The objective of this work is to provide teachers of mathematics disciplines with new tools developed with GeoGebra software. The motivation is the difficulty that students have to understand the abstract content of mathematical notions. In order to palliate this shortage, we create tools that may be used in class by teachers and by students for their autonomous learning.

Student's autonomous learning and tools for teachers by means of the use of GeoGebra.

Mathematics I and II subjects of the degrees in ADE and ECO of the University of Barcelona

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mathEopt

SUMMARY

The objective of our project has been to improve the teaching in mathematics subjects of the degrees in Economics (ECO) and Business management (ADE) by incorporating the GeoGebra free software.

The project started when university degrees were being implemented. At that time, we realized that we could no longer use the Derive software because the practical lectures in the computer rooms were cancelled and only the teacher's computer attached to a projector remained. On the top of that, the time devoted to lectures was reduced. Under those constraints, we thought that the ability of GeoGebra to give functions and vectors life could help us in explaining the theoretical concepts of the mathematics subjects in an appealing and intuitive manner.

We have created several applets for the subjects Mathematics I and Mathematics II of the degrees in ECO and ADE. Thanks to the Moodle platform we have been able to put these applets at students' and teachers' disposal.

Linked innovation lines: 216231 - Moodle / 201995 – Formative assessment / 235623 - Project-based learning, PBL / 020691 – Case study / 025424 – Simulations

Keywords: Mathematic **s**oftware; Theoretical concept; Applet.

Contents

INTRODUCTION

OBJECTIVES

Our main goal has been to enhance the ability for abstract comprehension of our students and to improve the understanding of our subjects.

GeoGebra allows to fulfil the wish of so many mathematics teachers to make abstract mathematical concepts tangible to their students. These concepts include the linear combination of vectors, the linear dependence and independence, the bases of a vector space, the coordinates of a vector in different bases, the topological notions of open, closed, and bounded sets, the level curves of a function of two variables, and the family of curves that solve a differential equation.

When teaching the subject of Mathematics I, we often face the lack of time available to cover all the topics in the syllabus. Consequently, many times we had to forgo the "luxury" of interacting with the applets in classroom and suggest our students to use it on their own autonomous work. Regarding the subject of Mathematics II, the applets that analyze the sensitivity when the coefficients of the objective function or the independent terms of the constraints change have been especially valuable.

We believe to have accomplished the following objectives that were initially considered:

- To improve the comprehension in the subjects involved.
- To stimulate the interest of the students in mathematical issues.
- To improve the teaching activity by incorporating new computer resources.
- To offer new tools for the autonomous learning.

Developing this project has been a very good experience for the involved teachers, even if it has been very time consuming. Getting familiar with applets has demanded quite some effort. However, we hope that our work will ease the explanation of the topics covered. Hopefully, the creation of the applets within this project has contributed to take a step forward in improving the learning process of our students.

DETECTED SHORTAGES

A main shortage that we have encountered is the limited capability of GeoGebra for computing and drawing functions of more than one variable. The graphs in three dimensions are often difficult to understand, even for teachers. This fact increases the challenge of making useful applets in $\mathbb{R}^3$.

A second obstacle that we have faced is the lack of time to use the applets during class hours. In this regard, we would like to stress the broad syllabus of the subject. Indeed, we consider that even if the use of applets has a reward in students understanding, it also has a time cost for showing its performance in class.

APPLETS

In what follows we present the applets (objectives, how they work, limitations, and kinds of problems for which each of them is appropriate). All of them form a GeoGebrabook which is available online.

