Measurement of the branching fraction, polarization, and $\boldsymbol{C P}$ asymmetries in $\boldsymbol{B}^{\boldsymbol{0}} \rightarrow \boldsymbol{\rho}^{\boldsymbol{0}} \boldsymbol{\rho}^{\mathbf{0}}$ decay, and implications for the CKM angle $\alpha$
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#### Abstract

We study $B^{0} \rightarrow \rho^{0} \rho^{0}$ decays in a sample of $465 \times 10^{6} \Upsilon(4 S) \rightarrow B \bar{B}$ events collected with the $B A B A R$ detector at the PEP-II asymmetric-energy $e^{+} e^{-}$collider located at the Stanford Linear Accelerator Center (SLAC). We measure the branching fraction $\mathcal{B}=(0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$ and longitudinal polarization fraction $f_{L}=0.75_{-0.14}^{+0.11} \pm 0.05$, where the first uncertainty is statistical and the second is systematic. The evidence for the $B^{0} \rightarrow \rho^{0} \rho^{0}$ signal has a significance of 3.1 standard deviations, including systematic uncertainties. We investigate the proper-time dependence of the longitudinal component in the decay and measure the $C P$-violating coefficients $S_{L}^{00}=(0.3 \pm 0.7 \pm 0.2)$ and $C_{L}^{00}=(0.2 \pm 0.8 \pm 0.3)$. We study the implication of these results for the unitarity triangle angle $\alpha$.


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Measurements of $C P$-violating asymmetries test the flavor structure of the standard model by overconstraining the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. The time-dependent $C P$ asymmetry in the decays of $B^{0}$ or $\bar{B}^{0}$ mesons to a $C P$ eigenstate dominated by the tree-level amplitude $b \rightarrow u \bar{u} d$ measures $\sin 2 \alpha_{\text {eff }}$, where $\alpha_{\text {eff }}$ differs from the CKM unitarity triangle angle $\alpha \equiv \arg \left[-V_{t d} V_{t b}^{*} / V_{u d} V_{u b}^{*}\right]$ by a quantity $\Delta \alpha$ accounting for the contributions from loop (penguin) amplitudes. The value of $\Delta \alpha$ can be extracted from an analysis of the full set of isospin-related channels [2].

Since the tree contribution to the $B^{0} \rightarrow \rho^{0} \rho^{0}[3]$ decay is color suppressed, the decay rate is much smaller than those of the $B^{0} \rightarrow \rho^{+} \rho^{-}$and $B^{+} \rightarrow \rho^{+} \rho^{0}$ channels [4-8], and is sensitive to the penguin amplitude. Therefore a stringent limit on $\Delta \alpha$ can be set [2,7,9]. This makes the $\rho \rho$ system particularly effective for measuring $\alpha$.

In $B \rightarrow \rho \rho$ decays the final state is a superposition of $C P$-odd and $C P$-even states. An isospin-triangle relation [2] holds for each of the three helicity amplitudes $A_{\lambda=-1,0,+1}$, which can be separated through an angular analysis. The longitudinal polarization fraction $f_{L}=$ $\left|A_{0}\right|^{2} / \sum\left|A_{\lambda}\right|^{2}$ can be determined through the measurement of the distribution of the helicity angles $\theta_{1}$ and $\theta_{2}$, defined as the angles between the direction of the $\pi^{+}$and the direction of the $B$ meson in the rest system of each of the $\rho^{0} \rightarrow \pi^{+} \pi^{-}$candidates.

In this paper, we update our previous measurement [4] of the branching fraction $\mathcal{B}$ and longitudinal polarization fraction $f_{L}$ in $B^{0} \rightarrow \rho^{0} \rho^{0}$ decays, along with $\mathcal{B}$ for $B^{0} \rightarrow$ $\rho^{0} f_{0}, f_{0} f_{0}, \rho^{0} \pi^{+} \pi^{-}$, and $\pi^{+} \pi^{-} \pi^{+} \pi^{-}$. In addition, we present the first study of the time-dependent $C P$ asymmetry $\mathcal{A}_{C P}$ in this mode. We determine the coefficients $C_{L}^{00}$ and $S_{L}^{00}$ of $\mathcal{A}_{C P}$ for the longitudinal component, expressed as a function of $\Delta t$, the proper-time difference between the
two $B$ decays in $\Upsilon(4 S) \rightarrow B^{0} \bar{B}^{0}$ :

