New constraints on cosmological parameters and neutrino properties using the expansion rate of the Universe to $z\sim 1.75$

Michele Moresco 1 , Licia Verde 2 , Lucia Pozzetti 3 , Raul Jimenez 2 and Andrea Cimatti 1

¹Dipartimento di Astronomia, Universitá degli Studi di Bologna, via Ranzani 1, I-40127, Bologna, Italy

²ICREA & Institute of Sciences of the Cosmos (ICC), Universitat de Barcelona (IEEC-UB), Mart i Franqus, 1, Barcelona 08028, Spain

³INAF - Osservatorio Astronomico di Bologna, via Ranzani 1, 40127 Bologna, Italy

E-mail: michele.moresco@unibo.it, licia.verde@icc.ub.edu, lucia.pozzetti@oabo.inaf.it, raul.jimenez@icc.ub.edu, a.cimatti@unibo.it

Abstract. We have assembled a compilation of observational Hubble parameter measurements estimated with the differential evolution of cosmic chronometers, in the redshift range 0 < z < 1.75. This sample has been used, in combination with CMB data and with the most recent estimate of the Hubble constant H_0 , to derive new constraints on several cosmological parameters. The new Hubble parameter data are very useful to break some of the parameter degeneracies present in CMB-only analysis, and to constrain possible deviations from the standard (minimal) flat ACDM model. The H(z) data are especially valuable in constraining Ω_k and $\Omega_{\rm DE}$ in models that allow a variation of those parameters, yielding constraints that are competitive with those obtained using Supernovae and/or baryon acoustic oscillations. We also find that our H(z) data are important to constrain parameters that do no affect directly the expansion history, by breaking or reducing degeneracies with other parameters. We find that $N_{\rm rel} = 3.45 \pm 0.33$ using WMAP 7-years data in combination with South Pole Telescope data and our H(z) determinations ($N_{\rm rel} = 3.71 \pm 0.45$ using Atacama Cosmology Telescope data instead of South Pole Telescope). We exclude $N_{\rm rel} > 4$ at 95% CL (74%) CL) using the same datasets combinations. We also put competitive limits on the sum of neutrino masses, $\Sigma m_{\nu} < 0.24$ eV at 68% confidence level. These results have been proven to be extremely robust to many possible systematic effects, such as the initial choice of stellar population synthesis model adopted to estimate H(z) and the progenitor-bias.

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1 Introduction

The accelerated expansion of the Universe [1, 2] has become, during the last decades, a stronghold of modern cosmology, witnessed by a lot of independent probes. However the real physical mechanism driving such accelerated expansion is still unknown; one possible explanation is that it could be due to an unknown form of energy, "dark energy", which in its simplest embodiment is a cosmological constant, but other possibilities could include a breakdown of general relativity on large scales or an effect of interpreting the observation using a metric which is not correct for our inhomogeneous Universe.

At present, the most economical framework to describe all available observations is the Λ CDM paradigm, in which the Universe is spatially flat, its energy budget is dominated by a cosmological constant, and there are three massless neutrinos. The Λ CDM model is able to describe the evolution of the Universe with a minimal number of cosmological parameters; current data constrain these parameters at the % level. However, this is just a phenomenological description, as we do not have full physical understanding of many of these parameters (despite their small errorbars).

One approach to advance on this issue is to constrain cosmological parameters that describe deviations from this "minimal" model. Several different probes may be – and have been – used to set constraints on cosmological parameters, and especially on dark energy parameters, each one having its strengths and weaknesses; among them, the most well-known are the Cosmic Microwave Background (hereafter CMB), the Baryonic Acoustic Oscillation (hereafter BAO), the Supernovae type Ia (hereafter SNe), plus probes of the growth of structure via weak lensing studies and cluster of galaxies abundance. A good reference that gives an overview of combining different probes to obtain cosmological parameters is the series of papers by the WMAP team (e.g., [3, 4]).

The above probes constrain the expansion history using geometric measurements: e.g., BAO use standard rulers and SNe standard(-izable) candles. A complementary technique to track directly the expansion history of the Universe is to use massive and passively evolving early-type galaxies (hereafter ETGs) as "cosmic chronometers" [5], thus providing standard(-izable) clocks in the Universe. The basic idea underlying this approach is based on the measurement of the differential age evolution as a function of redshift of these chronometers, which provides a direct estimate of the Hubble parameter $H(z) = -1/(1+z)dz/dt \simeq -1/(1+z)\Delta z/\Delta t$. The main strength of this approach is the reliance on the measurement of a differential quantity, $\Delta z/\Delta t$, which provides many advantages in minimizing many common issues and systematic effects (for a detailed discussion, see [6]). Moreover, compared with other techniques, this approach provides a direct measurement of the Hubble parameter, and not

of its integral, as in the case of e.g., SNe or angular/angle-averaged BAO. For this reason the Hubble parameter estimated with this approach has been referred as "observational Hubble parameter data" (OHD, see e.g. [7, 8]).

There has been significant progress both in the theoretical understanding of the method and control of systematics uncertainties [6, 9] and in the improvement of observational data [6]. This paper is the second in a series of three [6, 10] where we present new H(z) determinations [6] and interpret their result in terms of cosmological parameters constraints (this paper) and a model independent reconstruction of the dark energy potential [10].

The paper is organized as follows. In Sect. 2 we explore the sensitivity of the Hubble parameter to the various cosmological parameters and its statistical power in constraining them. The data sample comprising all the observational H(z) measurements used in this analysis is provided in Sect. 3; in this section we also describe the additional datasets used to obtain constraints on the cosmological parameters, i.e. the cosmic microwave background measurements from WMAP (7 years results) [11], the results obtained from baryonic acoustic oscillations analysis [12], and finally the ones obtained from Supernovae type Ia [13]. In Sect. 4 we present our results in constraining the deviations from the standard Λ CDM model, studying both cases in which the expansion history of the Universe helps in constraining those parameters, and cases in which the constraints are set just by breaking some degeneracies between the parameters. We also discuss the reliability of our constraints against the choice of stellar population synthesis model adopted to estimate the Hubble parameter, and estimate the effect of the progenitor bias. Finally, in Sect. 5 we will explore how the constraints on the cosmological parameters will be improved with more accurate measurements of H(z) over a wider redshift range, which future surveys will make soon available.

2 Theoretical background

The detailed formalism at the base of the *cosmic chronometers* approach is introduced in Ref. [5], and explained in detail in e.g. Ref. [6], considering also all the possible sources of uncertainty and related issues; we refer to those references for a comprehensive discussion.

