu i}=\partial_\mu+A^i_\mu,\quad A^i_\mu=A^i_{L\mu}p_L+A^i_{R\mu}p_R\ ,$$
$$A^i_{L\mu}=W_\mu+Y_iB_\mu+\delta^{iq}G_\mu\ , \tag{5.3}$$
$$A^i_{R\mu}=(Y_i+\tau^3)B_\mu+\delta^{iq}G_\mu\ ,$$

where W_μ, B_μ, and G_μ take values in the fundamental (anti-Hermitian) representations of the Lie algebras of SU(2), U(1), and SU(3), respectively, and represent the intermediate bosons, photon, and gluon fields. The δ^{iq} reminds that only the quarks interact with the gluons. $Y_q=\frac{1}{6}$ and $Y_l=-\frac{1}{2}$ are the hypercharges.

These fields have the following transformation properties under local gauge transformations:

$$\begin{aligned}&\Psi^p_i\to g_i\Psi^p_i,\quad \overline{\Psi}^p_i\to\overline{\Psi}^p_i\hat{g}_i\ ,\\ &W_\mu\to g_WW_\mu g_W^\dagger+g_W\partial_\mu g_W^\dagger\ ,\\ &B_\mu\to B_\mu-\partial_\mu\theta\ ,\\ &G_\mu\to g_GG_\mu g_G^\dagger+g_G\partial_\mu g_G^\dagger\ ,\\ &g_q=g_Gg_We^{\theta Y_q}p_L+g_Ge^{\theta(Y_q+\tau^3)}p_R\ ,\\ &g_l=gWe^{\theta Y_1}p_L+e^{\theta(Y_1+\tau^3)}p_R\ ,\end{aligned}\tag{5.4}$$

where g_W and g_G belong to the fundamental representation of SU(2) and SU(3), respectively, and θ is a purely imaginary function. $\tau_3=\mathrm{diag}(\frac{1}{2},-\frac{1}{2})$ and carets mean interchanging p_L and p_R.

Lists of dimension-six operators have been given before in the literature [26,27]. Here we list in a more compact notation only those relevant for the fermion-gauge-boson sector. Consider first the operators bilinear in the fermionic fields:

$$\overline{\Psi}_i\gamma^\mu(D_\rho F^i_{1\mu\rho})\Psi_i\ , \tag{5.5a}$$

$$i\overline{\Psi}_i\gamma^\mu\{D_\rho,F^i_{2\mu\rho}\}\Psi_i\ , \tag{5.5b}$$

$$\overline{\Psi}_i\epsilon^{\mu\nu\rho\sigma}\gamma^\mu F^i_{3\nu\rho}D_\sigma\Psi_i\ , \tag{5.5c}$$

where

$$\begin{aligned}&F^i_{n\mu\rho}=F^i_{nT\mu\rho}P_T,\quad T=L,+,-\ ,\\ &P_L=p_L\cdot 1,\quad P_+=p_R\tau^+,\quad P_-=p_R\tau^-\ ,\\ &\tau^+=\begin{bmatrix}1&0\\0&0\end{bmatrix},\quad \tau^-=\begin{bmatrix}0&0\\0&1\end{bmatrix}\ ,\\ &F^i_{nL\mu\nu}=c^i_{nW}W_{\mu\nu}+c^i_{nBL}B_{\mu\nu}+c_{nGL}\delta^{iq}G_{\mu\nu}\ ,\\ &F^i_{n\pm\mu\nu}=c^i_{nB\pm}B_{\mu\nu}+c_{nG\pm}\delta^{iq}G_{\mu\nu},\quad n=1,2,3\ ,\\ &W_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu+[W_\mu,W_\nu]\ ,\\ &B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu\ ,\\ &G_{\mu\nu}=\partial_\mu G_\nu-\partial_\nu G_\mu+[G_\mu,G_\nu]\ .\end{aligned}\tag{5.6}$$

