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Can infrared gravitons screen Λ?

Jaume Garriga[1] and Takahiro Tanaka[2]

[1]*Departament de Física Fonamental, Universitat de Barcelona, Diagonal 647j, 08028 Barcelona, Spain*
[2]*Department of Physics, Kyoto University, Kyoto, Japan*

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It has been suggested that infrared gravitons in de Sitter space may lead to a secular screening of the effective cosmological constant. This seems to clash with the naive expectation that the curvature scalar should stay constant due to the Heisenberg equation of motion. Here, we show that the tadpole correction to the local expansion rate, which has been used in earlier analyses as an indicator of a decaying effective Λ, is not gauge invariant. On the other hand, we construct a gauge-invariant operator which measures the renormalized curvature scalar smeared over an arbitrary window function, and we find that there is no secular screening of this quantity (to any given order in perturbation theory).

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I. INTRODUCTION

In de Sitter space, long wavelength gravitons are frozen in, producing a cumulative deformation of space-time on large scales. This can be seen, for instance, in the tree-level graviton two-point function. In the transverse traceless gauge, the behavior of gravitons is similar to that of massless minimally coupled scalars [1], and their two-point function $\langle h(x)h(x')\rangle$ grows logarithmically with scale. Globally, such increasing departure from a de Sitter metric cannot be undone by a gauge transformation. Nevertheless, infrared gravitons do not contribute to tidal forces on small scales. The tree-level two-point function for the Riemann tensor $\langle \mathcal{R}_{\mu\nu\rho\sigma}(x)\mathcal{R}_{\alpha\beta\gamma\delta}(x')\rangle$ is infrared finite, and the contribution of gravitons with wavelength much larger than the curvature scale H^{-1} is in fact negligible. Hence, to lowest order in perturbation theory, the local geometry remains everywhere close to the unperturbed de Sitter space.[1]

It has long been suggested that graviton interactions may dramatically alter this picture, potentially leading to infrared screening of the cosmological constant [2]. The basic idea is the following. Gravitons carry energy and hence they are a source of the gravitational field. Hence, it is conceivable that the accumulation of infrared modes crossing the horizon in the expanding de Sitter phase may backreact on the average expansion rate of the Universe. *A priori*, it is unclear whether infrared gravitons can have much of an effect, since the "energy" in the gravitational field is contained in derivatives of the metric. To make a quantitative estimate, the authors of [2] calculated the graviton tadpole $\langle h_{\mu\nu}\rangle$ at the two-loop order. From that, they obtained the "tadpole corrected" expansion rate of the Universe $H(\langle h_{\mu\nu}\rangle)$, which turned out to decrease quadratically with cosmic time. This slowing down of the expansion rate of the "averaged metric" was interpreted as a secular screening of Λ by the long wavelength modes. If true, this would be a spectacular effect of low energy quantum gravity, with implications for the cosmological constant problem [3].

The purpose of this paper is to reanalyze this problem, with an emphasis on gauge invariance. In Sec. II we show that the tadpole correction to the expansion rate, as defined in Ref. [2], is not gauge invariant (and can in fact be given an arbitrary time dependence). In Sec. III we discuss a physically motivated gauge-invariant definition of the expansion rate, which in the present context essentially links it to the local value of the Ricci scalar. In Sec. IV we calculate a gauge-invariant smeared expectation value of the Ricci scalar, suitably renormalized, showing that there is no infrared secular screening of this quantity. Our conclusions are summarized in Sec. IV.

II. ON THE TADPOLE CORRECTION TO THE EXPANSION RATE

The theory under consideration is pure gravity with a cosmological constant. The action is given by

$$S_{\rm gr} = \frac{1}{2\kappa}\int \sqrt{-g}(\mathcal{R} - 2\Lambda)d^4x, \tag{1}$$

where $\mathcal{R}$ is the Ricci scalar and $\kappa = 8\pi G$. Here, G is Newton's constant. We are interested in perturbations around the de Sitter space solution, and for definiteness we shall adopt the flat chart description. The perturbed metric can be written as

$$g_{\mu\nu}(x) = a^2(\eta)[\eta_{\mu\nu} + h_{\mu\nu}(x)]. \tag{2}$$

Here $\eta_{\mu\nu}$ is the Minkowski metric, $a(\eta) = -1/(H_0\eta)$, with $-\infty < \eta < 0$ the conformal time and H_0 the constant unperturbed expansion rate.

To perform a systematic perturbative expansion using the path integral, we must add to (1) a gauge fixing term $S_{\rm gf} = -(1/2)\eta^{\mu\nu}F_\mu[h]F_\nu[h]$, where the function F_μ is such that $F_\mu[h] = 0$ selects one representative out of a given gauge orbit. In Ref. [2] this function was chosen as

[1]Provided, of course that H is well below the Planck scale. The graviton power spectrum is scale invariant for wavelengths above H^{-1}, with amplitude of order $h \sim H/M_p$.

$$F_\mu[h] \equiv a\left(h^\nu_{\mu,\nu} - \frac{1}{2}h,_\mu + 2h^\nu_\mu \frac{a,_\nu}{a}\right). \quad (3)$$

Here, indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$. Suitable counterterms will be also be needed in order to remove divergences.[2] The total action takes the form

$$S_{\rm tot} = S_{\rm gr} + S_{\rm gf} + S_{\rm FP} + S_{\rm ct}, \quad (4)$$

where $S_{\rm FP}$ indicates the Faddeev-Popov (FP) ghost terms and $S_{\rm ct}$ the counterterms. The graviton tadpole is defined by

$$\langle h_{\mu\nu}\rangle_F = \int_{\rm CTP} \mathcal{D}\psi^+ \mathcal{D}\psi^- h^+_{\mu\nu} e^{iS_{\rm tot}[\psi^+]} e^{-iS_{\rm tot}[\psi^-]}, \quad (5)$$

where the subindex F refers to the gauge fixing function (3) and ψ indicates the set of dynamical variables: metric perturbations $h_{\mu\nu}$ and the FP ghosts and antighosts. The closed time path version of the path integral is indicated, since we are interested in expectation values (rather than matrix elements between in and out vacua).