$$
\begin{equation*}
\mathcal{A}_{C P}(\Delta t)=-C_{L}^{00} \cos \Delta m \Delta t+S_{L}^{00} \sin \Delta m \Delta t, \tag{1}
\end{equation*}
$$

where $\Delta m=(0.507 \pm 0.005) \hbar \mathrm{ps}^{-1}$ is the mass difference between two $B^{0}$ mass eigenstates [10]. When combined with measurements of $B^{0} \rightarrow \rho^{+} \rho^{-}$and $B^{+} \rightarrow \rho^{+} \rho^{0}$, $\mathcal{A}_{C P}, \mathcal{B}$, and $f_{L}$ in $B^{0} \rightarrow \rho^{0} \rho^{0}$ allow a complete isospin analysis and improve the constraints on the penguin contribution to $B \rightarrow \rho \rho$ decays. Changes with respect to our previous analysis [4] include a larger data sample, improved track selection and reconstruction techniques, inclusion of the $B$-decay time information, and improved handling of background events originating from $B$ decays.

We use a sample of $(465 \pm 5) \times 10^{6} \Upsilon(4 S)$ decays into $B \bar{B}$ pairs collected with the $B A B A R$ detector [11] at the PEP-II asymmetric-energy $e^{+} e^{-}$collider [12]. A detailed description of the BABAR detector is available elsewhere [4,11].

We select $B \rightarrow M_{1} M_{2} \rightarrow\left(\pi^{+} \pi^{-}\right)\left(\pi^{+} \pi^{-}\right)$candidates, where $M_{1,2}$ stands for a $\rho^{0}$ or $f_{0}(980)$ candidate, from neutral combinations of four charged tracks that are consistent with originating from a single vertex near the $e^{+} e^{-}$ interaction point. We veto tracks that are positively identified as kaons or electrons. The identification of signal $B$ candidates is based on several kinematic variables. The beam-energy-substituted mass $m_{\mathrm{ES}}=\left[\left(s / 2+\mathbf{p}_{i}\right.\right.$. $\left.\left.\mathbf{p}_{B}\right)^{2} / E_{i}^{2}-\mathbf{p}_{B}^{2}\right]^{1 / 2}$, where the initial $e^{+} e^{-}$four-momentum ( $E_{i}, \mathbf{p}_{\mathbf{i}}$ ) and the $B$ momentum $\mathbf{p}_{\mathbf{B}}$ are defined in the laboratory frame, is centered near the $B$ mass with a resolution of $2.6 \mathrm{MeV} / c^{2}$ for signal candidates. The difference $\Delta E=$ $E_{B}^{\mathrm{cm}}-\sqrt{s} / 2$ between the reconstructed $B$ energy in the center of mass (c.m.) frame and its known value $\sqrt{s} / 2$ has a maximum near zero with a resolution of 20 MeV for signal events. Four other kinematic variables describe two pos-
sible $\pi^{+} \pi^{-}$pairs: invariant masses $m_{1}, m_{2}$ and helicity angles $\theta_{1}, \theta_{2}$.

We use the kinematic selection of signal candidates described in [4]. We require $5.245<m_{\mathrm{ES}}<$ $5.290 \mathrm{GeV} / c^{2}, \quad|\Delta E|<85 \mathrm{MeV}, \quad 0.55<m_{1,2}<$ $1.050 \mathrm{GeV} / c^{2}$, and $\left|\cos \theta_{1,2}\right|<0.98$. The extended dipion invariant mass range allows us to study the nonresonant decays $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$and $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$, as well as $B^{0} \rightarrow \rho^{0} f_{0}$ and $B^{0} \rightarrow f_{0} f_{0}$. The contributions from the higher mass resonances in this range are relatively small. We suppress the dominant $e^{+} e^{-} \rightarrow q \bar{q}(q=u, d, s$, $c)$ continuum background using a neural network-based discriminant $\mathcal{E}$, which combines eight topological variables [4].

