

Measurements of branching fractions for  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$ , and  $B^0 \rightarrow \omega \gamma$ 

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We present branching fraction measurements for the radiative decays  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$ , and  $B^0 \rightarrow \omega \gamma$ . The analysis is based on a data sample of  $465 \times 10^6 B\bar{B}$  events collected with the *BABAR* detector at the PEP-II asymmetric-energy  $B$  Factory located at the Stanford Linear Accelerator Center. We find  $\mathcal{B}(B^+ \rightarrow \rho^+ \gamma) = (1.20_{-0.37}^{+0.42} \pm 0.20) \times 10^{-6}$ ,  $\mathcal{B}(B^0 \rightarrow \rho^0 \gamma) = (0.97_{-0.22}^{+0.24} \pm 0.06) \times 10^{-6}$ , and a 90% C.L. upper limit  $\mathcal{B}(B^0 \rightarrow \omega \gamma) < 0.9 \times 10^{-6}$ , where the first error is statistical and the second is systematic. We also measure the isospin-violating quantity  $\Gamma(B^+ \rightarrow \rho^+ \gamma)/2\Gamma(B^0 \rightarrow \rho^0 \gamma) - 1 = -0.43_{-0.22}^{+0.25} \pm 0.10$ .

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## I. INTRODUCTION

Within the standard model (SM), the radiative decays  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$ , and  $B^0 \rightarrow \omega \gamma$  proceed mainly through a  $b \rightarrow d \gamma$  electroweak penguin amplitude with a virtual top quark in the loop. Hence, the decay rates depend on the magnitude of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{td}$ . The branching fraction results from recent next-to-leading order calculations are listed in Table I. While these exclusive decay rates have a large theoretical uncertainty dominated by the imprecise knowledge of the form factors, some of this uncertainty cancels in the ratio of  $B \rightarrow \rho(\omega) \gamma$  to  $B \rightarrow K^* \gamma$  branching fractions. This ratio provides a constraint on the ratio of the CKM matrix elements  $|V_{td}/V_{ts}|$ , which can also be obtained from the ratio of  $B_d$  and  $B_s$  mixing frequencies [4]. Physics beyond the SM could affect differently  $B \rightarrow \rho(\omega) \gamma$  and  $B_d/B_s$  mixing, and hence create inconsistencies between the results obtained from the two methods.

The ratio of  $B \rightarrow \rho(\omega) \gamma$  to  $B \rightarrow K^* \gamma$  branching fractions is related to  $|V_{td}/V_{ts}|$  [5] via

$$\frac{\mathcal{B}[B \rightarrow \rho(\omega) \gamma]}{\mathcal{B}(B \rightarrow K^* \gamma)} = S \left| \frac{V_{td}}{V_{ts}} \right|^2 \left( \frac{1 - m_{\rho(\omega)}^2/m_B^2}{1 - m_{K^*}^2/m_B^2} \right)^3 \zeta_{\rho(\omega)}^2 \times [1 + \Delta R_{\rho(\omega)}]. \quad (1)$$

The coefficient  $S$  is 1 for  $\rho^+$  and  $\frac{1}{2}$  for  $\rho^0$  or  $\omega$ ,  $m$  is the particle mass,  $\zeta_{\rho(\omega)}$  is the ratio of the form factors for the decays  $B \rightarrow \rho(\omega) \gamma$  and  $B \rightarrow K^* \gamma$ , and  $\Delta R_{\rho(\omega)}$  accounts for differences in decay dynamics, including weak annihilation contributions. The precision of the  $|V_{td}/V_{ts}|$  determination can be improved by using an average branching fraction for  $B \rightarrow \rho(\omega) \gamma$  decays. Within the SM, the isospin asymmetry between  $B^+ \rightarrow \rho^+ \gamma$  and  $B^0 \rightarrow \rho^0 \gamma$  is dominated by weak annihilation contributions, and is expected

to be small; on the other hand, the asymmetry between  $B^0 \rightarrow \rho^0 \gamma$  and  $B^0 \rightarrow \omega \gamma$  can be sizable, due to the difference in the form factors [1,3].

We report an updated study of the decays  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$ , and  $B^0 \rightarrow \omega \gamma$  based on  $465 \times 10^6 B\bar{B}$  events, corresponding to an integrated luminosity of  $423 \text{ fb}^{-1}$ , a data sample 25% larger than that used in our previous publication [6]. In addition, we reduce backgrounds considerably by using a multivariate algorithm based on bootstrap-aggregated (bagged) decision trees (BDTs) [7] and additional discriminating variables to separate signal from background.

## II. THE *BABAR* DETECTOR AND DATA SET

The data sample is collected with the *BABAR* detector at the PEP-II asymmetric-energy  $e^+e^-$  storage ring at a center-of-mass (CM) energy near  $\sqrt{s} = 10.58 \text{ GeV}$ , corresponding to the  $Y(4S)$  resonance (on resonance). Charged particle trajectories and energy loss ( $dE/dx$ ) are measured with a five-layer silicon vertex tracker (SVT) and a 40-layer drift chamber (DCH) in a 1.5 T magnetic field. Photons and electrons are detected in a CsI(Tl) crystal electromagnetic calorimeter (EMC) with photon energy resolution  $\sigma_E/E = 0.023(E/\text{GeV})^{-1/4} \oplus 0.019$ . A ring-imaging Cherenkov detector based on the detection of internally reflected Cherenkov light (DIRC) provides information for charged particle identification. The  $K-\pi$

TABLE I. Recent predictions of the branching fractions.

