The Impact of Coordination and Targets on Monetary and Fiscal Policy

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Abstract

This paper analyzes the implications of coordination of fiscal and monetary policy. We construct a differential game between a government and a central bank, which are set with the task of stabilizing the economy after an external shock. The mechanics of the game are created by assuming that the authorities must balance the cost of using their policy instruments against the cost of a slower convergence to equilibrium. We compare the competitive and cooperative equilibria under two types of shocks and when the policy makers operate under inflation targeting and output targeting respectively. The main conclusion is that policy coordination can be important for allowing the authorities to reach their targets, especially when they operate under inflation targeting.

Keywords: monetary policy, fiscal policy, inflation targeting, output targeting, differential games.

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1. Introduction

Since the 1980´s, there has been a tendency for countries to move towards increased central bank independence. This "divorce" of monetary and debt management, as it's referred to in Laurens and Piedra (1998), was motivated primarily by the need for price stability. It was believed that by removing monetary policy decisions from the political process, inflation could be stabilized, since policy would no longer be subject to the whims of voters and electoral cycles. Then, to the extent that inflation is damaging the performance of the real economy, central bank independence would increase overall welfare. Nowadays there exists a large body of evidence, both theoretical and empirical, in support of the claim that central bank independence is an effective method for controlling inflation.[1]

Notwithstanding the positive consequences of the reforms, the separation of monetary and fiscal policy decisions also created reasons for concern. When the major instruments for stabilizing the economy - the fiscal deficit and the steering nominal interest rate - are controlled by two separate institutions that are (more or less) independent from each other, there arises a need for *policy coordination*. In deciding the proper fiscal policy, the government will need to take into consideration how the central bank will respond, and vice versa. Consequently, both the monetary and fiscal authorities might find it useful to consult each other before they pursue their individual objectives, that is, to coordinate their decisions. As noted in Lambertini and Rovelli (2003), the need for coordination arises primarily because the policy instruments of the government and the central bank have similar effects on aggregate demand, and hence also on output and inflation. Fiscal and monetary expansions alike will, everything else equal, increase output and inflation to levels above where they would have been otherwise. Contractions will have the opposite effect.

In general, the monetary and fiscal authorities will work under a set of objectives or guidelines, such as the commonly used inflation targeting. One may then measure the benefits of policy coordination by the extent to which it helps the authorities to reach those objectives. Moreover, the benefits will likely depend on the specific set of objectives that are being used, how easy they are to reach without coordination, and what type of disequilibrium the policy makers are responding to. Thus, Hanif and Arby (2003) make the point that the objectives of government and central bank policy may be contradictory, in the sense that the authorities, when they pursue policy independently without coordination, undermine the actions of each other. This concern is also mentioned shortly in Abdel-Haleim (2016). In such situations, coordination may have substantial benefits for both authorities.

In this paper, we analyze the argument that the need for policy coordination may crucially depend on the set of objectives that the policy authorities are operating under. By policy coordination we will mean a situation where the authorities are actively consulting and communicating with each other, with the purpose of achieving a pareto optimal outcome. We do not mean a situation where they act individually and obtain that same outcome because

[1] Overviews of the research on this topic since the 1980's and onwards can be found in Parkin (2012).

their individual objectives happen to produce it. In short, by coordination we mean cooperation.[2]

An example will illustrate the basic argument. Assume that the government and the central bank operate under output targeting. That is, their main concern is to achieve output stability rather than stability in some other aggregate such as inflation. Next, assume that the disequilibrium is such that inflation and output are both above their equilibrium levels. Then, the authorities will want to enact proper contractionary policies to produce a smooth and stable return of output to its equilibrium level. Since output can be affected by policy quite directly – for instance by changing public demand – this should be less complicated than producing an equivalent adjustment in inflation. Assume, on the other hand, that the main concern of the authorities is inflation targeting, as it often is for central banks. Then, the task of bringing inflation back to equilibrium is complicated by the fact that the impact of policy instruments is more indirect. Fiscal and monetary policy affect inflation mainly *through* output, and hence inflation is more difficult to control. Therefore, an objective that prioritizes inflation stability might pose a greater challenge for the authorities.

As discussed in Blanchard et al (2010), decreasing inflation might require driving output temporarily down *below* its equilibrium level. This is, for instance, what motivated the contractionary policies of the UK in the 1980's. Hence, in our example, it might not be enough for the inflation targeting authorities to steer output back to its equilibrium level. They will have to go further and decrease output below equilibrium if that is the only way to reduce inflation. Then, the task of policy making will be more complicated and the need for coordination of fiscal and monetary policy is likely to be higher.

To analyze the issue described above, we build a simple model in a differential game setting. Thus, we will model the government and the central bank as agents in a two-player policy game and solve for the competitive and cooperative equilibria, in the latter case by using Nash bargaining. Throughout the paper, we will think of the competitive solution to the policy game as the situation where the fiscal and monetary authorities act independently from one another and don't coordinate their actions. Likewise, we will think of the cooperative solution as reflecting policy coordination. We simulate the model using standard parameter values, and capture the gains from policy coordination under two types of policy objectives and two types of initial disequilibria or shocks. The gains from coordination are measured with loss functions of the sort that are commonly used in the literature on monetary and fiscal policy.

The rest of this paper is organized as follows. In Section 2, we present a short overview of the literature on differential policy games. We also discuss some central issues that arise in those types of models, and how they have typically been solved in the literature. In Section 3, we present our model and derive the competitive and cooperative solutions. In Section 4 we calibrate the model and present the results. Section 5 concludes.

[2] A somewhat different definition of policy coordination among the fiscal and monetary authorities is offered in Abdel-Haleim (2016): "Coordination is defined as the necessary arrangements that assure that the decisions taken by both authorities are not contradictory".

2. Literature Review and Related Issues

One early contribution to the literature on differential policy games is Turnovsky et al (1987). They build a two-country model to analyze the effects of monetary policy on welfare. The agents of the game are the monetary authorities of the two countries, and both are charged with steering their respective economies towards equilibrium. The initial disequilibrium is created by a shock to the real exchange rate, which creates an imbalance in trade flows and hence affects output. Since monetary policy affects the real exchange rate, the central banks can reduce the loss in welfare caused by the shock. The authors conclude that cooperation reduces welfare losses by between 6 and 10 percent.

An example of a game with interaction between both a monetary and a fiscal authority, acting in the same country, can be found in Dixit and Lambertini (2003). They build a model with monopolistic competition, which creates inefficiently low output, and gives policy makers an incentive to intervene in the economy even in the absence of any external shocks. Moreover, both the government and the central bank can operate under either discretion of commitment, and this has implications for the strategic interaction of the authorities. The game is solved for the Nash and leadership equilibria. One conclusion of the paper is that, from a welfare point of view, fiscal leadership is generally better than monetary leadership, although both may be worse than the Nash equilibrium.

Another paper with interaction between fiscal and monetary authorities is Di Bartolomeo and Di Gioacchino (2004). They too include a leadership version of the game, and solve for the cases of monetary leadership and fiscal leadership alongside the Nash equilibrium. The game is solved in two stages. The first one establishes whether the differential game is sequential and which of the two players that will have leadership. The paper also includes a discussion on what leadership scenario that may be most realistic.

A somewhat different – and creative – approach comes from Cellini and Lambertini (2003), where a two-player game is set up between the central bank and the representative household of the private sector. The paper is concerned with the discretion vs commitment debate in monetary policy, and attempts to analyze under what conditions the central bank can exploit a first-mover advantage over the private sector to gain a costless reduction in inflation. For this purpose, a leadership solution is found, with the central bank as the leader.

In the late 90's, the creation of the Euro raised interest in models for monetary unions. Van Aarle et al (2001) models two countries that have the same currency and hence the same monetary policy. A three-player game involving the joint monetary authority, and the fiscal authority for each country, is solved for a sophisticated macroeconomic model which includes dynamics for output, inflation, wages and unemployment. Moreover, the authors differentiate between a Neoclassical regime, in which excess unemployment is the result of high wages, and a Keynesian regime, in which unemployment is due to low aggregate demand. The game is solved for both the competitive and the cooperative cases, and the welfare effects of cooperation are analyzed. This paper has been a major source of ideas for the present study, both in terms of the model and the calibration.