The generation indices have not been explicitly displayed. The c's are Hermitian $N\times N$ matrices connecting the different generations. [The Hermiticity of the c's follows from the Hermiticity of (5.5) in Minkowski space.] Operators containing three derivatives are omitted since they must be chosen $O(\hbar)$ in order to avoid unphysical poles in the propagators (see Sec. II), and hence they do not contribute to the anomaly which is given entirely by one loop diagrams [19] (see also Sec. III). Consider next the four-fermion interactions. They can be written as

$$f_B{}^{TT'}_{ii'(pq)(rs)}\overline{\Psi}{}^p_i\gamma^\mu P_T\Psi^q_i\overline{\Psi}{}^r_{i'}\gamma^\mu P_{T'}\Psi^s_{i'}\ , \tag{5.7a}$$

$$f_G{}^{TT'}_{(pq)(rs)}\overline{q}{}^p\gamma^\mu\lambda^aP_Tq^q\overline{q}{}^r\gamma^\mu\lambda^aP_{T'}q^s\ , \tag{5.7b}$$

$$f_{W\,ii'(pq)(rs)}\overline{\Psi}{}^p_i\gamma^\mu\tau^Ip_L\Psi^q_i\overline{\Psi}{}^r_{i'}\gamma^\mu\tau^Ip_L\Psi^s_{i'}\ , \tag{5.7c}$$

$$f_{GW\,(pq)(rs)}\overline{q}{}^p\gamma^\mu\lambda^a\tau^Ip_Lq^q\overline{q}{}^r\gamma^\mu\lambda^a\tau^Ip_Lq^s\ , \tag{5.7d}$$

$$h_B{}^{hHH'}_{ii'(pq)(rs)}\overline{\Psi}{}^p_i\Psi^q_{i'}\overline{\Psi}{}^r_{i'}p_{H'}\tau^h\Psi^s_i\ , \tag{5.7e}$$

$$h_G{}^{hHH'}_{(pq)(rs)}\overline{q}{}^pp_H\lambda^a\tau^hq^q\overline{q}{}^rp_{H'}\lambda^a\tau^hq^s\ , \tag{5.7f}$$

$$d_B{}^{Hhh'}_{i(pq)(rs)}\epsilon\overline{\Psi}{}^p_ip_H\tau^h\Psi^q_i\overline{\Psi}{}^r_ip_H\tau^{h'}\Psi^s_i\ , \tag{5.7g}$$

$$d_G{}^{Hhh'}_{(pq)(rs)}\epsilon\overline{q}{}^pp_H\lambda^a\tau^hq^q\overline{q}{}^rp_H\lambda^a\tau^{h'}q^s\ , \tag{5.7h}$$

$h,h'=+,-;\tau^I$ and λ^a are the generators of the fundamental representation of the Lie algebra of SU(2) and SU(3), respectively, and ϵ is the antisymmetric tensor acting on the fundamental representation of SU(2). The hermiticity of these terms in Minkowski space implies that the coupling constants in (5.7) satisfy

$$\begin{aligned}&(f_B{}^{TT'}_{ii'(pq)(rs)})^*=f_B{}^{TT'}_{ii'(qp)(sr)},\quad f_B{}^{TT'}_{ii'(pq)(rs)}=f_B{}^{T'T}_{i'i(rs)(pq)}\ ,\\ &(f_G{}^{TT'}_{(pq)(rs)})^*=f_G{}^{TT'}_{(qp)(sr)},\quad f_G{}^{TT'}_{(pq)(rs)}=f^{T'T}_{G(rs)(pq)}\ ,\\ &(f_{W\,ii'(pq)(rs)})^*=f_{W\,ii'(pq)(sr)},\quad f_{W\,ii'(pq)(rs)}=f_{W\,i'i(rs)(pq)}\ ,\\ &(f_{GW}{}^{TT'}_{(pq)(rs)})^*=f_{GW}{}^{TT'}_{(qp)(sr)},\quad f_{GW}{}^{TT'}_{(pq)(rs)}=f_{GW}{}^{T'T}_{(rs)(pq)}\ ,\\ &(h_B{}^{hHH'}_{ii'(pq)(rs)})^*=h_B{}^{h\overline{H}\overline{H}'}_{i'i(qp)(sr)},\quad h_B{}^{hHH'}_{ii'(pq)(rs)}=h_B{}^{hH'H}_{i'i(rs)(pq)},\quad h_B{}^{hHH}_{ii'(pq)(rs)}=0\ ,\\ &(h_G{}^{hHH'}_{(pq)(rs)})^*=h_G{}^{h\overline{H}\overline{H}'}_{(qp)(sr)},\quad h_G{}^{hHH'}_{(pq)(rs)}=h_G{}^{hH'H}_{(rs)(pq)},\quad h_G{}^{hHH}_{(pq)(rs)}=0\ ,\\ &(d_B{}^{Hhh'}_{i(pq)(rs)})^*=d_B{}^{\overline{H}hh'}_{i(qp)(sr)},\quad d_B{}^{Hhh'}_{i(pq)(rs)}=-d_B{}^{Hh'h}_{i(rs)(pq)},\quad d_B{}^{Hhh}_{i(pq)(rs)}=0\ ,\\ &(d_G{}^{Hhh'}_{(pq)(rs)})^*=d_G{}^{\overline{H}hh'}_{(qp)(sr)},\quad d_G{}^{Hhh'}_{(pq)(rs)}=-d_G{}^{Hh'h}_{(rs)(pq)},\quad d_G{}^{Hhh}_{(pq)(rs)}=0\end{aligned}\tag{5.8}$$