Mode	Branching fraction ( $\times 10^{-6}$ )		
	Reference [1]	Reference [2]	Reference [3]
$B^+ \rightarrow \rho^+ \gamma$	$1.41 \pm 0.27$	$1.58_{-0.46}^{+0.53}$	$1.16 \pm 0.26$
$B^0 \rightarrow \rho^0 \gamma$	$0.69 \pm 0.12$	$0.76_{-0.23}^{+0.26}$	$0.55 \pm 0.13$
$B^0 \rightarrow \omega \gamma$	$0.55 \pm 0.09$		$0.44 \pm 0.10$

<sup>1</sup>Charge conjugate modes are implied throughout.

separation in the DIRC is above  $4\sigma$  at laboratory momenta up to  $3 \text{ GeV}/c$ . In order to identify muons, the magnetic flux return is instrumented with resistive plate chambers and limited streamer tubes. A detailed description of the detector can be found elsewhere [8].

We use a GEANT4-based [9] Monte Carlo (MC) simulation to model the BABAR detector response, taking into account the varying accelerator and detector conditions. Dedicated signal and background MC samples are used to optimize selection criteria, to obtain signal efficiencies, and to validate the analysis. Data control samples, including  $41 \text{ fb}^{-1}$  of data collected about  $40 \text{ MeV}$  below the  $B\bar{B}$  production threshold (off resonance), are used to study backgrounds coming from continuum  $e^+e^- \rightarrow q\bar{q}$ , with  $q = u, d, s, c$ .

### III. EVENT RECONSTRUCTION AND BACKGROUND SUPPRESSION

The decays  $B \rightarrow \rho(\omega)\gamma$  are reconstructed by combining a high-energy photon with a vector meson reconstructed in the decay modes  $\rho^+ \rightarrow \pi^+\pi^0$ ,  $\rho^0 \rightarrow \pi^+\pi^-$ , and  $\omega \rightarrow \pi^+\pi^-\pi^0$ . The dominant source of background is coming from continuum events that contain a high-energy photon from  $\pi^0$  or  $\eta$  decays or from initial-state radiation (ISR). There are also significant backgrounds from  $B$  meson decays. The decays  $B \rightarrow K^*\gamma$ ,  $K^* \rightarrow K\pi$  can mimic the signal when the kaon is misidentified as a pion. Decays of  $B \rightarrow (\rho/\omega)(\pi^0/\eta)$  with a high-energy photon from the  $\pi^0$  or  $\eta$  decay also mimic the signal. In addition, there are other  $B$  backgrounds originating mainly from  $B \rightarrow X_s\gamma$  and  $B \rightarrow X(\pi^0/\eta)$  decays.

The event selection and background suppression are performed in two steps. We apply a set of loose selection criteria to select well-measured photons and charged pions and to reject background events that are kinematically very different from the signal events. For events that pass the loose event selection criteria, we then use the BDT technique to further reduce background.

#### A. Loose selection

We reduce background contributions from continuum processes by considering only events for which the ratio  $R_2$  of second-to-zeroth Fox-Wolfram moments [10], calculated using the momenta of all charged and neutral particles in the event, is less than 0.7.

A photon candidate is identified as a cluster of energy deposited in contiguous EMC crystals, and not associated with any charged track. The high-energy photon must have energy  $1.5 < E_\gamma < 4.4 \text{ GeV}$  in the laboratory frame and  $1.5 < E_\gamma^* < 3.5 \text{ GeV}$  in the CM frame, be well contained within the EMC acceptance with polar angle  $-0.74 < \cos\theta < 0.93$ , and be isolated by at least  $25 \text{ cm}$  at the entrance of the EMC from any other photon candidate or charged track. The distribution of the energy deposition is required to be consistent with that of a photon shower.

Charged-pion candidates are selected from well-reconstructed tracks that have at least 12 DCH hits used in the track fit and a minimum momentum transverse to the beam direction of  $100 \text{ MeV}/c$ . The tracks are required to originate near the interaction point (IP): the distance of closest approach to the IP must be less than  $10 \text{ cm}$  along the beam direction and less than  $2 \text{ cm}$  in the plane perpendicular to the beam direction. The  $\pi^\pm$  identification is based on a likelihood  $L_i$  computed for particle hypothesis  $i (= \pi, K, p)$  using  $dE/dx$  measured in the SVT and DCH and the information of Cherenkov photons detected by the DIRC. The selection criteria are optimized to reject charged kaons produced in  $B \rightarrow K^*\gamma$  decays. The pion candidates in  $B^0 \rightarrow \omega\gamma$  must have  $L_K/(L_K + L_\pi) < 0.5$  and  $L_p/(L_p + L_\pi) < 0.98$  and must not be consistent with being an electron. The pion candidates in  $B \rightarrow \rho\gamma$  must have  $L_K/(L_K + L_\pi) < 0.2$  and  $L_p/(L_p + L_\pi) < 0.5$  and must not be consistent with either an electron or a muon candidate hypothesis; in addition, for all candidates with laboratory momenta above  $0.6 \text{ GeV}/c$ , the number of photons observed in the DIRC is required to be consistent with the number that is expected for the pion hypothesis. The performance of the pion identification requirements is evaluated with the decay  $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ , which provides a large, clean sample of  $\pi^\pm$  and  $K^\pm$ . Using the results shown in Fig. 1, we find that the pion identification requirement retains 85% of the pions from  $B \rightarrow \rho\gamma$  decays and rejects 99% of the kaons from  $B \rightarrow K^*\gamma$  decays.