In the papers mentioned above, there are certain “standard questions” faced by the authors. One such question is which specific objectives one assumes that the fiscal and monetary authorities follow. Another is which variables one assumes to be under the direct control of policy makers, as opposed to being steered only indirectly. For the government, an obvious and common choice for a control variable is the fiscal deficit. For the central bank, a common but in no way dominating approach is to let the central bank control the nominal interest rate. This approach is used in Saulo et al (2012), Galí (2008) and Woodford (2003).[3] On the other hand, Di Bartolomeo and Di Gioacchino (2004) and Van Aarle et al (1999) build models where the central bank controls the monetary base. Finally, in Cellini and Lambertini (2003), the central bank is given direct control over inflation itself.

As for the objectives, it is generally assumed that policy makers are attempting to minimize a loss or cost function, which depends on the deviation of a set of macroeconomic aggregates from their equilibrium levels. It is also standard to let the loss function be quadratic in all its arguments.[4] Typical arguments are output, inflation, the government deficit and the nominal interest rate. Hence, in a seminal paper by Kydland and Prescott (1977), the central bank’s loss function depends on the deviation of inflation and output from their equilibrium levels. The same setup is adopted in Dixit and Lambertini (2003) and Cellini and Lambertini (2003). As pointed out in Woodford (2003), this simple form is widely used in the literature on monetary policy.

Other setups are also common. In Lambertini and Rovelli (2003), the central bank’s loss function includes the real interest rate rather than output, while the government’s loss function includes both those variables along with the fiscal deficit. More extensive loss functions can be found in van Aarle et al (1999) where both the central bank and the government attempts to control deviations in output, inflation and unemployment. Another extensive setup is offered in Engwerda (2006), where the monetary base and the government debt enter the loss functions. In general, the choice of loss functions will depend on how much one assumes that the fiscal and monetary authorities “care” about each variable. For instance, since most central banks have the main objective of ensuring price stability, inflation is an obvious choice for the monetary authority’s loss function. The government, on the other hand, might be more concerned with unemployment and output.

As the preceding paragraphs make clear, it is common to include in the loss functions not just major aggregates like output and inflation, but also the policy instruments. What is the economic justification for this? In general, governments are advised not to deviate too far from running a balanced budget. Low deficits may require cuts in welfare services, which presumably is unpopular among voters, while high deficits tend to crowd out private investment. Moreover, as noted in van Aarle et al (1999), for members of the European Union large deficits may result in sanctions on the country, as specified in the union’s Stability and

[3] In Galí (2008), there’s also a short section discussing the case where the central bank controls the monetary base.

[4] This is convenient because, if in addition the dynamics are linear, there are well known methods for solving the optimization problem. There are also intuitive arguments for the quadratic form: the marginal cost of deviation increases in the deviation and, importantly, small deviations in several variables are preferable to a large deviation in a single variable.

Growth Pact. The assumption that monetary policy activism is costly is less clear, but nevertheless common in the literature. One can, perhaps, imagine that large volatility in the nominal interest rate make financial returns unpredictable and discourages investment, which hurts overall economic development. Van Aarle et al (1999), discussing the ECB, note that "other things equal policy makers prefer a constant level of their instrument rather than to undertake changes all the time".

Finally, an issue which is often ignored in the literature is what exactly the loss functions are a measure of. In models on only monetary policy, the loss functions of the central bank is sometimes derived from underlying optimization problems describing the behavior of households and firms. This is the case in Galí (2008), Woodford (2003) and more generally in the standard New Keynesian Model. In those cases, the loss function can be interpreted as reflecting welfare or social utility. However, problems arise when the model contains both a government and a central bank with separate loss functions, as is the case in policy games, since there is no obvious way to decide which of the two loss functions reflect welfare. In Dixit and Lambertini (2003) it is assumed that the government's loss function is the welfare loss, while the central bank has a loss function that differs from welfare. Van Aarle et al (2001) seem to assume that both loss functions capture welfare losses, although they do not explain whether this means that there are two separate measures of welfare. Another approach, however, is to assume that both loss functions reflect just the objectives that the authorities follow when deciding on policy, with no reference to welfare. The magnitude of the loss is then simply the extent to which an authority fails to reach its objectives. That is the interpretation we will use here.

3. The Model

First, we will define the loss functions and the dynamics of our model. We will assume that the government controls the fiscal deficit while the central bank controls the nominal interest rate. The two other variables in our model, which are output and inflation, will then be state variables that the authorities can impact indirectly.

The (instantaneous) loss functions for the fiscal authority and for the monetary authority are given by

$$L_F = \lambda_{xF} x^2 + \lambda_{\pi F} \pi^2 + \lambda_{gF} g^2, \tag{1}$$

$$L_M = \lambda_{xM} x^2 + \lambda_{\pi M} \pi^2 + \lambda_{iM} (i - \rho)^2. \tag{2}$$

Here, x is the real output, π is inflation, g is the real fiscal deficit, i is the nominal interest rate and ρ is the rate at which the authorities discount the future. Hence, our setup is the most standard one with inflation and output as arguments, except with the addition of the policy instruments. Note that each authority is considering only its own instrument, since it is not responsible for the actions of the other authority. As is standard, output and the fiscal deficit are measured in terms of log deviations from their equilibrium levels. This means that all four variables have a percentage interpretation and not a quantity interpretation. Henceforth when we write output or inflation, we are referring to the log deviations of those variables from their equilibrium levels. Note also that the nominal interest rate loss depends on deviations from the parameter ρ. In some economic models on monetary and fiscal policy, one can show that the equilibrium real interest rate must equal the discount rate.[5] We will assume that the inflation target, to which inflation will be steered after a shock, is equal to zero. Then, the nominal and real interest rates must be equal in equilibrium. The assumption of a zero inflation in equilibrium is in line with Galí (2008), where the inflation target is assumed to be "close to zero".[6]

Next, we will define the dynamics that govern how output and inflation evolve as functions of the policy decisions. For output, we will assume the relationship

$$\dot{x} = \beta g - \alpha(i - \pi - \rho) - \varepsilon x. \qquad (3)$$

This is essentially a continuous time Investment-Saving equation of the kind that is common in models for monetary policy evaluation.[7] The first two terms relate output to the fiscal deficit and the real interest rate: an increase in the fiscal deficit is expansionary while an increase in the real interest rate is contractionary. The third term captures the fact that eventually output will always return to its equilibrium level, even in the absence of policy activism.[8] The parameters α and β are essentially measures of the fiscal and monetary multipliers. We shall discuss their economic meaning further in Section 4.

We also need to define the evolution of inflation. For this we will use the simplified Phillips relation

$$\dot{\pi} = \kappa x. \qquad (4)$$

This tells us that inflation increases whenever output is above its equilibrium level, and vice versa. Our Phillips relation is based on the discrete time New Keynesian Philips Curve as derived in Blanchard et al (2010), which has the form $\pi_t = \mu\pi_{t-1} - \kappa(u_t - u_n)$, where u_t is the unemployment rate and u_n its equilibrium level. By assuming a simple linear relationship

[5] See Galí (2008) for an example of this.

[6] In practice, central banks tend to prefer inflation levels slightly above zero because deflation is viewed as more harmful than inflation. In our model, there is no difference between inflation and deflation in terms of their impact on the loss function, and hence a target of zero is natural.

[7] Most models of this type are built in discrete time and have no fiscal deficit in the IS equation, since the purpose is usually to analyze monetary policy only. See Clarida et al (1999) and Saulo et al (2012) for examples of this. For a version in continuous time, see Cochrane (2017).

