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OWA OPERATORS IN HUMAN RESOURCE MANAGEMENT

Abstract. *We develop a new approach that uses the ordered weighted averaging (OWA) operator in different methods for the selection of human resources. The objective of this new model is to manipulate the neutrality of the old methods, so the decision maker can select human resources according to his degree of optimism or pessimism. In order to develop this model, first, a short revision of the OWA operators is introduced. Next, we briefly explain the general model for the selection of human resources and suggest three new indexes for the selection of human resources that use the OWA operator and the hybrid average in the Hamming distance, in the adequacy coefficient and in the index of maximum and minimum level. The main advantage of this method is that it is more complete than the previous ones so the decision maker gets a better understanding of the decision problem. The work ends with an illustrative example that shows the results obtained by using different types of aggregation operators in the new approaches.*

Keywords: *OWA operator, Selection of human resources, Hamming distance, Adequacy coefficient.*

JEL Classification D81, M12, M51

1. INTRODUCTION

The selection of the most appropriate human resources for the company represents a fundamental problem for its good development. The enterprise needs to analyze how to select the best worker according to its interests. In order to solve this problem, the company has to develop a selection process in which it has to compare the different characteristics of each available candidate found in the market with its ideals. Among the great variety of studies existing in selection, this work will focus on the models developed by Gil-Aluja (1998), Kaufmann and Gil-Aluja (1986; 1987) and Merigó and Gil-Lafuente (2008a) about selection of human resources, the models developed by Gil-Lafuente (1990; 2005), Gil-Lafuente and Merigó (2010) and Merigó and Gil-Lafuente (2007; 2008b; 2008c; 2008d; 2010) about financial and strategic management, and the models developed by Gil-Lafuente (2001; 2002) about selection of players in sport management. Note that these methods are based on the use of fuzzy subsets. For other methods see, for example, Canós and Liern (2008), Figueira et al., (2005), Karayiannis (2000) and Xu and Chen (2008).

One problem about these selection indexes is that they are neutral against the attitudinal character of the decision maker. Thus, when developing the selection process, we cannot manipulate the results according to the interests of the decision maker. This problem becomes important in situations where we want to under estimate or over estimate the decisions in order to be more or less prudent against the uncertain factors affecting the future. One common method for aggregating the information considering the decision attitude of the decision maker is the ordered weighted averaging (OWA) operator introduced by Yager (1988). Since its appearance, the OWA operator has been studied by different authors such as (Merigó, 2007; Merigó and Casanovas, 2010a; 2010b; 2010c; 2010d; 2010e; 2010f; Merigó et al., 2010; Merigó and Gil-Lafuente, 2008b; 2008c; 2009; Wang et al., 2009; Xu, 2005; Xu and Hu, 2010; Yager, 1993; 2009a; 2009b; 2010).

Our objective in this paper consists in developing new selection indexes that include the attitudinal character of the decision maker for the selection of human resources. These new indexes consist in combining the classical selection methods with the OWA operator and the hybrid average because then, the neutrality of the classical methods will be changed by the OWA operator. We introduce in the selection of human resources, the ordered weighted averaging distance (OWAD) operator (Merigó and Gil-Lafuente, 2010; Xu and Chen, 2008), the hybrid averaging distance (HAD) operator (Xu, 2008), the ordered weighted averaging adequacy coefficient (OWAAC) (Merigó and Gil-Lafuente, 2008c; 2010), the hybrid averaging adequacy coefficient (HAAC), the ordered weighted averaging index of maximum and minimum level (OWAIMAM) and the hybrid averaging index of maximum and minimum level (HAIMAM) operator. We also develop an illustrative example of the new approach where we can see different results by using different particular cases of these new methods. Thus, we see that each method may lead to different decisions.