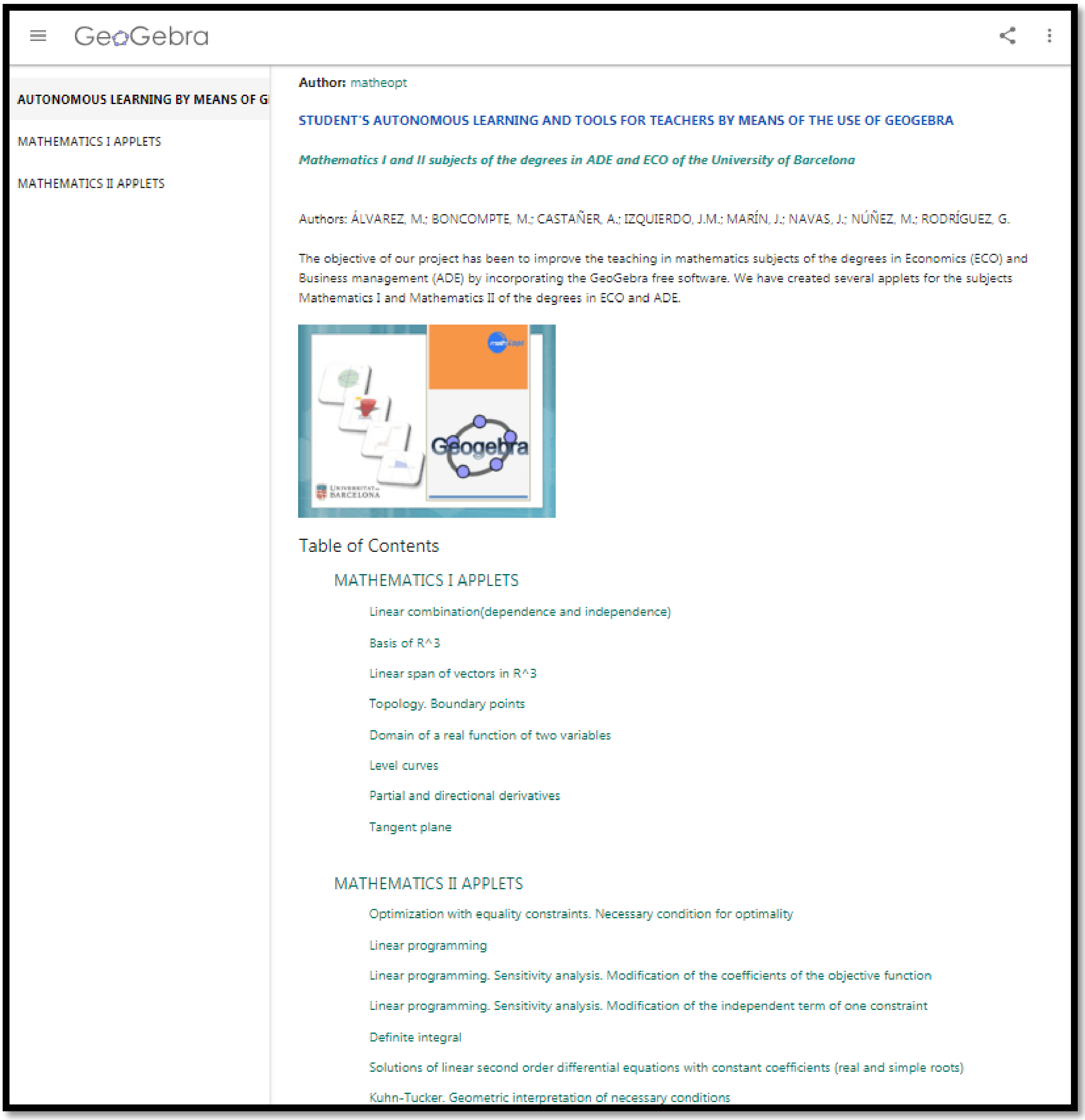


Figure 1: “Autonomous learning by means of GeoGebra” GeoGebrabook

First, we will introduce the applets built for the subject Mathematics I and next, the applets built for Mathematics II.

MATHEMATICS I APPLETS

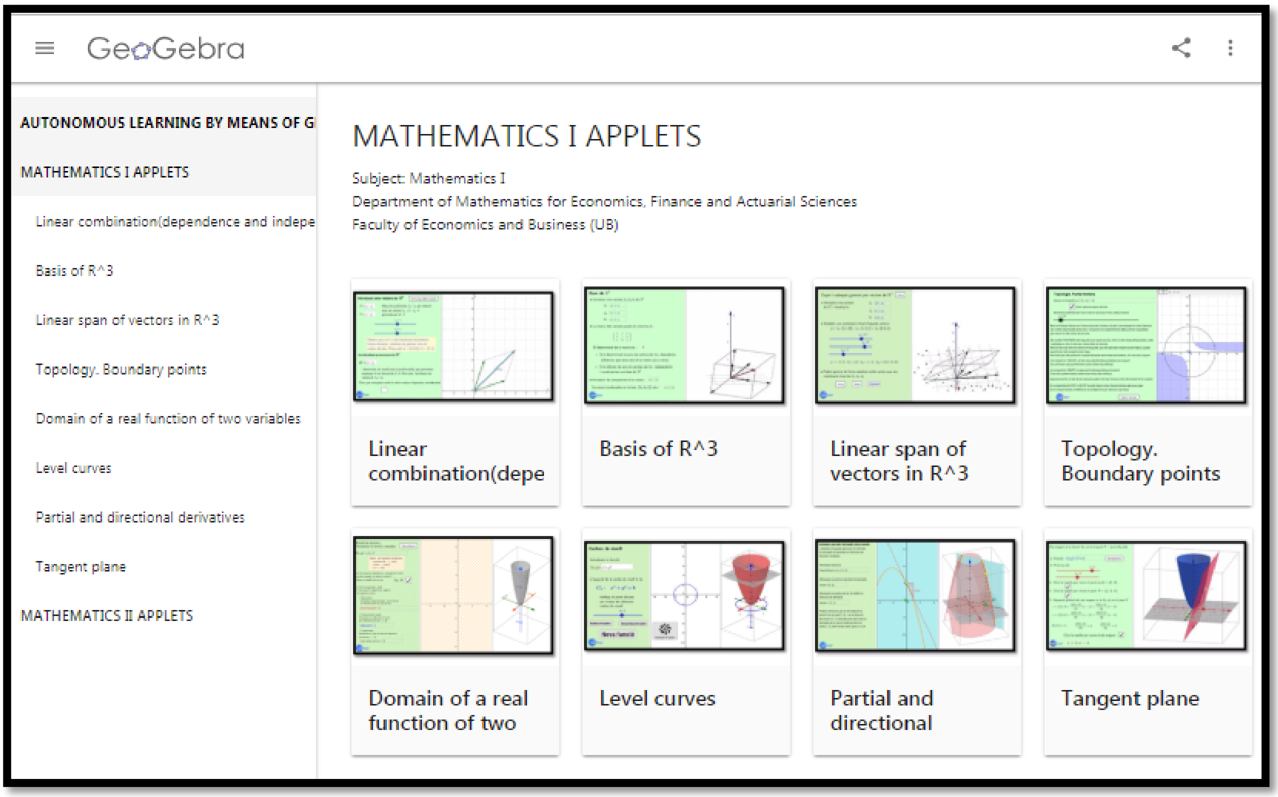
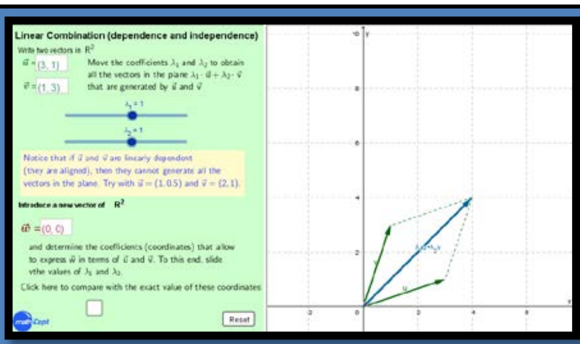


Figure 2: Mathematics I applets

LINEAR COMBINATION (DEPENDENCE AND INDEPENDENCE)



Objective

What does it teach to do?	What concepts does it explain?
This applet teaches us to graphically and analytically calculate the coordinates of a vector in a given basis.	*Linear combination of two vectors in the plane. Coordinates of a vector in a basis of* $\mathbb{R}^2$.

Explanation of the applet

How does it work?