[8] In the literature, it is often assumed that shocks to output (and consequently output deviation itself) converge towards zero over time. This is typically done by modeling the shock as a stationary $AR(1)$ process. Galí (2008) uses this method. The third term in (3) ensures that our model has this desirable property, even though we do not model the shock explicitly. For instance, assume that policy is conducted such that the first two terms are zero at all time. The IS equation then reduces to $\dot{x} = -\varepsilon x$, which can be solved for $x = x_0 e^{-\varepsilon t}$, where x_0 is the initial condition (the size of the shock). Hence, output converges to zero.

between unemployment deviation and output deviation, the continuous time version of the Philips relation of Blanchard et al becomes (4).[9]

Notice that the fiscal deficit and the nominal interest rate appear in equation (3), but not in equation (4). This implies that the policy makers have a rather direct control over output, while their control over inflation is only *through output*. In particular, inflation will decrease if and only if output is below zero, regardless of policy. Hence, in the case of excess inflation, the benefits of bringing inflation down must be balanced against the cost of low output. This trade-off will be important when we interpret the simulations of our model. We stress that this key assumption is not unique for our model. Similar assumptions can be found in Galí (2008), Woodford (2003) and Blanchard et al (2010).[10]

3.1. The Cooperative Solution

We shall first solve our model for the cooperative game (the case of policy coordination), as this will yield results that simplify solving the competitive game. Hence, we need to solve a single dynamic optimization problem, where the objective function is a convex combination of the loss functions for the authorities:

$$L_C = \theta L_F + (1-\theta)L_M. \tag{5}$$

Here, θ is a parameter of arbitrary value in the open unit interval. We will solve for it later when we find the Nash bargaining product. For the other parameters of the model, at this analytical stage, the only constraint we will impose is that they are all positive. This is sufficient for deriving the results we need.

Formally, the optimization problem we need to solve can be summarized as

$$max_{g,i} \int_0^\infty e^{-\rho t}(-(\theta\lambda_{xF} + (1-\theta)\lambda_{xM})x^2 - (\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M})\pi^2 \\ - \theta\lambda_{gF}g^2 - (1-\theta)\lambda_{iM}(i-\rho)^2)dt,$$

$$s.t. \quad \dot{x} = \beta g - \alpha(i - \pi - \rho) - \varepsilon x,$$

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[9] Assuming $\mu = 1$, which would mean rational expectations, we get the difference equation $\pi_t - \pi_{t-1} = -\kappa(u_t - u_n)$. To get from unemployment to output, we apply the following argument, which is also from Blanchard et al (2010). If output deviation is currently positive, unemployment deviation must be negative, since additional labor is needed to produce the extra goods. The opposite is true if output deviation is negative. This relation is generally known as Okun's Law. Now, a decrease in unemployment shifts the balance of power in the labor market in the favor of the suppliers of labor, which puts upwards pressure on wages. Firms must react to the higher wages by either lowering profits or increasing the growth rate of the prices on their goods. In competitive markets where profit margins are low, the latter option must dominate, and so the growth in prices accelerate. Hence, we may write the Philips relation as $\pi_t - \pi_{t-1} = -\kappa(-x_t)$, which in continuous time becomes (4).

[10] The trade-off is often captured mathematically by a "sacrifice ratio". It which denotes how much excess unemployment (or low output) is needed bring inflation down by one percent. See Blanchard et al (2010) for a discussion on this.

$$\dot{\pi} = \kappa x,$$

$$x(0) = x_0, \pi(0) = \pi_0.$$

This is a linear-quadratic optimization problem. To find the feedback solution, we adopt the Hamilton-Jacobi-Bellman procedure. Hence, we set up the HJB equation for the optimization problem,

$$\begin{aligned} \rho V = max_{g,i} \Big\{ &-(\theta\lambda_{xF} + (1-\theta)\lambda_{xM})x^2 - (\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M})\pi^2 \\ &- \theta\lambda_{gF} g^2 - (1-\theta)\lambda_{iM}(i-\rho)^2 \\ &+ (\beta g - \alpha(i - \pi - \rho) - \varepsilon x)\frac{\partial V}{\partial x} + \kappa x \frac{\partial V}{\partial \pi} \Big\}. \end{aligned} \tag{6}$$

Here, V is the value function (the negative of the cost or loss) and $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial \pi}$ are its derivatives with respect to output and inflation. First, we derive the optimal controls as functions of $\frac{\partial V}{\partial x}$,

$$g = \frac{\beta}{2\theta\lambda_{gF}}\frac{\partial V}{\partial x}, \tag{7}$$

$$i = \rho - \frac{\alpha}{2(1-\theta)\lambda_{iM}}\frac{\partial V}{\partial x}. \tag{8}$$

Substituting these back into the HJB equation yields $0 = -\rho V - (\theta\lambda_{xF} + (1-\theta)\lambda_{xM})x^2 - (\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M})\pi^2 + \sigma(\frac{\partial V}{\partial x})^2 + (\alpha\pi - \varepsilon x)\frac{\partial V}{\partial x} + \kappa x \frac{\partial V}{\partial \pi}$ where $\sigma = \frac{\beta^2}{4\theta\lambda_{gF}} + \frac{\alpha^2}{4(1-\theta)\lambda_{iM}}$. We now need a conjecture for the form of the value function. Since the objective function is quadratic, we conjecture a value function which is a quadratic form in output and inflation,[11]

$$V = Ax^2 + Bx + C\pi^2 + D\pi + Ex\pi + F. \tag{9}$$

Here, A to F are constants to be determined. Next, substitute this value function and its derivates with respect to output and inflation into the HJB equation, and group the terms based on powers of x and π. The HJB equation then tells us that the sum of these six groups of terms must be equal to zero. Since this must hold regardless of the values of output and inflation, all the groups themselves must equal zero. Then, we have a system of six equations in the six constants A to F,

$$4\sigma A^2 - (\rho + 2\varepsilon)A - (\theta\lambda_{xF} + (1-\theta)\lambda_{xM}) + \kappa E = 0, \tag{10}$$

$$4\sigma AB - (\rho + \varepsilon)B + \kappa D = 0, \tag{11}$$

$$\sigma E^2 + \alpha E - (\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M}) - \rho C = 0, \tag{12}$$

$$2\sigma BE + \alpha B - \rho D = 0, \tag{13}$$

[11] See Engwerda (2006) for a discussion on value function conjectures in linear-quadratic differential games.

$$4\sigma AE - (\rho + \varepsilon)E + 2\alpha A + 2\kappa C = 0, \tag{14}$$

$$\sigma B^2 - \rho F = 0. \tag{15}$$

To solve this system, solve (14) for C and substitute into (12). Then, (10) and (12) form a system of two nonlinear equations in A and E. Depending on parameter values, this system may have up to four solutions. Solving for them explicitly is not possible, so we will solve for them numerically once parameter values have been assigned. The question remains, however, of which of the four solutions to use. In Appendix A, we provide an economic argument for why all the solutions except one can be ignored. We also prove that in the remaining solution, and for all admissible parameter values, it is the case that $A < 0$ and $E < -\alpha/2\sigma$, which will be important later.

Substituting A and E into the equations (11) and (13-15), we can solve for $C = (\sigma E^2 + \alpha E - (\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M}))\rho^{-1}$, and we can easily find that B, D and F are all equal to zero. Hence, the value function for this problem simplifies to $V = Ax^2 + C\pi^2 + Ex\pi$. The two conditions for the value function to be concave are $2A < 0$ and $4AC - E^2 > 0$.[12] We have already concluded that A is negative. As for the second condition, whether it holds will depend on the parameter values we assign.[13]

Having found V, we can substitute $\frac{\partial V}{\partial x}$ into (7) and (8) to find the optimal controls,

$$g = \frac{\beta}{2\theta\lambda_{gF}}(2Ax + E\pi), \tag{16}$$

$$i = \rho - \frac{\alpha}{2(1-\theta)\lambda_{iM}}(2Ax + E\pi). \tag{17}$$

Next, substituting the optimal controls into the dynamics, we get an autonomous system of two differential equations in output and inflation,

$$\begin{pmatrix} \dot{x} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} 4\sigma A - \varepsilon & 2\sigma E + \alpha \\ \kappa & 0 \end{pmatrix} \begin{pmatrix} x \\ \pi \end{pmatrix}. \tag{18}$$

Clearly, the equilibrium point for this system is at the coordinates $x = 0$ and $\pi = 0$. Making use of our set of initial conditions, we can solve the system for the unique paths for output and inflation. Substituting those solutions back into (16) and (17), we also have unique paths for the fiscal deficit and the nominal interest rate. For future use, we will label these optimal paths in the cooperative game $x^*_{CP}(t)$, $\pi^*_{CP}(t)$, $g^*_{CP}(t)$ and $i^*_{CP}(t)$. We also need to show that the system (18) is globally stable.[14] To prove stability, we calculate the system's trace, which

[12] These conditions are derived from the principal minors of the Hessian matrix for the value function.