In this section we analyze, from the theoretical point of view, the statistical power in constraining several cosmological parameters of the measurements of the Hubble parameter H(z). This section provides physical understanding of how the change of different cosmological parameters affects H(z), which will then be useful to understand the degeneracies among cosmological parameters we can hope to break with the addition of accurate measurements of the expansion history of the universe to, e.g., CMB constraints.

Assuming a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, it is possible to write the Hubble parameter as a function of the cosmological parameters (including curvature and dark energy with redshift-dependent equation of state parameter) as follows:

$$H(z) = H_0 \left\{ \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{\rm DE} (1+z)^{3(1+w_0+w_a)} e^{-3w_a \frac{z}{1+z}} \right\}^{1/2}$$
(2.1)

where H_0 is the Hubble constant, Ω_i is the adimensional energy density parameter for each specie in the Universe (i.e. matter Ω_m , curvature Ω_k and dark energy Ω_{DE} ; we have neglected radiation since its contribution is irrelevant in the late Universe), and dark energy Equation of State (EoS) parameter as suggested in Ref. [14, 15], $w(z) = w_0 + w_a(z/(1+z))$.

To illustrate the dependence of the observed H(z) on cosmological parameters, we fix a fiducial cosmology, taken from the latest H_0 estimate [16] and from the analysis of the WMAP 7-years data [4] (i.e. $H_0 = 73.8 \text{km s}^{-1} \text{Mpc}^{-1}$, $\Omega_{k,0} = 0$, $\Omega_{m,0} = 0.27$, $\Omega_{\text{DE},0} = 0.73$, $w_0 = -1$, $w_a = 0$), and perturb it by varying one parameter at a time by $\pm 10\%$, 25%, 50% (in the case of Ω_k and w_a , we explored the values 0.1, 0.25 and 0.5). We then estimate the absolute and fractional difference on H(z), $(H(z)_{\text{ref}} - H(z)_{\text{pert}})/H(z)_{\text{ref}}$. The results are shown in Fig. 1: the upper plot shows the effect on H(z) of the adimensional density parameters Ω_m , Ω_{DE} and Ω_k , showing respectively the Hubble parameter (plots on upper rows) and the fractional difference (plots on lower rows). In the lower plot



Figure 1. The dependence of the Hubble parameter H(z) on the cosmological parameters. One cosmological parameter at the time is being changed with respect to the fiducial model by 10, 25 and 50%. In each of the two panels, the upper (lower) rows show the absolute (relative) effect of this variation on the Hubble parameter as a function of redshift.

we show, as in the upper plot, the dependence on the Hubble constant, dark energy equation-of-state parameter w_0 and w_a .

As it is evident from Eq. 2.1, the dependence on the Hubble constant H_0 is linear; what is less intuitive from the form of the equation is the detailed dependence on the other parameters. We found that the Hubble parameter is particularly sensitive to the curvature parameter, showing a percentage



Figure 2. Hubble parameter measurements. Open circle points correspond to H(z) estimates from Ref. [6], open triangle points from Ref. [17], and starred points are taken from Ref. [18]. We also show a fiducial Λ CDM cosmology, with $H_0 = 73.8$, $\Omega_{k,0} = 0$, $\Omega_{m,0} = 0.27$, $\Omega_{DE,0} = 0.73$, $w_0 = -1$, $w_a = 0$.

variation of ~ 30-40% in H(z) with $\Omega_k = 0.5$; the dependence on the other density parameter is less pronounced and redshift dependent, with a percentage variation up to 25% at $z \sim 2$ with a variation of 50% in Ω_m , and a percentage variation up to 25% at $z \sim 0$ with a variation of 50% in $\Omega_{\rm DE}$. The less trivial parameters to be constrained are, as already known, the dark energy equation-of-state parameters w_0 and w_a : for the first one, a variation of 50% produces a non-symmetric percentage variation, that could go up to 20% when moving towards less negative values of w_0 , but being less sensitive to more negative values of w_0 . The parameter w_a is clearly the most elusive one, showing a percentage variation of only 5% corresponding to the maximum value explored, i.e. $w_a = 0.5$.

3 Observational Data & Method

We combined the observational constraints on the Hubble parameter available from the literature to obtain the widest redshift lever arm.

We started from the dataset provided by Ref. [17], which obtained 9 determinations of the Hubble parameter in the range 0.1 < z < 1.75 from the relative dating of 32 passively evolving galaxies. Ref. [18] obtained high-quality spectra of red-envelope galaxies in 24 galaxy clusters in the redshift range 0.2 < z < 1.0, yielding 2 more observations of the Hubble parameter at $z \sim 0.5$ and $z \sim 0.9$. Finally, we added the latest 8 new high-accuracy estimates of H(z) provided by Ref. [6]. They analyzed the spectroscopic evolution of a large sample of 11324 early type galaxies, extracted from several spectroscopic surveys to ensure a high statistic and uniform coverage in the redshift range 0.15 < z < 1.1.

There are many systematic sources of uncertainty that can bias this kind of analysis; they can all be understood in terms of degeneracies between parameters which can in some way mimic an age-effect (e.g. the stellar metallicity). In the following we list them, and explain how they have been treated and their effect propagated in the total error budget of the H(z) measurements. We refer to the original papers [6, 17, 18] for the detailed discussion of how they have been taken into account.