This paper is organized as follows. In Section 2 we briefly describe the OWA operator. Section 3 explains the basic aspects of the selection of human resources with fuzzy techniques. In Section 4, we develop the process to follow when using the OWA operator with the Hamming distance in the selection of human resources. Section 5 analyzes the combination between the OWA operator and the adequacy coefficient and Section 6 the combination between the OWA operator and the index of maximum and minimum level. Finally, Section 7 gives an illustrative example of the suggested approach and Section 8 ends the paper with the main conclusions.

2. OWA OPERATORS

The OWA operator (Yager, 1988) provides a parameterized family of aggregation operators which have been used in many applications (Beliakov et al., 2007; Liu et al., 2010; Karayiannis, 2000; Merigó, 2007; 2010; Merigó and Casanovas, 2009; 2010; Merigó and Gil-Lafuente, 2009b; 2010; Wei et al., 2010;

Xu, 2005; Yager, 1993; Yager and Kacprzyk, 1997). In the following, we provide a definition of the OWA operator as introduced by Yager (1988).

Definition 1. An OWA operator of dimension n is a mapping F: $R^n \rightarrow R$ that has an associated weighting vector W of dimension n having the properties:

1) $w_j \in [0, 1]$
2) $\sum_{j=1}^{n} w_j = 1$

and such that

$$f(a_1, a_2,\ldots, a_n) = \sum_{j=1}^{n} w_j b_j \quad (1)$$

where b_j is the jth largest of the a_i.

A fundamental aspect of this operator is the reordering of the arguments, based upon their values. That is, the weights rather than being associated with a specific argument, as in the case of the usual weighted average, are associated with a particular position in the ordering. This reordering introduces nonlinearity into an otherwise linear process.

If B is a vector corresponding to the ordered arguments, we shall call this the ordered argument vector, and W^T is the transpose of the weighting vector, then the OWA aggregation can be expressed as:

$$f(a_1, a_2,\ldots, a_n) = W^T B \quad (2)$$

The OWA operator is a mean or averaging operator. This is a reflection of the fact that the operator is commutative, monotonic, bounded and idempotent. It is commutative because any permutation of the arguments has the same evaluation. It is monotonic because if $a_i \geq d_i$ for all i, then, $f(a_1,\ldots, a_n) \geq f(d_1,\ldots, d_n)$. It is bounded because $\text{Min}\{a_i\} \leq f(a_1,\ldots, a_n) \leq \text{Max}\{a_i\}$. It is idempotent because if $a_i = a$, for all i, then, $f(a_1,\ldots, a_n) = a$.

By choosing a different manifestation of the weighting vector, we are able to obtain different types of aggregation operators such as the maximum, the minimum, the average and the weighted average (Yager, 1988). For example, the maximum is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The minimum is obtained when $w_n = 1$ and $w_j = 0$ for all $j \neq n$. The average is found when $w_j = 1/n$ for all j and the weighted average when the ordered position of i is the same than the ordered position of j for all i and j. Note that other families of OWA operators are found in Karayiannis (2000), Merigó (2010), Merigó and Gil-Lafuente (2008b; 2008c; 2009b), Xu (2005), Yager (1993; 2009a) and Yager and Kacprzyk (1997).

Another factor to consider, are the two measures introduced by Yager (1988) for characterizing a weighting vector and the type of aggregation it performs. The first measure $\alpha(W)$, the attitudinal character, is defined as:

$$\alpha(W) = \sum_{j=1}^{n} \left(\frac{n-j}{n-1} \right) w_j \quad (3)$$

It can be shown that $\alpha \in [0, 1]$. The more of the weight located near the top of W, the closer α to 1 and the more of the weight located toward the bottom of W, the closer α to 0. Note that for the optimistic criteria $\alpha(W) = 1$, for the pessimistic criteria $\alpha(W) = 0$, and for the average criteria $\alpha(W) = 0.5$.

The second measure (Yager, 1988) is called the entropy of dispersion of W. It is defined as:

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j) \quad (4)$$

This can be used to provide a measure of the information being used. That is, if $w_j = 1$ for some j, known as step-OWA (Yager, 1993), then $H(W) = 0$, and the least amount of information is used. Note that other measures are studied by Yager (1993; 2009).