[13] For all the sets of parameter values we use in this paper, the condition is satisfied.

[14] Stability is important because having a system which does not return to its equilibrium after a shock makes no sense economically. To make sense of the system, we need a unique equilibrium point which is a sink, so that whatever initial conditions we choose for output and inflation, the economy will eventually converge to the equilibrium.

is $4\sigma A - \varepsilon < 0$, and its determinant which is $-\kappa(2\sigma E + \alpha) > 0$. The inequalities follow from the constraints on the solutions for A and E as described above. This is sufficient for stability.

It remains to find θ, the weight on the loss functions for the cooperative game. For that purpose, we first need to calculate the discounted stream of instantaneous costs for each agent as functions of the optimal strategies, as evaluated at time zero. To do this, we simply insert the optimal equilibrium paths for the variables into the loss functions of the fiscal and monetary authorities,

$$L_F^* = \int_0^\infty e^{-\rho t}(-\lambda_{xF} x_{CP}^*(t)^2 - \lambda_{\pi F}\pi_{CP}^*(t)^2 - \lambda_{gF} g_{CP}^*(t)^2)dt, \qquad (19)$$

$$L_M^* = \int_0^\infty e^{-\rho t}(-\lambda_{xM} x_{CP}^*(t)^2 - \lambda_{\pi M}\pi_{CP}^*(t)^2 - \lambda_{iM}(i_{CP}^*(t) - \rho)^2)dt. \qquad (20)$$

The equilibrium paths for the four variables all depend on A and E, both of which depend on the parameter θ. As we move θ over the unit interval, we trace out a correspondence in the (L_F^*, L_M^*) space. Since we cannot solve for A and E explicitly, we cannot know this correspondence. The Nash bargaining solution is the value θ^* that maximizes the product of the deviations of L_F^* and L_M^* from some threat point with the values L_F' and L_M'. That is, we want to solve the problem[15]

$$max_\theta \ (L_F^* - L_F')(\ L_M^* - L_M'). \qquad (21)$$

In our case, the threat point is defined by the costs that the fiscal and monetary authorities would incur without cooperation. The intuition is that each authority wishes to cooperate if and only if it will reduce its cost by doing so. Thus, to find the threat point we need to solve the competitive game.

3.2. The Competitive Solution

We will now solve the game for when the fiscal and monetary authorities act independently, without cooperating. In doing so we must solve a dynamic optimization problem for each agent separately, derive their best response functions and solve for the Nash equilibrium of the game.

For the fiscal authority, the problem can be formally summarized as

$$max_g \int_0^\infty e^{-\rho t}(-\lambda_{xF} x^2 - \lambda_{\pi F}\pi^2 - \lambda_{gF} g^2)dt,$$

$$s.t. \ \ \dot{x} = \beta g - \alpha(i - \pi - \rho) - \varepsilon x,$$

$$\dot{\pi} = \kappa x,$$

[15] Without explicit values for A and E, this optimization problem must be solved numerically. Of course, the solution will depend on what values we choose for the parameters of the model.

$$x(0) = x_0, \pi(0) = \pi_0.$$

The HJB equation for this problem has almost the same structure as the one in the cooperative game. Again, the appropriate conjecture for the value function is a quadratic form in output and inflation, $V_F = A_F x^2 + B_F x + C_F \pi^2 + D_F \pi + E_F x\pi + F_F$, where the subscript F is used to differentiate the value function of the fiscal authority's problem from that in the cooperative game. The optimal control for the fiscal authority is

$$g = \frac{\beta}{2\lambda_{gF}} \frac{\partial V_F}{\partial x} = \frac{\beta}{2\lambda_{gF}} (2A_F x + B_F + E_F \pi). \tag{22}$$

It differs from the optimal control in the cooperative game only in that the parameter θ is missing. As before, the value function can be substituted into the HJB equation to produce a system of six equations in six unknown variables,

$$4\sigma_F A_F^2 - (\rho + 2\varepsilon)A_F - \lambda_{xF} + \kappa E_F = 0, \tag{23}$$

$$4\sigma_F A_F B_F - (\rho + \varepsilon)B_F - 2\alpha A_F (i - \rho) + \kappa D_F = 0, \tag{24}$$

$$\sigma_F E_F^2 + \alpha E_F - \lambda_{\pi F} - \rho C_F = 0, \tag{25}$$

$$2\sigma_F B_F E_F + \alpha B_F + \alpha E_F (i - \rho) - \rho D = 0, \tag{26}$$

$$4\sigma_F A_F E_F - (\rho + \varepsilon)E_F + 2\alpha A_F + 2\kappa C_F = 0, \tag{27}$$

$$\sigma_F B_F^2 - \alpha B_F (i - \rho) - \rho F = 0. \tag{28}$$

Here, $\sigma_F = \beta^2 / 4\lambda_{gF}$. To solve the system, first solve for A_F and E_F using, as before, the method outlined in Appendix A. We can prove restrictions on the parameters that are analogous to those in the cooperative game, i.e. $A_F < 0$ and $E_F < -\alpha/2\sigma_F$. Then the remaining variables form a system of the four equations (24) and (26-28) in four unknown variables, which can easily be solved. Unlike in the cooperative game, none of the parameters A_F to F_F are zero. This is because of the minor but crucial difference that system (23-28) contains terms with the nominal interest rate, i. We are especially interested in B_F, since we need it to derive the best response function. In solving the system, we get

$$B_F = \frac{-(2\rho A_F + \kappa E_F)\alpha(i - \rho)}{\rho^2 - 4\rho\sigma_F A_F + \rho\varepsilon - \kappa(2\sigma_F E_F + \alpha)} = f_{B_F}(A_F, E_F)(i - \rho). \tag{29}$$

Given the restrictions on A_F and E_F, as described in the previous paragraph, it must hold that $f_{B_F}(A_F, E_F) > 0$. The best response function for the fiscal authority is then

$$BR_F = \frac{\beta}{2\lambda_{gF}} \big(2A_F x + f_{B_F}(A_F, E_F)(i - \rho) + E_F \pi\big). \tag{30}$$

Notice that since an explicit analytical solution of A_F and E_F cannot be derived, this is as far as we can go without assuming values for the parameters.

Some remarks on the properties of the best response function are in order. Firstly, an increase in output yields a contractionary response from the fiscal authority. The reason is that the output deviation is costly, and the government will want to bring it back to zero. Secondly, an increase in the nominal interest rate (a contractionary monetary policy) will yield an expansionary fiscal response. This is because, everything else equal, the monetary contraction will decrease output, which prompts the government to increase output by raising the deficit. Thirdly, an increase in inflation will yield a contractionary response. Here the reason is that given no change in the nominal interest rate, higher inflation will decrease the real interest rate, which acts expansionary on the economy. Hence, the government must respond with a contractionary fiscal policy to stabilize output. Finally, note that the magnitude of the best response decreases in the parameter λ_{gF}: neither A_F nor E_F depends on λ_{gF} so the effect is unambiguous. Recall that λ_{gF} denotes the cost associated with deviating from a balanced budget. If the cost is high, the fiscal authority will be more hesitant in using its policy instrument, and this is reflected in the best response function.