- Stellar Metallicity and Star Formation History. It is well known that the effect of a change in stellar metallicity (Z) or in Star Formation History (SFH) is degenerate with the effect of a change in age, so that the multi-band spectral energy distribution of a galaxy with a given age and metallicity may be very similar to that of a galaxy with lower age and higher metallicity. Analogous degeneracies are present when analyzing galaxy spectra, especially if wavelengths below 4000 Å are not observed (see [18]). All the samples considered take into account this systematic effect in the total H(z) error. In Ref. [17], the sample has been selected by choosing ETGs with spectra best-fitted by a passive population, and their ages have been estimated marginalizing over the metallicity and star formation history. In Ref. [18] to estimate the best age a χ^2 fit to the spectra has been performed using a large grid of ages and metallicities, and the errorbars have been evaluated also in this case by marginalizing over the metallicity; the star formation has been assumed to occur in a single burst of short duration, and galaxies with poor values of χ^2 have been discarded. In Ref. [6] the "cosmic chronometers" approach has been developed to be even less dependent on all the degeneracies previously discussed. The ETGs sample has been selected to represent the most massive $(loq_{10}(\mathcal{M}/\mathcal{M}_{\odot}) \geq 11)$ and passive galaxies. Instead of using the age estimated from the best-fit to the spectral energy distribution or to the spectrum, the 4000 Å break (D4000) has been studied, which is a direct observable present in ETGs spectra known to correlate with age (and metallicity) of the galaxy. To calibrate this index, two different theoretical stellar population synthesis models have been studied. Finally, since in Ref. [9] it has been noted the existence of a clear dependence of the D4000-z relation on the stellar mass, two different mass subsamples have been obtained, and the Hubble parameter obtained separately from the two. Since the obtained results show a good agreement, the H(z) measurements obtained from the two mass-subsamples have been averaged. The total errorbar of the Hubble parameter has been estimated by considering both the statistical errors and the systematic uncertainty due to star formation history and metallicity.
- Stellar Population Synthesis Models. Also the choice of different Stellar Population Synthesis (SPS) models, used to estimate the age or to calibrate the D4000-age relation, may bias the estimate of H(z), and hence be a possible source of systematic uncertainty. Ref. [18] explored the sensitivity of the age estimate obtained by fitting galaxy spectra, finding a general agreement between the measurements performed using different models in particular when is available the wavelength coverage below 4000 Å. In the work of Ref. [6] it is discussed in detail the effect of considering different stellar population synthesis models, by studying both Bruzual & Charlot (2003) ([20], hereafter BC03) and Maraston & Strömbäck (2011) ([21], hereafter MaStro) models. Since all the other previous analysis in literature provided H(z) measurements only for BC03 models. In Sect. 4.3 we discuss the reliability of our results by analyzing also the constraints obtained with the MaStro models from Ref. [6], to estimate the sensitivity of our results to different stellar population synthesis models.
- **Progenitor Bias** The progenitor bias (e.g. see [19]) refers to the effect for which high-redshift samples of ETGs might not be statistically equivalent to the low-redshift samples, but a subset of the low-redshift samples biased toward higher ages. The measurements of Ref. [17] and Ref. [18] are based on the estimate of the upper and redder envelope of the age-z relations, probing the oldest galaxies at each redshift. Hence they are by definition not affected by such a problem. On the contrary, Ref. [6] estimates H(z) from the median D4000 z relation of a mass-selected sample, and hence may be biased; in Sect. 4.4 we quantify this effect, and its influence on the total errors.

z	H(z)	$\sigma_{H(z)}$	Ref.
0.090	69	12	[17]
0.170	83	8	[17]
0.179	75	4	[6]
0.199	75	5	[6]
0.270	77	14	[17]
0.352	83	14	[6]
0.400	95	17	[17]
0.480	97	62	[18]
0.593	104	13	[6]
0.680	92	8	[6]
0.781	105	12	[6]
0.875	125	17	[6]
0.880	90	40	[18]
1.037	154	20	[6]
1.300	168	17	[17]
1.430	177	18	[17]
1.530	140	14	[17]
1.750	202	40	[17]

Table 1. H(z) measurements (in units [km s⁻¹Mpc⁻¹]) and their errors. This dataset can be downloaded at the address http://www.astronomia.unibo.it/Astronomia/Ricerca/Progetti+e+attivita/cosmic_chronometers.htm (alternatively http://start.at/cosmicchronometers).

In summary, we built a sample of 19 observational H(z) measurements in the range 0.1 < z < 1.75, spanning almost 10 Gyr of cosmic time; these values are reported in Tab. 1 and shown in Fig. 2. This dataset represents the widest sample of observational constraints on the Hubble parameter available.

To set constraints on cosmological parameters, we decided to combine the OHD just described with the latest results from the WMAP 7-year release (hereafter WMAP7yr) [11] and with South Pole Telescope [22] and Atacama Cosmology Telescope [23, 24] (alternatively) measurements of the CMB temperature power spectrum damping tail. To complete the OHD, we will also use the latest and most precise estimate of the Hubble constant H_0 [16], which provides the local value of the Hubble parameter. Our aim, as stated before, is to constrain cosmological parameters using only CMB temperature data and OHD, thus combining only two probes. For a comparison, we will also show constraints from the combination of CMB [11], BAO [12] and SNe data [13]. To estimate the relative importance of the H(z) measurements and of the Hubble constant, we will also analyze the constraints obtained by combining to the CMB data individually the Hubble constant H_0 and the OHD.

Monte Carlo Markov (MCMC) chains for the WMAP 7yr data have been obtained from the LAMBDA web site¹ and they have been importance-sampled with the H(z) and H_0 data. We considered different cosmologies, to see how much the use of the OHD can help in improving the constraints on different cosmological parameters. The only case not covered by the LAMBDA web site is the open w(z)CDM model, i.e. where the dark-energy equation of state is parameterized as $w(z) = w_0 + w_a(z/(1+z)))$; we decided, therefore, to study the MCMC run specifically for this case by Ref. [18], to explore how much the new H(z) data help in constraining specifically the dark energy parameters. Those chains, built allowing for arbitrary curvature and for an evolving dark energy equation of state parametrized as described above, run on WMAP 5yr data and assume a constraint on H_0 from the Hubble Key Project [27].

 $^{^{1}}$ www.lambda.gsfc.gov

4 Results

For a simple flat Λ CDM, the addition of OHD measurements does not change appreciably the parameters constraints: within this model the parameters are tightly constrained by CMB data alone. Still we note that OHD and CMB data are nicely consistent. As anticipated, OHD measurements are however extremely useful in breaking CMB parameters degeneracies for models beyond the "minimal" Λ CDM model. In particular we will show their fundamental importance in models which allow a time-variability of the dark energy EoS, where despite its extreme accuracy, the H_0 alone is not able to set precise constraints to the cosmological parameters. We therefore explore two different scenarios. In Sect. 4.1 we show how H(z) measurements can set constraints on the parameters which can influence the expansion history of the Universe, such as Ω_m , $\Omega_{\rm DE}$, Ω_k , w_0 , and w_a ; in Sect. 4.2 we show the importance of the OHD in breaking degeneracies between parameters that have an effect on H(z) and parameters that do not. This allow us to use our H(z) data to constrain parameters which do not have a direct effect of H(z) (but that are somewhat degenerate with the expansion history), such as the number of the relativistic species in the Universe $N_{\rm rel}$ and the sum of the neutrino masses Σm_{ν} .