To start with, the applet requests that two vectors (which we draw in green) are introduced. Then, moving two sliding points, the vector of the plane generated by these two vectors is drawn in blue. In particular, if moving the sliding points, we observe that we fill the entire drawing, then the two given vectors are linearly independent and span $\mathbb{R}^2$. *Therefore, they form a basis of* $\mathbb{R}^2$. *On the other hand, if the two vectors that we begin with are aligned, we will only generate a straight line.*

The second part of the applet is aimed at thinking about the inverse problem: instead of moving the sliding points to generate different vectors in the plane, we can write either one vector (which is drawn in red) to find its coordinates in the basis chosen at the beginning. We will determine these coordinates geometrically with the sliding points until the generated blue vector matches our red vector. If we want to check the accuracy of the coordinates obtained geometrically, we will click in the white box at the end and get the exact coordinates; in fact, this box can be used as a coordinate calculator. Note that the values obtained with the sliding points are approximations of the real value of these coordinates

Limitations:

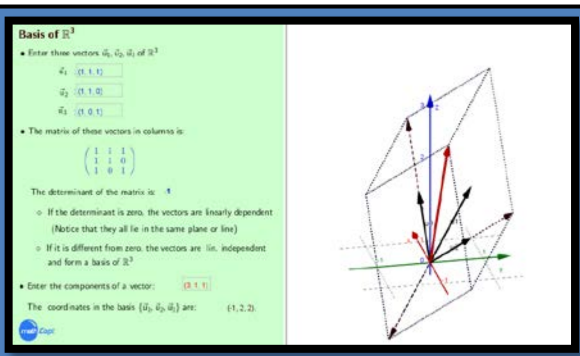
Sometimes it is not possible to calculate the coordinates exactly with the sliding points of the entered red vector. However, the exact values can always be obtained by clicking on the last box of the applet.

Another limitation is that this applet only works for vectors of the plane, that is, for vectors with two components.

Kinds of problems for which it is appropriate

This applet is intended not so much as a tool to solve exercises, but to geometrically visualize the concepts of spanning sets, linear dependence and coordinates of a vector of $\mathbb{R}^2$ *in a basis. For this reason, it may be useful to present the definition of these concepts in class.*

BASIS OF $\mathbb{R}^3$



Objective

What does it teach to do?	What concepts does it explain?
This applet determines whether a set of vectors forms a basis of $\mathbb{R}^3$ by analyzing the matrix built from these vectors, and graphically confirms the result obtained with the arrangement of the vectors in space.	Coordinates and components of a vector in a basis.

Explanation of the applet

How does it work?	Limitations:
The applet starts with the cleaning of the data of the previous problem and has three functional parts. The first part consists of introducing three vectors of $\mathbb{R}^3$ and observing if they are linearly independent, that is, if they form a basis of $\mathbb{R}^3$. The second part is automatic and generates the matrix of the vectors arranged in columns and calculates their determinant to find out if they are linearly independent. The third and last part proposes entering a vector: the applet automatically calculates the coordinates of the vector in the entered basis.	It does not work for determining bases of vector subspaces.

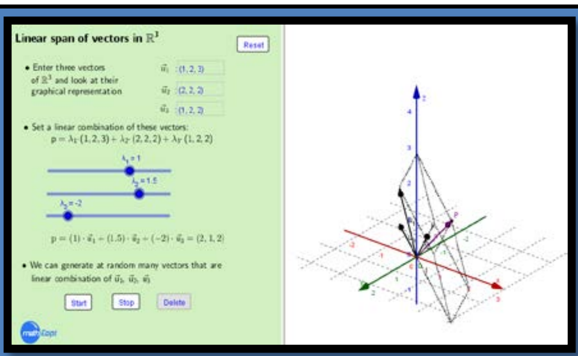
Kinds of problems for which it is appropriate

The applet is designed to help you understand, analytically and geometrically in $\mathbb{R}^3$, the concepts of linear combination of vectors, linear independent vectors and basis of a vector space.

Exercises:

1. *Determine whether the following sets of vectors are linearly independent or not: {(1,0,0),(0,1,0),(0,0,1)}, {(1,1,0),(0,1,1),(-1,0,1)}, {(1,1,0),(-2,1,1),(1,1,1)}.*
2. *Determine the coordinates of the vector (3,1,1,1) in the different bases found in the previous exercise.*
3. *Is the set of vectors {(1,-3,2),(5,2,-1),(7,13,-8),(13,-5,4)} a spanning set of $\mathbb{R}^3$? Is it a linearly independent set? Is it a basis of $\mathbb{R}^3$?*

LINEAR SPAN OF VECTORS IN $\mathbb{R}^3$



Objective

What does it teach to do?	*What concepts does it explain?*
This applet aims to give the student a graphic idea of the following concepts: linear combination of $\mathbb{R}^3$ *vectors, linearly independent vectors and linearly dependent vectors*	*Vector subspace described as the linear span of a set of vectors.*

Explanation of the applet

How does it work?

At first, the "start" button clears the data of the previous problem. Operationally speaking, this applet has three parts:

- *The first part consists of introducing three vectors of* $\mathbb{R}^3$ *and observing which are linearly dependent.*
- *The second part consists of entering, by means of three sliding points, three values of the parameters used in the linear combination of the previous vectors and viewing and analyzing the resulting vector.*
- *The third part proposes us to activate an animation process where different linear combination vectors are randomly generated from the three initials. The student is expected to observe which subset of vectors is generated. The "Start" button starts the process and the "Stop" button stops it.*

Limitations:

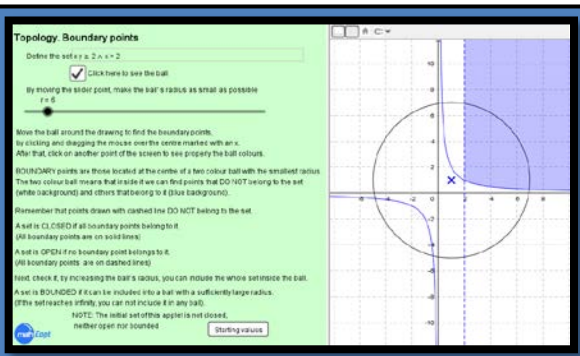
It serves to illustrate the concept of linear combination and linear span of a set of vectors. It does not determine whether a set of vectors is linearly independent, it only offers a graphical intuition. For the analytical determination it is recommended to use the applet "Basis of $\mathbb{R}^3$*"*

Kinds of problems for which it is appropriate

Exercises: *Graphically study if the sets of vectors below are linearly independent and find the dimension of the generated vector subspaces (their linear span):*

a. $\{(1,0,0),(0,1,0),(2,0.5,0)\}$
b. $\{(1,1,0),(0,1,1),(1,1,1)\}$
c. $\{(1,2,0),(0,2,-1),(1,0,1)\}$
d. $\{(1,1,0),(-2,1,-3),(1,0,1)\}$

TOPOLOGY. BOUNDARY POINTS



Objective

What does it teach to do?	What concepts does it explain?
This applet teaches us to distinguish in $\mathbb{R}^2$ the points in the boundary of a set from those that are not, which will allow us to state whether a particular set is open, closed or bounded.	*Boundary points of a set; closed set; open set; bounded set.*