3. SELECTION OF HUMAN RESOURCES WITH THE OWA OPERATOR

The motivation for using the OWA operator in the selection of human resources appears because the decision maker wants to take the decision with a certain degree of optimism or pessimism rather than with a neutral position. Due to the fact that the traditional methods in the selection of human resources (Gil-Aluja, 1998; Kaufmann and Gil-Aluja, 1986; 1987) are neutral against the attitude of the decision maker, the introduction of the OWA operator in these models can change the neutrality and reflect decisions with different degrees of optimism and pessimism. These techniques can be used in a lot of situations but the general ideas about it is the possibility of under estimate or over estimate the problems in order to get results that reflects this change in the evaluation phase. This can be useful in a lot of situations, for example, in situations where the decision maker wants to over estimate the results in order to take a more risky decision than in normal cases. Obviously, this increase in the risk can affect our decision doing that we select a different person than we would have chosen with a neutral criteria.

The process to follow in the selection of human resources (Dobre and Alexandru, 2010; Lefter et al., 2010) with the OWA operator, is similar to the process developed in Gil-Aluja (1998) and Kaufmann and Gil-Aluja (1986; 1987) with the difference that the instruments used will include the OWA operator in the selection process. Note that similar models that use the OWA operator have been developed for other selection processes (Gil-Lafuente and Merigó, 2010; Merigó and Gil-Lafuente, 2007; 2008b; 2008c; 2010). The 5 steps to follow are:

Step 1*:* Analysis and determination of the significant characteristics of the available candidates for the company. Theoretically, it will be represented as: $C = \{C_1, C_2, \ldots, C_i, \ldots, C_n\}$, where C_i is the ith characteristic to consider of the candidate and we suppose a limited number n of required characteristics.

Step 2*:* Fixation of the ideal levels of each significant characteristic in order to form the ideal worker. That is:

Table 1. Ideal worker

	C_1	C_2	…	C_i	…	C_n
$P =$	μ_1	μ_2	…	μ_i	…	μ_n

where P is the ideal worker expressed by a fuzzy subset, C_i is the ith characteristic to consider and $\mu_i \in [0, 1]$; $i = 1, 2, \ldots, n$, is the valuation between 0 and 1 for the ith characteristic.

Step 3*:* Fixation of the real level of each characteristic for all the different candidates considered. That is:

Table 2. Available candidates

	C_1	C_2	…	C_i	…	C_n
$P_k =$	$\mu_1^{(k)}$	$\mu_2^{(k)}$	…	$\mu_i^{(k)}$	…	$\mu_n^{(k)}$

with $k = 1, 2, \ldots, m$; where P_k is the kth candidate expressed by a fuzzy subset, C_i is the ith characteristic to consider and $\mu_i^{(k)} \in [0, 1]$; $i = 1, \ldots, n$, is the valuation between 0 and 1 for the ith characteristic of the kth candidate.

Step 4*:* Comparison between the ideal worker and the different candidates considered, and determination of the level of removal using the OWA operator. That is, changing the neutrality of the results to over estimate or under estimate them. In this step, the objective is to express numerically the removal between the ideal worker and the different candidates considered. For this, it can be used the different available selection indexes such as the Hamming distance, the adequacy coefficient, the index of maximum and minimum level, etc. (Merigó and Gil-Lafuente, 2007).

Step 5*:* Adoption of decisions according to the results found in the previous steps. Finally, we should take the decision about which person select. Obviously, our decision will consist in choose the candidate with the best results according to the index used.

4. USING THE OWAD OPERATOR IN THE SELECTION OF HUMAN RESOURCES

In this Section we introduce a new index for the selection of human resources that uses the OWA operator in the Hamming distance. We call it the ordered weighted averaging distance (OWAD) operator (Merigó and Gil-Lafuente, 2008b; 2010; Xu and Chen, 2008). It can be defined as follows.