4.1 Constraints on deviations from Λ CDM that affect directly the expansion history

We started by considering simple generalizations of the "minimal" Λ CDM where the additional parameter affects directly the late-time expansion history. For a model with an additional curvature parameter Ω_k (o Λ CDM) we obtain the constraints shown in Fig. 3 and Tab. 2. Figure 3 shows the 1 and 2 sigmas joint constraints in the Ω_m and Ω_{DE} plane; all other cosmological parameters are marginalized over. Table 2 shows constraints on Ω_m and Ω_{DE} marginalized over all other cosmological parameters. For comparison we show both CMB-only constraints from WMAP7yr data and constraints for WMAP7yr combined with OHD and H_0 measurements; to study the relative improvements to the constraints due to the contribution of the Hubble constant and of the Hubble parameter alone, we also show in the table the effect of combining WMAP7yr+ H_0 and WMAP7yr+OHD: clearly the direct constraint on the expansion history provided by OHD breaks the CMB degeneracy shrinking the errors on Ω_m , Ω_{DE} , and Ω_k by about an order of magnitude. Table 2 shows that the inclusion of the Hubble constant in the analysis (which alone has the same power of OHD in constraining these cosmological parameters) used as a pivot for OHD, is useful to set the final mean values of the parameters.

We next added yet another parameter describing deviations from the standard cosmological model; we allowed the dark energy to deviate from a cosmological constant in the form of a fluid with equation of state parameter w_0 considered constant (owCDM). The resulting constraints are reported in Tab. 3 and in Fig. 4. We define an effective $\Delta \chi^2$ as $-2 \ln P/P_{max}$, where P denotes the posterior and P_{max} its maximum; given a uniform prior the posterior can be identified with the likelihood. The effective $\Delta \chi^2$ is used only for plotting purposes.

As Fig. 4 shows, the limits obtained on both Ω_k and on the dark energy parameter w_0 by WMAP7yr data alone (dotted lines) are not very tight; if we consider the joint analysis of WMAP7yr with OHD or H_0 , the limits obtained on the cosmological parameters are significantly improved, where H_0 is best suited to constrain the adimensional density parameters while OHD statistical power is in

MARGINALIZED 1D CONSTRAINTS
oACDM model

	Ω_{m}	$\Omega_{\rm DE}$	Ω_k			
WMAP7yr	0.40 ± 0.19	0.63 ± 0.18	-0.0008 ± 0.068			
WMAP7yr+OHD	0.280 ± 0.032	0.725 ± 0.031	-0.0039 ± 0.0076			
WMAP7yr+ H_0	0.245 ± 0.030	0.752 ± 0.024	0.0020 ± 0.0072			
WMAP7yr+OHD+ H_0	0.261 ± 0.026	0.743 ± 0.020	0.0011 ± 0.0067			

Table 2. Constraints on $\Omega_{\rm m}$, $\Omega_{\rm DE}$ and Ω_k at 1- σ obtained for an open ACDM cosmology.



Figure 3. 1 and 2 sigma joint constraints in the $\Omega_m - \Omega_{DE}$ plane obtained for an open-ACDM model with WMAP7yr data alone (red shaded areas) and WMAP7yr+OHD+ H_0 data (blue shaded areas).

constraining Ω_k and the dark energy EoS. When we combine WMAP7yr data with both H(z) and H_0 , we find that the constraint on Ω_k is improved of almost one order of magnitude, and the accuracy on the estimate of w_0 is also about three times better (see Tab. 3 and the solid lines of Fig. 4). As a comparison, we also show how much the constraint are improved by adding to WMAP7yr data informations from BAO and SNe. We find that the combination of WMAP7yr+OHD+ H_0 has almost the same statistical power in constraining Ω_k as WMAP7yr+BAO+SNe. In the case of the dark energy parameter w_0 , the WMAP7yr+BAO+SNe constraints are about 3 times better compared to the constraint obtained with the OHD; however we point out that the constraints on w_0 from OHD may be improved if (when) future surveys will provide an higher statistics of cosmic chronometers not only in the local Universe, but also at higher redshifts, where the difference in H(z) due to w_0 becomes more significant (see Fig. 1).

Finally, we further generalized our approach by exploring the constraints that can be set when

MARGINALIZED 1D CONSTRAINTS owCDM model

owCDM model							
$\Omega_{\rm m}$ $\Omega_{\rm DE}$				w_0			
WMAP7yr	0.58 ± 0.31	> 0.52	-0.013 ± 0.071	-0.65 ± 0.61			
WMAP7yr+OHD	0.277 ± 0.045	0.726 ± 0.042	0.0019 ± 0.0085	-0.90 ± 0.18			
WMAP7yr+ H_0	0.246 ± 0.021	0.769 ± 0.029	-0.010 ± 0.014	-1.29 ± 0.31			
WMAP7yr+OHD+ H_0	0.251 ± 0.017	0.751 ± 0.019	0.0024 ± 0.008	-1.06 ± 0.15			
WMAP7yr+BAO+SNe	0.288 ± 0.017	0.718 ± 0.015	-0.0055 ± 0.0064	-0.997 ± 0.060			

Table 3. Constraints on $\Omega_{\rm m}$, $\Omega_{\rm DE}$, Ω_k and on the equation of state parameter w_0 at 1- σ obtained for an open wCDM cosmology.



Figure 4. Effective $\Delta \chi^2$ as a function of Ω_k (left panel) and w_0 (right panel) for an open wCDM cosmology. The dotted lines show the constraint obtained with WMAP7yr data alone, the dashed line the constraints obtained from the combined WMAP7yr+BAO+SNe, and the solid lines the constraints from the combined WMAP7yr+OHD+ H_0 .

not only the dark energy equation-of-state is let free as a constant, but also allowed to vary with time, with w(z) parametrized with w_0 and w_a (ow(z)CDM). We already discussed that such a model is not already available in the LAMBDA website, and that we decided to study the MCMC privately run from Ref. [18] (see Sect. 3). Since this chain has been run with a prior on H_0 [27], in this case we studied the effect of adding to this chain respectively the Hubble parameter measurements, the much tighter prior on H_0 provided by Ref. [16], and both OHD and H_0 . The 1 and 2 sigma joint constraints showing the $\Omega_{DE} - \Omega_m$, $\Omega_k - w_0$, and $w_0 - w_a$ planes are displayed in Fig. 5 and Fig. 6, and the



Figure 5. Constraints obtained for an open-w(z)CDM model. Left panel: 1 and 2 sigma joint constraints in the $\Omega_m - \Omega_{\rm DE}$ plane obtained with WMAP5yr+ H_0 [27] data (red shaded areas), WMAP5yr+ H_0 [16] data (light-blue shaded areas), and WMAP5yr+OHD+ H_0 [16] data (blue shaded areas). Right panel: 1 and 2 sigma joint constraints in the $w_0 - \Omega_k$ plane obtained with WMAP5yr+ H_0 [27] (red shaded areas), WMAP5yr+ H_0 [16] data (light-blue shaded areas), and WMAP5yr+OHD+ H_0 [16] (blue shaded areas).