We now turn to the monetary authority's optimization problem, which can be formally summarized as

$$max_i \int_0^\infty e^{-\rho t}(-\lambda_{xM}x^2 - \lambda_{\pi M}\pi^2 - \lambda_{iM}(i-\rho)^2)dt,$$

$$s.t.\;\; \dot{x} = \beta g - \alpha(i - \pi - \rho) - \varepsilon x,$$

$$\dot{\pi} = \kappa x,$$

$$x(0) = x_0, \pi(0) = \pi_0.$$

This problem is structurally the same as that of the fiscal authority, and can be solved, step by step, in the same way. Thus, we can derive a system of six equations in the six unknown variables A_M to F_M, solve the system and derive the best response function for the monetary authority:

$$BR_M = \rho - \frac{\alpha}{2\lambda_{iM}}\big(2A_M x + f_{B_M}(A_M, E_M)g + E_M\pi\big). \tag{31}$$

Here, A_M and E_M are both negative parameters, and $f_{B_M}(A_M, E_M)$ is positive. Again, some remarks will show how the best response function captures our intuition for monetary policy. The central bank will respond to an increase in output by raising the nominal interest rate, with the purpose of pushing output deviation back towards zero. Likewise, a fiscal expansion will also yield a contractionary response from the central bank. The same is true for an increase in inflation, the reason being that when inflation increases, the nominal interest rate must also increase so that the real interest rate is stabilized. Also, like in the case of the fiscal authority, we note that the cost of using the policy instrument, which for the central bank is captured by λ_{iM}, reduces the use of that instrument.

The best response functions are linear, and can therefore easily be combined to solve for the fiscal deficit and the nominal interest rate as functions of output and inflation only. Inserting those two equations into the dynamics of the problem, equations (3) and (4), we have an autonomous system in output and inflation. Along with the initial conditions, the system can be solved for the equilibrium paths. However, unlike in the cooperative game, there's no guarantee that the system will be globally stable. That will depend on the parameter values that we choose in calibration. Fortunately, it turns out that for the set of parameter values used in this paper, and for the other "economically reasonable" parameter values that we have checked, the system will be stable.

Finally, we need to derive the threat point for the Nash bargaining solution. That is, we need to find the values of L'_F and L'_M in (21). To do this, we proceed as in the cooperative game by inserting the equilibrium paths for the variables into the loss functions and integrate them over the planning horizon with discounting. However, note that unlike the costs of the cooperative game, which depend on θ, the costs of the competitive game depend only on parameters for which we assign values directly. Once L'_F and L'_M have been found, the Nash bargaining solution can be derived by solving problem (21). Hence, we have found θ.

4. The Simulation Study

In this section, we assign parameter values to our model and investigate how the economy responds to disequilibrium. We will analyze two sets of objectives for the authorities: in the first, the authorities prioritize inflation stability over output stability, and in the second vice versa. We can think of these sets as corresponding to inflation targeting and output targeting. As explained in the introduction, we should expect policy coordination to be more important in the first case.

We will also analyze two sets of initial conditions. First, the case where output and inflation both start *above* their equilibrium levels. We can think of this as a demand shock: an increase in demand has raised output, which in turn has raised inflation. The government and the central bank are faced with the problem of steering both variables back towards the preferred levels. The second set of initial conditions we study is that when output is initially *below* but inflation is *above* the equilibrium levels. We can think of this as a supply shock, since such shocks typically move output down and inflation up.[16] As we shall see, the demand shock scenario is the more interesting case. For both types of shocks, we will assume that the initial deviation of the variables from the equilibrium levels is two percent.

[16] The terms "demand shock" and "supply shock" refer, of course, to whether the underlying source of the shock comes from the demand side or the supply side of the economy. They do not refer to the way in which the shocks affect the economic variables. In this paper, we make no assumption about the sources of the shocks, we are only interested in how they affect inflation and output. Hence, the interpretations we make are convenient but not necessary.

The parameter values for our simulations are as follows. Following Galí (2008), we set $\varepsilon = 0.69$.[17] For the coefficients on the fiscal deficit, we set $\beta = 0.5$. Thus, a one percent increase in the fiscal deficit increases output by half a percent. This is not unrealistic: if we imagine that the government's share of the economy is 50 percent, our value of β implies that the fiscal deficit increases the output one-for-one in terms of actual units of output. This fits with the common assumption that total spending in the economy, and hence total output, equals the sum of public and private spending.[18]

For the coefficient on the nominal interest rate we set $\alpha = 0.5$. In Galí (2008), where the IS equation is derived rather than assumed, it turns out that α is the intertemporal elasticity of substitution in the optimization problem of the household. Hence, we should set α close to empirical estimates of that preference parameter. In a meta-analysis by Havanek et al (2013) it is concluded that a typical value of the intertemporal elasticity of substituion is around one half, which motivates our decision.[19]

Consequently, our calibration satisfies $\alpha = \beta$, which means that the fiscal and monetary authorities are "equally efficient" in stabilizing the economy. This assumption will be convenient because it implies that no aspects of the adjustment towards equilibrium arise because one authority is more efficient than the other. Recall that what we are interested in analyzing is the impact of the objectives.

The remaining parameter values are all taken from van Aarle et al (1999) and van Arlee et al (2001). We set $\kappa = 0.25$, $\rho = 0.1$, $\lambda_{gF} = 5$ and $\lambda_{iM} = 2.5$. For the case when the authorities prioritize inflation we set $\lambda_{\pi F} = 5$, $\lambda_{xF} = 2$, $\lambda_{\pi M} = 2.5$ and $\lambda_{xM} = 1$. For the case where they prioritize output we set $\lambda_{\pi F} = 2$, $\lambda_{xF} = 5$, $\lambda_{\pi M} = 1$ and $\lambda_{xM} = 2.5$. Note that the parameter values are greater for the government than they are for the central bank. This reflects an assumption that the government is "more responsible" than the central bank for the overall state of the economy.

In summary, we have four scenarios to consider: two types of shocks and for each shock two sets of objectives. In the analysis that follows, figures 1 to 4 provide impulse response functions for the four variables in our model, for each scenario. Table 1 provides the value of θ and the gains from coordination. We present the gain for each authority, and the average of the two. We will be particularly interested in the average gain, since that is our measure of the overall improvement in the outcome under coordination. Following Turnovsky et al (1987), we present the gains in terms of the percentage deviation of the loss in the cooperative solution from the loss in the competitive solution.

[17] This means that about 50 percent of the shock fades away each period, as $e^{-0.69*1} \cong 0.5$. It is the same level as that in Galí (2008), where a different method is used for modeling the shock.

[18] Another way to think of β is as a measure of how well the economy obeys Ricardian equivalence. That is, it measures to what extent households respond to an increase in the fiscal deficit by simply saving more rather than consuming, in the knowledge that a decrease in the fiscal deficit must come at some point in the future. The more our economy obeys Ricardian equivalence, the smaller will the effect of a government expansion be on output, and hence the smaller is β.

[19] In van Arlee et al (2001), from where we take most other parameter values for our model, the value of α is 0.4.

	Gov. Gain (%)	CB Gain (%)	Average Gain (%)	NB Weight
Demand shock under Inflation Targeting	66.05	65.84	65.96	0.22
Demand shock under Output Targeting	24.93	4.71	14.82	0.25
Supply shock under Inflation Targeting	44.29	35.16	39.73	0.13
Supply shock under Output Targeting	0.01	15.91	7.96	0.35

Table 1: The gains from coordination under two types of shocks and objectives.

4.1 The Demand Shock

We begin with the case where the authorities prioritize inflation over output, displayed in Figure 1. Notice first that in the competitive solution, the government reacts to the shock by lowering the fiscal deficit, that is, by enacting a contractionary policy. The purpose is to reduce the costly deviation of output and bring it down towards the equilibrium level. The central bank, on the other hand, reacts by lowering the nominal interest rate, which has an expansionary effect. Hence, in the competitive solution the authorities enact policies that are contradictory. This situation continues throughout the whole adjustment process. Indeed, after ten periods the fiscal deficit and the nominal interest rate are still two and one percent below the equilibrium levels. Now, since the fiscal contraction is more aggressive than the monetary expansion, as measured by the deviation from the equilibrium levels, the overall effect of policy on output is contractionary. Because of this, and because the shock itself is fading away, output decreases. As for inflation, it increases for about one period, and then begins a slow convergence towards zero.