Definition 2. An OWAD operator of dimension n, is a mapping $OWAD$: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W, with the sum of the weights equal to 1 and $w_j \in [0, 1]$ such that:

$$OWAD\,(P, P_k) = \sum_{j=1}^{n} w_j D_j \quad (5)$$

where D_j represents the jth largest of the $|\mu_i - \mu_i^{(k)}|$, μ_i and $\mu_i^{(k)}$ are the ith arguments of the sets P and P_k, and $k = 1, 2, \ldots, m$.

By choosing a different manifestation of the weighting vector, we are able to obtain different types of aggregation operators. For example, the maximum distance is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The minimum is found when $w_n = 1$ and $w_j = 0$ for all $j \neq n$. The normalized Hamming distance is obtained when $w_j = 1/n$ for all j. Note that in the case of tie in the final result, it could be used in the decision the second best or worst result, and so on.

Note that further families of OWAD operators could be developed by using the same methodology as it has been used in the OWA operator (Merigó, 2007; Xu, 2005; Yager, 1993).

Additionally, we can present an equivalent removal index that it is a dual of the OWAD because $OWADD(P, P_k) = 1 - OWAD(P, P_k)$. We call it the ordered weighted averaging dual distance (OWADD) operator.

Furthermore, we can extend the OWAD operator by using the hybrid average (Wei, 2009; Xu and Da, 2003; Zhao et al., 2009; 2010). Thus, we are able to assess the information by using weighted averages and OWA operators in the same formulation. We call it the hybrid averaging distance (HAD) operator (Xu, 2008). It can be defined as follows.

Definition 3. A HAD operator of dimension n, is a mapping HAD: $R^n \times R^n \rightarrow R$ that has an associated weighting vector W, with the sum of the weights equal to 1 and $w_j \in [0, 1]$ such that:

$$HAD(P, P_k) = \sum_{j=1}^{n} w_j D_j \tag{6}$$

where D_j represents the jth largest of the $|\mu_i - \mu_i^{(k)}|^* = nv_i|\mu_i - \mu_i^{(k)}|$, v_i is the weight of the weighted average such that the sum of the weights equal to 1 and $v_i \in [0, 1]$, μ_i and $\mu_i^{(k)}$ are the ith arguments of the sets P and P_k, and $k = 1, 2, \ldots, m$.

Note that in this case we can also consider the dual by using $HADD(P, P_k) = 1 - HAD(P, P_k)$.

5. USING THE OWAAC OPERATOR IN THE SELECTION OF HUMAN RESOURCES

In this Section, we introduce the use of the OWA operator in the selection of human resources with the adequacy coefficient. We call it the ordered weighted averaging adequacy coefficient (OWAAC) (Merigó and Gil-Lafuente, 2008c; 2010). It can be defined as follows.

Definition 4. An OWAAC operator of dimension n, is a mapping $OWAAC$: $[0, 1]^n \times [0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $w_j \in [0, 1]$ and the sum of the weights is equal to 1, such that:

$$OWAAC(P_k \rightarrow P) = \sum_{j=1}^{n} w_j K_j \tag{7}$$

where K_j represents the jth largest of the $[1 \wedge (1 - \mu_i + \mu_i^{(k)})]$, and $k = 1, 2, \ldots, m$.

Note that $\wedge$ refers to the minimum and $\vee$ to the maximum. In this case, the reordering step is done in a decreasing order as the best result is the largest number. Thus, the type of OWA operator used in the adequacy coefficient is the DOWA operator: $K_1 \geq K_2 \geq \ldots \geq K_n$. The final result will be a number between [0, 1], being the maximum possible result 1.

By choosing a different manifestation of the weighting vector, we are able to obtain different types of aggregation operators. For example, the normalized adequacy coefficient is obtained when $w_j = 1/n$ for all j.

Analogously to the OWAAC operator, we can suggest an equivalent removal index that it is a dual of the OWAAC because $OWADAC\ (P_k \rightarrow P) = 1 - OWAAC\ (P_k \rightarrow P)$. We call it the ordered weighted averaging dual adequacy coefficient (OWADAC). It can be defined as follows.