We form  $\pi^0$  candidates from pairs of photons with energies greater than  $50 \text{ MeV}$  in the laboratory frame and an invariant mass  $m_{\gamma\gamma}$  in the range  $115\text{--}150 \text{ MeV}/c^2$ . We combine the identified pions into vector-meson candidates requiring  $630 < m_{\pi^+\pi^-} < 960 \text{ MeV}/c^2$ ,  $640 < m_{\pi^+\pi^0} < 930 \text{ MeV}/c^2$ , and  $760 < m_{\pi^+\pi^-\pi^0} < 790 \text{ MeV}/c^2$  for  $\rho^0$ ,  $\rho^+$ , and  $\omega$ , respectively. The charged-pion pairs are required to originate from a common vertex.

The photon and  $\rho/\omega$  candidates are combined to form the  $B$  meson candidates. We define  $\Delta E \equiv E_B^* - \sqrt{s}/2$ , where  $E_B^* = E_{\rho/\omega}^* + E_\gamma^*$  is the CM energy of the  $B$  meson candidate. The  $\Delta E$  distributions of signal events are expected to peak near zero with a resolution of about  $50 \text{ MeV}$  dominated by the photon energy resolution, and to have a tail in the negative region due to photon energy loss in the detector. We also define the beam-energy-substituted mass  $m_{\text{ES}} \equiv \sqrt{s/4 - \mathbf{p}_B^{*2}}$ , where  $\mathbf{p}_B^*$  is the CM momentum of the  $B$  candidate modified by scaling the photon momentum so that  $E_{\rho/\omega}^* + E_\gamma^* - \sqrt{s}/2 = 0$ . This procedure improves the  $m_{\text{ES}}$  resolution for the signal events in the  $\Delta E$  negative tail. Signal events are expected to have an  $m_{\text{ES}}$  distribution centered at the mass of the  $B$  meson  $m_B$  with a resolution of  $3 \text{ MeV}/c^2$ . We consider candidates with  $m_{\text{ES}} > 5.22 \text{ GeV}/c^2$  and  $-0.3 < \Delta E < 0.3 \text{ GeV}$  for further analysis. This region includes sidebands that allow the continuum background yields to be extracted from a fit to the data.

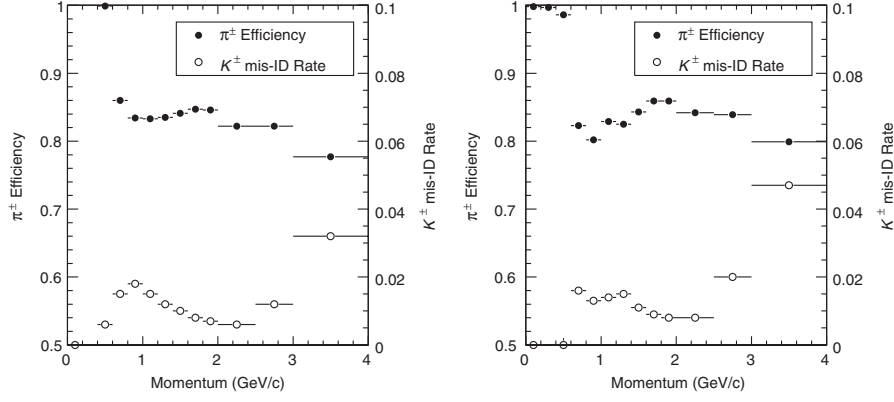


FIG. 1. Performance of the charged-pion identification requirement applied to  $B \rightarrow \rho\gamma$  decays, evaluated using the  $D^*$  control sample. Filled circles are for  $\pi^\pm$  efficiency and use the left-hand scale. Open circles are for  $K^\pm$  misidentification and use the right-hand scale. The plot on the left shows results for continuum MC events and the plot on the right shows results for data.

The signal efficiencies for the loose selection described above are 22% for  $B^+ \rightarrow \rho^+\gamma$ , 25% for  $B^0 \rightarrow \rho^0\gamma$ , and 17% for  $B^0 \rightarrow \omega\gamma$ .

### B. Bagged decision tree

The bagged decision trees are trained separately for the  $B^+ \rightarrow \rho^+\gamma$ ,  $B^0 \rightarrow \rho^0\gamma$ , and  $B^0 \rightarrow \omega\gamma$  channels with MC simulated signal and background samples of about 60 000 and 90 000 events, respectively, and then validated with independent samples of the same size. The background sample consists of a  $B\bar{B}$  MC sample that is about 3 times larger than the data and of a continuum MC sample that is about 1.5 times larger. For the input classifiers, we choose approximately 60 event quantities that characterize the kinematics of the  $\pi^0$  candidates, the high-energy photon, the vector meson, the  $B$  meson, and the rest of the event (ROE), which are the particles that are not used to reconstruct the  $B$  candidate. These quantities all have distributions that agree well between off-resonance data and continuum MC events.