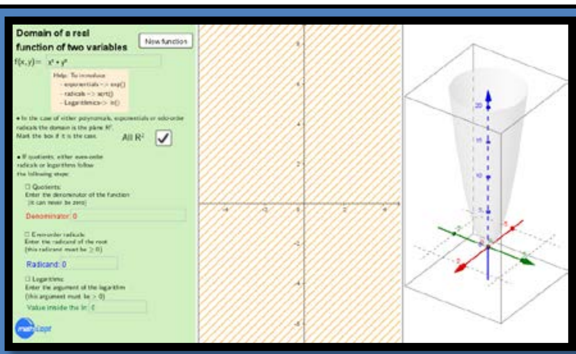
Explanation of the applet

How does it work?	Limitations:
The applet prompts you to enter a set of $\mathbb{R}^2$ and then click a button to display a ball on the screen. This ball, whose radius can be adjusted by means of a sliding point, can be moved by clicking and dragging its center on the graph. *The applet explains that the boundary points are those where, when a ball is placed on top of them with the smallest possible radius, two colors appear: blue for those points of the ball that belong to the set, and white for the ones that do not belong to the set. It should also be borne in mind that the points that are on dashed lines, corresponding to strict inequalities, do not belong to the set.* *Next, it is explained what a closed set is and what an open set is. The student will learn to distinguish them in the sense that, if all the boundary points of the set belong to it, we have a closed set, and if none belong to it, we have an open set.* *Finally, the concept of the bounded set is explained, asking the student to increase the radius of the ball as much as necessary in order to include the set within the ball. If this is possible, the set is bounded and, if not, the set is expected to be not bounded.*	*The applet is can be used only in two variable problems.* *The applet allows you to vary the set of feasible solutions, but only using inequalities and basic logical relations.* *Note that when the original values are restored, the set is not recovered.*

Kinds of problems for which it is appropriate

This applet is designed to graphically interpret the concepts of open and closed sets in relation to the corresponding boundary points, as well as the concept of a bounded set by means of sufficiently large radius balls.

DOMAIN OF A REAL FUNCTION OF TWO VARIABLES



Objective

What does it teach to do?	What concepts does it explain?
This applet graphically describes in the XY base plane the domains of a very wide class of real functions of two variables.	*The domain of a real function of two variables and, indirectly, that of an elementary functional expression.*

Explanation of the applet

How does it work?	Limitations:
After a series of steps that allow us to introduce the function we want to study taking into account the elementary functions that are composed within it, this applet draws in a coordinate system the function in $\mathbb{R}^3$ and its domain on an additional plane.	*The real function to be studied must be expressed by means of certain elementary functions.*

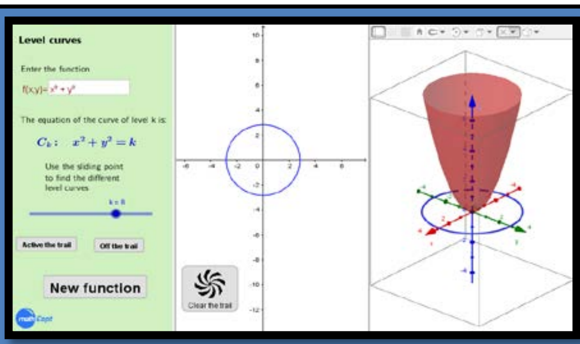
Kinds of problems for which it is appropriate

Drawing the domains of real functions of two variables.

Exercises: *Find the domains of the real functions of two variables below:*

1. $f(x,y) = -x^2 - y^2 + 10$.
2. $f(x,y) = \ln(x^2 - y^2)$.
3. $f(x,y) = 3 - \exp(x - y)$.

LEVEL CURVES



Objective

What does it teach to do?	What concepts does it explain?
This applet graphically determines the contour lines of a two-variable scalar function that has previously been represented. The applet analyzes how the graph of a level curve varies and moves as, with a sliding point, we change the value of the parameter associated with that curve.	*Domain, graph, and level curve of a real function of two variables.*

Explanation of the applet

How does it work?	Limitations:
The applet draws the graph of a real function of two variables. Once it has been drawn, the level curve of a parameter is entered and, thanks to a sliding point that gives specific values to this parameter, the level curves for the different values are displayed in two scenarios: on the XY base plane associated with the three-dimensional graph of the function, and on an additional XY plane.	*The three-dimensional graph of some real functions of two variables.*

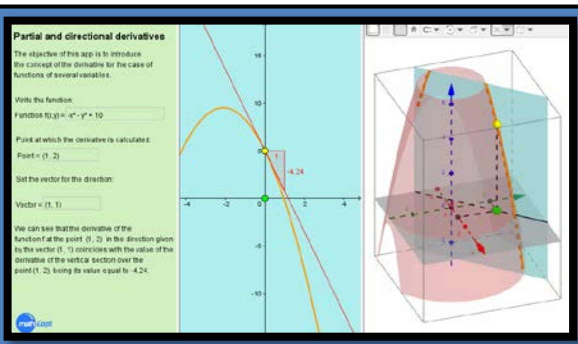
Kinds of problems for which it is appropriate

Graphs of level curves of real functions of two variables.

Exercises: *Study and draw the level curves of the functions below:*

1. $f(x,y) = 2x \cdot y$
2. $f(x,y) = \ln(x+y)$
3. $f(x,y) = x/3y$

PARTIAL AND DIRECTIONAL DERIVATIVES



Objective

What does it teach to do?	What concepts does it explain?
This applet provides us with a geometric explanation of the concept of derivative in functions of two variables by extending interpretation of a function of one variable (as a measure of the change of the function with respect to changes in the independent variable). Since we now have different directions in which to do this change, it is necessary to set a direction.	*Directional derivatives and partial derivatives of real functions of two variables.*

Explanation of the applet

How does it work?	Limitations:
Once a real function has been introduced, the applet asks for the point where we want to calculate the derivative and a vector that will set the direction along which we differentiate (it does not need to be normalized). The applet performs both analytical and graphical (in two and three dimensions) calculation of the resulting directional derivative. The applet has three screens: a first one where we enter the problem data and where the directional derivative is calculated analytically, and two more, of a graphical nature. The first of these graphs shows, in the Cartesian plane, the vertical section of the function (curve-section), the image of the point where the derivative on this curve is calculated (yellow point), as well as the associated derivative. In the second graphic screen, and in three dimensions, a representation of the original function appears, the vertical plane defined according to the direction of the vector, and the section product of the intersection between the vertical plane and the graph of the function.	*Since it is a question of working on the geometric understanding of the concept of directional differentiation, simple functions, planes or paraboloids of revolution must be used to clearly observe the vertical section and the resulting graphic illustration.*

