D-String on Near Horizon Geometries and Infinite Conformal Symmetry

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We show that the symmetries of effective D-string actions in constant dilaton backgrounds are directly related to homothetic motions of the background metric. In the presence of such motions, there are infinitely many nonlinearly realized rigid symmetries forming a loop (or looplike) algebra. Near horizon (anti–deSitter) D3 and D1 + D5 backgrounds are discussed in detail and shown to provide 2D interacting field theories with infinite conformal symmetry. [S0031-9007(98)06979-8]

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The recent past has seen an increasing interest in the conjecture of a correspondence between large N limits of certain d-dimensional conformal field theories and supergravity on the product of $(d+1)$-dimensional anti–de Sitter (AdS) space with a compact manifold [1,2]. This suggested consideration of world-volume brane actions on near horizon backgrounds. M2-, M5-, and D3-branes have been studied [1,3,4] and interacting $(p+1)$-dimensional theories in Minkowski space-time with conformal $SO(2,p+1)\times SO(d-p-1)$ symmetry were found [5]. The conformal symmetries of these branes reflect the isometries of $AdS_{p+2}\times S^{d-p-2}$. The case of a D-string in the near horizon geometry of a (D1 + D5)-brane was also considered in [5].

In this work we study the rigid symmetries of effective D-string actions of the Born-Infeld type on curved backgrounds with constant dilaton. We find that the symmetries are related with homothetic motions of the background metric. Each of these motions gives rise to infinitely many nonlinearly realized rigid symmetries, with the Born-Infeld gauge field transforming in a nontrivial way. The algebra of these symmetries is a loop generalization of the algebra associated with the homothetic motions. We spell out the symmetry transformations before gauge fixing and in the static gauge for the world-sheet diffeomorphisms. The gauged fixed transformations generate infinitely many symmetries of interacting $(1+1)$-dimensional field theories in a flat space-time.

We then specify these general results for particularly interesting D3- and (D1 + D5)-brane backgrounds and show that the gauge fixed field theories in the respective near horizon (AdS) backgrounds have infinite conformal symmetry. In the case of the D3 background the symmetry group is a loop generalization of $ISO(1,3)\times SO(6)$. In the near horizon limit there is an enhancement of the symmetry to the loop generalization of conformal $SO(2,4)\times SO(6)$ due to the AdS geometry. The symmetry group contains as a subgroup a loop version of conformal $SO(2,2)$ with nonlinearly realized special conformal transformations.

In the case of a D-string on a near horizon (D1 + D5) background we get an interacting theory with infinite conformal $SO(2,2)\times SO(4)\times ISO(4)$ loop symmetry. The zero modes of the loop algebra reproduce the corresponding results of [5].

We remark that these structures are not restricted to Dirac-Born-Infeld actions. Rather, they are present in a more general set of models studied here. Hence, in appropriate backgrounds one gets a set of conformal field theories. This does not exclude that kappa-invariant extensions of our formulation and/or T duality properties may select the Dirac-Born-Infeld action.

It is natural to wonder how these results extend to Dp-branes with $p>1$. This is not known; a complete classification of the symmetries for $p>1$ has not been carried out so far. Of course, the presence of infinitely many symmetries may well be restricted to the case $p=1$, as the two-dimensional case is often special. On the other hand, the presence of a Kac-Moody version of the conformal group SO(2,4) for D3-branes in the near horizon geometry has been conjectured recently in [6] and would be reminiscent of our result for $p=1$. Work in this direction is in progress.

Symmetries and homothetic motions.—The effective Born-Infeld actions for D-strings considered here can be cast in a form similar to the familiar sigma model formulation of the Nambu-Goto action. In this form they are contained in a more general class of models with an action of the form

$$S=\frac{1}{2}\int d^2\sigma\,\{\sqrt{\gamma}\;\gamma^{\mu\nu}f(\varphi)g_{mn}(x)\partial_\mu x^m\partial_\nu x^n+\epsilon^{\mu\nu}\times[b_{mn}(x)\partial_\mu x^m\partial_\nu x^n+D(\varphi)F_{\mu\nu}]\},\tag{1}$$

where $\gamma_{\mu\nu}$ is an auxiliary world-sheet metric, φ is an auxiliary scalar field, $\epsilon^{\mu\nu}$ is the usual Levi-Civita tensor density, and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is an Abelian field strength. g_{mn} and b_{mn} are to be thought of as target space metric and 2-form, respectively. We do not impose