To reduce combinatorial background in the reconstructed  $\pi^0$  candidates, we use in the BDT the invariant mass  $m_{\gamma\gamma}$  and  $\cos\theta_{\gamma\gamma}$ , the cosine of the opening angle between the photons in the laboratory frame.

We associate the high-energy photon candidate  $\gamma$  with each of the other photons  $\gamma'$  in the event and calculate the likelihood ratio

$$\mathcal{LR}_i = \frac{\mathcal{P}(m_{\gamma\gamma'}, E_{\gamma'}|i)}{\mathcal{P}(m_{\gamma\gamma'}, E_{\gamma'}|\text{signal}) + \mathcal{P}(m_{\gamma\gamma'}, E_{\gamma'}|i)}, \quad (2)$$

where  $i = \pi^0, \eta$  and  $\mathcal{P}$  is the probability density function (PDF) defined in terms of the energy of the second photon in the laboratory frame  $E_{\gamma'}$  and the invariant mass of the pair  $m_{\gamma\gamma'}$ . The PDFs are determined from simulated signal and continuum background events. The likelihood ratios  $\mathcal{LR}_{\pi^0}$  and  $\mathcal{LR}_\eta$  are used in the BDT to reject high-energy photons from  $\pi^0$  and  $\eta$  decays.

To reject background events from  $B \rightarrow \rho(\pi^0/\eta)$  and  $B \rightarrow \omega(\pi^0/\eta)$ , we also use the vector-meson helicity angle  $\theta_H$ , which is defined as the angle between the  $B$  momentum vector and the  $\pi^+$  track calculated in the  $\rho$  rest frame for a  $\rho$  meson, or the angle between the  $B$  momentum vector and the normal to the  $\omega$  decay plane for an  $\omega$  meson. This variable is useful because in signal events the vector meson is transversely polarized, while in the background events it is longitudinally polarized.

Variables used in the BDT to reduce continuum background include  $R_2$ , the significance of the separation of the two  $B$  vertices along the beam axis ( $S_{\Delta z}$ ), the polar angle of the  $B$  candidate momentum in the CM frame with respect to the beam axis ( $\theta_B^*$ ), and  $R_2'$ , which is  $R_2$  in the frame recoiling against the photon momentum. We compute the moments  $M_i \equiv \sum_j p_j^* \cdot |\cos\theta_j^*|^i / \sum_j p_j^*$  with  $i = 1, 2, 3$ , where  $p_j^*$  is the momentum of each particle  $j$  in the ROE and  $\theta_j^*$  is the angle of the momentum with respect to an axis. We use the  $M_i$  with respect to the photon direction and the ROE thrust axis. We also include flavor-tagging variables [11] to exploit the differences in lepton and kaon production between background and  $B$  decays.

While we find that all the variables contribute to the sensitivity of the analysis, the most effective ones are  $S_{\Delta z}$ ,  $\cos\theta_{\gamma\gamma}$ ,  $R_2$ ,  $\cos\theta_B^*$ ,  $M_3$  with respect to the photon direction, the missing mass of the ROE,  $\cos\theta_H$ , and  $\mathcal{LR}_{\pi^0, \eta}$ . The distribution of the BDT output for the decay  $B^0 \rightarrow \rho^0\gamma$  is shown in Fig. 2. We require the BDT output to be greater than 0.94 (0.93) for  $B \rightarrow \rho\gamma$  ( $B^0 \rightarrow \omega\gamma$ ). These selection requirements have been optimized for maximum statistical signal significance with assumed signal branching fractions of  $1.0 \times 10^{-6}$  and  $0.5 \times 10^{-6}$  for the charged and neutral modes, respectively. The signal significance is determined from a fit described in the next section. For the signal events that pass the loose selection criteria, the BDT requirements have an efficiency of 19% for  $B^+ \rightarrow \rho^+\gamma$ , 31% for  $B^0 \rightarrow \rho^0\gamma$ , and 34% for  $B^0 \rightarrow \omega\gamma$ .

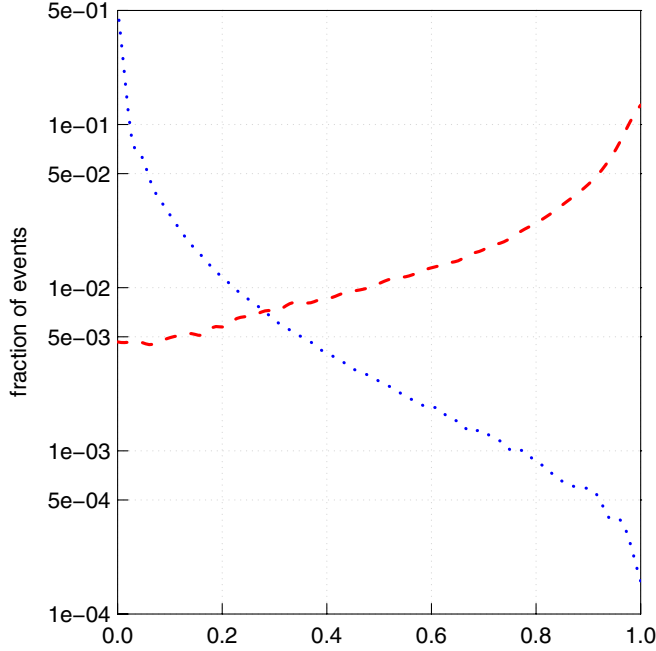


FIG. 2 (color online). Distributions of the BDT output for  $B^0 \rightarrow \rho^0 \gamma$  in signal (dashed line) and background (dotted line) MC samples. The distributions are normalized to the same area.