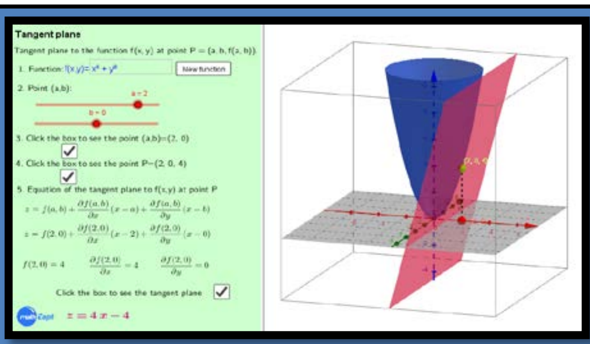
Kinds of problems for which it is appropriate

The objective of this applet, although it calculates the directional derivative, is to help understand the concept of directional derivative from its geometric interpretation.

Exercises: *Compute the directional derivative along the vector v of the function f at the point P in the following cases:*

1. $f(x,y) = -x^2 - y^2 + 10$; $P = (1,2)$, $v = (1,1)$.
2. $f(x,y) = -x^2 - y^2 + 0{,}1\cdot\exp(x + y)$; $P = (1,1)$, $v = (1,0)$.

TANGENT PLANE



Objective

What does it teach to do?	What concepts does it explain?
This applet finds and graphically represents the equation of the tangent plane of a function of two variables at a point in its domain and analyzes how this plane varies analytically and geometrically when the coordinates of the point vary.	Tangent plane of a real function of two variables; tangency point.

Explanation of the applet

How does it work?	Limitations:
The applet calculates and graphically represents the equation of the tangent plane of a scalar function of two variables at a given point. The applet allows us to visualize how this plane varies when the coordinates of the point move within a previously established numerical range. It is requested that the student finds the equation of the tangent planes on his/her own in certain cases and compares the results obtained with those provided by the applet.	Those of the GeoGebra application, especially on a graphical level.

Kinds of problems for which it is appropriate

Those in which you want to do a linear approximation of a real function around a point. Remember that the tangent plane solves this problem in the most satisfactory way possible.

MATHEMATICS II APPLETS

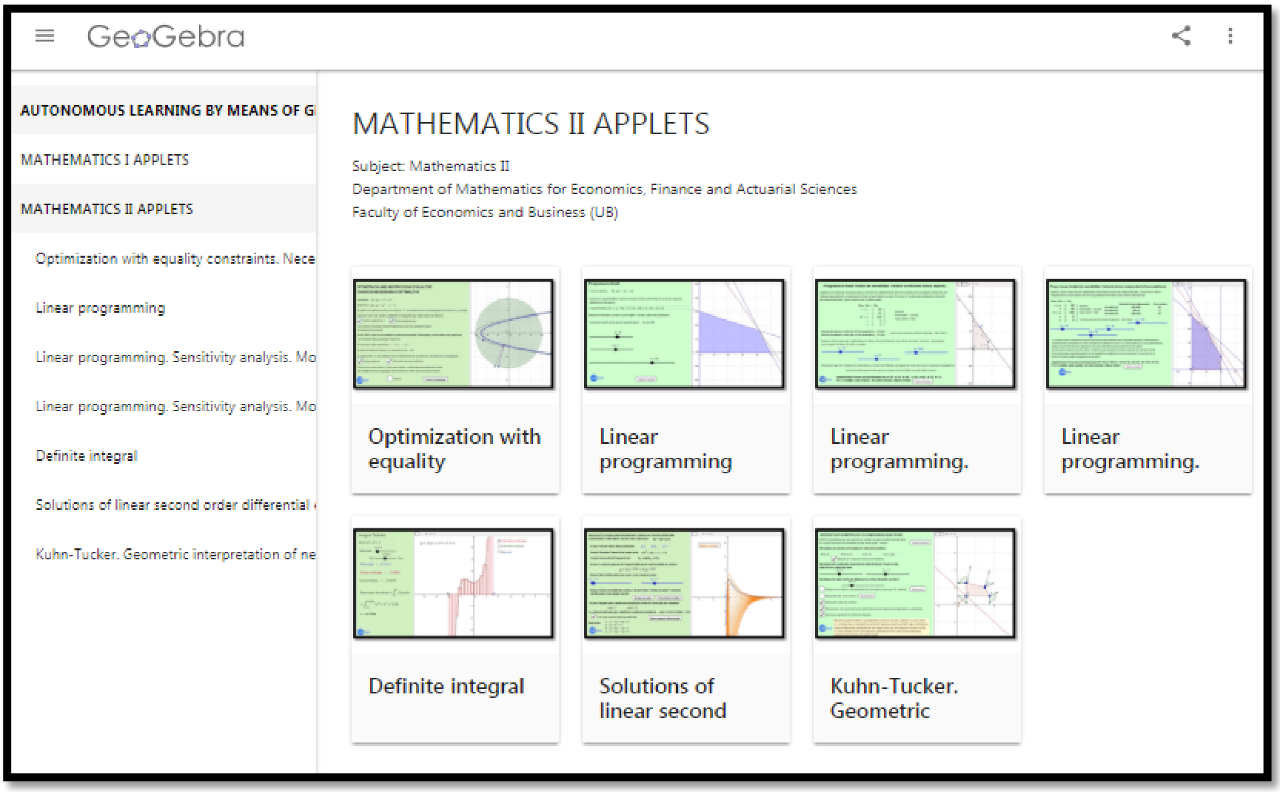
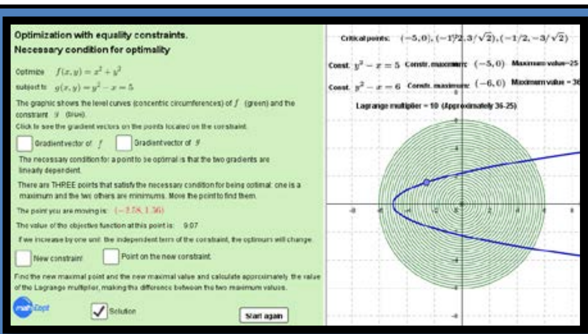


Figure 3: Mathematics II applets

OPTIMIZATION WITH EQUALITY CONSTRAINTS. NECESSARY CONDITION FOR OPTIMALITY



Objective

What does it teach to do?