Definition 5. An OWADAC operator of dimension n, is a mapping $OWADAC$: $[0, 1]^n \times [0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $w_j \in [0, 1]$ and the sum of the weights is equal to 1, then:

$$OWADAC\ (P_k \rightarrow P) = 1 - \sum_{j=1}^{n} w_j K_j \tag{8}$$

where K_j represents the jth largest of the $[1 \wedge (1 - \mu_i + \mu_i^{(k)})]$, and $k = 1, 2, \ldots, m$.

The final result will be a number between [0, 1]. Note that in this case we usually select the lowest value as the best result.

It is also possible to obtain different families of aggregation operators with the OWADAC operator by using different manifestations of the weighting vector such as the maximum, the minimum, the normalized dual adequacy coefficient (NDAC) and the weighted dual adequacy coefficient (WDAC). Note that the NDAC is obtained when $w_j = 1/n$ for all j.

Another interesting issue to consider is the unification point in the selection of human resources. As it has been explained in Merigó and Gil-Lafuente (2007), the unification point appears when the results obtained in the Hamming distance are the same than the results obtained in the adequacy coefficient. In the new methods suggested in this paper, we also find the unification point when the OWAD and the OWAAC accomplish the theorems explained in Merigó and Gil-Lafuente (2007). Note that it is possible to find a total unification point or a partial unification point and we could generalize it for all the human resources considered in the decision problem. The theorem that explains this generalization is very similar with the difference that now we consider all the characteristics i and all the human resources k.

Following Xu and Da (2003), we can extend the OWAAC operator by using the hybrid average. Thus, we are able to consider weighted averages and OWA operators in the adequacy coefficient. We call it the hybrid averaging adequacy coefficient (HAAC). It can be defined as follows.

Definition 6. A HAAC operator of dimension n, is a mapping $HAAC$: $[0, 1]^n \times [0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with the sum of the weights equal to 1 and $w_j \in [0, 1]$ such that:

$$HAAC\,(P, P_k) = \sum_{j=1}^{n} w_j D_j \tag{9}$$

where D_j represents the jth largest of the $[1 \wedge (1 - \mu_i + \mu_i^{(k)})]^* = nv_i[1 \wedge (1 - \mu_i + \mu_i^{(k)})]$, v_i is the weight of the weighted average such that the sum of the weights equal to 1 and $v_i \in [0, 1]$, μ_i and $\mu_i^{(k)}$ are the ith arguments of the sets P and P_k, and $k = 1, 2, \ldots, m$.

Note that in this case we can also consider the dual that we call the hybrid averaging dual adequacy coefficient (HADAC), by using $HADAC\,(P, P_k) = 1 - HAAC\,(P, P_k)$. It is also worth noting the possibility of distinguishing between descending and ascending orders by using $w_j = w^*_{n+1-j}$, where w_j is the jth weight of the DHAAC operator and w^*_{n+1-j} the jth weight of the AHAAC operator.

6. USING THE OWAIMAM OPERATOR IN THE SELECTION OF HUMAN RESOURCES

In this Section, we develop an index for the selection of human resources that uses the OWA operator in the index of maximum and minimum level. We call it the ordered weighted averaging index of maximum and minimum level (OWAIMAM). It can be defined as follows.

Definition 7. An OWAIMAM operator of dimension n, is a mapping $OWAIMAM$: $[0, 1]^n \times [0, 1]^n \rightarrow [0, 1]$ that has an associated weighting vector W, with $w_j \in [0, 1]$ and the sum of the weights is equal to 1, such that:

$$OWAIMAM\,(P_k \rightarrow P) = \sum_{j=1}^{n} w_j S_j \tag{10}$$

where $S_j = \left\lfloor w_j^{*}[0 \vee (\mu_j - \mu_j^{(k)})] + w_j^{'}\left|\mu_i - \mu_i^{(k)}\right|\right\rfloor$ represents the jth smallest of all the $|\mu_i - \mu_i^{(k)}|$ and the $[0 \vee (\mu_i - \mu_i^{(k)})]$; with $k = 1, 2, \ldots, m$, μ_i and $\mu_i^{(k)}$ are the ith arguments of the sets P and P_k, and the weighting vector W is divided in w_j^{*}, that affects the arguments that use the dual adequacy coefficient and $w_j^{'}$ that affects the arguments that use the Hamming distance.