The striking result in the competitive solution is that the authorities are, between themselves, engaging in policy that is costly, but that has no effect on the stabilization. When one authority enacts expansionary policy and the other enacts contractionary policy, the overall effect will be the net effect of the two. Clearly, then, the competitive solution is inefficient, in the sense that it has room for Pareto improvements.

How can we understand this contradictory behavior of the authorities, and why is it the government who acts expansionary rather than the central bank? Mathematically, the answer can be found in the best response functions, equations (30) and (31). In both equations, the fiscal deficit and the nominal interest rate have a positive relationship. Hence, all other things equal, a reduction in the fiscal deficit causes a nominal expansion.

More importantly, what is the economic interpretation? Notice first that both authorities have an interest in decreasing output, since the positive deviation produced by the shock is costly. However, we have assumed that the government has a larger overall responsibility for the economy than the central bank has. Therefore, the government perceives deviation in output and inflation as costlier relative to deviation in its instrument than the central bank does. This implies that the government prefers a faster adjustment and is willing to incur more cost from using its instrument. Consequently, the government's aggressive policy results in a faster and more volatile adjustment than what the central bank would consider optimal. The central bank then responds by "canceling out" some of the government´s policy, which is done by lowering the nominal interest rate. This situation, in which the authorities attempt to steer the economy in different directions, is of course a result of the lack of coordination.

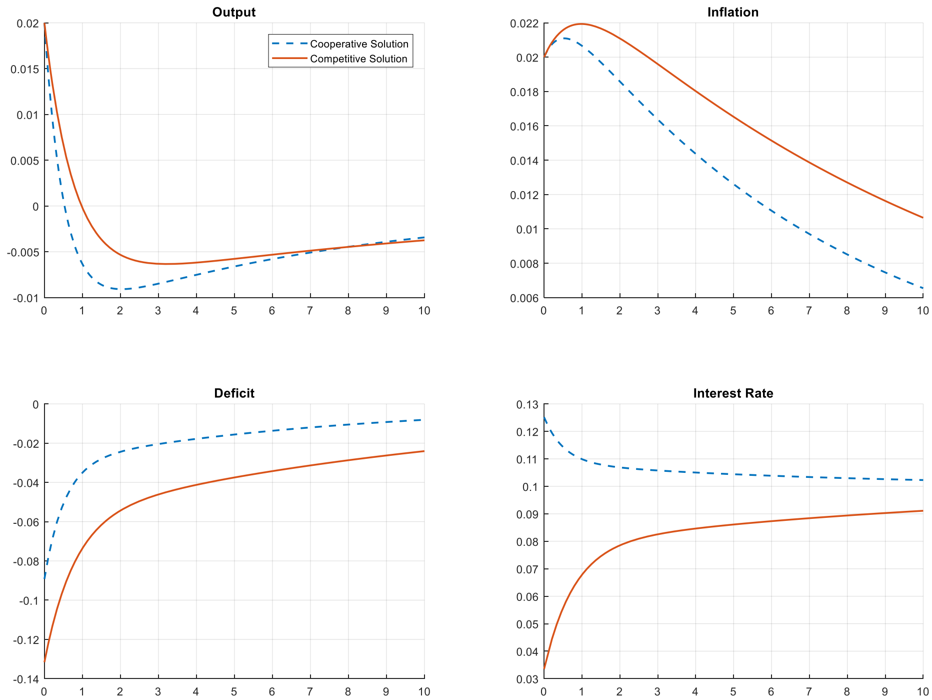

Figure 1: A demand shock when the authorities prioritize inflation over output.

Looking next at the cooperative solution, we see that the problem of contradictory policies has been removed entirely. Now both authorities are enacting contractionary measures, as would be natural under a positive demand shock. We also see that the deviation of both instruments from their equilibrium levels are smaller than in the competitive solution, which lowers the cost inherent in policy activism.

When both authorities are contracting the economy, the response of output is much more aggressive. However, as in the competitive solution, the decrease in output does not stop at the equilibrium level, but is continuing down into the negative. The reason is that, given how we defined our Phillips relation, inflation will decline if and only if output is negative. Hence, to bring inflation down, the authorities must in effect bring output down below its equilibrium level. Notice how, once output is in the negative, inflation immediately start declining and continues to do so for the rest of the adjustment.

Finally, we point out the important difference that the induced decrease of output below its equilibrium level is considerably larger in the cooperative solution. To understand this, recall that in the scenario described in Figure 1, the main concern of the authorities is inflation. For this reason, it is rational to suffer some extra cost from a negative output to have the benefits of lower inflation. Notice how in the cooperative solution, inflation returns to the equilibrium level faster. This is a direct result of the fact that the authorities were willing to suffer a larger decrease in output a few periods prior. Now, coordination allows the authorities to enact greater control over the adjustment, and they use that to increase deviation of the variable they care less about (output) and decrease the deviation of the variable they care more about (inflation). That is, they adjust the trade-off in a way that they both benefit from.

We now turn to the second demand shock scenario, where the authorities prioritize output over inflation, displayed in Figure 2. Here, the overall picture is much like that in the previous scenario: the competitive solution has contradictory policies which are fully resolved in the cooperative solution, and the authorities bring down inflation by inducing a decrease of output below the equilibrium level. However, the gains from coordination are only a fraction of what they were with inflation targeting, as can be seen in Table 1. The reasons for this difference are as follows. First, notice how in the competitive solution, monetary policy is only expansionary for the first period. After that, the nominal interest rate is *above* its equilibrium level, and the policies are no longer contradictory. Hence, the problems of no coordination are smaller than they were with inflation targeting. Next, notice that the drop of output down below zero is of approximately the same size with or without coordination. This contrasts with the scenario with inflation targeting, where the induced recession was larger with coordination.

The underlying cause of these differences is that when the initial disequilibrium is a demand shock, one must fully stabilize output before one can start stabilizing inflation. That is, only after output has been brought down to zero is it possible for inflation to start declining. In short, output is an “easier target”. It then follows that, to the extent that the authorities prioritize output over inflation, the overall difficulty in handling the shock is reduced. When the task of stabilization is easier, the cost from not coordinating that task is smaller, and hence the benefits from coordination are also smaller, which is what we see in Table 1. To better understand this aspect of our model, it is useful to return to the dynamics in equations (3) and (4). We recall that the instruments only appear in the dynamics for output, not for inflation. Hence, the authorities can only affect inflation through output, and in this sense, they have a more direct control over output than they do over inflation.

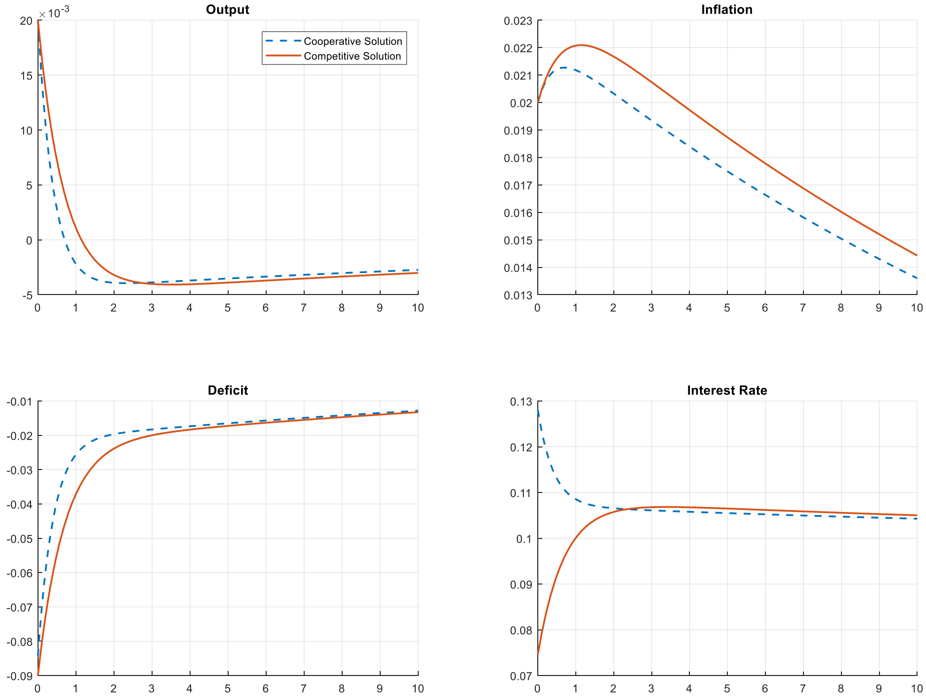

Figure 2: A demand shock when the authorities prioritize output over inflation.