This applet interprets geometrically the necessary condition of an equality-constrained optimum by explaining what the Lagrange multiplier value represents. It is therefore a geometric applet and not a calculation applet.

What concepts does it explain?

Gradient of a constraint; level curves of the objective function; optimization with equality constraints; Lagrange multipliers.

Explanation of the applet

How does it work?

The applet presents a specific problem in which the objective function is $\boldsymbol{f}(x,y) = x^2 + y^2$ *and the equality constraint is* $g(x,y) = y^2 - x = 5$. *The level curves of the objective function and the constraint appear on the graphical display.*
The applet asks the student to click in the control boxes to draw the gradient of the objective function on a given point of the feasible curve. The student can move the point and check that the gradient of the objective function is perpendicular to its level curves and that the gradient of the constraint function is perpendicular to the feasible curve.
It then asks you to find the points where the two gradient vectors are linearly dependent, reporting that there are three such points. These are the points where the necessary condition for constrained optimality is met. By looking at the level curves of the objective function and the constraint, the student can see which of these points are the two minima and which is the maximum. In addition, the applet indicates the coordinates of the moving point and the value of the objective function at that point.
Finally, the applet asks the student to increase the independent term of the constraint in one unit and draws the associated constraint. It is very easy to guess, now without the help of the applet, how the maximum is shifted and calculate what the new maximum value will be. The difference between the new maximum value and the previous one will be approximately the value of the Lagrange multiplier. The applet ends up discovering the solution to the problem.

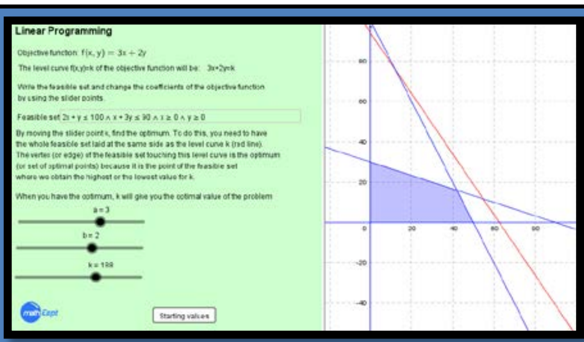
Limitations:

The applet does not allow you to vary the objective function or the constraint. In spite of this, there is no loss of generality in the geometric visualization of the necessary condition for optimality in problems with equality constraints.

Kinds of problems for which it is appropriate

This applet has an illustrative purpose.

LINEAR PROGRAMMING



Objective

What does it teach to do?	*What concepts does it explain?*
This applet interprets geometrically the concept of the optimum of a linear program and allows us to visualize how the slope of the level curves of the objective function varies when its coefficients vary.	*Set of feasible solutions; level curves of the objective function; optimum of a linear program; slope of the level curves.*

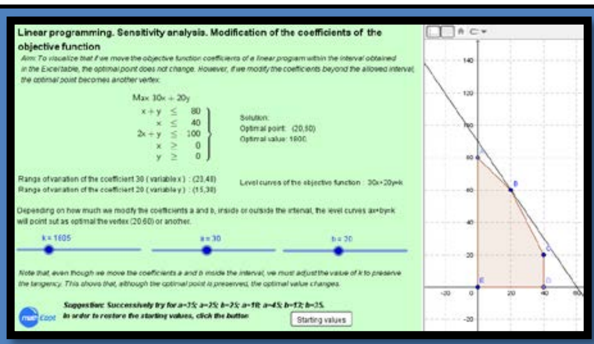
Explanation of the applet

How does it work?	*Limitations:*
The applet allows us to change the coefficients of the objective function and see how these changes affect the slope of the level curves. It also makes it possible to change the set of feasible solutions. *Once the coefficients of the objective function have been set using two sliding points and the set of feasible solutions has been set, the applet invites the student to find the optimum solution to the problem by varying the parameter of the level line corresponding to the objective function with another sliding point. The teacher should explain that the optimum is that vertex of the set of feasible solutions for which the whole set lies on one side of the level line.* *The student can also check that the optimum must necessarily be at a vertex or at an edge of the set of feasible solutions.*	*The applet is limited to two variable problems.* *The applet allows you to vary the set of feasible solutions, but only using inequalities and simple logical relations.* *It should be borne in mind that when the original values of the three sliding points are restored, the set of feasible solutions is not recovered.*

Kinds of problems for which it is appropriate

The applet is designed to help you understand the geometric meaning of the optimum of a linear program. It is for illustrative purposes rather than for calculation purposes.

LINEAR PROGRAMMING. SENSITIVITY ANALYSIS. MODIFICATION OF THE COEFFICIENTS OF THE OBJECTIVE FUNCTION



Objective

What does it teach to do?

This applet shows us how the optimum of a linear program can vary when the coefficients of the associated objective function change. It also graphically interprets the intervals of variation of these coefficients for which this optimum does not change, although the associated optimum values do. This means that the applet also allows us to calculate the variation of these optimal values.

What concepts does it explain?

Sensitivity analysis when the coefficients of the objective function vary; geometric effect of the variation of the coefficients of the objective function in a linear program; meaning of the ranges of variation of the coefficients of the objective function obtained with Excel.

Explanation of the applet

How does it work?

The applet is related to the following problem:

A toy factory produces two types of wooden toys: cars and trains. The car sells for 54€ and requires 10€ of wood. In addition, each car that is produced increases the cost of labor by 14€. The train sells for €39, uses €9 of wood and increases the cost of labor by €10. The production of these two toys requires specialized labor: finishing and carpentry. A car needs 2 hours of finishing work and 1 hour of carpentry. A train needs 1 hour of finishing and 1 hour of carpentry. This factory has all the wood you may want, but only 100 hours of finishing work per week and 80 hours of carpentry per week. The demand for trains is unlimited, but at most 40 cars a week are sold. It is requested: (a) Determine the number of cars and trains to be produced per week in order to maximize the profits made from these two toys. (b) Would the optimum change if the labor cost to produce a car were to be reduced by €5, and what would be the new optimum value? (c) Would the optimum change if the sale price of the car were 69€? (d) Would the optimum change if the wood to make a train cost €16? The result provided by Excel is:

Variables cells

Cell	Name	Final Valor	Reduced Cost	Objective Coefficient	Permissible Augment	Permissible Reduce
C3	Variables x1	20	0	30	10	10
D3	Variables x2	60	0	20	10	5

Limitations:

The applet is intended to interpret geometrically the sensitivity of the optimum of a linear program when the coefficients of the objective function change but is not intended to calculate it.