We use multivariate $B$-flavor tagging algorithms trained to identify primary leptons, kaons, soft pions, and highmomentum charged particles from the other $B$, called $B_{\mathrm{tag}}$ [13]. The effective tagging efficiency is $(31.1 \pm 0.3) \%$. Additional background discrimination power arises from the difference between the tagging efficiencies for signal and background in seven tagging categories $\left(c_{\text {tag }}\right)$. We determine $\Delta t$ and its error $\sigma_{\Delta t}$ from the spatial separation between the decay vertices of the signal $B$ and $B_{\text {tag }}$ and require $|\Delta t|<15 \mathrm{ps}$ and $\sigma_{\Delta t}<2.5 \mathrm{ps}$.

After application of all selection criteria, 72154 events are retained. On average, each selected event has 1.05 signal candidates, while in Monte Carlo (MC) [14,15] samples of longitudinally (transversely) polarized $B^{0} \rightarrow$ $\rho^{0} \rho^{0}$ decays, we find 1.15 (1.03) candidates. When more than one candidate is present in the same event, the candidate that yields the smallest $\chi^{2}$ for the four-pion vertex is selected. Simulation shows that $18 \%$ of longitudinally and $4 \%$ of transversely polarized $B^{0} \rightarrow \rho^{0} \rho^{0}$ events are misreconstructed with one or more tracks that do not originate from the $B^{0} \rightarrow \rho^{0} \rho^{0}$ decay. These are mostly lowmomentum tracks from the other $B$ meson in the event. Such partially reconstructed candidates are included in the definition of the signal probability density functions (PDFs). The rate of events in which all pions from $B^{0} \rightarrow$ $\left[\pi^{+} \pi^{-}\right]_{\rho}\left[\pi^{+} \pi^{-}\right]_{\rho}$ are found but incorrectly paired in the $\rho^{0}$ candidates is negligibly small.

We use an unbinned extended maximum likelihood fit to extract the $B^{0} \rightarrow \rho^{0} \rho^{0}$ event yield, $f_{L}, C_{L}^{00}$, and $S_{L}^{00}$. We also fit for the event yields of $B^{0} \rightarrow \rho^{0} f_{0}, B^{0} \rightarrow f_{0} f_{0}$, $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$, and $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$decays. The likelihood function includes the background components from nonsignal $B$ decays and continuum. The PDFs for each component depend on ten discriminating variables: $m_{\mathrm{ES}}$, $\Delta E, \mathcal{E}, m_{1}, m_{2}, \cos \theta_{1}, \cos \theta_{2}, c_{\mathrm{tag}}, \Delta t$, and $\sigma_{\Delta t}$.

Since the statistical correlations among the variables are found to be negligibly small, we take the total PDF as the product of the PDFs for the separate variables. Exceptions are the kinematic correlation between the two helicity angles in signal, and mass-helicity correlations in several $B$-decay classes and misreconstructed signal.

We use double-Gaussian functions to parametrize the $m_{\mathrm{ES}}$ and $\Delta E$ PDFs for signal, and relativistic Breit-Wigner functions for the resonance line shapes of $\rho^{0}$ and $f_{0}(980)$, with the $f_{0}(980)$ mass and width taken from [16]. The angular distribution for $B$ decays [14] (expressed as a function of $f_{L}$ for $B^{0} \rightarrow \rho^{0} \rho^{0}$ ) is multiplied by a detector acceptance function determined from simulations. We assume that the $\rho^{0}$ in $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$is longitudinally polarized (i.e., $\pi^{+} \pi^{-}$are produced in the $S$ wave), and we use the phase-space distributions for $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$. The $(\pi \pi)$ invariant mass and helicity distributions of misreconstructed signal events are parametrized with empirical shapes in a way similar to that used for $B$ background discussed below. The neural network discriminant $\mathcal{E}$ is described by the sum of three asymmetric Gaussian functions with different parameters for signal and background distributions.