In events where multiple candidates are present, we select the one with the reconstructed vector-meson mass closest to the nominal mass. This criterion is chosen because the mass of the vector meson is found to be uncorrelated with the variables used in the fit. After applying all the selection criteria described above to signal MC samples, we find signal efficiencies of 4.2% for  $B^+ \rightarrow \rho^+ \gamma$ , 7.7% for  $B^0 \rightarrow \rho^0 \gamma$ , and 5.2% for  $B^0 \rightarrow \omega \gamma$  [taking into account the branching fraction  $\mathcal{B}(\omega \rightarrow \pi^+ \pi^- \pi^0) = 0.892 \pm 0.007$  [12]], while backgrounds are reduced by  $O(10^{-5})$ .

#### IV. MAXIMUM LIKELIHOOD FIT

We determine signal yields from an unbinned maximum likelihood fit to  $m_{ES}$  and  $\Delta E$ . The likelihood function for a signal mode  $k$  ( $= \rho^+ \gamma, \rho^0 \gamma, \omega \gamma$ ) with a sample of  $N_k$  events is defined as

$$\mathcal{L}_k = \exp\left(-\sum_{i=1}^{N_{\text{hyp}}} n_i\right) \left[ \prod_{j=1}^{N_k} \left( \sum_{i=1}^{N_{\text{hyp}}} n_i \mathcal{P}_i(\tilde{x}_j; \tilde{\alpha}_i) \right) \right], \quad (3)$$

where  $N_{\text{hyp}}$  is the number of event hypotheses, and  $n_i$  is the yield for each. For  $B^0 \rightarrow \omega \gamma$ , three event hypotheses are considered: signal, continuum background, and combinatorial  $B$  backgrounds. For  $B^0 \rightarrow \rho^0 \gamma$ , a  $B^0 \rightarrow K^{*0} \gamma$  background hypothesis is also included, while for  $B^+ \rightarrow \rho^+ \gamma$ , a combined  $B^+ \rightarrow K^{*+} \gamma / \rho^+ \pi^0$  hypothesis is included. Since the correlations between  $m_{ES}$  and  $\Delta E$  are found to be negligible in MC event samples, we define the probability density function  $\mathcal{P}_i(\tilde{x}_j; \tilde{\alpha}_i)$  as the product of individ-

ual PDFs for each observable  $x_j = m_{ES}, \Delta E$ , given the set of parameters  $\tilde{\alpha}_i$ .

The individual PDFs are determined from fits to dedicated MC event samples. The signal  $m_{ES}$  PDFs are parametrized by a Crystal Ball (CB) function [13] and the  $\Delta E$  PDFs are parametrized as

$$f(\Delta E) \propto \exp\left(\frac{-(\Delta E - \mu)^2}{2\sigma_{L,R}^2 + \alpha_{L,R}(\Delta E - \mu)^2}\right), \quad (4)$$

where  $\mu$  is the peak position of the distribution,  $\sigma_L$  and  $\sigma_R$  are the widths on the left and right of the peak, and  $\alpha_L$  and  $\alpha_R$  are measures of the tails on the left and right of the peak, respectively. The peak positions and widths of the signal  $m_{ES}$  and  $\Delta E$  PDFs are corrected for the observed difference between data and MC samples of  $B \rightarrow K^* \gamma$  decays. The PDFs for the remaining  $B^0 \rightarrow K^{*0} \gamma$  and combined  $B^+ \rightarrow K^{*+} \gamma / B^+ \rightarrow \rho^+ \pi^0$  backgrounds are determined from dedicated MC samples that are 100 times larger than the data. These PDFs are described by a CB function for  $m_{ES}$  with a peak position the same as that of the signal PDF—but with a much larger width—and a CB function for  $\Delta E$  with a peak position near  $-80$  MeV. The negative  $\Delta E$  peak position is due either to a kaon misidentified as a pion in  $B \rightarrow K^* \gamma$  or to a single missing photon in  $B^+ \rightarrow \rho^+ \pi^0$ . The  $m_{ES}$  and  $\Delta E$  PDFs for all other  $B$  backgrounds are determined from the  $B\bar{B}$  MC sample. The  $m_{ES}$  spectra peak slightly in the signal region, and therefore are parametrized by a CB function, while the  $\Delta E$  spectra are parametrized by an exponential function. The continuum  $m_{ES}$  and  $\Delta E$  PDFs are parametrized by an ARGUS threshold function [14] and a first order polynomial, respectively.