The left-hand side of Eq. (5) can be computed diagrammatically order by order in perturbation theory. In the gauge (3), the propagator is infrared divergent in the limit of infinite volume [2], so it is convenient to compactify the spatial directions and consider a finite (although in principle arbitrarily large) comoving volume. If we choose a spatially homogeneous initial state, symmetry requires that

$$\langle h_{\mu\nu}\rangle_F = A_F(\eta)\eta_{\mu\nu} + B_F(\eta)t_\mu t_\nu, \quad (6)$$

where, $t^\mu = (\partial_\eta)^\mu$.

From (2) and (6), the "averaged" metric $\langle g_{\mu\nu}\rangle_F$ is a flat Friedmann-Robertson-Walker (FRW) metric, with expansion rate given by

$$H_F \equiv H(\langle h_{\mu\nu}\rangle_F) = \frac{d\ln[a(1+A_F)^{1/2}]}{a(1+A_F-B_F)^{1/2}d\eta} = \frac{H_0}{(1+A_F-B_F)^{1/2}}\left[1 - \frac{1}{2}\frac{\eta A'_F}{(1+A_F)}\right], \quad (7)$$

where the prime indicates derivative with respect to η.

In Ref. [2], Tsamis and Woodard calculated A_F and B_F at the two-loop order. Upon substitution in (7) they obtained

$$H_F = H_0\left[1 - 4\kappa^2\left(\frac{H_0}{4\pi}\right)^4\left[\frac{1}{6}(H_0 t)^2 + \mathcal{O}(H_0 t)\right] + \mathcal{O}(\kappa^6)\right], \quad (8)$$

which decreases quadratically with cosmological time $t = -H^{-1}\ln(H_0\eta)$. As mentioned in the introduction, this result was interpreted in [2] as a secular screening of the effective cosmological constant by the infrared gravitons. However, as we shall now discuss, H_F is not invariant under generic gauge transformations, and so the above interpretation seems rather questionable.

Let us consider a new gauge fixing function $G[h]$ in the vicinity of $F[h]$. If $F[h] = 0$, then we can find a new metric perturbation

$$\tilde{h}_{\mu\nu} = h_{\mu\nu} + \delta_\chi h_{\mu\nu},$$

related to $h_{\mu\nu}$ by a gauge transformation $\delta x^\mu = x'^\mu - x^\mu = \chi^\mu$, such that $G[\tilde{h}] = 0$. Here

$$\delta_\chi h_{\mu\nu} = 2a^{-2}\nabla_{(\mu}\chi_{\nu)} + \mathcal{O}(\chi^2),$$

where ∇_μ is the covariant derivative with respect to the full metric $g_{\mu\nu}$ and $\chi_\nu = g_{\nu\lambda}\chi^\lambda$.

Note that the gauge transformation will in general depend on $h_{\mu\nu}$,

$$\chi^\mu = \chi^\mu[h], \quad (9)$$

and even for simple changes of the gauge function $F[h]$ the dependence of χ^μ on h can be quite nontrivial. The point, however, is that for every h this transformation will exist. In what follows, we shall consider the class of gauge functions G in the neighborhood of F which are defined through the equation

$$F[h] = G[h + \delta_\chi h],$$

for some χ.

Both $S_{\rm gr}$ and $S_{\rm ct}$ are gauge invariant, $S_{\rm gr}[h] = S_{\rm gr}[h + \delta_\chi h]$, and $S_{\rm ct}[h] = S_{\rm ct}[h + \delta_\chi h]$. Moreover $(S_{\rm gf})_F \times [h] = (S_{\rm gf})_G[h + \delta_\chi h]$ and $(S_{\rm FP})_F[h] = (S_{\rm FP})_G[h + \delta_\chi h]$. It is then straightforward to show, by changing variables in (5), that

$$\langle h_{\mu\nu}\rangle_G = \langle h_{\mu\nu} + \delta_\chi h_{\mu\nu}\rangle_F. \quad (10)$$

The variation of the tadpole under gauge transformations is thus given by

$$\begin{aligned}\langle h_{\mu\nu}\rangle_G - \langle h_{\mu\nu}\rangle_F &= \langle \delta_\chi h_{\mu\nu}\rangle \equiv (\delta_\chi A)\eta_{\mu\nu} + (\delta_\chi B)t_\mu t_\nu \\ &= \Big\langle [\eta_{\lambda\nu} + h_{\lambda\nu}]\chi^\lambda_{,\mu} + [\eta_{\lambda\mu} + h_{\lambda\mu}]\chi^\lambda_{,\nu} \\ &\quad + h_{\mu\nu,\lambda}\chi^\lambda - \frac{2}{\eta}[\eta_{\mu\nu} + h_{\mu\nu}]\chi^0\Big\rangle \\ &\quad + O(\chi^2). \end{aligned} \quad (11)$$

Here $\delta_\chi A$ and $\delta_\chi B$ represent the changes in $A(\eta)$ and $B(\eta)$ defined in (6). In the above expression χ is treated as a small quantity, but $h_{\mu\nu}$ is not necessarily small.

In the case when χ^μ is a c-number (and by this we mean a function independent of h), the only transformation compatible with the symmetries of a flat FRW is

[2] General relativity is nonrenormalizable, and the number of counterterms needed in $S_{\rm ct}$ increases with the number of loops at which we calculate our observables. However, the number is finite at any order, as it is usually the case in effective field theories.

$$\chi^\mu = f(\eta) t^\mu. \tag{12}$$

If we neglect $h_{\mu\nu}$ in Eq. (11), the expansion rate H is invariant under gauge transformations. The reason is that for a flat FRW the expansion rate is given in terms of the temporal component of the Einstein tensor

$$H^2 = \tfrac{1}{3} G^0_0. \tag{13}$$

The background is such that

$$G^\mu_\nu - \Lambda \delta^\mu_\nu = 0. \tag{14}$$

It follows that $\delta_\chi G^\mu_\nu = 0$, and from (13) $\delta_\chi H = 0$. More explicitly, from (11) we have $\delta_\chi A = -(2f/\eta)$ and $\delta_\chi B = -2f'$, and linearizing (7) we have[3]

$$\delta_\chi H \equiv H_G - H_F = (H_0/2)[\delta_\chi B - \delta_\chi A - \eta(\delta_\chi A)'] = 0. \tag{15}$$

Provided that χ^μ is independent of h, the above consideration can be extended to the case when the tadpole is nonvanishing. Using (6) in (11) we find $\delta_\chi A = -(2f/\eta)(1 + A) + A' f$ and $\delta_\chi B = -2f'(1 + A - B) - (2f/\eta)B + B' f$. Substituting these variations in (7), it is straightforward to check that

$$\delta_\chi H(\eta) = \frac{dH(\eta)}{d\eta} f(\eta) + \cdots. \tag{16}$$

Here, $H(\eta)$ represents the right-hand side of (7), which depends on time through A and B, and the ellipsis denote higher orders in χ.