The PDFs for nonsignal $B$-decay modes are generally modeled with empirical analytical distributions. Several variables have distributions identical to those for signal, such as $m_{\mathrm{ES}}$ when all four tracks come from the same $B$, or $m_{1,2}$ when both tracks come from a $\rho^{0}$ meson. Also, for some of the modes the two $\pi^{+} \pi^{-}$pairs can have different mass and helicity distributions, e.g., when only one of the two combinations comes from a genuine $\rho^{0}$ or $f_{0}$ meson, or when one of the two pairs contains a high-momentum pion (as in $B \rightarrow a_{1} \pi$ ). In such cases, we use a fourvariable correlated mass-helicity PDF. The proper-time distribution for signal and background $B$ decays is convolved with a resolution function [13], while the time distribution of continuum background is assumed to have zero lifetime.

The signal and $B$-background PDF parameters are extracted from simulation. The MC parameters for the $m_{\mathrm{ES}}$, $\Delta E$, and $\mathcal{E}$ PDFs are adjusted by comparing data and simulation in control channels with similar kinematics and topology, such as $B^{0} \rightarrow D^{-} \pi^{+}$with $D^{-} \rightarrow$ $K^{+} \pi^{-} \boldsymbol{\pi}^{-}$. The continuum background PDF shapes are extracted from on-resonance sideband data ( $m_{\mathrm{ES}}<$ $5.27 \mathrm{GeV} / c^{2}$ ), with parameters of most PDFs (for $m_{\mathrm{ES}}$, $\Delta E, \mathcal{E}, \theta_{1}, \theta_{2}$, and $\left.\Delta t\right)$ left free in the final fit. The tagging efficiencies, mistag fractions, and the parameters of the proper-time distributions for signal modes are obtained in dedicated fits to events with identified exclusive $B$ decays [13]. For inclusive $B$ backgrounds these parameters are determined by MC and their systematic uncertainties are evaluated in data.

We study the contributions of the dominant backgrounds by using high-statistics exclusive MC samples. We single out the $B^{0} \rightarrow a_{1}^{ \pm} \pi^{\mp}, \quad B^{0} \rightarrow \rho^{0} K^{* 0}$, and $B^{0} \rightarrow f_{0} K^{* 0}$ modes, which have kinematic distributions similar to those of the signal events, and parametrize their PDFs individually. The event yield for $B^{0} \rightarrow a_{1}^{ \pm} \pi^{\mp}$ is allowed to vary in the fit, while the yields for $B^{0} \rightarrow \rho^{0} K^{* 0}, B^{0} \rightarrow f_{0} K^{* 0}$ are fixed to the expected values [17].

TABLE I. Event yields; fraction of longitudinal polarization $\left(f_{L}\right)$; selection efficiency corresponding to measured polarization; branching fractions $(\mathcal{B})$; branching fraction upper limits at $90 \%$ confidence level (CL); and significance $\mathcal{S}$ including systematic uncertainties. The first uncertainty is statistical and the second is systematic.

| Mode | Yield | $f_{L}$ | Efficiency $(\%)$ | $\mathcal{B}\left(10^{-6}\right)$ | Upper limit $\left(10^{-6}\right) \mathcal{S}(\sigma)$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $B^{0} \rightarrow \rho^{0} \rho^{0}$ | $99_{-34}^{+35} \pm 15$ | $0.75_{-0.14}^{+0.11} \pm 0.05$ | $23.28 \pm 0.07$ | $0.92 \pm 0.32 \pm 0.14$ | $\ldots$ | 3.1 |
| $B^{0} \rightarrow \rho^{0} f_{0} \rightarrow \rho^{0}\left[\pi^{+} \pi^{-}\right]_{f_{0}}$ | $3_{-20}^{+22} \pm 12$ | $\ldots$ | $24.16 \pm 0.09$ | $0.03 \pm 0.19 \pm 0.10$ | $<0.40$ |  |
| $B^{0} \rightarrow f_{0} f_{0} \rightarrow\left[\pi^{+} \pi^{-}\right]_{f_{0}}\left[\pi^{+} \pi^{-}\right]_{f_{0}}$ | $7_{-5}^{+8} \pm 7$ | $\ldots$ | $27.22 \pm 0.07$ | $0.05_{-0.04}^{+0.06} \pm 0.05$ | $<0.19$ |  |
| $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$ | $-13_{-35}^{+39} \pm 18$ | $\ldots$ | $1.68 \pm 0.01$ | $-1.6_{-4.5}^{+5.0} \pm 2.2$ | $<8.8$ |  |
| $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ | $8_{-25}^{+30} \pm 11$ | $\cdots$ | $0.55 \pm 0.01$ | $3.0_{-9.9}^{+11.6} \pm 4.1$ | $<23.1$ |  |