($\overline{L}=R$, $\overline{R}=L$). By introduction of the auxiliary fields $b^i_{(pq)T\mu}$, $g^a_{(pq)T\mu}$, $w^{iI}_{(pq)_\mu}$, $r^{aI}_{(pq)\mu}$, $\varphi_B{}^{hH}_{ii'(pq)_\mu}$, and $\varphi_G{}^{ahH}_{ii'(pq)\mu}$, (5.7) reads

$$\overline{\Psi}{}_i^p\gamma^\mu b^i_{(pq)\mu}\Psi_i^q-\tfrac{1}{4}b^i_{(pq)T\mu}f_B^{-1}{}^{TT'}_{ii'(pq)(rs)}b^{i'}_{(rs)T'\mu},\quad b^i_{(pq)\mu}=P_T b^i_{(pq)T\mu}\ , \tag{5.9a}$$

$$\overline{q}^{\,p}\gamma^\mu g_{(pq)\mu}q^q-\tfrac{1}{4}g^a_{(pq)T\mu}f_G^{-1}{}^{TT'}_{(pq)(rs)}g^a_{(rs)T'\mu},\quad g_{(pq)\mu}=\lambda^a P_T g^a_{(pq)T\mu}\ , \tag{5.9b}$$

$$\overline{\Psi}{}_i^p\gamma^\mu w^i_{(pq)\mu}\Psi_i^q-\tfrac{1}{4}w^{iI}_{(pq)\mu}f_W^{-1}{}_{ii'(pq)(rs)}w^{i'I}_{(rs)\mu},\quad w^i_{(pq)\mu}=w^{iI}_{(pq)\mu}\tau^I p_L\ , \tag{5.9c}$$

$$\overline{q}^{\,p}\gamma^\mu r_{(pq)\mu}q^q-\tfrac{1}{4}r^{aI}_{(pq)\mu}f_{GW}^{-1}{}_{(pq)(rs)}r^{aI}_{(rs)\mu},\quad r_{(pq)\mu}=\lambda^a\tau^I p_L r^{aI}_{(pq)\mu}\ , \tag{5.9d}$$

$$\begin{aligned}&\overline{\Psi}{}_i^p\Phi_{B\,ii'(pq)}\Psi_{i'}^q-\tfrac{1}{4}\varphi_B{}^{hH}_{ii'(pq)}(M_B^{-1})_{ii'(pq)(rs)}\varphi_B{}^{h'H'}_{ii'(rs)},\quad \Phi_{B\,ii'(pq)}=p_H\tau^h\varphi_B{}^{hH}_{ii'(pq)}\ ,\\ &M_B{}^{hh'HH'}_{ii'(pq)(rs)}=\delta^{hh'}h_B{}^{hHH'}_{ii'(rs)}1+\delta^{HH'}\delta^{ii'}d_B{}^{Hhh'}_{i(pq)(rs)}\epsilon,\quad (\varphi_B{}^{hH}_{ii'(pq)})^*=-\varphi_B{}^{h\overline{H}}_{i'i(qp)}\ ,\end{aligned} \tag{5.9e}$$

$$\begin{aligned}&\overline{q}^{\,p}\Phi_{G\,(pq)}q^q-\tfrac{1}{4}\varphi_G{}^{ahH}_{(pq)}M_G^{-1}{}^{hh'HH'}_{(pq)(rs)}\varphi_G{}^{ah'H'}_{(rs)},\quad \Phi_{G\,(pq)}=p_H\lambda^a\tau^h\varphi_G{}^{ahH}_{(pq)}\ ,\\ &M_G{}^{hh'HH'}_{(pq)(rs)}=\delta^{hh'}h_G{}^{hHH'}_{(pq)(rs)}1+\delta^{HH'}d_G{}^{Hhh'}_{(pq)(rs)}\epsilon,\quad (\varphi_G{}^{ahH}_{(pq)})^*=\varphi_G{}^{ah\overline{H}}_{(qp)}\ .\end{aligned} \tag{5.9f}$$