The simple form of (16) is easily understood. To lowest order in χ, the variation $\delta_\chi h_{\mu\nu}$ is *exactly* linear in $h_{\mu\nu}$ [the metric perturbation $h_{\mu\nu}$ is *not* treated as a small parameter in Eq. (11)]. Because of that, $\langle h_{\mu\nu}\rangle$ transforms like a classical metric under infinitesimal c-number gauge transformations χ^μ,

$$\langle \delta_\chi h_{\mu\nu}\rangle = \delta_\chi \langle h_{\mu\nu}\rangle + \cdots. \tag{17}$$

Equation (16) follows immediately by noting that $H^2 = (1/3)G^0_0$ is a scalar under redefinitions of the time coordinate (because it has mixed temporal indices). The gauge dependence (16) is therefore rather irrelevant: it indicates that we have changed the parametrization of a time-dependent function, but this does not change the value of the expansion rate H as a function of proper time t [defined by $dt = a(1 + A - B)^{1/2} d\eta$].

[3] A derivation of (15) under similar assumptions was given in [2]. This, however, does not establish that $H(\langle h_{\mu\nu}\rangle)$ will be invariant. As shown below, Eq. (15) does not hold for generic gauge transformations.

However, the above conclusions do not apply to the case where the gauge transformations depend on the metric [4][4]

$$\chi^\mu = \chi^\mu[h]. \tag{20}$$

For generic choices of $\chi[h]$, we should expect

$$\langle h\chi\rangle \neq \langle h\rangle\langle\chi\rangle,$$

and from Eq. (11), we should likewise expect that

$$\langle \delta_\chi h_{\mu\nu}\rangle \neq \delta_{\langle\chi\rangle}\langle h_{\mu\nu}\rangle + \cdots.$$

We may thus anticipate that, in general, the expectation value of the gauge transformed metric perturbation $\langle h + \delta_\chi h\rangle$ will not be gauge equivalent to the original one $\langle h\rangle$.

The expansion rate $H(\langle h_{\mu\nu}\rangle)$ as a function of proper time t would be gauge invariant (and therefore meaningful) if and only if $\langle h + \delta_\chi h\rangle$ is related to $\langle h\rangle$ by a time reparametrization [see e.g. the discussion around Eq. (17)]. In equations, this means that for each $\chi^\mu[h]$ we should be able to find a vector ξ^μ such that

$$\langle \delta_\chi h_{\mu\nu}\rangle = \delta_\xi \langle h_{\mu\nu}\rangle + \cdots, \tag{21}$$

where, by symmetry, $\xi^\mu = g(\eta) t^\mu$. However, an equation of this sort cannot hold for a generic $\chi[h]$. To illustrate the point, we may restrict ourselves to *lowest order* in perturbation theory, where the expectation value of odd functions of h will vanish. Let us therefore take χ to be an odd function of the metric perturbation h. In this case Eq. (21) to lowest order in perturbation theory reads

$$\left\langle 2h_{\lambda(\mu}\chi^\lambda_{,\nu)} + h_{\mu\nu,\lambda}\chi^\lambda - \frac{2}{\eta} h_{\mu\nu}\chi^0 \right\rangle = -\frac{2g}{\eta}\eta_{\mu\nu} - 2g' t_\mu t_\nu. \tag{22}$$

[4] For illustration, note that even the simplest changes in the gauge function F may lead to a complicated dependence of χ on h (21). Consider for instance the one parameter class of gauges

$$F^{(\alpha)}_\mu = F_\mu[h] \equiv a\left(h^\nu_{\mu,\nu} + \alpha h,_\mu + 2h^\nu_\mu \frac{a,_\nu}{a}\right), \tag{18}$$

out of which (3) corresponds to $\alpha = -1/2$. A change of α corresponds to $\delta F_\mu = a h,_\mu \delta\alpha$, and we have

$$\int dx' \frac{\delta F_\mu[h]}{\delta h_{\rho\sigma}(x')} \delta_\chi h_{\rho\sigma}(x') = a h,_\mu \delta\alpha.$$

By introducing the explicit expression of F and $\delta_\chi h_{\mu\nu}$, this equation takes the form

$$\mathcal{O}_{\mu\nu}[h]\chi^\nu = h,_\mu, \tag{19}$$

where $\mathcal{O}_{\mu\nu}[h]$ is a second order differential operator whose coefficients depend on $h_{\mu\nu}$ and its first and second derivatives. Equation (19) should in principle be solved in order to find χ in terms of h. It is clear that in this case the dependence will be highly nonlocal (and difficult to find explicitly), but the point is that we cannot restrict consideration to c-number gauge transformations, since generically χ depends on h.

Here we have used $\xi^\mu = g(\eta)t^\mu$, as dictated by symmetry. When we consider rescaling given by $\chi^\mu[h] \to k(\eta)\chi^\mu[h]$ with an arbitrary function k, we have the terms with $k'(\eta)$ and those with $k(\eta)$ on the left-hand side of Eq. (22). In order to satisfy this equality for an arbitrary function k, the right-hand side also has to have the terms with $k'(\eta)$ and those with $k(\eta)$. This requires that $g(\eta)$ should also transform to $k(\eta)g(\eta)$. Comparing the term proportional to $k'(\eta)$, we immediately find

$$g = \langle h_{0\lambda}\chi^\lambda\rangle. \tag{23}$$

Then, if we choose $\chi^\mu = \chi^0[h]t^\mu$, where χ^0 is an arbitrary odd function of $h_{\mu\nu}$ (including possible arbitrary explicit dependence on η and contraction of internal indices with the vector t^μ), the $\mu = \nu = 0$ component of (22) reads

$$\langle h_{00,0}\chi^0\rangle = 0. \tag{24}$$

Clearly, this equation does not hold in generic gauges and for generic choices of χ^0. In particular, it does not hold in the gauge defined in Eq. (3), which completes our proof.[5]

It follows that the "tadpole corrected" expansion rate is not physically meaningful. Rather, given the enormous freedom in choosing χ^μ (which can include arbitrary functions of h and η) it appears that $H(\langle h_{\mu\nu}\rangle)$ can in fact be given arbitrary dependence on proper time t.