Figure 6. 1 and 2 sigma joint constraints in the $w_0 - w_a$ plane obtained for an open-w(z)CDM model with WMAP5yr+ H_0 [27] data (red shaded areas), WMAP5yr+ H_0 [16] data (light-blue shaded areas), and WMAP5yr+OHD+ H_0 [16] data (blue shaded areas).

corresponding estimates are reported in Tab. 4.

The red shaded areas of Fig. 5 and Fig. 6 show the constraints obtained by the original MCMC, which combined WMAP5yr data with the H_0 estimate of Ref. [27]. Also in this case, it is evident the major improvement that is obtained by adding to the previous dataset the H(z) measurements and the H_0 estimate of Ref. [16] by looking at the blue shaded areas of the same figures. To highlight the improvement provided by the H(z) measurements, we also show in light-blue shaded areas the constraints obtained by combining WMAP5yr data with the extremely accurate estimate of H_0 from Ref. [16]. We gain a factor three in accuracy in the constraint of Ω_m and Ω_{DE} , a factor 40 for Ω_k , and about a factor two for w_0 . Moreover a comparison with the constraints on w_0 and w_a obtained by Ref. [18] highlights how improved measurements of H(z) [6] and H_0 [16] help to shrink the confidence region especially around w_a . Including these new measurements, we set a limit of $w_a = 0.56^{+0.92}_{-0.96}$ (68% C.L.). It is important to stress that compared to the measurement of Ref. [18], the major

MARGINALIZED 1D CONSTRAINTS ow(z)CDM model

	Ω_{m}	$\Omega_{ m DE}$	$\Omega_{\mathbf{k}}$	w_0	w_a
WMAP5yr+ H_0 [27]	0.291 ± 0.064	0.709 ± 0.059	-0.01 ± 0.42	-0.74 ± 0.68	< -0.01
WMAP5yr+OHD+ H_0 [27]	0.298 ± 0.042	0.697 ± 0.044	0.005 ± 0.012	-0.89 ± 0.41	$0.56^{+0.92}_{-0.96}$
WMAP5yr+ H_0 [16]	0.261 ± 0.025	0.734 ± 0.035	-0.01 ± 0.45	-1.06 ± 0.65	< 0.05
WMAP5yr+OHD+ H_0 [16]	0.270 ± 0.025	0.725 ± 0.026	0.0040 ± 0.0093	-1.19 ± 0.27	$0.56\substack{+0.92\\-0.96}$

Table 4. Constraints on $\Omega_{\rm m}$, $\Omega_{\rm DE}$, $\Omega_{\rm k}$, w_0 and w_a at 1- σ obtained for a open ACDM cosmology with equation-of-state parameter for dark energy parametrized as $w(z) = w_0 + w_a(z/(1+z))$.

contribution in the improved constraints is given by the new H(z) measurements of Ref. [6], since the same analysis repeated with the H_0 estimate of Ref. [28] (the same used in Ref. [18]), still gave compatible results with the ones discussed above.

4.2 Constraints on deviations from ΛCDM that affect the expansion history only through parameter degeneracies

It has been noted already in the literature that neutrino properties can be constrained by combining CMB data with low redshift observations of the expansion rate (e.g. see Refs. [16, 25, 26]).

We started by considering a simple flat Λ CDM Universe where the effective number of relativistic species $N_{\rm rel}$ is let free, and not fixed to the standard value of $N_{\rm rel} = 3.04$; we analyzed this model studying the combined dataset of WMAP7yr results [11] with the South Pole Telescope (SPT) dataset [22], and of WMAP 7-years results with the Atacama Cosmology Telescope (ACT) dataset [23, 24].

The effective $\Delta \chi^2$ as a function of $N_{\rm rel}$, for CMB data alone is shown with the dotted line of Fig. 7. CMB data yield a $N_{\rm rel} = 3.84 \pm 0.62$ (at 68% C.L.) in the case of WMAP7yr+SPT, and $N_{\rm rel} = 5.19 \pm 1.28$ (at 68% C.L.) in the case of WMAP7yr+ACT.

These constraint are improved by adding BAO data from Ref. [12]. In fact the combinations WMAP7yr+SPT+BAO+ H_0 and WMAP7yr+ACT+BAO+ H_0 yield $N_{\rm rel} = 3.84 \pm 0.42$ and $N_{\rm rel} = 4.52 \pm 0.74$ (at 68% C.L.) respectively. The fact that these measurements are skewed towards $N_{\rm rel} = 4$ has led some authors to claim the existence of an extra relativistic specie (also called "dark radiation") in both cases. The analysis of the combination WMAP7yr+SPT(/ACT)+OHD+ H_0 does not provide evidence for deviations from the standard value of three neutrinos as $N_{\rm rel} = 3.04$ is within the 2σ errorbar; more precisely we find $N_{\rm rel} = 3.45 \pm 0.33$ ($N_{\rm rel} = 3.71 \pm 0.45$, both at 68%CL), and our data are able to exclude $N_{\rm rel} > 4$ at 95% (74%) C.L. All the constraints on $N_{\rm rel}$ are presented in Tab. 5.

Of course OHD does not constrain $N_{\rm rel}$ directly but it breaks the CMB degeneracy between $N_{\rm rel}$ and the parameters setting matter-radiation equality and governing the expansion history (see e.g., [25]). We note how, in this case, almost all the statistical power of constraining $N_{\rm rel}$ is given by



Figure 7. Effective $\Delta \chi^2$ as a function of $N_{\rm rel}$. The left panel shows the constraints obtained with WMAP7yr+SPT (dotted line), WMAP7yr+SPT+BAO+ H_0 (dashed line), and WMAP7yr+SPT+OHD+ H_0 (solid line); the right panel shows the constraints obtained with WMAP7yr+ACT (dotted line), WMAP7yr+ACT+BAO+ H_0 (dashed line), and WMAP7yr+ACT+OHD+ H_0 (solid line). As a reference, the value $N_{\rm rel} = 3.04$ is shown with a vertical dashed line.