The 1 and the antisymmetric tensor ϵ in (5.9e) and (5.9f) act on the fundamental representation of SU(2). The auxiliary fields satisfy

$$\begin{aligned}&b^{i\dagger}_{(pq)\mu}=-b^i_{(pq)\mu},\quad g^\dagger_{(pq)\mu}=-g_{(qp)\mu},\quad w^{i\dagger}_{(pq)\mu}=-w^i_{(qp)\mu}\ ,\\ &r^\dagger_{(pq)\mu}=-r_{(qp)\mu},\quad \Phi_B{}^\dagger_{ii'(pq)}=-\hat{\Phi}_{B\,i'i(qp)},\\ &\Phi_G{}^\dagger_{(pq)}=-\hat{\Phi}_{G\,(qp)}\ ,\end{aligned} \tag{5.10}$$

so that (5.9) is Hermitian in Minkowski space. In (5.9) we assumed that the matrices formed with the coupling constants appearing in (5.7) are invertible. If this is not so, some coupling constants are necessarily combinations of the rest. In this case, it is not difficult to see that (5.7) can be written as an invertible matrix coupled to suitable combinations of fermionic bilinears. One auxiliary field is then introduced for each one of these combinations. In fact, the final outcome can be read off the invertible case (5.9) by taking the auxiliary fields of a particular form and dropping in the inverse matrices the rows and columns corresponding to those which have been taken as combinations of the rest in the original matrix. We can then proceed without loss of generality as if the matrices (5.7) were indeed invertible. We have just to remember that forcing particular forms of the auxiliary fields translates into having relationships between the coupling constants (5.7).

After all these steps have been carried out, the bilinear fermionic part of the Lagrangian can be accommodated to the form $\overline{\Psi}\mathrm{D}\Psi$, where

$$\begin{aligned}\mathrm{D}&=\gamma^\mu\left[\partial_\mu+\mathrm{A}_\mu+\frac{i}{\Lambda^2}\{D_\rho,F_{2\mu\rho}\}+\frac{1}{\Lambda^2}\epsilon^{\mu\nu\rho\sigma}F_{3\nu\rho}D_\sigma\right]\\ &\quad+\frac{1}{\Lambda^2}\ ,\\ \mathrm{A}_\mu&=A_\mu+\frac{1}{\Lambda^2}(D_\rho F_{1\mu\rho})+\frac{1}{\Lambda^2}K_\mu\ .\end{aligned} \tag{5.11}$$

The quark-lepton and generation indices have not been explicitly displayed. Recall that A_μ is diagonal in the quark-lepton and generation indices and $F_{n\mu\rho}$, $n=1,2,3$, is diagonal in the quark-lepton indices only. The explicit expressions for K_μ and Φ are

$$\begin{aligned}&K_{i(rs)\mu}=b^i_{(rs)\mu}+w^i_{(rs)\mu}p_L+\delta^{iq}(g_{(rs)\mu}+r_{(rs)\mu}p_L)\ ,\\ &\Phi_{ii'(rs)}=\Phi_{Bii'(rs)}+\delta^{iq}\delta^{i'q}\Phi_{G\,(rs)}\ .\end{aligned} \tag{5.12}$$

Then K_μ is diagonal in the quark-lepton indices only and Φ is off diagonal in both quark-lepton and generation indices. Recall also that A_μ takes values in the direct sum of representations of the Lie algebras of U(1), SU(2), SU(3), and SU(2)⊗SU(3), and Φ in the direct sum of 1 and the fundamental representation of the Lie algebra of SU(3). The notation is intended to be a mnemonic. W's (w's) are always related to SU(2), G's (g's) to SU(3), and B's (b's) to Abelian pieces.