In summary, the main conclusions from the demand shock scenarios are the following:

1. There is gain from coordination. The gain arises primarily because coordination eliminates contradictory policies that are costly but don't contribute to stabilization.
2. The gains from coordination are larger when the authorities exercise inflation targeting. The reason is that the control of policy makers over inflation is more indirect than their control over output, which increases the need for coordination.
3. The objectives matter for how fast output and inflation return to their equilibrium levels. Both variables converge faster when they are prioritized.

4.2. The Supply Shock

We will now look at the situation where the initial disequilibrium is such that inflation starts above and output starts below the equilibrium levels. As before, we shall first look at the scenario where the authorities prioritize inflation over output, displayed in Figure 3.

In the competitive solution, the government responds to the shock by lowering the fiscal deficit. The contraction is quite small though: at most, the deficit is only three percent below the equilibrium level. The government is attempting to decrease output, despite that it's already negative. The reason is that the main concern of the government is inflation, which will decrease faster if output is lowered further. Also, since the shock is fading away, there is upward pressure on output, so contractionary policies are needed just to maintain the same level. As can be seen in Figure 3, output does indeed increase despite the negative fiscal deficit.

The central bank initially raises the nominal interest rate, but then moves it down below the equilibrium level, where it stays for the rest of the adjustment. Hence, the competitive solution suffers from the same problem that we discussed in the demand shock scenario: the government and the central bank are working against each other by enacting policies that are contradictory. Again, the problem is resolved in the cooperative solution, in which both authorities work towards decreasing the speed at which output converges to the equilibrium level. The result is a slower convergence of output, which results in a faster convergence of inflation. Since the authorities prioritize inflation, this trade-off is beneficial.

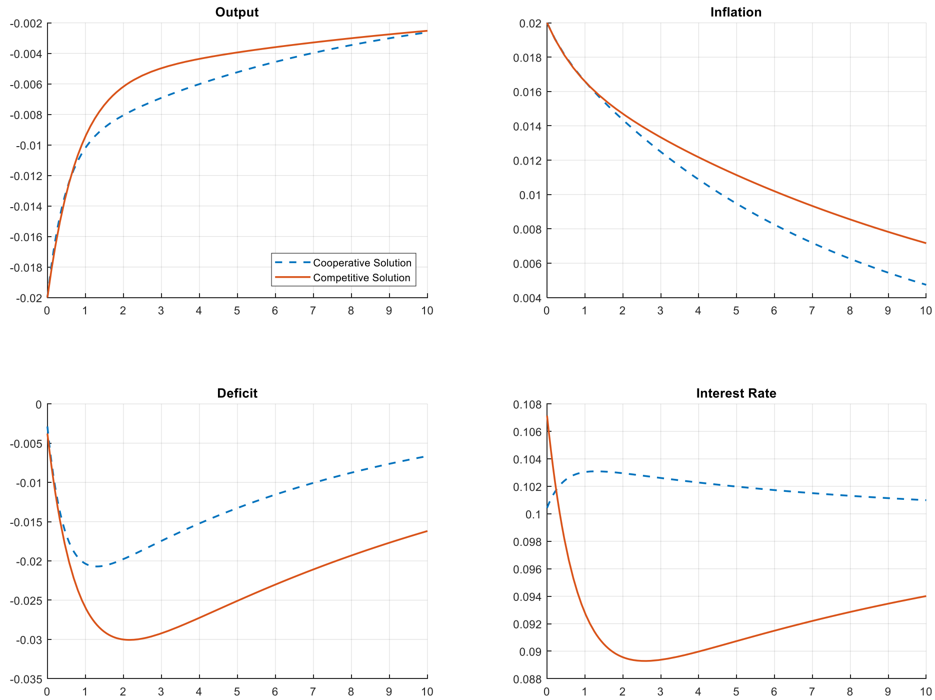

Figure 3: A supply shock when the authorities prioritize inflation over output.

One major difference compared to the demand shock scenario, is that there is no need for the authorities to drive output to the other side of the equilibrium level to stabilize inflation. Output is already negative, which is necessary and sufficient for inflation to decline. Thus, what we see is a gradual adjustment in which both variables slowly converge to their equilibrium levels. In this sense, the supply shock is easier for the authorities to handle than the demand shock. Then, we should expect the need for – and gain from – coordination to be lower in supply shock scenario. This is indeed what we find in Table 1: the gain from coordination under inflation targeting is 66 percent in the demand shock scenario and 40 percent in the supply shock scenario.

Let's now consider the supply shock scenario under output targeting, displayed in Figure 4. The general pattern is a lot like that with inflation targeting. Notice, however, that in the competitive solution, the central bank is consistently contracting the economy, while it is the government that moves from an initial expansion to a contraction: the roles have been reversed compared to when they exercised inflation targeting. As always, the cooperative solution removes contradictory policies: both authorities begin with an expansion and then switch to a contraction at the same time (after about one third of the first period). Thus, they are always steering the economy in the same direction.

Next, recall that under inflation targeting, output converged slower and inflation converged faster in the cooperative solution. With output targeting, we see the opposite: coordination causes an overall less contractionary policy, which allows output to return towards its equilibrium level faster. Consequently, the return of inflation is slower. This, of course, is to be expected.

As for the gain from cooperation, we see that it is the smallest in this scenario, about eight percent. This is reflected in Figure 4. For instance, the fiscal deficit develops in almost the same way with or without cooperation, and the paths for output and inflation are also quite similar. Indeed, the gain from cooperation for the government is virtually zero: the overall gain comes mostly from less volatility in the nominal interest rate.

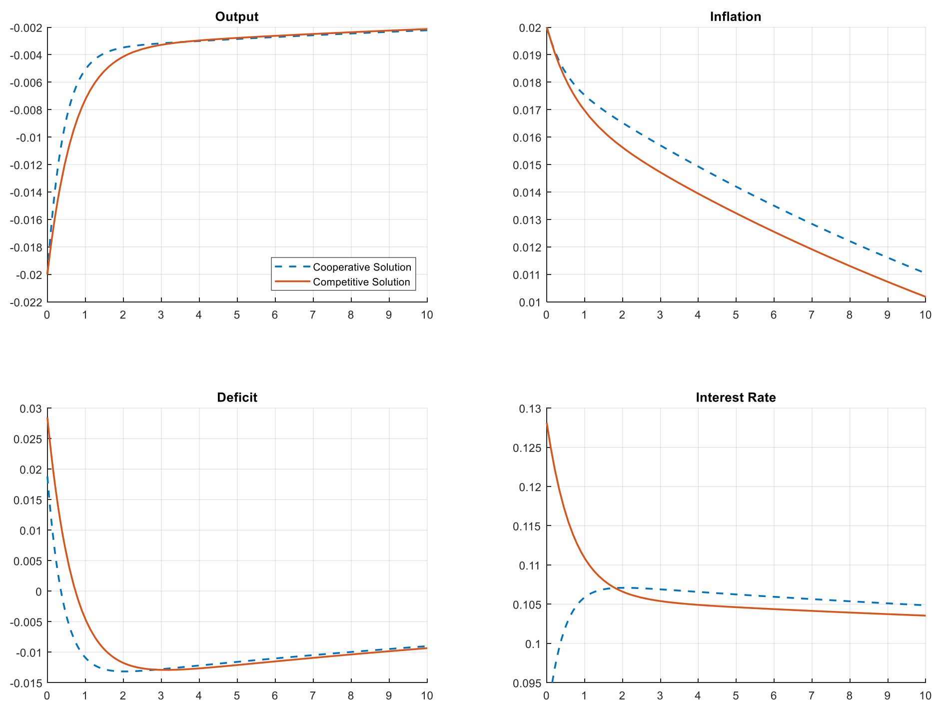

Figure 4: A supply shock when the authorities prioritize output over inflation.