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restrictions on f and D apart from $f, D \neq$ const, but we note that one of them may be chosen conveniently (only the relative choice of f and D characterizes a particular model). Born-Infeld actions arise for

$$f^2(\varphi) - D^2(\varphi) = 1\,. \tag{2}$$

Indeed, eliminating the auxiliary fields $\gamma_{\mu\nu}$ and φ using the equations of motion, the Lagrangian turns for (2) into

$$L_{\rm BI} = \sqrt{-\det(\mathcal{G}_{\mu\nu} + F_{\mu\nu})} + \tfrac{1}{2}\,\epsilon^{\mu\nu}\mathcal{B}_{\mu\nu}\,, \tag{3}$$
$$\mathcal{G}_{\mu\nu} = g_{mn}(x)\partial_\mu x^m \partial_\nu x^n, \qquad \mathcal{B}_{\mu\nu} = b_{mn}(x)\partial_\mu x^m \partial_\nu x^n.$$

This represents Born-Infeld models with a "Wess-Zumino term" determined by b_{mn} and a constant dilaton which may be made explicit by rescaling g_{mn} and A_μ. More general Born-Infeld models, in particular, models with nonconstant dilaton, can also be cast in a sigma model form [7,8], but are not considered here.

In [8] we have shown among others that all of the rigid symmetries of actions (1) (and generalizations thereof) are determined by generalized Killing vector equations. An analysis of these equations, similar to the one performed for the example treated in [7], shows that the rigid symmetries of models (1) are generated by

$$\begin{aligned} \Delta x^m &= \xi_i^m(x)\,\lambda^i(\varphi), \qquad \Delta\gamma_{\mu\nu} = 0\,, \\ \Delta\varphi &= K_i\,\lambda^i(\varphi)\,f(\varphi)/f'(\varphi)\,, \\ \Delta A_\mu &= (1/D')[\,f\,\sqrt{\gamma}\,\epsilon_{\mu\nu}\gamma^{\nu\varrho}\,g_{mn}(\Delta x^n)'\partial_\varrho x^m \\ &\quad + (b_{nm}\Delta x^n + \lambda^i q_{mi})'\partial_\mu x^m - A_\mu(D'\Delta\varphi)'\,]. \end{aligned} \tag{4}$$

Here, prime denotes differentiation with respect to φ, the $\lambda^i(\varphi)$ are *arbitrary* functions of φ, and $\{\xi_i^m(x), q_{mi}(x)\}$ denotes a complete set of inequivalent solutions of

$$\mathcal{L}_i g_{mn}(x) = -K_i g_{mn}(x), \qquad K_i = \text{const}\,, \tag{5}$$
$$\mathcal{L}_i b_{mn}(x) = \partial_n q_{mi}(x) - \partial_m q_{ni}(x)\,, \tag{6}$$

where $\mathcal{L}_i$ is the Lie derivative along ξ_i. Using (5) and (6), it is not difficult to verify that the above transformations Δ generate indeed symmetries of an action (1).

The symmetries of Born-Infeld actions (3) are obtained from the above formulas by eliminating the auxiliary fields $\gamma_{\mu\nu}$ and φ, resulting in

$$\begin{aligned} \Delta x^m &= \xi_i^m(x)\lambda^i(\mathcal{F})\,, \\ \Delta A_\mu &= [V_{\mu i}(1 - \mathcal{F}^2) + W_{\mu i}(1 - \mathcal{F}^2)^{3/2}]\,\frac{d\lambda^i(\mathcal{F})}{d\mathcal{F}} + A_\mu K_i\Big[\mathcal{F}^{-2} - 2 + (\mathcal{F} - \mathcal{F}^{-1})\,\frac{d}{d\mathcal{F}}\Big]\lambda^i(\mathcal{F})\,, \end{aligned} \tag{7}$$

where

$$\begin{aligned} V_{\mu i} &= -\sqrt{\mathcal{G}}\,\epsilon_{\mu\nu}\mathcal{G}^{\nu\varrho}\,\xi_i^m(x)g_{mn}(x)\partial_\varrho x^n, \\ W_{\mu i} &= [b_{mn}(x)\xi_i^n(x) - q_{mi}(x)]\partial_\mu x^m, \\ \mathcal{F} &= \tfrac{1}{2}\,\mathcal{G}^{-1/2}\epsilon^{\mu\nu}F_{\mu\nu}, \qquad \mathcal{G} = -\det(\mathcal{G}_{\mu\nu})\,. \end{aligned} \tag{8}$$