OHD, since the constraints obtained with WMAP7yr+SPT(/ACT)+OHD+ H_0 and with WMAP7yr+SPT(/ACT)+OHD are almost identical.

$\Lambda CDM + N_{rel} model$					
	N_{rel}	N_{rel}			
	using SPT data	using ACT data			
WMAP7yr+SPT(/ACT)	$3.84 \pm 0.62 (\pm 1.22)$	$5.19 \pm 1.28 (\pm 2.52)$			
WMAP7yr+SPT(/ACT)+OHD	$3.37 \pm 0.34 (\pm 0.67)$	$3.59 \pm 0.48 (\pm 0.94)$			
WMAP7yr+SPT(/ACT)+ H_0	$3.72 \pm 0.46 (\pm 0.89)$	$4.37 \pm 0.74 (\pm 1.46)$			
WMAP7yr+SPT(/ACT)+OHD+ H_0	$3.45 \pm 0.33 (\pm 0.65)$	$3.71 \pm 0.45 (\pm 0.88)$			
WMAP7yr+SPT(/ACT)+BAO+ H_0	$3.84 \pm 0.42 (\pm 0.83)$	$4.52 \pm 0.74 (\pm 1.47)$			

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Table 5. Constraints on $N_{\rm rel}$ at 1- σ (2- σ) obtained for a flat Λ CDM cosmology.

We then considered a flat Λ CDM Universe where the total neutrino mass is not set to be zero but is left as a free parameter, and looked both at the constraints on the sum of neutrino masses² Σm_{ν} and at the contour plot $\Sigma m_{\nu} - \sigma_8$. Figure 8 shows that WMAP7yr alone is not able to set stringent constraints on the neutrino mass (dotted line of the left plot of Fig. 8), with $\Sigma m_{\nu} < 0.76 \text{ eV}$ (at 68% C.L.) and presents a clear degeneracy with the σ_8 parameter (see the red shaded areas of the right plot of Fig. 8). When WMAP7yr is combined with BAO and Supernovae Type Ia (SN) datasets [13], part of the degeneracy is lifted, and there is a consequent improvement also in the neutrino mass constraints, with $\Sigma m_{\nu} < 0.42 \text{ eV}$ (at 68% C.L.) (see the dashed line and the green shaded areas respectively of the left and right plots of Fig. 8). Also in this case, more stringent constraints are obtained by combining WMAP7yr+OHD+ H_0 (see the blue shaded area and the continuous line of the left and right plots of Fig. 8 respectively), with a tight constraint on the neutrino mass $\Sigma m_{\nu} < 0.24$ eV (at 68% C.L.). However, we note that for this particular model, with the currently achieved precision in H(z), the final constraints are mostly driven by the H_0 measurement. In Sect. 5 we will discuss the effect of more precise OHD measurements in constraining cosmological parameters.

Similarly to the $N_{\rm rel}$ case, OHD do not constrain the neutrino mass directly, but break CMB degeneracies (as for example explored in Ref. [26] and references therein). All the constraints on Σm_{ν} and σ_8 are reported in Tab. 6.

MARGINALIZED 1D C	CONSTRAINTS
$\Lambda \text{CDM} + \Sigma m_{\nu}$	model

	Σm_{ν}	σ_8			
WMAP7yr	< 0.76	0.687 ± 0.075			
WMAP7yr+OHD	< 0.40	0.742 ± 0.057			
WMAP7yr+ H_0	< 0.24	0.755 ± 0.045			
WMAP7yr+OHD+ H_0	< 0.24	0.759 ± 0.045			
WMAP7yr+BAO+SNe	< 0.42	0.734 ± 0.060			

Table 6. Constraints on Σm_{ν} (in units of eV) and σ_8 at 1- σ obtained for a flat Λ CDM cosmology.

 $^{^{2}}$ We assume three degenerate neutrino species; current data have no sensitivity on the hierarchy, so this is a very good approximation.



Figure 8. Left panel: effective $\Delta \chi^2$ as a function of Σm_{ν} obtained with WMAP7yr alone (dotted line), WMAP7yr+SN+BAO (dashed line), and WMAP7yr+OHD+ H_0 (solid line). Right panel: 1 and 2 sigma joint constraints in the $\Sigma m_{\nu} - \sigma_8$ plane obtained with WMAP7yr alone (red shaded areas), WMAP7yr+SN+BAO (green shaded areas), and WMAP7yr+OHD+ H_0 (blue shaded areas).

4.3 Dependence of the results on the assumed stellar population synthesis model

One of the most important sources of uncertainty in the estimate of H(z) is the stellar population synthesis (SPS) model assumed. This issue has not be extensively studied yet, due to the intrinsic difficulties in developing new models. Nevertheless, it has to be taken into account, because, in principle, different models may yield to different age (or differential age) estimates, therefore biasing the measurements of the Hubble parameter.

As already discussed, to date most of the H(z) estimates have been obtained using BC03 models [20], which are at present time the most widely used. In Ref. [18] an effort has been done in investigating the dependence of the age estimate on the assumed SPS model, by building a mock catalog with one library and trying to recover the age with another one; in their analysis they found a general agreement, but also noted that the wavelength coverage is fundamental, since the agreement worsen noticeably in the case that the spectra analyzed do not include light bluewards of the 4000 Å break. In Ref. [9] the differential evolution of the 4000 Å break feature as a function of redshift has been studied in a sample of ETGs extracted from SDSS to provide estimates of the Hubble constant and of the dark energy parameter w_0 . These estimates have been obtained either with BC03 models and the new MaStro models, leading to the conclusion that there is no significant difference between BC03 and MaStro models, since the values for both H_0 and w_0 are compatible within the 1 σ errors. At the

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	$\Omega_{\rm m}$	Ω_{DE}	$\Omega_{\mathbf{k}}$	w_0	w_a		
WMAP5yr+ $H_0[27]$	0.291 ± 0.064	0.709 ± 0.059	-0.01 ± 0.42	-0.74 ± 0.68	< -0.01		
WMAP5yr+ $H_0[16]$ +OHD(BC03)*	0.269 ± 0.025	0.729 ± 0.027	0.0004 ± 0.011	-1.25 ± 0.28	$0.85^{+1.15}_{-1.23}$		
$WMAP5yr + H_0[16] + OHD(MaStro)^*$	0.257 ± 0.023	0.740 ± 0.026	0.0011 ± 0.0094	-1.01 ± 0.27	< -0.23		

Table 7. Comparison of the constraints on $\Omega_{\rm m}$, $\Omega_{\rm DE}$, $\Omega_{\rm k}$, w_0 and w_a obtained for a open ΛCDM cosmology with equation-of-state parameter for dark energy parametrized as $w(z) = w_0 + w_a(z/(1+z))$ assuming different SPS models. *For this comparison, the H(z) measurements are taken from just Ref. [6].

present time, only Ref. [6] provides measurements of H(z) using the "cosmic chronometers" approach with two different models; in their work they found that, with their approach, the Hubble parameter estimates have proven to be extremely robust even changing SPS model, with values always in agreement (except for one estimate at high redshift) well within the 1σ errors.