Notice that (5.11) has the same form as (4.7) and $\mathrm{D}^\dagger$ satisfies the same properties as (4.8). Therefore the discussion after (4.8) applies to the SM as well. The possible contributions to $A^\epsilon_{\mathrm{cov}}(u,\mathrm{D})$ are a generalization of (3.9):

$$r_{1ij}\mathrm{tr}(uD_\rho F_i D_\rho F_j)\ , \tag{5.13a}$$

$$r_{2ij}\mathrm{tr}(uD_\rho D_\rho F_i F_j)\ , \tag{5.13b}$$

$$r_{3ij}\mathrm{tr}(uF_i D_\rho D_\rho F_j)\ , \tag{5.13c}$$

$$r_{4ijk}\mathrm{tr}(uF_i F_{\rho j}F_{\rho k})\ . \tag{5.13d}$$

The F_i and $F_{\rho i}$ are given in (5.6) for the SM (in this case the traces now must be understood for generation indices as well) or in the discussion after (4.4) for a general semisimple gauge group G. The counterterms which account for (5.13d) read

$$\begin{aligned}\tfrac{1}{8}r_{4ijk}\mathrm{tr}\{u[&-\tfrac{1}{2}D_\rho(F_{ij}D_\rho F_k)+D_\rho(F_iD_\rho F_{jk})+D_\rho(D_\rho F_{ij}F_k)-\tfrac{1}{2}D_\rho(D_\rho F_iF_{jk})+D(F_{\rho ij}D_\rho F_k)-2D(D_\rho F_{\rho ij}F_k)\\ &+D(D_\rho F_{\rho i}F_{jk})+D(F_{ij}D_\rho F_{\rho k})-2D(F_iD_\rho F_{\rho jk})+D(D_\rho F_iF_{\rho jk})]\}\end{aligned} \tag{5.14}$$

where $F_{ij}=\sum_{n=\mathrm{ideals}}c_i^{(n)}c_j^{(n)}F^{(n)}$ for G semisimple and the obvious generalization for the SM (where the $c^{(n)}$'s are matrix valued in generation space). A simpler set of counterterms accounts for (5.13a)–(5.13c). These are

$$\tfrac{1}{4}(3r_{3ij}+r_{1ij}-r_{2ij})\mathrm{tr}[uD_\rho(F_iD_\rho F_j)]+\tfrac{1}{4}(3r_{2ij}+r_{1ij}-r_{3ij})\mathrm{tr}[uD_\rho(D_\rho F_iF_j)]$$
$$+\tfrac{1}{2}(r_{3ij}+r_{2ij}-r_{1ij})\mathrm{tr}[uD(F_{\rho i}D_\rho F_j)+uD(F_iD_\rho F_{\rho j})+uD(D_\rho F_{\rho i}F_j)+uD(D_\rho F_iF_{\rho j})]\ . \quad (5.15)$$

The possible terms in $A^{\epsilon}_{\mathrm{cov}}(u,\mathrm{D})$ depending on K as well as the corresponding counterterms are an immediate generalization of those in (3.8), (3.9) and have not been explicitly displayed. This concludes our proof that there are no genuine contributions to the anomaly at $O(\Lambda^{-2})$, nor for an arbitrary semisimple gauge group G neither for the N-generation SM.

VI. DISCUSSION

We have presented a perturbative framework in which finite cutoff effects can be taken into account in a regularization-independent, and hence unitarity-preserving, way. We have focused on the occurrence of local gauge anomalies, but presumably many other questions can be addressed. One may be worried, however, about the consistency of the framework itself, as keeping a cutoff is traditionally believed to bring in any sort of trouble. The obvious objection, which is the regularization dependence, is overcome by allowing arbitrary couplings in the dimension-six operators as discussed in Sec. II. Lorentz covariance and the internal symmetries can be kept manifest by using ϵ regularization. Terms with higher derivatives are a potential source of problems related to unitarity (see [28] for a recent discussion). In order to avoid unphysical poles in the propagators they must be treated as perturbations and this is achieved by taking them to be $O(\hbar)$. Notice also that keeping terms $O(\Lambda^{-2})$ at the end of the calculations does not violate locality (keeping terms at any order in Λ does). Consequently, we do not expect problems related to unitarity, though this point should be checked eventually.