III. OBSERVABLES

Gravitational radiation of wavelength shorter than the Hubble radius has an impact on the background expansion rate. Formally, this can be accounted for in the so-called Isaacson approximation (see e.g. [5]), where the Einstein tensor is split into a background "long wavelength" contribution and the contribution from short-wavelength gravitational waves. A classical bath of short-wavelength gravity waves does modify the time evolution of the scale factor, much like a bath of radiation would. However, this does not mean that it screens the cosmological constant, which also contributes to the expansion rate as usual.

On the other hand, here we are not interested in the effect of short-wavelength modes but in the collective effect of infrared graviton modes and their interactions. Could these cause a secular screening of the cosmological constant? What we mean by this is an adiabatic erosion of the expansion rate, such as the one suggested by Eq. (8), which would lead to an initial quasi-de Sitter phase with

$$|\dot{H}| \ll H^2. \tag{25}$$

As emphasized in the previous subsection, instead of calculating the expectation value of the metric, it is important to look for some gauge-invariant characterization of the expansion rate. One such observable was suggested by Abramo and Woodard [6]. The basic idea is to consider the value of a scalar field φ conformally coupled to gravity, with a constant source term J:

$$\left[\Box + \frac{\mathcal{R}}{6}\right]\phi = J. \tag{26}$$

In a flat FRW universe, the Ricci scalar is given by

$$\mathcal{R} = 12H^2(t) + 6\dot{H}(t). \tag{27}$$

For instance, when the scale factor takes the form $a \propto t^p$, we have $H(t) = pt^{-1}$ and $\mathcal{R} = (12p^2 - 6p)t^{-2}$. The general solution of Eq. (26) in this case is of the form

$$\phi(t) = A_+t^{\alpha_+} + A_-t^{\alpha_-} + \left(\frac{2p^2}{2p^2 + 5p + 2}\right)\frac{J}{2H^2(t)}, \tag{28}$$

where $A_\pm$ are arbitrary constants and

$$\alpha_\pm = \frac{-3p \pm \sqrt{p^2 + 4p}}{2}.$$

The last term in Eq. (28) is proportional to t^2. As we have $\Re\alpha_\pm < 2$ for $p > -4/3$, the two first terms in (28) decay faster than the last term. Therefore, at late times only the third term will be important. The limit of quasi-de Sitter expansion ($\dot{H} \ll H^2$) corresponds to $p \gg 1$, and in this case we have

$$\phi(t) \approx \frac{J}{2H^2(t)} \qquad (t \to \infty,\ p \gg 1). \tag{29}$$

We may interpret this result in the following way. The source J creates the field ϕ. In turn, the field is diluted by the expansion, which causes the amplitude to fall off as the inverse of the scale factor. At late times, the value of ϕ is dominated by the field created by the source J during the last expansion time, on the surface of a sphere of radius H^{-1}, while the initial conditions set by the coefficients $A_\pm$ become irrelevant. Thus, the late time asymptotic behavior of $\phi(t)$ sourced by a constant J is a measure of the time-dependent expansion rate $H(t)$, through Eq. (29). Let us now consider the quantum theory. To avoid the introduction of a new quantum scalar field in the system, it was suggested in Ref. [6] that $\phi(t)$ be defined as the expectation value of inverse of the retarded conformal propagator acting on the constant source

$$\phi(t) = \left\langle \frac{1}{\Box + \frac{1}{6}\mathcal{R}} J \right\rangle. \tag{30}$$

This quantity would then be a characterization of the inverse of the square of the expansion rate, through the identification (29). Although this seems to be an appropriate definition, it is certainly somewhat complicated be-

[5] Equation (24) may accidentally hold in some gauge for all possible functions χ^0. For instance, it holds in the transverse and traceless gauge, since $\langle h_{00}\rangle$ vanishes to lowest order. In that case we should examine the other components of Eq. (22) to see whether they can hold for generic χ.

cause of the nonlocal character of the operator within brackets.

In the present context, however, there is a much simpler alternative which is equally well motivated. Rather than keeping a constant source and looking for the field it creates at a given point, we may ask the converse: What source $J(x)$ will give a constant field $\phi(t) \to 1$ at late times? Classically, if the expansion rate is a constant, $H = H_0$, then a constant source will produce an asymptotically constant field $\phi(t) \to J/(2H_0^2)$ at $t \to \infty$. In the quasi-de Sitter limit ($p \gg 1$), a constant field $\phi \to 1$ is caused by a source of the form $J \to 2H^2(t)$. In this sense, the source which is needed in order to keep $\phi \to 1$ can be used as a measure of the local expansion rate. Needless to say, this measure reduces trivially to the curvature scalar

$$2H^2(t) \approx J(t) = (\Box + \tfrac{1}{6}\mathcal{R})1 = \tfrac{1}{6}\mathcal{R}. \tag{31}$$

This illustrates the fact that in a quasi-de Sitter phase we may adopt $\sqrt{\mathcal{R}/12}$ as a *local* definition of the expansion rate, because the second term in (27) is negligible. The expansion rate can change by a large amount in the course of time, but as long as it does so adiabatically, the curvature scalar will be a good tracer of $H(t)$.

Now, in the quantum theory, the metric fluctuates. If we adopt the expectation value of the classical expression as our definition of $H(t)$, we have

$$H^2(t) \approx \tfrac{1}{12}\langle\mathcal{R}\rangle. \tag{32}$$

Naively, in pure gravity with a cosmological constant, we may expect to have a relation of the form

$$\langle\mathcal{R}(x)\rangle = 4\Lambda, \tag{33}$$

which would readily imply the constancy of $H(t)$, with no room for a secular screening. Intuitively, Eq. (33) seems to follow from the Heisenberg equation of motion, which the field operator is supposed to satisfy identically. Nevertheless, the definition of a physically meaningful $\langle\mathcal{R}\rangle$, and the proof of an equation of the form (33), involves a number of subtleties related to gauge invariance and renormalization (see also [7,8]). A full discussion of this point is postponed to the next section.