Here, we want to study the effect on the cosmological parameters estimate of the difference in H(z) due to the SPS model assumed. We, therefore, decided to compare the constraints obtained with OHD obtained with BC03 models (see Tab. 1) with the ones obtained using OHD which assume MaStro SPS models; in order to perform a meaningful comparison, here we use as H(z) measurements only those obtained from Ref. [6], not adding data from Ref. [17] and Ref. [18] which have been obtained with BC03 models. We show here as an illustrative example the case of the ow(z)CDM model, which is the most general. The results are reported in Tab. 7, where as a comparison are provided also the constraints obtained with just WMAP 5 years data in addition to H_0 estimate from Ref. [27].

We find the estimates of the adimensional density parameters $\Omega_{\rm m}$, $\Omega_{\rm DE}$, and $\Omega_{\rm k}$ to be extremely robust against the change of SPS model, testifying the strength and the stability of OHD in constraining the curvature and matter content in the Universe. The estimates of w_0 and w_a are also fully compatible within the 1σ errors, although the errors on w_a are still large.

We therefore conclude that, given the current level of statistical errors, the analysis performed with our method is reliable, and robust against a possible bias to to the different choice of SPS model.

4.4 The effect of the progenitor bias on the cosmological parameters

The "cosmic chronometers" approach is based on the assumption that the chronometers that are used to probe the relative age evolution of the Universe are well synchronized, i.e. they have all formed at the same redshift. The progenitor bias is an effect which may falsify this assumption, altering the correct age-redshift relation by flattening it, since it adds to the sample an increasing younger population with decreasing redshift. It is straightforward to understand that the resulting Hubble parameter obtained from this flattened relation result biased toward higher values. This effect, if accurately estimated, may be corrected to recover the unbiased H(z).

However, there are a few caveat before naively correcting for this effect. The first (and most important) one, is that there is no evidence that our analysis is biased in that sense. As discussed before, part of our sample [17, 18] is based on estimates obtained from the upper-envelope of the agez relation, and therefore, by definition, is not affected by progenitor bias. Moreover, in Ref. [6] the upper-envelope of the D4000 - z relation has also been analyzed, as a check, and the H(z) obtained (which should not be biased by the progenitor bias effect) is in good agreement with the Hubble parameter estimated from the median relations, showing that the data selected in this way are still well compatible with a null contribution of the progenitor bias. This mainly reflects the fact that the progenitor bias is supposed to be significant especially for less massive ETGs, in which the star formation, even if limited, is still ongoing; on the contrary, the sample analyzed here has been carefully selected to keep only the extremely red, passive and massive ETGs exactly to avoid this issue. The other point is that the correction to be applied is model-(and also cosmology-)dependent, and until a more detailed knowledge of the mechanisms of ETGs formation and evolution is achieved it is difficult (if not impossible) to estimate accurately this effect, and then to correct for it.

Nevertheless, we decided to estimate approximately the maximum possible magnitude of this effect assuming a conservative contribution due to progenitor bias, and its impact on the total error of cosmological parameters. In this sense, we warn the readers that this estimate represents an upper limit to the effect of the progenitor bias, since as discussed above our data are well compatible with being unbiased.

In Ref. [6] is provided the theoretical formalism to correctly estimate the effect of this bias on the Hubble parameter, and a rough estimate is given on the basis of the actual observational constraints. On average it has been found that it is roughly half the statistical error (~ $0.6\sigma_{\text{stat}}$), but adding only ~ 1% on average to the total error budget, if also this contribution is summed in quadrature. Since, for the purpose of this paper, we are just interested in the effect that the progenitor bias has on the estimated cosmological parameters, we proceeded as follows: we considered two possible H(z) relations, one which is assumed unbiased and one which we corrected taking into account the



Figure 9. Simulated H(z) with 5% and 2.5% uncertainty, and relative constraints on various cosmological parameters. The red contours are obtained from WMAP data alone (WMAP+ H_0 [27] in the case of the $w_0 - w_a$ and $w_0 - \Omega_k$ plots), the blue contours from the analysis of WMAP+OHD+ H_0 [16] using the observed H(z) measurements, the light-green contours using the simulated H(z) measurements assuming a 5% error on the Hubble parameter and the violet contours using the simulated H(z) measurements assuming a 2.5% error on the Hubble parameter.

progenitor bias effect as estimated in [6]. The difference between the values estimated from the two relations will quantify the systematic error due to the progenitor bias. We compared the original total error with a new total error obtained by summing in quadrature also the progenitor bias contribution, and estimated its impact by evaluating for each cosmological parameter A the percentage difference between the old and the new error, $(\sigma_{A,prog bias} - \sigma_A)/|A|$.

We found that the impact of considering the progenitor bias in the total errorbars is, for each model analyzed, less than 2% for Ω_m , and less than 1% for $\Omega_{\rm DE}$; the effect is higher for Ω_k simply because this value is compatible with being null. Concerning the dark energy EoS parameters the impact is higher, but still always less than 10% for w_0 (~ 5% when considering also the prior from the Hubble constant of Ref. [16]). Concerning the constraints on $N_{\rm rel}$, the impact is always less than 1%. Finally the limit given to the sum of the neutrino masses is worsened of about 20% (40% if not considering the prior on H_0).

5 Future perspectives

At the present day, the studied OHD represent the most precise and with the widest redshift coverage H(z) measurements available. However, it has to be stressed that, unlike all the other kind of cosmological probes which nowadays can be based on dedicated surveys (e.g. BOSS [29] for the BAO)

these results have been obtained mainly as by-product of spectroscopic surveys devoted to other kind of studies. In this section we therefore decided to explore the possible improvement to the constraints of cosmological parameters if (when) a much higher statistic over a wider redshift coverage will be achieved; note that this study represents more than a theoretical exercise, since Euclid [30] and BOSS [29] will provide thousands of passive galaxies at z > 0.5, and this will help to improve significantly the accuracy in H(z). With the increasing statistic, it will become crucial to understand if the systematic effects may be controlled and minimized. In Ref. [6] it is shown that the systematic error due to metallicity and star formation uncertainty can be reduced at < 1% for a sample of ETGs large enough. Moving at higher redshift, hence moving closer to the redshift of formation of these galaxies, the study of the effective SFH and of the progenitor bias will be more important, even if some methods have already been suggested to minimize this issue (e.g. see Ref. [6, 18]).