Note that w_j^{*} and $w_j^{'}$ is an artificial construction of the weighting vector W in order to identify which arguments use the dual adequacy coefficient and which ones the Hamming distance. In this case, an AOWA operator is used in the reordering step ($S_1 \leq S_2 \leq \ldots \leq S_n$) with the particularity that it always selects the jth smallest of all the possible values, independently if it is a result coming from the Hamming distance or from the removal index of the adequacy coefficient.

Note that in this case we are also able to obtain different types of aggregation operators by using a different weighting vector. For example, the maximum is found when $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$. The minimum when $w_n = 1$ and $w_j = 0$ for all $j \neq n$ and the normalized index of maximum and minimum level when $w_j = 1/n$ for all j. Note that in the case of tie in the final result, especially for the maximum and the minimum, it could be used in the decision the second best or worst result, and so on.

Analogously to the OWAIMAM operator, we can suggest an equivalent removal index that it is a dual of the OWAIMAM because $OWADIMAM\ (P_k \rightarrow P) = 1 - OWAIMAM\ (P_k \rightarrow P)$. We call it the ordered weighted averaging dual index of maximum and minimum level (OWADIMAM).

Another interesting issue to consider is the unification point in the selection of human resources for the index of maximum and minimum level. As it has been explained in Merigó and Gil-Lafuente (2007), in these situations, the index of maximum and minimum level becomes the Hamming distance. Note that it is possible to find a total unification point or a partial unification point (Merigó and Gil-Lafuente, 2007). In the following, we show the main proposition when using the OWA operator.

Proposition 1. Assume $OWAD\ (P, P_k)$ is the selection of human resources with the OWAD operator and $OWAIMAM\ (P_k \rightarrow P)$ the selection of human resources with the OWAIMAM operator. If $\mu_i \geq \mu_i^{(k)}$ for all i, then:

$$OWAD\ (P, P_k) = OWAIMAM\ (P_k \rightarrow P) \tag{11}$$

Proof. Let

$$OWAD\ (P, P_k) = \sum_{j=1}^{n} w_j \mid \mu_i - \mu_i^{(k)} \mid$$ and

$$OWAIMAM\ (P_k \rightarrow P) = \sum_{j=1}^{n} [w_j * [0 \vee (\mu_i - \mu_i^{(k)})] + w_j' \mid \mu_i - \mu_i^{(k)} \mid]$$

Since $\mu_i \geq \mu_i^{(k)}$ for all i, $[0 \vee (\mu_i - \mu_i^{(k)})] = (\mu_i - \mu_i^{(k)})$ for all i, then

$$OWAIMAM\ (P_k \rightarrow P) = \sum_{j=1}^{n} w_j (\mu_i - \mu_i^{(k)}) = OWAD\ (P, P_k) \qquad \blacksquare$$

Note that $w_j^* + w_j' = w_j$.

Analysing this proposition, we could generalize it for all the human resources considered in the decision problem. The proposition that explains this generalization is very similar to Proposition (1) with the difference that now we consider all the characteristics i and all the human resources k.

Finally, note also that we can also extend the OWAIMAM operator in a similar way as we have done in Section 4 and 5 by using the hybrid average. Thus, we get the hybrid averaging IMAM (HAIMAM) operator. Furthermore, we can also develop other extensions by using induced and generalized aggregation operators (Merigó and Gil-Lafuente, 2009b), mixture operators and multi-person operators (Merigó and Casanovas, 2010f).