There may also be small accuracy errors in obtaining the optimum value inherent in the characteristics of the sliding point.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint Right side	Permissible Augment	Permissible Reduce
E10	x<=40	20	0	40	1E+30	20
E8	2x+y<=100	100	10	100	20	20
E9	x+y<=80	80	10	80	20	20

The applet provides the solution: the optimal point and the optimal value. It also indicates the intervals provided by Excel. It draws the domain and the level curve of the objective function.

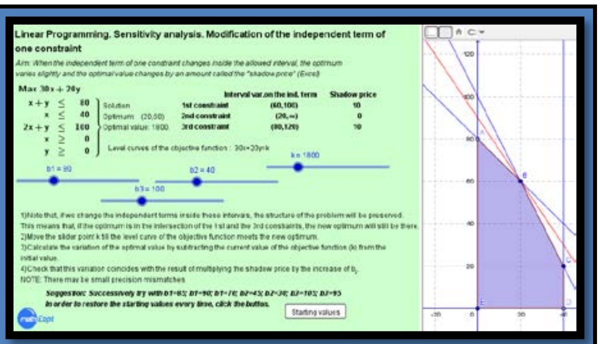
The applet offers three sliding points, one for each coefficient, and one for the level curve value of the objective function that will indicate the optimal value of the problem.

The student, by moving the sliding points corresponding to the coefficients, sees how the slope of the level curve of the objective function changes on the graphical display. You can check that if you keep the coefficient within the range, the optimum point will remain the same vertex. You will notice, however, that you will have to move the slider point of the level curve of the objective function to adjust it back to the vertex. This means that the optimum value has changed. If you look at the previous optimum value and the one you now must set to adjust the drawing, you will know how much the optimum value has changed. You can also check that, if you change the coefficients beyond the values indicated by the interval obtained with Excel, when you adjust the level curve, you will find another vertex to be optimal.

Kinds of problems for which it is appropriate

This is an applet designed to graphically interpret the solutions, i.e. the optimum and the optimum value, of a specific linear program.

LINEAR PROGRAMMING. SENSITIVITY ANALYSIS. MODIFICATION OF THE INDEPENDENT TERM OF ONE CONSTRAINT



Objective

What does it teach to do?

This applet shows us how the optimum of a linear program can vary when the independent terms of the constraints change. It also graphically interprets the intervals of variation of these independent terms within which the shadow price does not vary.

What concepts does it explain?

Sensitivity analysis when varying the independent terms of the constraints of a linear program; meaning of the intervals of variation of independent terms obtained with Excel; shadow price.

Explanation of the applet

How does it work?

The applet is related to the following problem:

A toy factory produces two types of wooden toys: cars and trains. The car sells for 54€ and requires 10€ of wood. In addition, each car that is produced increases the cost of labor by 14€. The train sells for €39, uses €9 of wood and increases the cost of labor by €10. The production of these two toys requires specialized labor: finishing and carpentry. A car needs 2 hours of finishing work and 1 hour of carpentry. A train needs 1 hour of finishing and 1 hour of carpentry. This factory has all the wood you may want, but only 100 hours of finishing work per week and 80 hours of carpentry per week. The demand for trains is unlimited, but at most 40 cars a week are sold. It is requested: (a) Determine the number of cars and trains to be produced per week in order to maximize the profits made from these two toys. (b) How much money would the producer be willing to pay to increase the total number of finishing hours per week (101 finishing hours) by 1 hour? (c) How much money would the producer be willing to pay to increase the total number of hours of carpentry per week (81 hours of finishing) by 1 hour? (d) Would you improve your profits if you could sell 41 cars a week? The result provided by Excel is:

Variable cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Permissible Augment	Permissible Reduce
C3	Variables x1	20	0	30	10	10
D3	Variables x2	60	0	20	10	5

Limitations:

The applet is designed to interpret geometrically the sensitivity of the optimum of a linear program when the independent terms of the constraints change, but not to calculate them.

The applet is related to a specific problem and cannot be changed.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint Right side	Permissible Augment	Permissible Reduce
E10	x<=40	20	0	40	1E+30	20
E8	2x+y<=100	100	10	100	20	20
E9	x+y<=80	80	10	80	20	20

The applet provides the solution: the optimal point and the optimal value. It also indicates the intervals provided by Excel and the shadow prices. The domain is drawn, the blue lines are the constraints and the red line is the level curve of the objective function.

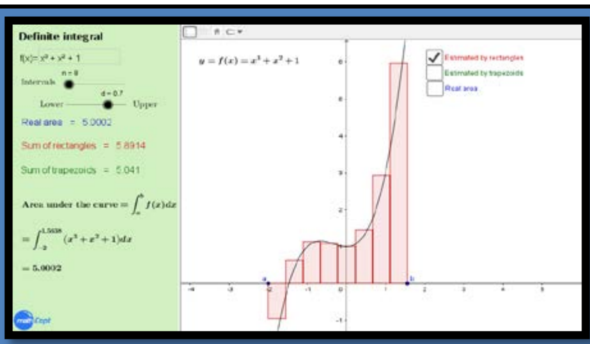
The applet offers four sliding points, one for each independent term, and one for the optimal value of the problem.

The student, by moving the sliding points corresponding to the constraints, sees how the constraints change on the graphical screen. You can see that if you keep the independent term within the range, the optimal point will vary slightly, and the basic structure of the problem will not change (i.e. if the optimum was the intersection of the 1st and 3rd constraint, it will still be). If you leave the interval, however, the problem will change radically. You can also check that if you keep the independent term within the range, the optimum point will vary slightly. You will notice that you will have to move the sliding point from the optimal value to the new vertex. This means that the optimum value has also changed. If you look at the previous optimal value and the one you now must set to adjust the drawing, you will know how much the optimal value has changed. You can also check that this variation is precisely the value of the shadow price multiplied by the variation of the term independent.

Kinds of problems for which it is appropriate

This is an applet designed to graphically interpret how the optimum and the optimal value vary when the term changes independently of a constraint, and to understand the meaning of the shadow price associated with this constraint.