Before closing, one comment is in order about the classical backreaction. Classically, in a flat FRW universe, we have

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}(1 + w_{\rm eff}), \tag{34}$$

where $w_{\rm eff} = p/\rho$ is the ratio of pressure p to energy density ρ. As mentioned at the beginning of this section, the classical backreaction due to short-wavelength gravitational waves modifies the expansion law like a usual radiation field, with $w_{\rm rad} = 1/3$. If the density in radiation is comparable to the cosmological term, then $(1 + w_{\rm eff})$ will not be small, and from (27), $\mathcal{R}$ will not be a good tracer of $H(t)$. At the classical level, $\mathcal{R}$ is constant, and hence it is completely insensitive to the classical backreaction effect. In this respect, the observable originally proposed in Ref. [6], given in Eq. (30), cannot sense the traceless component of the energy momentum tensor, either. Clearly, $\phi = \text{const}$ is one of the solutions when we assume $J = \text{const}$. Although there are various solutions for ϕ for a constant J, this variety is due to the degrees of freedom of the initial conditions, which are supposed to be irrelevant at late times. The study of alternative observables which are, at least, sensitive to the backreaction effect caused by classical gravitational waves, is postponed for future work.

IV. IS THERE A SECULAR SCREENING?

As discussed above, the Ricci scalar $\mathcal{R}$ is a good indicator of the adiabatic evolution of the expansion rate in a quasi-de Sitter phase. Here, we will discuss the calculation of its renormalized expectation value in the theory of pure gravity. For this purpose, it will be quite important to work with quantities which are invariant under diffeomorphisms. When we consider the gauge transformation

$$x^\mu \to \bar{x}^\mu = x^\mu - \chi^\mu, \tag{35}$$

$\mathcal{R}(x)$ transforms like $\mathcal{R}(x) \to \bar{\mathcal{R}}(x) \approx \mathcal{R}(x) + \mathcal{R}_{,\mu}\chi^\mu$. In this sense $\mathcal{R}$ is not invariant. However, we do not really need to measure the value of curvature at a specified point in space-time. Rather, it will be sufficient for our purposes to consider its value smeared over a certain sample volume. Let us introduce a window function $W(x)$ which by definition transforms as a space-time scalar.[6] Then, for any space-time scalar operator $\mathcal{O}(x)$, the integral $\int d^4x\sqrt{-g}W(x)\mathcal{O}(x)$ is manifestly gauge invariant.[7] It will be useful to introduce the following notation for the expectation value of this quantity:

$$\langle\sqrt{-g}\mathcal{O}\rangle_W \equiv \langle{\rm phys}|\int d^4x\sqrt{-g}W(x)\mathcal{O}(x)|{\rm phys}\rangle.$$

[6]This window function should transform as a scalar under the action of BRST (Becchi, Rouet, Stora, and Tyutin) charge, and hence it cannot be just a c-number. We may think of $W(x)$ as an operator made out of other fields, which are used to probe the gravitational field. Our assumption is that the presence of these other fields should not disturb the results of the purely gravitational calculation significantly. For present purposes, such fields should not induce infrared effects which might enhance or cancel out the cumulative screening which we are investigating. For instance, we may consider a sector with very massive particles, much larger than H, whose contribution to the energy momentum tensor does not contain any cumulative infrared effects.

[7]Note that $W(x) = W_z(x) \equiv \delta^{(4)}(x^\mu - z^\mu)/\sqrt{-g(x)}$ is *not* a suitable window function. Although this is a scalar with respect to the transformation of x, it is also so with respect to the transformation of z. The bi-scalar transforms as $\delta W_z(x) = \chi^\mu(x)(\partial W_z(x)/\partial x^\mu) + \chi^\mu(z)(\partial W_z(x)/\partial z^\mu)$, and for that reason $\int d^4x\sqrt{-g}\mathcal{R}(x)W_z(x) = \mathcal{R}(z)$ is not gauge invariant [in agreement with the discussion below Eq. (35)].

A precise definition of the arbitrary physical state $|\text{phys}\rangle$ will be given below.

Taking into account the fact mentioned above, we find that a gauge-invariant quantum version of Eq. (31) should be understood as

$$12H^2(t) \approx \langle\sqrt{-g}\mathcal{R}\rangle_W/\langle\sqrt{-g}\rangle_W.$$

[An implicit assumption is that the timescale at which $H(t)$ changes is much longer than the width of the window function W.] Therefore the basic goal of this section is to show that the equation

$$\langle(\sqrt{-g}\mathcal{R})_{\text{ren}}\rangle_W = 4\Lambda\langle\sqrt{-g}_{\text{ren}}\rangle_W, \tag{36}$$

holds for any scalar window function W, which means that there is no secular evolution of the expansion rate. The definition of the operators $(\sqrt{-g}\mathcal{R})_{\text{ren}}$ and $\sqrt{-g}_{\text{ren}}$ appearing in (36) requires explanation. In the path integral approach, we can calculate the n-point functions $\langle h_{\mu\nu}(x_1)\cdots h_{\rho\sigma}(x_n)\rangle$ to arbitrary loop order. All divergences in this calculation can be reabsorbed by diagrams involving the vertices generated by the counterterms in S_{ct}. Still, such renormalized n-point functions will not be free from divergences in the coincidence limit, when two or more of the n points are brought to sit on top of each other. Since $\sqrt{-g}\mathcal{R}$ contains the coincidence limit of n-point functions of $h_{\mu\nu}$, the counterterms in S_{ct} will fail to render a finite expectation value for $\sqrt{-g}\mathcal{R}$. This situation, of course, is not specific to gravity, and the problem is remedied once we introduce a probe field which couples to the composite operator of interest. This will allow us to define a suitable regularized operator $(\sqrt{-g}\mathcal{R})_{\text{ren}}$ whose renormalized expectation value is finite.