Let us now comment on the nature of the above symmetries. The occurrence of arbitrary functions $\lambda^i(\varphi)$ in (4) implies that each nontrivial solution to (5) and (6) gives rise to *infinitely many rigid symmetries*. Equation (5) defines so-called homothetic motions of g_{mn} and the K_i are called homothetic constants [9]. Homothetic motions with nonvanishing homothetic constants are called proper because the others are just isometries of the metric. One can always choose a basis of homothetic motions such that at most one of them is proper. Without loss of generality, we can thus use $i = 1, 2, \ldots$ for isometries of the metric, reserve $i = 0$ for a proper homothetic motion (if any), and normalize ξ_0 such that $K_i = \delta_i^0$.

The commutator of a proper homothetic motion and an isometry of the metric is always again an isometry, as (5) implies $[\mathcal{L}_0, \mathcal{L}_i]g_{mn} = 0$. The algebra of homothetic motions is thus of the form

$$[\mathcal{L}_i, \mathcal{L}_j] = c_{ij}{}^k\mathcal{L}_k, \quad [\mathcal{L}_0, \mathcal{L}_i] = c_i{}^j\mathcal{L}_j \quad (i,j,k \geq 1)\,, \tag{9}$$

where $c_{ij}{}^k$ and $c_i{}^j$ are structure constants.

The presence of arbitrary functions of φ in (4) (which turn into functions of $\mathcal{F}$ upon elimination of φ) implies that the algebra of the corresponding symmetries is a loop version of (9), the role of the loop variable being played by φ (or a function thereof). This is seen by expanding the functions λ^i in a suitable basis for functions of φ. A particularly nice form of the algebra emerges in a basis consisting of powers of the function $f(\varphi)$ occurring in (1). We denote the corresponding basis of symmetries by $\{\Delta_i^\alpha\}$, where α indicates the power of $f(\varphi)$,

$$\Delta_i^\alpha x^m = -\xi_i^m(x)f^\alpha(\varphi), \qquad \Delta_i^\alpha\varphi = -\delta_i^0\,\frac{f^{\alpha+1}(\varphi)}{f'(\varphi)}\,. \tag{10}$$

It is now straightforward to verify that in this basis the symmetry algebra reads, on x^m and φ,

$$[\Delta_i^\alpha, \Delta_j^\beta] = c_{ij}{}^k\Delta_k^{\alpha+\beta} \qquad (i,j,k \geq 1)\,, \tag{11}$$
$$[\Delta_0^\alpha, \Delta_i^\beta] = (c_i{}^j - \beta\delta_i^j)\Delta_j^{\alpha+\beta} \qquad (i,j \geq 1)\,, \tag{12}$$
$$[\Delta_0^\alpha, \Delta_0^\beta] = (\alpha - \beta)\Delta_0^{\alpha+\beta}. \tag{13}$$

Note that (11) is a loop algebra associated with the isometries of the metric. Hence, if there is no proper homothetic motion, the symmetry algebra is a true loop algebra. In the presence of a proper homothetic motion, it turns into the semidirect sum of the loop algebra (11) and the Witt algebra (13). We note that, in general, the algebra has on A_μ the above form only up to gauge transformations and on-shell trivial symmetries.

D3 and $D1 + D5$ backgrounds.—We treat now two particularly interesting curved backgrounds and give the symmetry transformations before gauge fixing.

First we consider a D3-brane supergravity background with target space metric and 2-form given by

$$ds^2 = H^{-1/2}\eta_{ab}dx^a dx^b + H^{1/2}\delta_{AB}dx^A dx^B,$$
$$b_{mn} = 0, \qquad H = 1 + (R/r)^4, \tag{14}$$

where $r^2 = \delta_{AB}x^A x^B$, $a = 0,\ldots,3$, and $A = 4,\ldots,9$. The rigid symmetries are obtained from (4) by solving (5) and (6). Because $b_{mn} = 0$, the solution of (6) is trivial; i.e., we can choose $q_{mi} = 0$ without loss of generality. An analysis of (5) shows that in this case we have $K_i = 0$; i.e., there is no proper homothetic motion. Hence, the solutions of (5) are exhausted by the Killing vector fields of the metric in (14). The latter correspond to Poincaré transformations in the 4-space parallel to the D3-brane, and rotations in the transverse directions. The symmetry transformations of φ and x^m read thus in this case:

$$\Delta x^a = \lambda^a(\varphi) + \lambda^{ab}(\varphi)\eta_{bc}x^c, \qquad \lambda^{ab} = -\lambda^{ba},$$
$$\Delta x^A = \lambda^{AB}(\varphi)\delta_{BC}x^C, \qquad \lambda^{AB} = -\lambda^{BA}, \tag{15}$$
$$\Delta\varphi = 0.$$

The transformations of A_μ are then obtained from (4). Equations (15) imply that the symmetry group is in this case a loop version of ISO(1,3) $\times$ SO(6).