In addition, we construct a charmless event category consisting of $B^{0} \rightarrow \rho^{+} \rho^{-}, B^{0} \rightarrow \rho^{ \pm} \pi^{\mp}, B^{+} \rightarrow \rho^{+} \rho^{0}$, $B^{+} \rightarrow a_{1}^{0} \pi^{+}, B^{+} \rightarrow a_{1}^{+} f_{0}, B \rightarrow \eta^{\prime} K$, and $B^{+} \rightarrow \rho^{0} \pi^{+}$ backgrounds. Kinematic distributions in these events, especially events in which at least one charged particle is not correctly associated to the $B$ candidate, are similar to each other, and also to those of other, poorly measured, charmless decays. We allow the overall event yield for this category of events to vary in the fit, after fixing the relative weight of each mode to the expected value $[10,18]$. The remaining events, which mostly originate from open $b \rightarrow c$ transitions, are parametrized as a separate background component, with its yield left free in the fit.

Table I summarizes the results of the fit. The $B^{0} \rightarrow \rho^{0} \rho^{0}$ decay is observed with a significance of 3.1 standard deviations $(\sigma)$, as determined by the quantity $\mathcal{S}=$ $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\text {max }}\right)}$, where $\mathcal{L}_{\text {max }}$ is the maximum likelihood value, and $\mathcal{L}_{0}$ is the likelihood for a fit with the signal contribution set to zero. Both likelihoods include systematic uncertainties, which are assumed to be Gaussian distributed and are discussed below. This significance level corresponds to a probability of $1.0 \times 10^{-3}$ that the observed signal yield is consistent with a background fluctuation. We do not observe significant event yields for $B^{0} \rightarrow \rho^{0} f_{0}$ or $B^{0} \rightarrow f_{0} f_{0}$ decays, nor for the nonresonant decays $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$and $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$. We find $280 \pm 53, \quad 670 \pm 96, \quad 2329 \pm 147, \quad$ and $68701 \pm 281$ events for the $B^{0} \rightarrow a_{1}^{ \pm} \pi^{\mp}$, charmless, $b \rightarrow c$, and continuum backgrounds, respectively, consistent with expectations. In Fig. 1 we show the projections of the fit results onto $m_{\mathrm{ES}}$ and $m_{\pi \pi}$.

From the fit to the proper-time distribution of the data sample, we determine the $C P$-violating parameters

$$
\begin{aligned}
& \left.S_{L}^{00}=0.3 \pm 0.7(\text { stat. }) \pm 0.2 \text { (syst. }\right) \\
& C_{L}^{00}=0.2 \pm 0.8(\text { stat. }) \pm 0.3 \text { (syst.) }
\end{aligned}
$$

for the longitudinal component of the $B^{0} \rightarrow \rho^{0} \rho^{0}$ sample. The $C P$ asymmetry projection is shown in Fig. 2. The statistical correlations between fit parameters for $B^{0} \rightarrow$ $\rho^{0} \rho^{0}$ are given in Table II.