It is instructive to start by considering the simpler example of an interacting scalar field ψ in Minkowski spacetime [9]. In this case, the two-point function $\langle\psi(x)\psi(x')\rangle$ is finite after renormalization, but its coincidence limit $\langle\psi^2(x)\rangle$ is still divergent. In the context of a single free scalar field, there is no counterterm to renormalize the value of $\langle\psi^2(x)\rangle$. On the other hand, we can only measure this seemingly divergent quantity through some interaction. Let us therefore introduce a coupling to a probe scalar field ϕ via the interaction Lagrangian $-\lambda\psi^2\phi^2/2$. Now, we can "measure" $\lambda\langle\psi^2(x)\rangle$ as a contribution to the mass of the probe field ϕ. Here, the probe field is treated as classical, meaning that we neglect all the loop diagrams containing its propagator. We also assume that its amplitude is infinitesimally small. Now, $g\langle\psi^2(x)\rangle$ can be renormalized because the divergence in $\langle\psi^2(x)\rangle$ can be absorbed by a mass counterterm of the ϕ-field. Hence, we have found a regularized operator $\lambda\psi^2_{\text{ren}}(x) = \lambda\psi^2(x) + \delta m^2_\phi$ whose renormalized expectation value

$$m^2_{\phi(\text{ren})} \equiv \langle\lambda\psi^2_{\text{ren}}(x)\rangle,$$

is finite by virtue of the probe field counterterm $\delta\mathcal{L}_\phi = -(\delta m^2_\phi)\phi^2/2$.

The same argument works for $\sqrt{-g}\mathcal{R}$. We consider a probe massless scalar field ϕ with the curvature coupling as we discussed in the preceding section. The action we add is

$$S^\phi + S^\phi_{\text{ct}} = -\frac{1}{2}\int d^4x\sqrt{-g}(g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \xi\mathcal{R}\phi^2 + \delta\mathcal{M}^2_\phi[g]\phi^2), \tag{37}$$

where the mass counterterm $\delta\mathcal{M}^2_\phi[g]$ is made up of curvature invariants and will be further specified below. We may now define

$$(\sqrt{-g}\xi\mathcal{R})_{\text{ren}} \equiv \sqrt{-g}(\xi\mathcal{R} + \delta\mathcal{M}^2_\phi), \tag{38}$$

whose renormalized expectation value may be thought of as the local value of the mass of the ϕ field

$$m^2_{\phi(\text{ren})}[W] \equiv \frac{\langle\xi(\sqrt{-g}\mathcal{R})_{\text{ren}}\rangle_W}{\langle\sqrt{-g}_{\text{ren}}\rangle_W}. \tag{39}$$

Since the volume $\langle\sqrt{-g}\rangle_W$ contains polynomials of $h_{\mu\nu}$, this quantity is also divergent. Hence, we have to renormalize this expression by adding a counterterm $\delta\sqrt{-g}$, i.e. $\langle\sqrt{-g}_{\text{ren}}\rangle_W = \langle\sqrt{-g} + \delta\sqrt{-g}\rangle_W$. To be more precise, in order to renormalize the volume, we need to add another probe field to measure it. For example, we can consider a scalar field with $\xi = 0$ as a probe. In this case the renormalized volume integral of its mass will measure the renormalized volume.

The value of $m^2_{\phi(\text{ren})}[W]$ will change depending on the choice of the finite part of counterterms. However, if there is a choice of counterterms in which the relation $m^2_{\phi(\text{ren})}[W] = 4\xi\Lambda$ is maintained independently of the window function W, it is such renormalization conditions that are natural and appropriate for the theory that we are considering. Finite renormalization of local counterterms will correspond to introducing new interactions between the probe field and gravity, different from the original curvature coupling term. We shall not pursue the consideration of such interactions here. They correspond to higher-order irrelevant operators (which are *not* expected to lead to infrared effects of the sort we are interested in). Thus, the basic question is whether we can choose local counterterms which make the renormalized value of $m^2_{\phi(\text{ren})}$ to be constant.

The key identity is

$$\begin{aligned} 0 &= -i\int_{\phi=0}\mathcal{D}\psi\int d^4x\frac{\delta}{\delta\tilde{h}_{\mu\nu}(x)}W(x)g_{\mu\nu}(x)e^{iS_{\text{tot}}} \\ &= \left\langle\frac{\sqrt{-g}}{2\kappa}(\mathcal{R} - 4\Lambda) + g_{\mu\nu}\frac{\delta(S_{gf+FP} + S_{\text{ct}})}{\delta\tilde{h}_{\mu\nu}}\right\rangle^{(\phi=0)}_W, \end{aligned} \tag{40}$$

where we assumed that the window function $W(x)$ is independent of $\tilde{h}_{\mu\nu} \equiv g_{\mu\nu} - g^{(0)}_{\mu\nu}$. $g^{(0)}_{\mu\nu}$ is the background metric which can be different from the de Sitter one as long as it solves the Einstein equations. The first equality follows from functional integration by parts. In the second equality, we have dropped the term $-16i\delta(0)$ which arises from functional differentiation $\delta g_{\mu\nu}(x)/\delta\tilde{h}_{\mu\nu}(x)$. It is clear that this naively divergent term is local and can be grouped together with $S_{\rm ct}$. In fact, such a functional derivative vanishes in dimensional regularization. All the variables in the path integral are to be understood as $(+)$-fields and the integral over $(-)$-fields has been abbreviated, since the closed time path formalism is not essential for the current discussion.

We submit that the appropriate choice of $\delta\mathcal{M}^2_\phi$ which implements our renormalization condition is given by

$$\sqrt{-g}\delta\mathcal{M}^2_\phi = 2\kappa\xi g_{\mu\nu}\frac{\delta S_{\rm ct}}{\delta\tilde{h}_{\mu\nu}} + 4\sqrt{-g}\xi\Lambda\delta_{\rm vol}. \qquad (41)$$

Here, $S_{\rm ct}$ is the counterterm action for the theory of pure gravity, without the ϕ field. Thus, $\sqrt{-g}\delta\mathcal{M}^2_\phi$ is local as long as $S_{\rm ct}$ is so. For the reason explained above, we have chosen a specific form of the counterterms. The other part $4\sqrt{-g}\Lambda\delta_{\rm vol}$ is also local. Then, substituting in Eq. (38), we have

$$\langle(\sqrt{-g}\mathcal{R})_{\rm ren}\rangle_W = 4\Lambda\langle\sqrt{-g_{\rm ren}}\rangle_W - 2\kappa\left\langle g_{\mu\nu}\frac{\delta S_{\rm gf+FP}}{\delta\tilde{h}_{\mu\nu}}\right\rangle^{(\phi=0)}_W, \qquad (42)$$

where we have used Eq. (40).