Next we discuss the near horizon geometry of (14) due to its importance for the conjectures in [1]. Close to the horizon ($r \to 0$) one can neglect the constant in the harmonic function H and end up with

$$(ds^2)_{\text{hor.}} = \frac{r^2}{R^2}\,\eta_{ab}dx^a dx^b + \frac{R^2}{r^2}\,\delta_{AB}dx^A dx^B. \tag{16}$$

Again one finds that the solutions of (5) are exhausted by the Killing vector fields. However, the asymptotic metric has more isometries than the original one,

$$\Delta x^a = \lambda^a(\varphi) + \lambda^{ab}(\varphi)\eta_{bc}x^c + \lambda_D(\varphi)x^a$$
$$- 2\lambda_S^b(\varphi)\eta_{bc}x^a x^c + \lambda_S^a(\varphi)(\eta_{bc}x^b x^c + R^4 r^{-2}),$$
$$\Delta x^A = [2\lambda_S^a(\varphi)\eta_{ab}x^b - \lambda_D(\varphi)]x^A + \lambda^{AB}(\varphi)\delta_{BC}x^C,$$
$$\Delta\varphi = 0. \tag{17}$$

The additional isometries, corresponding to λ_D and λ_S, are indeed reminiscent of dilatations and special conformal transformations in $(1+3)$-dimensional flat space. The symmetry group is now a loop version of SO(2,4) $\times$ SO(6). This symmetry enhancement originates from the anti–de Sitter geometry and corresponds to the supersymmetry enhancement discussed in [10].

Finally, we consider the near horizon geometry of a (D1 + D5) supergravity background. The target space metric and 2-form are given by

$$ds^2 = \frac{r^2}{R_1R_5}\,\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R_1}{R_5}\,\delta_{ab}dx^a dx^b$$
$$+ \frac{R_1R_5}{r^2}\,\delta_{AB}dx^A dx^B, \tag{18}$$

$$b = \frac{r^2}{R_1^2}\,dx^0 \wedge dx^1 + 2R_5^2\sin^2\theta_1\sin\theta_2\theta_3 d\theta_1 \wedge d\theta_2,$$

where $r^2 = \delta_{AB}x^A x^B$, $\mu = 0,1$, $a = 2,\ldots,5$, $A = 6,\ldots,9$ and the θ_i are spherical coordinates for the x^A as in [5]. Again there are no proper homothetic motion; i.e., the solutions of (5) are exhausted by the Killing vector fields of the metric in (18). The 2-form b is not invariant under all of these isometries but it is still invariant up to exact forms, as required by (6). The symmetries form a loop version of SO(2,2) $\times$ SO(4) $\times$ ISO(4) through

$$\Delta x^\mu = \lambda^\mu(\varphi) + \lambda^{\mu\nu}(\varphi)\eta_{\nu\varrho}x^\varrho + \lambda_D(\varphi)x^\mu + \lambda_S^\nu(\varphi)[\delta_\nu^\mu(\eta_{\varrho\sigma}x^\varrho x^\sigma + R_1^2R_5^2r^{-2}) - 2\eta_{\nu\varrho}x^\mu x^\varrho],$$
$$\Delta x^A = [2\lambda_S^\mu(\varphi)\eta_{\mu\nu}x^\nu - \lambda_D(\varphi)]x^A + \lambda^{AB}(\varphi)\delta_{BC}x^C, \qquad \Delta x^a = \lambda^a(\varphi) + \lambda^{ab}(\varphi)\delta_{bc}x^c, \qquad \Delta\varphi = 0, \tag{19}$$

where $\lambda^{mn} = -\lambda^{nm}$. The corresponding transformations ΔA_μ are obtained from (4), with $q_{\mu i} = q_{ai} = 0$ and

$$\lambda^i q_{Ai} = 2\lambda_S^\nu\epsilon_{\nu\mu}x^\mu\delta_{AB}x^B R_5^2 r^{-2} + \lambda^{BC}x^D(b_{AB}\delta_{CD} - \tfrac{1}{2}\epsilon_{ABCD}R_5^2r^{-2}). \tag{20}$$