We have seen that no genuine contribution to the anomaly arises at $O(\Lambda^{-2})$. Consequently, anomaly cancellation at $O(\Lambda^{-2})$ does not relate coupling constants of dimension-six operators as one might naively expect, and hence does not constrain the physics beyond the SM.

If one is not worried about having a unitary framework to calculate Λ^{-2} effects, but confines oneself to find out what the effects $O(\Lambda^{-2})$ in the anomaly are in a given regularization for the power-counting renormalizable Lagrangian, then our conclusion still holds. In fact the implementation is much simpler in this case. We just have to drop the terms proportional to Λ^{-2} in the operator (4.7). Then the result in the Appendix consists only of the terms (A1a), (A1d), and (A1i).

Let us also mention that fundamental scalar fields may also contribute to the covariant anomaly at $O(\Lambda^{-2})$. Again, it is not difficult to see that all possible contributions amount to gauge-covariant renormalizations of the current, so no genuine anomaly is left.

Although the motivation for our analysis as presented in the introduction was the triviality of $\lambda\Phi^4$, the actual scope of our work is wider. In fact, from the phenomenological point of view, the basic assumptions are two: (i) there are new unknown interactions which become relevant at some scale Λ (the cutoff), and (ii) the physics at energies approaching the scale Λ (but still small in comparison to it) can be well described by adding to the power-counting renormalizable Lagrangian higher-dimensional operators which respect the symmetries of that Lagrangian. (These assumptions have been used before [11,26].) The triviality of $\lambda\Phi^4$ just provides a pure quantum-field-theoretical understanding of why the built-in cutoff must be there. It has been useful to us in order to present our ideas in a, we hope, coherent way.

In summary, we have described a perturbative procedure which allows to take into account effects due to a finite built-in cutoff in a regularization-independent, and hence unitarity-preserving, way in perturbatively renormalizable theories. We have proven that at leading order there are no genuine contributions to the local gauge anomaly.

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APPENDIX

We display here the result of the calculation of the covariant anomaly at $O(\Lambda^{-2})$:

$$A^{\mathrm{cov}}(u)=\frac{1}{32\pi^2}\Big[\tfrac{1}{4}\mathrm{tr}\gamma_5u\{\tilde F_{\mu\nu},F_{\mu\nu}\} \quad (\mathrm{A1a})$$
$$+\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5u\{\tilde F_{\mu\nu},D_\mu K_\nu\} \quad (\mathrm{A1b})$$
$$+2\frac{1}{\Lambda^2}\mathrm{tr}u\{\tilde F_{3\mu\nu},D_\mu K_\nu\} \quad (\mathrm{A1c})$$
$$+\frac{1}{12}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5u\{D_\rho D_\rho F_{\mu\nu},\tilde F_{\mu\nu}\} \quad (\mathrm{A1d})$$
$$-\frac{1}{3}\frac{1}{\Lambda^2}\mathrm{tr}u\{D_\rho D_\rho F_{\mu\nu},\tilde F_{3\mu\nu}\} \quad (\mathrm{A1e})$$
$$-\frac{4}{3}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5u\{D_\rho D_\rho F_{3\mu\nu},\tilde F_{3\mu\nu}\} \quad (\mathrm{A1f})$$
$$-\frac{1}{2}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5u\{D_\rho D_\rho F_{1\mu\nu},\tilde F_{\mu\nu}\} \quad (\mathrm{A1g})$$
$$-\frac{1}{\Lambda^2}\mathrm{tr}u\{D_\rho D_\rho F_{1\mu\nu},\tilde F_{3\mu\nu}\} \quad (\mathrm{A1h})$$

$$+\frac{1}{24}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u\{D_\rho\tilde{F}_{\mu\nu},D_\rho F_{\mu\nu}\} \quad \text{(A1i)}$$

$$-\frac{1}{2}\frac{1}{\Lambda^2}\mathrm{tr}u\{D_\rho\tilde{F}_{\mu\nu},D_\rho F_{3\mu\nu}\} \quad \text{(A1j)}$$

$$-\frac{3}{2}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u\{D_\rho\tilde{F}_{3\mu\nu},D_\rho F_{3\mu\nu}\} \quad \text{(A1k)}$$

$$+\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u\{D_\rho\tilde{F}_{2\mu\nu},D_\rho F_{2\mu\nu}\} \quad \text{(A1l)}$$