DEFINITE INTEGRAL



Objective

What does it teach to do?	*What concepts does it explain?*
The objective of this applet is to approximate the area between the graph of a function and the abscissa axis (definite integral) by means of sums of rectangle areas (approximation 1) and trapezoid areas (approximation 2).	*Definite integral of a real function of continuous real variable.*

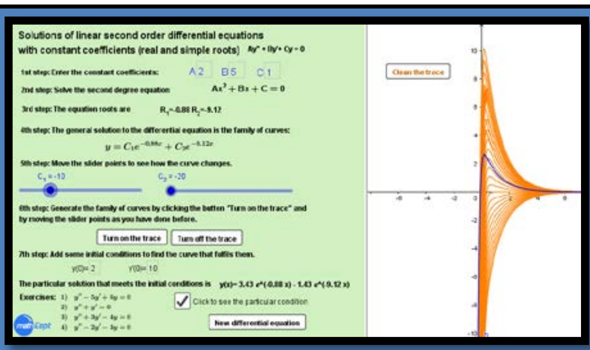
Explanation of the applet

How does it work?	*Limitations:*
The applet approximates the area bounded by the graph of a real continuously variable function and the abscissa axis between two given points. The area is calculated in the applet with the corresponding definite integral. Both approaches (approximation 1 by rectangles and approximation 2 by trapezoids) begin by dividing the segment between the two points on the abscissa axis into a series of subintervals that will form the basis of the rectangles and/or trapezoids; in other words, we can vary the values of the two points by clicking on them with the mouse, as well as the number of subintervals with a first sliding point. As for the approach 1, the applet allows us to choose the height of the rectangles with a second sliding point that takes values between 0 (lower sum) and 1 (upper sum). For the trapezes, the upper segment of each trapezoid is fixed for each sub-interval. Finally, the applet shows us graphically the progressive and staggered distribution of the adjacent rectangles and/or trapezoids that approximate the area; as the number of subintervals grows, we can see how the set of rectangular and/or trapezoidal areas progressively covers the area in question,	*Those of the GeoGebra application, especially at the graphics level. Moreover, the method used (exhaustive method) does not fully prefigure the fundamental theorem of the calculation of an integral of Newton and Leibniz.*

Kinds of problems for which it is appropriate

Those in which you want to approximate the area of certain flat figures that take the form of a rectangle with a top/bottom side that is not rectilinear given by a continuous function. It should be remembered that the method used is an approximate method (exhaustive method).

SOLUTIONS OF LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS (REAL AND SIMPLE ROOTS)



Objective

What does it teach to do?	*What concepts does it explain?*
This applet finds the family of curves that are the general solution of a second-order linear Ordinary Differential Equation (ODE) with constant coefficients and calculates the particular solution that satisfies certain initial conditions.	*Family of curves, general and particular solutions of an EDO.*

Explanation of the applet

How does it work?	*Limitations:*
The applet allows us to enter the constant coefficients of a second order linear ODE and shows us how to construct the characteristic equation, as well as to find analytically the general solution of the ODE as long as the roots of this characteristic equation are real and simple (and thus different). At the same time, and by activating the trace, the family of curves that are the general solution of the ODE appears on the graphic screen. Finally, the applet also allows the introduction of initial conditions in the ODE so that the particular solution curve (integral curve) that meets them appears on the graphical screen.	*This applet is limited to working with those second order linear ODEs with constant coefficients that have associated characteristic equations with real and simple roots.*

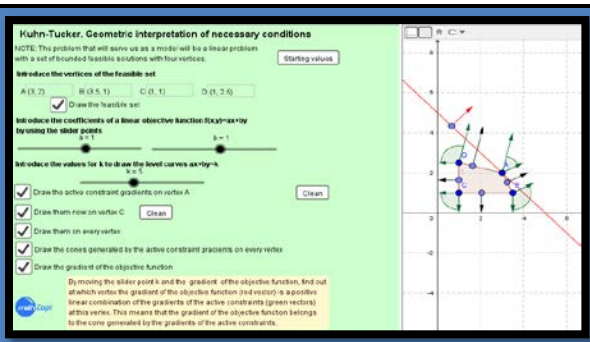
Kinds of problems for which it is appropriate

The applet is designed to help you understand that the general solution of an ODE is a family of curves and that the particular solution that meets certain initial conditions is a curve of this family. Its purpose is to illustrate rather than to calculate.

Exercises:

1) *y''-3y'+2y=0 with y(0)=0 and y'(0)=2*
2) *y''-y=0 with y(0)=1 and y'(0)=0*

KUHN-TUCKER. GEOMETRIC INTERPRETATION OF NECESSARY CONDITIONS



Objective

What does it teach to do?	*What concepts does it explain?*
This applet graphically interprets the necessary condition for local maximum in an optimization problem with inequality constraints (Kuhn-Tucker conditions) in a simple case of linear programming.	*Active constraint at a vertex; gradient of a constraint; level curve of the objective function; cone generated by the gradients of the active constraints; necessary condition for optimality of a problem with inequality constraints.*

Explanation of the applet

How does it work?

The applet, on the first place, asks us to enter the vertices of the feasible set and draws it. The coefficients of the two-variable linear objective function and the value of the parameter associated with the level curve of the objective function must then be entered using the sliding points.

Then, by clicking on the control box, it draws the gradients of the active constraints at the vertices. Two vectors appear, one on each of the active constraints. The student clearly sees what these active constraints are and that the gradient vectors are perpendicular to the constraints. The student repeats this process for each of the vertices and, for clarity, is told to erase the gradients on the previous vertex.

The cones generated by these constraints are shown below. When asked to draw the gradient of the objective function as well, a vector appears in red on the level curve of the objective function that the student will have to move to each vertex, changing the level curve parameter if necessary.

If the gradient of the objective function is within the cone generated by the active constraint gradients, the vertex meets the necessary Kuhn-Tucker conditions. This vertex is therefore a global maximum (in the case of linear programming, the Kuhn-Tucker conditions are necessary and sufficient conditions for global optimization). If the gradient vector is outside the cone, the associated vertex does not satisfy the necessary Kuhn-Tucker conditions of maximum and therefore is not local maximum.

Limitations:

The domains of the objective functions must be polygons.

The applet interprets the local maximum condition, but it can easily be used to explain the minimum case if you consider that minimizing one function is equivalent to maximizing the opposite.

Kinds of problems for which it is appropriate

The applet is intended to help you understand the geometric meaning of Kuhn-Tucker conditions. It has an illustrative rather than calculating purpose.

mathEopt