7. ILLUSTRATIVE EXAMPLE

The information of the example follows the methodology explained by Gil-Lafuente (2005) although we have made some changes in the paper and applied it in human resource management.

Step 1: Analysis and determination of the significant characteristics for the company. Assume that a company wants to select a worker for a vacant and it has 3 candidates P_1, P_2, P_3, with different characteristics. It is considered for each characteristic a property.

Step 2: Fixation of the ideal level for each significant characteristic. It is defined the ideal worker for the company as:

Table 3. Characteristics of the ideal worker

	C_1	C_2	C_3	C_4	C_5
P^* =	0.9	0.8	0.6	0.8	0.3

Step 3: Fixation of the real level of each characteristic for all the different candidates considered. For each of these characteristics, it is found the following information:

Table 4. Available candidates

	C_1	C_2	C_3	C_4	C_5
P_1 =	0.8	0.7	0.3	1	1
P_2 =	0.8	1	0.6	0.3	0.3
P_3 =	1	0.6	1	1	0.2

Step 4: Comparison between the ideal worker and the different candidates considered, and determination of the level of removal using the OWA operators. We consider the normalized Hamming distance, the weighted Hamming distance, the OWAD operator and the AOWAD operator. In this example, we assume that the company decides to use the following weighting vector: W = (0.1, 0.1, 0.2, 0.3, 0.3). With this weighting vector, we can calculate the degree of optimism of the decision, by using Eq. (3), as:

$$\alpha(W) = \sum_{j=1}^{n}\left(\frac{n-j}{n-1}\right)w_j = \frac{1}{4}\left[4\cdot 0.1 + 3\cdot 0.1 + 2\cdot 0.2 + 0.3\right] = 0.35\,,$$

and the degree of dispersion, by using Eq. (4), as:

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j) = -\left[2\cdot\left(0.1\cdot\ln\left(0.1\right)\right) + 0.2\cdot\ln\left(0.2\right) + 2\cdot\left(0.3\cdot\ln\left(0.3\right)\right)\right] = 1.504\,.$$

If we elaborate the selection process with the Hamming distance, we get the following. First, we have to calculate the individual distances of each characteristic to the ideal value of the corresponding characteristic forming the fuzzy subset of individual distances for each candidate. The results are shown in Table 5.

Table 5. Individual distances

	C_1	C_2	C_3	C_4	C_5
$P_1 =$	0.1	0.1	0.3	0.2	0.7
$P_2 =$	0.1	0.2	0	0.5	0
$P_3 =$	0.1	0.2	0.4	0.2	0.1

Once obtained all the distances, we go for the aggregation. Then, we reorder the different values of each fuzzy subset using equation (5) and considering the type of aggregation we are developing (that is, $D_i^{(k)} = \left|\mu_i - \mu_i^{(k)}\right|$ for $i = 1, ..., 5$ and $k = 1, 2, 3$). In this case, we use the normalized Hamming distance (NHD), the weighted Hamming distance and the OWAD operator.

For example, if we use the NHD for P_1, we get:

$$NHD^{(1)} = \frac{1}{5}\sum_{i=1}^{5} D_i^{(1)} = \frac{1}{5}\cdot\left[0.1+0.1+0.3+0.2+0.7\right] = 0.28 .$$

If we use the WHD for P_2, we get:

$$WHD^{(2)} = \sum_{i=1}^{5} w_i D_i^{(2)} = 0.1\cdot 0.1 + 0.1\cdot 0.2 + 0.2\cdot 0 + 0.3\cdot 0.5 + 0.3\cdot 0 = 0.18 .$$

And if we use the OWAD for P_3, we get:

$$OWAD^{(3)} = \sum_{i=1}^{5} w_i D_{(5-i+1)}^{(3)} = 0.1\cdot\left[0.4+0.2\right] + 0.2\cdot 0.2 + 0.3\cdot\left[0.1+0.1\right] = 0.16 .$$

In this way, we could develop all the calculations for all the available candidates. The results are shown in Table 6. Note that we also include the results with the OWADD operator.