$$-\frac{1}{3}\frac{1}{\Lambda^2}\mathrm{tr}u\{\tilde{F}_{3\mu\nu},[F_{\mu\rho},F_{\nu\rho}]\} \quad \text{(A1m)}$$

$$+\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u(-3\{\tilde{F}_{\mu\nu},[F_{3\mu\rho},F_{3\nu\rho}]\} + \tfrac{44}{3}\{\tilde{F}_{3\mu\nu},[F_{\mu\rho},F_{3\nu\rho}]\}) \quad \text{(A1n)}$$

$$+\frac{16}{3}\frac{1}{\Lambda^2}\mathrm{tr}u\{\tilde{F}_{3\mu\nu},[F_{3\mu\rho},F_{3\nu\rho}]\} \quad \text{(A1o)}$$

$$+\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u\{\tilde{F}_{\mu\nu},[F_{1\mu\rho},F_{\nu\rho}]\} \quad \text{(A1p)}$$

$$+2\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u\{\tilde{F}_{3\mu\nu},[F_{1\mu\rho},F_{\nu\rho}]\} \quad \text{(A1q)}$$

$$-\frac{i}{6}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u[D_\rho D_\rho F_{2\mu\nu},\tilde{F}_{\mu\nu}] \quad \text{(A1r)}$$

$$+\frac{i}{3}\frac{1}{\Lambda^2}\mathrm{tr}u[D_\rho D_\rho F_{2\mu\nu},\tilde{F}_{3\mu\nu}] \quad \text{(A1s)}$$

$$-\frac{1}{3}\frac{1}{\Lambda^2}\mathrm{tr}\gamma_5 u[D_\rho\tilde{F}_{2\mu\nu},D_\rho F_{\mu\nu}] \quad \text{(A1t)}$$

$$+\frac{1}{3}\frac{1}{\Lambda^2}\mathrm{tr}u[D_\rho\tilde{F}_{3\mu\nu},D_\rho F_{2\mu\nu}] \quad \text{(A1u)}$$

$$+\frac{1}{6}\frac{1}{\Lambda^2}\mathrm{tr}u[\tilde{F}_{3\mu\nu},[F_{\mu\rho},F_{\nu\rho}]] \quad \text{(A1v)}$$

$$+\frac{1}{\Lambda^2}\mathrm{tr}u(\tfrac{5}{3}[\tilde{F}_{\mu\nu},[F_{2\mu\rho},F_{3\nu\rho}]] + 4[\tilde{F}_{3\mu\nu},[F_{\mu\rho},F_{2\nu\rho}]]) \quad \text{(A1w)}$$

$$+\frac{1}{\Lambda^2}\mathrm{tr}u(-\tfrac{5}{3}\{\tilde{F}_{\mu\nu},\{F_{2\mu\rho},F_{3\nu\rho}\}\} - 16\{\tilde{F}_{3\mu\nu},\{F_{\mu\rho},F_{2\nu\rho}\}\})\Bigg], \quad \text{(A1x)}$$

$$\tilde{F}_{\mu\nu}=\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} .$$

The traces over Dirac matrices just keep track of the relative signs between left and right contributions in a compact way. The covariant derivatives in (A1) must be understood as acting on the closest object on the right only. Contributions $\mathrm{tr}\gamma_5 u\{\tilde{F}_{3\mu\nu},F_{3\mu\nu}\}$ may have occurred but cancel against each other. For G simple $F_{\mu\nu}F_{n\rho\sigma}=F_{n\mu\nu}F_{\rho\sigma}=c_nF_{\mu\nu}F_{\rho\sigma}=F_{\mu\nu}F_{\rho\sigma}c_n$, $n=1,2,3$ and hence (A1t)–(A1x) vanish. For G semisimple (A1t)–(A1w) also vanish because the commutators split the contribution into a sum over the contribution of each ideal which vanishes.

The result for the SM can be obtained from (A1) by undoing the definitions (5.6) and (5.12), taking $u=u_Lp_L+u_Rp_R$, $u_L=v+Y_i\theta$, $u_R=(Y_i+\tau^3)\theta$ [v takes values in the Lie algebra of SU(2) and θ is a purely imaginary function], and considering the traces over the generation indices as well.

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