The remaining task is to show that the second term in the right-hand side of (42) vanishes when the expectation value is taken for physical states. To show this, the essential point is to understand what is meant by the physical state. Since ϕ is set to 0, we neglect it completely in the following discussion. It will be very convenient for our purposes to follow the standard construction for gauge fixing based on the BRST invariance [10]. The gauge transformation changes $\tilde{h}_{\mu\nu}(x) \to \bar{\tilde{h}}_{\mu\nu}(x) = \tilde{h}_{\mu\nu}(x) + \delta\tilde{h}_{\mu\nu}(x)$ with

$$\delta\tilde{h}_{\mu\nu}(x) = g_{\mu\rho}\chi^\rho{}_{,\nu} + g_{\nu\rho}\chi^\rho{}_{,\mu} + g_{\mu\nu,\rho}\chi^\rho. \qquad (43)$$

The BRST transformation δ_B of $\tilde{h}_{\mu\nu}$ is obtained by simply replacing χ^μ with a Grassmanian field c^μ in Eq. (43). The BRST transformation of c^μ is determined by requiring the nilpotency of the BRST transformation, $\delta_B^2\tilde{h}_{\mu\nu} = 0$. Different from the usual gauge theory, this equation does not determine $\delta_B c^\mu$ locally. The obtained equation contains derivatives of $\delta_B c^\mu$. Hence, we do not give an explicit expression for $\delta_B c^\mu$, which is not required below. We add the antighost field $\bar{c}^\mu$ and its BRST transformation introduces B^μ-field as $\delta_B\bar{c}^\mu = iB^\mu$. From the requirement of nilpotency of the BRST transformation, we have $\delta_B B^\mu = 0$. After these preparations, for an arbitrary gauge fixing function $F_\mu[\tilde{h}_{\alpha\beta}]$, the gauge fixing term and the Faddeev-Popov ghost term are simultaneously given by

$$S_{\rm gf+FP} = \int d^4x\mathcal{L}_{\rm gf+FP},$$

with

$$\mathcal{L}_{\rm gf+FP} = -i\delta_B\left[\bar{c}^\mu\left(F_\mu + \frac{1}{2}\alpha B_\mu\right)\right] = B^\mu\left(F_\mu + \frac{1}{2}\alpha B_\mu\right) - i(\delta_B\tilde{h}_{\alpha\beta})\frac{\delta F_\mu}{\delta\tilde{h}_{\alpha\beta}}\bar{c}^\mu, \qquad (44)$$

where indices in $\mathcal{L}_{\rm gf+FP}$ are raised and lowered by using the background metric $g^{(0)}_{\mu\nu}$. Since $F_\mu[\tilde{h}_{\alpha\beta}]$ may contain differentiation of $\tilde{h}_{\alpha\beta}$, $\delta F_\mu/\delta\tilde{h}_{\alpha\beta}$ is understood as the derivative operator that is obtained by the usual variational principle. In the present case it acts on $\bar{c}^\mu$. For simplicity, we assume that $F_\mu[\tilde{h}_{\alpha\beta}]$ is linear in $\tilde{h}_{\alpha\beta}$. Hence, $\delta F_\mu/\delta\tilde{h}_{\alpha\beta}$ is an operator solely written in terms of the background quantities.

Let us now consider the physical observables and the physical states. In the BRST formalism, observables are BRST invariant quantities. This corresponds to the usual notion of gauge-invariant variables such as the Bardeen parameter at the linear order. We should note that $\delta_B s(x) \neq 0$ for a scalar $s(x)$, which is the reason why we had to introduce a window function $W(x)$ to evaluate the expectation value of $\mathcal{R}$. An observable $\mathcal{O}$ satisfies $[Q_B, \mathcal{O}\} = 0$, where Q_B is the BRST charge defined in such a way that $\delta_B * = [Q_B, *\}$. Correspondingly, physical states are also required to be BRST invariant. Hence, they must satisfy

$$Q_B|\text{phys}\rangle = 0.$$

Therefore, for physical states, any operator that can be written in the "exact" form $[Q_B, *\}$ has vanishing expectation value.

Then, using $\delta_B W(x) = c^\mu\partial_\mu W(x)$, which is the standard transformation rule for any scalar quantity, it is straightforward to show that

$$\begin{aligned}\int d^4x W g_{\mu\nu}\frac{\delta}{\delta\tilde{h}_{\mu\nu}}\int d^4x'\mathcal{L}_{\rm gf+FP} &= \int d^4x W\left(g_{\mu\nu}\frac{\delta F_\alpha}{\delta\tilde{h}_{\mu\nu}}B^\alpha - i(\delta_B\tilde{h}_{\rho\sigma})\frac{\delta F_\alpha}{\delta\tilde{h}_{\rho\sigma}}\bar{c}^\alpha + i\partial_\mu\left(c^\mu g_{\rho\sigma}\frac{\delta F_\alpha}{\delta\tilde{h}_{\rho\sigma}}\bar{c}^\alpha\right)\right)\\ &= \left[Q_B, -i\int d^4x W g_{\mu\nu}\frac{\delta F_\alpha}{\delta\tilde{h}_{\mu\nu}}\bar{c}^\alpha\right\}. \qquad (45)\end{aligned}$$

Hence, the contribution from $S_{\rm gf+FP}$ vanishes when the expectation value is calculated for physical states. This finally establishes our claim that we can choose local counterterms such that $\langle(\sqrt{-g}\mathcal{R})_{\rm ren}\rangle_W = 4\Lambda\langle\sqrt{-g}_{\rm ren}\rangle_W$

holds for an arbitrary scalar window function $W(x)$. This simply means that $m^2_{\phi(\mathrm{ren})}[W]$, as a measure of the effect of the scalar curvature on a probe scalar field, stays a constant over the entire space-time.

V. CONCLUSION

A secular screening of the cosmological constant by infrared quantum effects would represent a very spectacular phenomenon in low energy quantum gravity. In this paper, we have reanalyzed the issue of gauge invariance in the definition of the expansion rate $H(t)$ which was used in the original analysis of this problem [2] [see Eq. (7)].