2D conformal field theories.—We now discuss the interacting conformal field theories obtained in the static gauge $x^\mu = \sigma^\mu$ ($\mu = 0,1$) for world-sheet diffeomorphisms. Before eliminating the auxiliary fields $\gamma_{\mu\nu}$ and φ, the action in the static gauge is thus a functional of

$$\{\phi\} = \{A_\mu, \gamma_{\mu\nu}, \varphi, x^2, x^3, \ldots\}.$$

This action is, of course, not invariant anymore under the transformations Δ given above. Rather it is invariant under particular combinations of these transformations and compensating world-sheet diffeomorphisms preserving the static gauge. These combinations are

$$\delta\phi = \mathcal{L}_\epsilon\phi - [\Delta\phi]_{x^\mu=\sigma^\mu}, \qquad \epsilon^\mu = [\Delta x^\mu]_{x^\mu=\sigma^\mu}, \tag{21}$$

where $\mathcal{L}_\epsilon$ is the world-sheet Lie derivative along ϵ^μ. The algebra of the δ's coincides with the algebra of Δ's. Hence, only the realization of these symmetries changes, but not the corresponding symmetry group.

Let us now illustrate this procedure for the near horizon D3-brane supergravity background (16). The corresponding action (1) reads, in the static gauge,

$$S = \frac{1}{2}\int d^2\sigma\{\sqrt{\gamma}\,\gamma^{\mu\nu}f(\varphi)r^2R^{-2}[\eta_{\mu\nu} + \delta_{\hat a\hat b}\partial_\mu x^{\hat a}\partial_\nu x^{\hat b} + R^4r^{-4}\delta_{AB}\partial_\mu x^A\partial_\nu x^B] + \epsilon^{\mu\nu}D(\varphi)F_{\mu\nu}\}, \tag{22}$$

where $\hat a, \hat b = 2,3$ correspond to the parallel D3-brane directions which have not been gauge fixed. The symmetries of (22) are now obtained from (21) using (4) and (17). For instance, a dilatation symmetry corresponding to λ_D involves a compensating diffeomorphism with parameter $\epsilon_D^\mu = \lambda_D(\varphi)\sigma^\mu$ and is now realized by

$$
\begin{aligned}
\delta_D x^{\hat a} &= \epsilon_D^\mu \partial_\mu x^{\hat a} - \lambda_D(\varphi) x^{\hat a}, \\
\delta_D x^A &= \epsilon_D^\mu \partial_\mu x^A + \lambda_D(\varphi) x^A, \qquad \delta_D \varphi = \epsilon_D^\mu \partial_\mu \varphi, \\
\delta_D \gamma_{\mu\nu} &= \epsilon_D^\varrho \partial_\varrho \gamma_{\mu\nu} + \gamma_{\varrho\nu} \partial_\mu \epsilon_D^\varrho + \gamma_{\mu\varrho} \partial_\nu \epsilon_D^\varrho, \qquad (23) \\
\delta_D A_\mu &= \epsilon_D^\varrho \partial_\varrho A_\mu + A_\varrho \partial_\mu \epsilon_D^\varrho - \lambda'_D(\varphi) f(\varphi) [D'(\varphi)]^{-1} \\
&\quad \times \sqrt{\gamma}\, \epsilon_{\mu\nu} \gamma^{\nu\varrho} R^{-2} r^2 [\eta_{\varrho\sigma} \sigma^\sigma + \delta_{\hat a \hat b} x^{\hat a} \partial_\varrho x^{\hat b} \\
&\quad - r^{-4} R^4 \delta_{AB} x^A \partial_\varrho x^B].
\end{aligned}
$$

These transformations generate symmetries of (22) for any choice of $\lambda_D(\varphi)$. This includes dilatations of the standard form for the special choice $\lambda_D = 1$,

$$\lambda_D = 1: \qquad \delta_D \phi = \sigma^\mu \partial_\mu \phi + w(\phi)\phi, \qquad (24)$$

where the Weyl weights $w(\phi)$ are given by

$$
\begin{gathered}
w(x^{\hat a}) = -1, \qquad w(x^A) = w(A_\mu) = 1, \\
w(\varphi) = 0, \qquad w(\gamma_{\mu\nu}) = 2.
\end{gathered}
$$