Table 6. Aggregated results with the Hamming distance

	NHD	*WHD*	*OWAD*	*NHDD*	*WHDD*	*OWADD*
$P_1 =$	0.28	0.35	0.2	0.72	0.65	0.8
$P_2 =$	0.16	0.18	0.09	0.84	0.82	0.91
$P_3 =$	0.2	0.2	0.16	0.8	0.8	0.84

In this case, our decision consists in selecting the candidate with the smallest distance. Thus, we select P_2 as it gives us the lowest distance.

If we develop the selection process with the adequacy coefficient, we get the following. First, we have to calculate how close the characteristics are to the ideal worker in a similar way as it has been done in Table 5. Once calculated all the different individual values, we construct the aggregation. In this case, the arguments will be ordered using equation (7). The results are shown in Table 7. Note that we also include the results with the OWADAC operator.

Table 7. Aggregated results with the adequacy coefficient

	NAC	*WAC*	*OWAAC*	*NDAC*	*WDAC*	*OWADAC*
$P_1 =$	0.9	0.92	0.86	0.1	0.08	0.14
$P_2 =$	0.88	0.84	0.82	0.12	0.16	0.18
$P_3 =$	0.94	0.95	0.91	0.06	0.05	0.09

The decision consists in selecting the candidate with the highest result because this means a higher approximation to the ideal worker. Thus, we select P_3 because it gives us the highest result for all the cases.

Finally, if we use the index of maximum and minimum level in the selection process as a combination of the normalized Hamming distance and the normalized adequacy coefficient, we get the following. In this example, we assume that the characteristics C_1 and C_2 have to be treated with the adequacy coefficient and the other three characteristics have to be treated with the Hamming distance. First, we calculate the individual removal of each characteristic to the ideal, independently that the instrument used is the Hamming distance or the adequacy index, in a similar way as it has been done in Table 5. Once calculated all the values for the individual removal, we construct the aggregation using equation (10). Here, we note that in the reordering step, it will be only considered the individual value obtained for each characteristic, independently that the value has been obtained with the adequacy coefficient or with the Hamming distance. The results are shown in Table 8. Note that we also include the results with the OWADIMAM operator.

Table 8. Aggregated results with the index of maximum and minimum level

	NIMAM	*WIMAM*	*OWAIMAM*	*NDIMAM*	*WDIMAM*	*OWADIMAM*
P_1	0.28	0.35	0.2	0.72	0.65	0.8
P_2	0.12	0.16	0.06	0.88	0.84	0.94
P_3	0.18	0.19	0.13	0.82	0.81	0.87

Thus, our decision consists in select P_2 because it is the candidate with the smallest removal to the ideal.

8. CONCLUSIONS

We have studied a large number of instruments for the selection of human resources. Due to the neutrality in the attitudinal character of the classical methods, we have suggested the use of the OWA operator in the selection process. As we have seen, the OWA operator permits to under estimate or over estimate the selection process according to a degree of optimism. With this in mind, we have suggested three new instruments for the selection of human resources that uses the OWA operator in the Hamming distance, in the adequacy coefficient and in the index of maximum and minimum level. We have called them the OWAD operator, the OWAAC operator and the OWAIMAM operator. Thus, we have obtained a new method that permits reflect the attitude of the decision makers in the selection process of human resources. We have further extended this approach by using the hybrid average obtaining the HAD operator, the HAAC operator and the HAIMAM operator.

We have also presented an application of the new approach in a decision making problem concerning the selection of human resources. We have studied the different results obtained by using different types of OWAD, OWAAC and OWAIMAM operators. We have seen that depending on the method and the particular case used, the results may be different leading to different decisions.

In future research, we expect to develop further extensions on these methods by using other types of OWA operators such as the use of order-inducing variables, quasi-arithmetic means and probabilistic information, and applying it in different decision making problems.

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