We have shown that such definition is only invariant under c-number gauge transformations, but not under generic changes of the gauge fixing term. Such changes correspond to gauge transformations where the gauge parameter χ^μ depends on the operator $h_{\mu\nu}$. Because of that, they introduce arbitrary time dependence in the expansion rate $H(t)$ as defined in Eq. (7). Hence, the interpretation of the results in Ref. [2] as a physical screening of Λ seems very questionable.

A truly gauge-invariant definition of $H(t)$ was introduced in Ref. [6]. This definition was motivated on physical grounds as follows. A constant source J in a quasi-de Sitter universe coupled to a conformal scalar field ϕ will produce a field $\phi(t)$. The amplitude of a free conformal scalar in quasi-de Sitter decays with time like the inverse of the scale factor. Hence, the late time behavior of $\phi(t)$ is dominated by the contribution of the source during the last e-folding time, and is therefore proportional to the surface of a sphere of Hubble size

$$\phi(t) \propto JH^{-2}(t).$$

It was proposed in [6] that such auxiliary field be used as a measure of the local expansion rate. The field ϕ is given by the inverse of the perturbed wave operator acting on the constant source. This is a nonlocal and rather cumbersome expression to deal with in the quantum theory. On the other hand, we have argued that there is an alternative definition which is equally useful if we wish to monitor an adiabatic change in the expansion rate [such as the one which would be suggested by Eq. (8)]. Indeed the curvature scalar $\mathcal{R}$ is proportional to $H^2(t)$ plus corrections of order $\dot{H}$ which are negligible in the adiabatic limit. So the question is whether the value of this scalar (or a suitable smearing of it) can change in the course of time. Classically, for the system of pure gravity coupled to a cosmological constant, this is impossible. By using the path integral approach, we confirmed that this conclusion is not altered when we take into account the subtleties associated with gauge invariance and renormalization. Therefore, according to this definition, we find no evidence of a secular screening of the cosmological constant, to all orders in perturbation theory.

It should be stressed that these arguments apply only to the case of pure gravity with a cosmological constant, and they do not exclude the possibility of interesting infrared effects in theories with a different field content [9,11–13] or due to nonperturbative effects [14]. Our considerations focused on the renormalized expectation value of the scalar curvature $\mathcal{R}$, which is insensitive to the classical back-reaction effect due to a bath of gravitons. In future work, we would like to examine different gauge-invariant indicators of the expansion rate [15], which give a nonvanishing result depending on the choice of the initial state.

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Note added.—After this paper was submitted to the archives, Tsamis and Woodard wrote a reply to it [16], expressing some points of view which we do not share.

First, they claim that we did not show that the renormalized Ricci scalar is constant, and that our Eq. (36) is completely consistent with screening. The observable we calculate is the expectation value of the integral of the Ricci scalar over a region of space-time. This operator is divergent, and so we define the corresponding renormalized operator by standard techniques. We show that this agrees with the expectation value of the integral of a constant, over the same region. The equality holds order by order in the loop expansion. The region of space-time is itself arbitrary, as long as the same one is used on both sides of the equation. In our view, this means that the renormalized Ricci scalar, as measured by its effect on a probe scalar field, stays constant, in as precise a sense as can be made. Notice that this is exactly the condition for J to be constant with constant ϕ in Eq. (26).

The authors of Ref. [16] object that we use an external scalar window function $W(x)$ in our definition of the gauge invariant operator (The reason for that is explained in our footnote [7]). This scalar is not constructed from the metric, and hence, the integral of $\sqrt{-g}W(x)\mathcal{R}$ does not correspond to any observable of the theory: it depends on the particular choice of $W(x)$. However, the statement that our equality (36) holds for any $W(x)$ is, of course, independent of this choice, and hence it is a physically meaningful statement. They also object that even if we show that $\langle(\sqrt{-g}\mathcal{R})_{\mathrm{ren}}\rangle_W = 4\Lambda\langle\sqrt{-g}_{\mathrm{ren}}\rangle_W$, both sides of the equation can evolve secularly in the same way. Even if that were the case, this would not imply any secular evolution of their ratio, which is the quantity of our interest (for constant ϕ, the ratio is proportional to J at the classical level). They also claim, at the beginning of Sec. III that our renormalization scheme is ''peculiar.'' We disagree with this appreciation. What we do is standard renormalization in low energy effective theory. We do make a particular choice for the finite parts of the local counterterms which need to be subtracted. This is explained in detail in the paragraphs between our Eqs. (36) and (40). The important point is that there is a choice of counterterms for which there is no secular screening of the renormalized operator. If a change in the local counterterms happened to give rise to some additional effect, then this would be an effect due to local physics (or, conceivably, to the secular evolution of the added higher-order local counterterms, although this seems unlikely), but it would be unrelated to the infrared secular evolution of $\mathcal{R}$.

The authors of [16] also purport that if we are allowed arbitrary subtractions in order to construct the renormalized operator $(\sqrt{-g}\mathcal{R})_{\rm ren}$, then we could absorb in its definition things like the one-loop effective potential of a scalar field. If so, they argue, we would reach the conclusion that $(\sqrt{-g}\mathcal{R})_{\rm ren}$ stays constant even in a theory like ''new inflation,'' where the potential is due to one-loop corrections. Of course this would not be correct, and it has nothing to do with the method we are using in the present paper. Arbitrary subtractions are simply not allowed. At each order in the loop expansion, we only allow as counterterms a finite number of higher dimension local operators, suppressed by corresponding powers of the Planck mass $M_{\rm pl}$. The number of counterterms will be larger if we work at a higher order, because this is unavoidable in nonrenormalizable theories. But these higher order counterterms can never absorb the lower order loop corrections since the power of $M_{\rm pl}$ is different.

We would agree that there are other observables one can look at. Our claim is that we see no evidence for a secular infrared screening in the observable we have analyzed. We should add that this is a better defined observable than the spatially averaged Hubble rate used in [2]. The authors of [16] claim in Sec. 2 of their reply that gauge-dependent quantities can have some physical content. While this is debatable, their discussion does not seem to warrant the preference of a gauge-dependent result over the gauge-invariant one we presented in this paper.

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