FIG. 1 (color online). Projections of the multidimensional fit onto the (a) $m_{\mathrm{ES}}$, (b) $\Delta E$, (c) di-pion invariant mass $m_{\pi \pi}$ (average of $m_{1}$ and $m_{2}$ distributions is shown), (d) cosine of the helicity angle $\cos \theta_{\pi \pi}$ (average of $\cos \theta_{1}$ and $\cos \theta_{2}$ distributions is shown), (e) event shape discriminant $\mathcal{E}$, and (f) $\Delta t$, after a requirement on the signal-to-background probability ratio with the plotted variable excluded, which enhances the fraction of signal events in the sample. This selection has $40 \%-60 \%$ efficiency for signal, depending on the projection. The data points are overlaid by the full PDF projection (solid line). Also shown are the $B^{0} \rightarrow \rho^{0} \rho^{0}$ PDF component (dotted line) and the sum of all other PDFs (dashed line).


FIG. 2 (color online). $\quad C P$ asymmetry $\mathcal{A}_{C P}$ after a requirement on the signal-to-background probability ratio, which enhances the fraction of signal events in the sample. The solid line represents the projection of the total PDF; the dotted line is the $B^{0} \rightarrow \rho^{0} \rho^{0}$ contribution.

Dominant systematic uncertainties in the fit originate from statistical errors in the PDF parametrizations due to the limited number of events in the control samples, variations in the $B$ background branching ratios fixed in the fit, and from potential fit bias. The PDF parameters are varied by their respective uncertainties to derive the corresponding systematic errors. The fit bias is studied in a large number ( $\sim 1000$ ) of MC experiments, in which signal and dominant charmless $B$ background events are fully simulated, while other backgrounds are sampled from their respective PDFs. We correct for the bias of $7.9 \pm 2.0$ events for $B^{0} \rightarrow \rho^{0} \rho^{0}, 0.00 \pm 0.07$ for $C_{L}^{00}$, and $-0.02 \pm$ 0.08 for $S_{L}^{00}$. The uncertainties associated with the $B$ background model are 4 events for the signal yield, 0.01 for $f_{L}$, 0.01 for $C_{L}^{00}$, and 0.11 for $S_{L}^{00}$. The systematic uncertainties due to the charmless background composition, arising from the uncertainties in the individual branching ratios and the $C P$ content of the $B$ background [18,19], are 5 events for the signal yield, 0.01 for $f_{L}, 0.18$ for $C_{L}^{00}$, and 0.14 for $S_{L}^{00}$. The above systematic uncertainties do not scale with event yield and are included in the calculation of the significance of the result.

We estimate the systematic uncertainty due to the interference between all $B^{0} \rightarrow 4 \pi$ modes using simulated samples in which the decay amplitudes for $B^{0} \rightarrow \rho^{0} \rho^{0}$, $B^{0} \rightarrow \rho^{0} f_{0}, \quad B^{0} \rightarrow f_{0} f_{0}, \quad B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}, \quad$ and $\quad B^{0} \rightarrow$ $\pi^{+} \pi^{-} \pi^{+} \pi^{-}$are generated according the measurements presented here, and those for $B^{0} \rightarrow a_{1}^{ \pm} \pi^{\mp}$ correspond to a

TABLE II. Correlation matrix for $B^{0} \rightarrow \rho^{0} \rho^{0}$ parameters.

| Parameter | Yield | $f_{L}$ | $S_{L}^{00}$ | $C_{L}^{00}$ |
| :--- | :---: | :---: | :---: | :---: |
| Yield | 1.000 | 0.086 | -0.136 | -0.273 |
| $f_{L}$ |  | 1.000 | -0.006 | -0.174 |
| $S_{L}^{00}$ |  |  | 1.000 | -0.035 |
| $C_{L}^{00}$ |  |  |  | 1.000 |

branching fraction of $(33.2 \pm 4.8) \times 10^{-6}[20]$. The values of the individual branching ratios are varied within their respective uncertainties, and the strong phases and $C P$ content of the interfering amplitudes are varied between zero and $2 \pi$ using uniform distributions. We take the rms variation of the fitted values ( 14 events for the $\rho^{0} \rho^{0}$ yield, 0.04 for $f_{L}$, and 0.07 for $S_{L}^{00}$ and $C_{L}^{00}$ ) as a systematic uncertainty. The interference with $B^{0} \rightarrow a_{1}^{ \pm} \pi^{\mp}$ is most important for $B^{0} \rightarrow \rho^{0} \rho^{0}$ (14 events) and $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$ (13 events), while for the nonresonant modes, the largest effects are between $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$and $B^{0} \rightarrow \rho^{0} f_{0}$ (9 events), and between $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$and $B^{0} \rightarrow$ $f_{0} f_{0}$ (5 events).