Analogously one determines the other symmetries in the static gauge. Altogether they form, as before, a loop generalization of SO(2,4) $\times$ SO(6) with a loop version of conformal SO(2,2) as a subgroup. This subgroup corresponds to λ^μ, $\lambda^{\mu\nu}$, λ_D, and λ_S^μ, and the parameters of the compensating world-sheet diffeomorphisms for this subgroup are thus

$$
\begin{aligned}
\epsilon_C^\mu &= \lambda^\mu(\varphi) + \lambda_D(\varphi)\sigma^\mu + [\lambda^{\mu\nu}(\varphi) - 2\sigma^\mu \lambda_S^\nu(\varphi)] \\
&\quad \times \eta_{\nu\varrho}\sigma^\varrho + \lambda_S^\mu(\varphi) \\
&\quad \times (\sigma^\nu \sigma^\varrho \eta_{\nu\varrho} + x^{\hat a} x^{\hat b} \delta_{\hat a \hat b} + R^4 r^{-2}). \qquad (25)
\end{aligned}
$$

The corresponding conformal transformations of $x^{\hat a}$, x^A. and φ can be written compactly as

$$
\begin{aligned}
\delta_C x^{\hat a} &= \epsilon_C^\mu \partial_\mu x^{\hat a} - \tfrac{1}{2}(\partial_\mu^{\text{exp.}} \epsilon_C^\mu) x^{\hat a}, \\
\delta_C x^A &= \epsilon_C^\mu \partial_\mu x^A + \tfrac{1}{2}(\partial_\mu^{\text{exp.}} \epsilon_C^\mu) x^A, \qquad (26) \\
\delta_C \varphi &= \epsilon_C^\mu \partial_\mu \varphi,
\end{aligned}
$$

where $\partial_\mu^{\text{exp.}}$ denotes differentiation only with respect to explicit σ^μ. Note that even the zero modes of the special conformal transformations ($\lambda_S^\mu = \text{const}$) are nonlinearly realized.

If we consider (22) in the Born-Infeld action case and expand in low velocities we get

$$
\begin{aligned}
L_{\text{BI}} &= \frac{r^2}{R^2} + \frac{r^2}{2R^2} \delta_{\hat a \hat b} \partial^\mu x^{\hat a} \partial_\mu x^{\hat b} \\
&\quad + \frac{R^2}{2r^2} \delta_{AB} \partial^\mu x^A \partial_\mu x^B \\
&\quad + \frac{R^2}{4r^2} F^{\mu\nu} F_{\mu\nu} + \dots, \qquad (27)
\end{aligned}
$$

where μ, ν are raised with $\eta^{\mu\nu}$.

The case of a D-string in the near horizon (D1 + D5) supergravity background (18) is treated analogously. The resulting symmetry transformations establish a loop generalization of the conformal SO(2,2) $\times$ SO(4) $\times$ ISO(4) symmetry found in [5]. The Weyl weights are again easily obtained from the special dilatation with $\lambda_D = 1$ which has again the form (24) and yields

$$
\begin{gathered}
w(x^A) = w(A_\mu) = 1, \qquad w(x^a) = w(\varphi) = 0, \\
w(\gamma_{\mu\nu}) = 2.
\end{gathered}
$$

Comments.—The symmetries of D-string actions described above may be viewed as generalizations of the familiar target space symmetries of the string. There are two important differences to the string case which are both direct consequences of the presence of the Born-Infeld gauge field. First, each target space symmetry gives rise to a family of infinitely many symmetries of the D-string action, whereas it yields only one rigid symmetry of the (Nambu-Goto or Polyakov) string action. Second, there is an additional infinite family of symmetries of the D-string action if the target space metric admits a proper homothetic motion. The latter are dilatational symmetries without any counterpart in the string case (see [7] for an example).

We stress that all of these infinitely many symmetries are present *in addition* to the world-sheet symmetries and must not be confused with the latter. Indeed, the action (1) is, of course, also gauge invariant both under world-sheet diffeomorphisms and under Weyl-transformations of $\gamma_{\mu\nu}$, as its string counterpart, the Polyakov action. In particular, one may consider the action (1) in a conformal gauge for these world-sheet symmetries (rather than in the static gauge considered above). That action has infinitely many conformal world-sheet symmetries on top of the symmetries discussed above. In particular, it may thus serve as a starting point for quantization, along the lines of string quantization based on the Polyakov action in a conformal gauge.

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