The fit to the data determines the signal yield  $n_{\text{sig}}$ , the continuum yield, and the shape parameters of the continuum PDFs. The shape parameters of the signal and  $B$  background PDFs are fixed in the fit. The relative yield between the peaking and the other  $B$  backgrounds is fixed to the value obtained from known branching fractions [12] and selection efficiencies determined from MC event samples. The overall yields of the  $B$  backgrounds are also fixed. All fixed parameters are later varied to evaluate systematic errors in  $n_{\text{sig}}$ .

We validate the fitting procedure using ensembles of signal and background events, with signal contributions of zero, 1, and 2 times the rate expected from previous measurements of the  $\rho^+ \gamma, \rho^0 \gamma$ , and  $\omega \gamma$  branching fractions. Two types of ensembles are produced: one with both signal and background events generated using the PDFs described above, and the other with signal events randomly sampled from the GEANT4 MC events and background events generated using the corresponding PDFs. No bias is found in the fit to these event samples.

Figure 3 shows the data points and the projections of the fit results for  $\Delta E$  and  $m_{ES}$  separately for each decay mode.

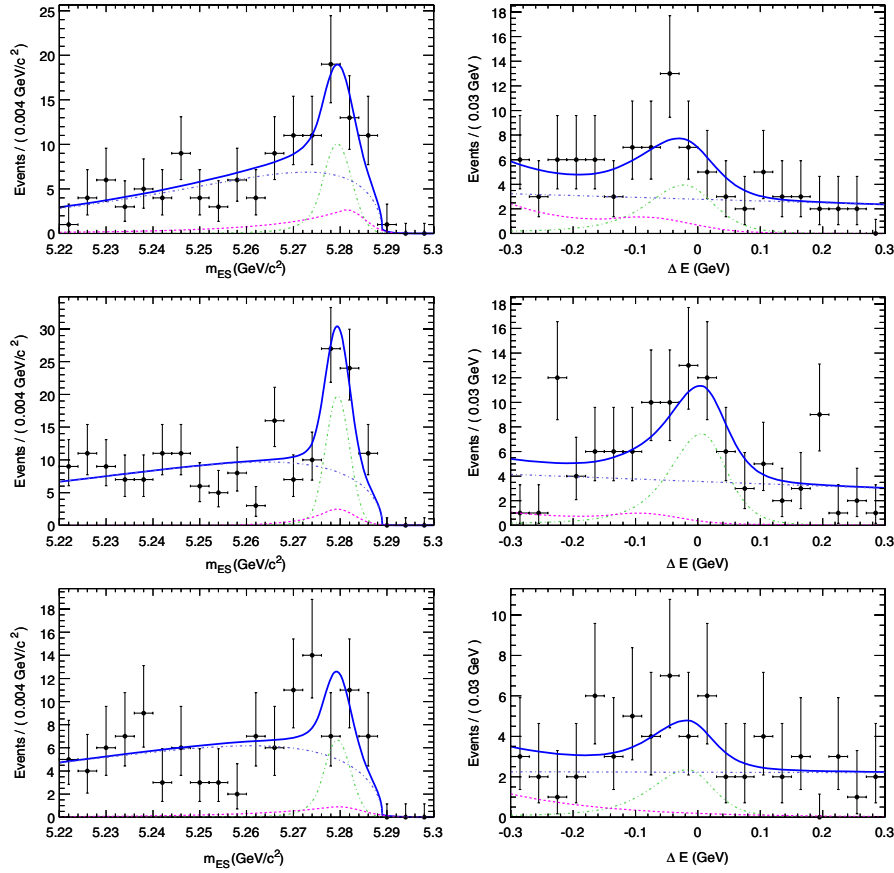


FIG. 3 (color online).  $\Delta E$  and  $m_{ES}$  projections of the fits for the decay modes  $B^+ \rightarrow \rho^+ \gamma$  (top panels),  $B^0 \rightarrow \rho^0 \gamma$  (middle panels), and  $B^0 \rightarrow \omega \gamma$  (bottom panels). For illustrative purposes only, these plots are made by requiring  $-0.2 < \Delta E < 0.1$  GeV for the  $m_{ES}$  projections and  $m_{ES} > 5.27$  GeV/ $c^2$  for the  $\Delta E$  projections. The points are data, the solid line is the fit result, the dashed line is the sum of  $B$  backgrounds, and the dash-dotted and dotted lines are the contributions from continuum background and signal, respectively.

The signal yields are reported in Table II. The significance is computed as  $\sqrt{2\Delta \ln \mathcal{L}}$ , where  $\Delta \ln \mathcal{L}$  is the log-likelihood difference between the best fit and the null-signal hypothesis. To take into account the systematic error in  $n_{\text{sig}}$ , the likelihood function is convolved with a Gaussian distribution that has a width equal to the systematic error.

TABLE II. The signal yield  $n_{\text{sig}}$ , significance  $\Sigma$  in standard deviations including the systematic error in  $n_{\text{sig}}$ , efficiency  $\epsilon$ , and branching fraction  $\mathcal{B}$  for each mode. The first error is statistical and the second is systematic. The branching fractions for  $B \rightarrow (\rho/\omega)\gamma$  and  $B \rightarrow \rho\gamma$  are obtained with the assumption of isospin and  $SU(3)_F$  symmetries; see Sec. VI.