Uncertainties in the reconstruction efficiency arise from track finding and particle identification, and are determined by dedicated studies on control data samples. Uncertainties due to other selection requirements, such as vertex probability, track multiplicity, and thrust angle, amount to $2.4 \%$ for the event yields, and are negligible for the polarization and $C P$ observables.

We perform an isospin analysis of $B \rightarrow \rho \rho$ decays, by minimizing a $\chi^{2}$ that includes the measured quantities expressed as the lengths of the sides of the isospin triangles [2]. We use the measured branching fraction and fraction of longitudinal polarization of $B^{+} \rightarrow \rho^{+} \rho^{0}$ decays [6]; the measured branching fraction, polarization, and $C P$ parameters $S_{L}^{+-}$and $C_{L}^{+-}$determined from the time evolution of the longitudinally polarized $B^{0} \rightarrow \rho^{+} \rho^{-}$decays [7]; and the branching fraction, polarization, and $C P$ parameters $S_{L}^{00}$ and $C_{L}^{00}$ of $B^{0} \rightarrow \rho^{0} \rho^{0}$ reported here. We assume uncertainties to be Gaussian distributed and neglect $I=1$ isospin contributions, electroweak loop amplitudes, nonresonant, and isospin-breaking effects.

We obtain a $68 \%(90 \%)$ CL limit $\left|\alpha-\alpha_{\text {eff }}\right|<15.7^{\circ}$ $\left(<17.6^{\circ}\right)$ where $\alpha_{\text {eff }}$ is defined by $\sin \left(2 \alpha_{\text {eff }}\right)=S_{L}^{+-} /(1-$ $\left.C_{L}^{+-2}\right)^{1 / 2}$. Figure 3 shows the confidence level with and without use of $S_{L}^{00}$ and $C_{L}^{00}$ in the isospin analysis fit. We observe four solutions near zero, as in the $B \rightarrow \pi \pi$ isospin


FIG. 3 (color online). CL on $\alpha-\alpha_{\text {eff }}$ determined from the isospin analysis. The long-dashed curve is obtained without the two $C P$ parameters $S_{L}^{00}$ and $C_{L}^{00}$. The dotted curve corresponds to the isospin analysis without $S_{L}^{00}$, and the solid curve CL includes both $C_{L}^{00}$ and $S_{L}^{00}$ in the fit. The horizontal dotted lines correspond to the $68 \%$ (top) and $90 \%$ (bottom) CL intervals.
analysis [21]. The additional constraint from $S_{L}^{00}$ provides some discrimination among the four solutions.

In summary, we confirm our earlier evidence [4] for $B^{0} \rightarrow \rho^{0} \rho^{0}$ decays with a significance of $3.1 \sigma$ and measure the branching fraction, longitudinal polarization fraction, and $C P$ asymmetries in these decays. Our results are statistically consistent with the previous measurement [4] and supersede it. These measurements combined with those for $B^{+} \rightarrow \rho^{0} \rho^{+}$and $B^{0} \rightarrow \rho^{+} \rho^{-}$decays provide a constraint on the penguin uncertainty in the determination of the CKM unitarity angle $\alpha$. Thus, this first measurement of the $C P$ asymmetries in $B^{0} \rightarrow \rho^{0} \rho^{0}$ decays establishes the technique that, with significantly larger data sets, would help determine $\alpha$ with high precision. We find no significant evidence for the decays $B^{0} \rightarrow \rho^{0} f_{0}, B^{0} \rightarrow f_{0} f_{0}$, $B^{0} \rightarrow \rho^{0} \pi^{+} \pi^{-}$, or $B^{0} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$.

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Note added in proof.-After our paper was submitted, we became aware of the measurement [22] by the Belle Collaboration. Although Ref. [22] does not find significant evidence for $B^{0} \rightarrow \rho^{0} \rho^{0}$, the branching fractions are consistent with our results.
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