Summarizing the supply shock scenarios, we see that they satisfy the same three points that we observed for the demand shock scenarios. We may add a fourth observation.

4. The gains from coordination are larger in the demand shock scenarios than in the supply shock scenarios. This is because the demand shock is more difficult to handle, i.e. it requires more fine tuning in the policy response, which increases the need for coordination.

Finally, we notice that in all four scenarios described above, the Nash bargaining weight θ is smaller than one half. Recall that θ determines the weights attached to the loss function of each agent in the solution to the cooperative game. If θ is small (below one half), the fiscal authority is "favored" in the cooperative solution and will suffer a smaller individual loss in stabilizing the economy after a shock. This would imply a large response in the fiscal deficit and a small response in the nominal interest rate (which is costly only for the monetary authority). What we generally find in Figures 1 to 4 is that the fiscal deficit is more volatile than the nominal interest rate in the cooperative solution (and in the competitive solution). The government is taking on more of the burden of stabilization, and this is reflected in the value of θ.

5. Conclusion

In this paper, we build a simple macroeconomic model to investigate how the government and the central bank react to disequilibria under inflation targeting or output targeting. Our purpose is to measure to what extent policy coordination can help the policy makers to reach their objectives. We solve the model in a game theoretic setting and compare the competitive solution with the Nash bargaining cooperative solution. In our simulation we use realistic parameter values from well-known papers in the literature. We find that policy coordination is important because it allows the government and central bank to avoid hindering the actions of one another. We also find that the gains from coordination are larger under inflation targeting and under a demand shock.

The paper follows in the tradition of the literature that investigates the interaction of fiscal and monetary policy by using the theory of differential games. We have especially drawn inspiration from van Aarle et al (2001) and from Lambertini and Rovelli (2003). We have also made use of the literature on the New Keynesian model for monetary policy, in particular Galí (2008) and Blanchard et al (2010). However, the objective of this paper and its conclusions differ from those of the above-mentioned authors.

In conclusion, we will shorty address the question of welfare implications from policy coordination. In this paper, we have ignored the welfare effects of monetary and fiscal policy. We have only focused on the extent to which the policy makers are able to reach their objectives. Although those objectives are usually, at least in some sense, related to the overall welfare of the economy, we have made no such assumption here. In principle, one could incorporate a welfare loss function, which would be *separate* from the loss functions of the policy makers. This seems to have rarely been attempted in the literature of differential policy games, nor in the (mostly discrete time) literature on monetary policy, where the loss function of the central bank is often equated with welfare. One reason for this is surely that there's no clear answer to how much the variables in our model should affect welfare. For instance, does a negative fiscal deficit reduce welfare more than a positive fiscal deficit? How costly are deviations in the nominal interest rate relative to deviations in inflation? These questions are complicated. To answer them satisfactorily one would probably have to derive the welfare loss function (and the rest of the model too) from the behavior and objectives of households and firms, as is often done in models on monetary policy only. It is therefore convenient to ignore the concept of welfare and interpret the policy maker's loss functions simply as objectives, as we have done in this paper. The incorporation of a welfare function into our model could be the subject of future research.

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Appendix A

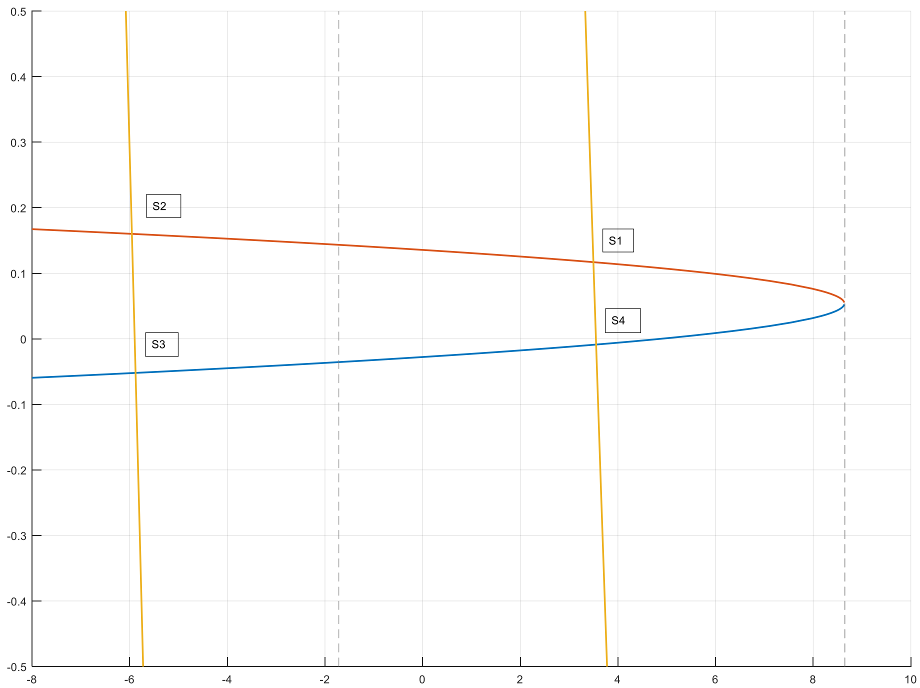
The system in A and E can be written as the following functions of E.

$$A = f_1(E) = \frac{(\rho + 2\varepsilon) + \sqrt{(\rho + 2\varepsilon)^2 + 16\sigma((\theta\lambda_{xF} + (1-\theta)\lambda_{xM}) - \kappa E)}}{4\sigma}$$

$$A = f_2(E) = \frac{(\rho + 2\varepsilon) - \sqrt{(\rho + 2\varepsilon)^2 + 16\sigma((\theta\lambda_{xF} + (1-\theta)\lambda_{xM}) - \kappa E)}}{4\sigma}$$

$$A = f_3(E) = \frac{E(\rho^2 + \rho\varepsilon - 2\kappa\alpha) + 2\kappa(\theta\lambda_{\pi F} + (1-\theta)\lambda_{\pi M}) - 2\kappa\sigma E^2}{2\rho(\alpha + 2\sigma E)}$$

Notice that the first two functions form the inverse of a parabola and that the third has a vertical asymptote at $E = -\alpha/2\sigma$. These functions are graphed below with the parameter values from the case with inflation targeting in Section 4. There are four intersection points, which we label S1 to S4.



First, we acknowledge that the solution for A must be negative. To see this, recall that the value function must be concave, which implies that $\frac{\partial^2 V}{\partial x^2} = 2A < 0$. Also, notice that the solution points S1 and S2 must have a positive A. This follows from the fact that $f_1(E) > 0$ for all E for which $f_1(E)$ is defined. But then, S1 and S2 cannot be solutions.

Next, we look at the solution S4. This solution must be to the right of the vertical asymptote of $f_3(E)$, and hence it holds that $E > -\alpha/2\sigma$. But this implies that the system (18) is not stable, which makes the solution economically unsuited. We conclude that only the solution S3 remains.

Finally, two aspects of the solution S3 should be noted. First, there is no guarantee that $A < 0$ in this solution. Hence, there's no guarantee that the solution must not be ruled out for the same reason as S1 and S2. It will depend on the parameter values that we assign. Secondly, having solved for A and E, we should make sure that $C < 0$, which also is not guaranteed. (The argument for why C must be negative is analogous to the same argument for A).