Mode	$n_{\text{sig}}$	$\Sigma$	$\epsilon(\%)$	$\mathcal{B}(10^{-6})$
$B^+ \rightarrow \rho^+ \gamma$	$23.3^{+8.1}_{-7.3} \pm 3.1$	$3.2\sigma$	4.2	$1.20^{+0.42}_{-0.37} \pm 0.20$
$B^0 \rightarrow \rho^0 \gamma$	$34.9^{+8.6}_{-7.9} \pm 1.2$	$5.4\sigma$	7.7	$0.97^{+0.24}_{-0.22} \pm 0.06$
$B^0 \rightarrow \omega \gamma$	$12.4^{+6.6}_{-5.7} \pm 2.0$	$2.2\sigma$	5.2	$0.50^{+0.27}_{-0.23} \pm 0.09$
$B \rightarrow (\rho/\omega)\gamma$		$6.5\sigma$		$1.63^{+0.30}_{-0.28} \pm 0.16$
$B \rightarrow \rho\gamma$		$6.0\sigma$		$1.73^{+0.34}_{-0.32} \pm 0.17$

## V. SYSTEMATIC UNCERTAINTIES

Table III gives the contributions to the systematic uncertainties. The systematic error affecting the signal efficiency includes uncertainties on tracking, particle identification,  $\gamma$  and  $\pi^0$  reconstruction, and BDT selection. The modeling of signal and background in the fit contributes to the uncertainties on the signal yields.

TABLE III. Fractional systematic errors (in %) of the measured branching fractions.

Source of error	$\rho^+ \gamma$	$\rho^0 \gamma$	$\omega \gamma$	$\rho \gamma$	$(\rho/\omega)\gamma$
Tracking efficiency	0.4	0.4	0.4	0.4	0.4
Particle identification	1.0	2.0	1.0	1.4	1.2
Photon selection	2.8	2.8	2.8	2.8	2.8
$\pi^0$ reconstruction	3.0	...	3.0	1.7	2.0
BDT efficiency	9.3	4.2	5.1	7.0	7.5
Signal model	7.1	2.1	16.3	3.0	3.0
Background model	10.9	2.8	2.7	4.3	3.6
$B\bar{B}$ counting	1.1	1.1	1.1	1.1	1.1
$\mathcal{B}(\omega \rightarrow \pi^+ \pi^- \pi^0)$	...	...	0.8	...	0.1
Sum in quadrature	16.7	6.6	17.9	9.5	9.5



The errors in BDT selections are determined from a control sample of the decay  $B^0 \rightarrow K^{*0}(\rightarrow K^+ \pi^-)\gamma$  for the  $\rho^0$  mode and a sample of  $B^+ \rightarrow K^{*+}(\rightarrow K^+ \pi^0)\gamma$  for the  $\rho^+$  and  $\omega$  modes. These  $B \rightarrow K^* \gamma$  decays are kinematically similar to  $B \rightarrow (\rho/\omega)\gamma$  decays. The events are required to pass all applicable loose selection criteria, except the pion identification requirements. We also require the invariant mass  $0.80 < m_{K^+ \pi^-} < 1.0$  GeV/ $c^2$  and  $0.82 < m_{K^+ \pi^0} < 0.96$  GeV/ $c^2$ . The BDT output classifiers are computed from the decision trees trained for the corresponding signal modes. The differences in the BDT selection efficiencies between the  $B \rightarrow K^* \gamma$  data and MC samples are used to correct the signal efficiencies. The efficiency correction factor is  $0.88 \pm 0.09$  for  $B^+ \rightarrow \rho^+ \gamma$ ,  $0.91 \pm 0.04$  for  $B^0 \rightarrow \rho^0 \gamma$ , and  $0.90 \pm 0.05$  for  $B^0 \rightarrow \omega \gamma$ . The uncertainty of the correction is taken as the systematic error. The large BDT systematic error for the decay  $B^+ \rightarrow \rho^+ \gamma$  is due to the limited size of the  $B^+ \rightarrow K^{*+}(\rightarrow K^+ \pi^0)\gamma$  sample. As a means of validating the BDT technique, we apply the same analysis technique to the  $B \rightarrow K^* \gamma$  data control samples and measure the branching fractions for  $B \rightarrow K^* \gamma$ . The results are consistent with the world averages [12].

The error in the pion identification requirements is estimated using the  $D^*$  control sample as shown in Fig. 1. Based on the difference of a momentum-weighted efficiency between the continuum MC sample and data, a 1% systematic error per charged pion is assigned to the  $B \rightarrow \rho \gamma$  decays. The MC sample is in better agreement with data for the looser pion identification criteria applied to  $B^0 \rightarrow \omega \gamma$ , and a 0.5% error per charged pion is assigned. The uncertainties from tracking,  $\pi^0$  reconstruction, and photon selection are also determined from suitable independent data control samples.

To estimate the uncertainty related to the modeling of the signal and background, we vary the parameters of the PDFs that are fixed in the fit within their errors. We vary the relative and absolute normalizations of  $B$  background components that are fixed in the fit based on a kaon misidentification study using the  $D^*$  control sample as shown in Fig. 1. We find that the difference in the momentum-weighted kaon misidentification rates between the data and MC samples is 23%, and we conservatively vary the  $B \rightarrow K^* \gamma$  background yield by 30%. The effect of the uncertainty of  $\mathcal{B}(B^+ \rightarrow \rho^+ \pi^0)$  [12] is also considered for the decay  $B^+ \rightarrow \rho^+ \gamma$ . For all the variations, the corresponding changes in the extracted signal yield are taken as systematic uncertainties, which are then combined, taking into account correlations. The error on background modeling for  $B^+ \rightarrow \rho^+ \gamma$  is dominated by uncertainties in  $B$  background PDFs.