We simulated two measurements of H(z) at 0.1 < z < 2 with an accuracy respectively of 5% and 2.5%, which are shown in the upper-left panel of Fig. 9. These data have been simulated assuming a flat Λ CDM cosmology, with $H_0 = 73.8 \text{ kms}^{-1} \text{Mpc}^{-1}$ [16]. The remaining three plots of Fig. 9 show the enhancement of the constraints that can be achieved with such measurements: the red contours represent the case of WMAP alone (WMAP+ H_0 [27] for the $w_0 - w_a$ and $w_0 - \Omega_k$ plots), the blue contours the case of WMAP+OHD+ H_0 [16] with the H(z) measurements used in this analysis, the light-green contours show how the constraints are shrunk with a 5% error on H(z) up to $z \sim 2$, and the violet contours with a 2.5% error.

Compared to the actual measures, with a 5% error it will be possible to reduce the errorbar on Ω_k of about 50%, on w_0 of about 30%, and on w_a of about 15%; similarly, with a 2.5% error the errorbar on w_0 will be reduced of about 65%, and on w_a of about 60%. The constraints on Σm_{ν} will be even more stringent: with a 5%-2.5% error on H(z) up to $z \sim 2$ it will be possible to reduce them of about 55% and 60% respectively.

This forecast has been performed assuming that the errors on H(z) are uncorrelated, which is the assumption that has been made in all analyses so far. As the statistical errors on the galaxy age determinations shrink, systematic errors become important and, in principle, they might induce correlations among H(z) estimates at different redshifts. However, preliminary analyses performed so far (see sect. 3 of Ref. [18]) do not find any evidence for this effect for spectra with sufficiently high spectral coverage and signal to noise. This is, however, an issue to consider and quantify carefully when analyzing future data. Similarly all tests reported in Sect. 4.3 and 4.4 indicate that the listed systematics are under control at the level of current data. A similar analysis will have to be performed on future data and of course it remains to be seen wether systematics errors can be kept under control so that our estimated forecasted H(z) errors can indeed be achieved.

The estimate of the Hubble parameter using the "cosmic chronometers" appears to be a promising avenue to be further explored and investigated as a cosmological probe complementary to the standard methods.

6 Conclusions

We have presented a compilation of direct measurements of the Hubble parameter as a function of redshift which includes the latest determinations and which has the widest redshift coverage (0.1 < z < 1.75) to date. The H(z) measurements are based on the use massive and passively evolving early-type galaxies as "cosmic chronometers", providing standard(-izable) clocks in the universe to trace its late-time expansion history. These measurements have been referred to as observational Hubble determination, OHD. The z = 0 measurement is provided by Ref. [16] using a different technique. This compilation has then be used in combination with CMB data to constrain cosmological parameters. As expected, the OHD are useful to break CMB cosmological parameters degeneracies only for models that deviate from the standard ("minimal") flat Λ CDM model.

The cosmological parameters that most directly affect the late-time expansion history are Ω_m and Ω_k (and thus Ω_{DE}). If dark energy is not assumed to be a cosmological constant but a fluid with equation of state parameter either constant in redshift or with a linear dependence on the scale factor, the equation of state parameters also affect the expansion history directly and can thus directly be constrained by the OHD.

We reported constraints on deviations from a simple "minimal" flat Λ CDM model that allow for spatial curvature and models that allow for spatial curvature and a dark energy parameterized by its equation of state parameter. We found that OHD are crucial in breaking CMB parameter degeneracies and the resulting constraints are competitive (and in broad agreement) with those obtained from BAO and/or Supernovae data.

We have also considered extensions of the "minimal" flat Λ CDM model where the extra parameters do not affect the expansion history directly but which are constrained by combining CMB with OHD through reduction of degeneracies with other parameters. We obtain competitive errors on the effective number of relativistic species, $N_{\rm rel} = 3.45 \pm 0.33$ at 1- σ using as external dataset also measurements from SPT, and $N_{\rm rel} = 3.71 \pm 0.45$ at 1- σ using as external dataset also measurements from ACT; our analysis did not find evidence for extra neutrino species, excluding $N_{\rm rel} > 4$ at 95% C.L. using SPT data (74% using ACT data). We also found very competitive limits on the sum of neutrino masses $\Sigma m_{\nu} < 0.24$ eV at 1- σ .

We studied separately the combination of CMB data with OHD and H_0 , to establish the relative contribution to the constraints given by these two measurements: we find that while for many models their statistical power in constraining cosmological parameters is comparable, the H(z) is particularly fundamental to constrain models in which a time-variability of the parameters (e.g. dark energy EoS) is allowed. On the other hand, at the state of the art of the achievable precision on H(z), the constraints on the sum of the neutrino mass is almost completely given by the combination of CMB data with H_0 .

The reliability and robustness of those constraints have been checked against the change of SPS model assumed to estimate H(z); we find that all the cosmological parameters are almost insensitive to the SPS model chosen, only the parameters associated to dark energy presenting a mild dependence, but still well within the 1σ errorbars. We also explored the effect that the progenitor bias may have on our estimates, and concluded that, even if present, given the actual statistical errors this effect should not significantly affect our estimate on most of the parameters.

Finally, we demonstrated that future measurements, which will provide an higher accuracy on the Hubble parameter (e.g. Euclid, BOSS), will allow to achieve an extreme accuracy for a variety of cosmological parameters. When measurements with an accuracy of ~ 5% or lower up to $z \sim 1 - 1.5$ will be available, the progenitor bias and the detailed star formation history of the galaxy will become more important in the total systematic error budget, and should be carefully taken into account, and their effect quantified.

Measurements of the expansion history are the only direct indication that the Universe is undergoing an accelerated expansion and therefore that it should be dominated by a dark energy component. Given the magnitude of the implications of the accelerated expansion, it is important to have different, independent and complementary observations of the expansion history. In addition to supernovae, providing the standard candles approach, and BAO, providing the standard ruler approach, the OHD yields another independent and direct measurement of the low redshift expansion history.

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