## VI. RESULTS

To calculate the branching fractions from the measured signal yields, we assume  $\mathcal{B}(Y(4S) \rightarrow B^0 \bar{B}^0) =$

$\mathcal{B}(Y(4S) \rightarrow B^+ B^-) = 0.5$ . The results are listed in Table II. For  $B^0 \rightarrow \omega \gamma$ , we also compute the 90% C.L. upper limit  $\mathcal{B}(B^0 \rightarrow \omega \gamma) < 0.9 \times 10^{-6}$  using a Bayesian technique, assuming a prior that is flat in the branching fraction and taking into account the systematic uncertainty.

We test the hypothesis of isospin symmetry by measuring the quantity

$$\Delta_\rho = \frac{\Gamma(B^+ \rightarrow \rho^+ \gamma)}{2\Gamma(B^0 \rightarrow \rho^0 \gamma)} - 1 = -0.43_{-0.22}^{+0.25} \pm 0.10.$$

Most theoretical calculations [1–3,15] predict small  $\Delta_\rho$ . For example, the estimate in Ref. [3] is  $-0.05 \pm 0.03$  for  $\gamma = 60^\circ$  and  $-0.10 \pm 0.02$  for  $\gamma = 70^\circ$ , where  $\gamma$  is the phase of  $V_{ub}^*$ . Our result is consistent with these predictions within the large experimental errors. However, it is worth noting that a recent calculation [16] indicated that non-perturbative charming penguin contributions can accommodate large  $\Delta_\rho$ . We also measure the SU(3)<sub>F</sub>-violating quantity

$$\Delta_\omega = \frac{\Gamma(B^0 \rightarrow \omega \gamma)}{\Gamma(B^0 \rightarrow \rho^0 \gamma)} - 1 = -0.49_{-0.27}^{+0.30} \pm 0.10,$$

which is consistent with the theoretical calculations.

We extract average branching fractions using a simultaneous fit to all the relevant decay modes with the constraints on the widths of the decay modes:  $\Gamma_{B^+ \rightarrow \rho^+ \gamma} = 2\Gamma_{B^0 \rightarrow \rho^0 \gamma} = 2\Gamma_{B^0 \rightarrow \omega \gamma}$ . The average branching fractions are defined as

$$\mathcal{B}(B \rightarrow \rho \gamma) \equiv \frac{1}{2} \left[ \mathcal{B}(B^+ \rightarrow \rho^+ \gamma) + 2 \frac{\tau_{B^+}}{\tau_{B^0}} \mathcal{B}(B^0 \rightarrow \rho^0 \gamma) \right] \quad (5)$$

and

$$\mathcal{B}[B \rightarrow (\rho/\omega)\gamma] \equiv \frac{1}{2} \left\{ \mathcal{B}(B^+ \rightarrow \rho^+ \gamma) + \frac{\tau_{B^+}}{\tau_{B^0}} [\mathcal{B}(B^0 \rightarrow \rho^0 \gamma) + \mathcal{B}(B^0 \rightarrow \omega \gamma)] \right\}, \quad (6)$$

where  $\tau_{B^+}/\tau_{B^0}$  is the measured ratio between the charged and neutral  $B$  meson lifetimes, for which the current world average is  $1.071 \pm 0.009$  [12]. Our measurements of the individual branching fractions are consistent with this hypothesis, with a  $\chi^2$  of 2.3 for 2 degrees of freedom. We find

$$\mathcal{B}(B \rightarrow \rho \gamma) = (1.73_{-0.32}^{+0.34} \pm 0.17) \times 10^{-6},$$

$$\mathcal{B}[B \rightarrow (\rho/\omega)\gamma] = (1.63_{-0.28}^{+0.30} \pm 0.16) \times 10^{-6}.$$

Using the world average value of  $\mathcal{B}(B^+ \rightarrow K^{*+} \gamma) = (4.03 \pm 0.26) \times 10^{-5}$ ,  $\mathcal{B}(B^0 \rightarrow K^{*0} \gamma) = (4.01 \pm 0.2) \times 10^{-5}$  [12], and the isospin averaged branching fraction  $\mathcal{B}(B \rightarrow K^* \gamma) = (4.16 \pm 0.17) \times 10^{-5}$ , we calculate

$$\begin{aligned}
R_{\rho^+} &= \frac{\mathcal{B}(B^+ \rightarrow \rho^+ \gamma)}{\mathcal{B}(B^+ \rightarrow K^{*+} \gamma)} = 0.030_{-0.011}^{+0.012}, \\
R_{\rho^0} &= \frac{\mathcal{B}(B^0 \rightarrow \rho^0 \gamma)}{\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)} = 0.024 \pm 0.006, \\
R_{\omega} &= \frac{\mathcal{B}(B^0 \rightarrow \omega \gamma)}{\mathcal{B}(B^0 \rightarrow K^{*0} \gamma)} = 0.012_{-0.006}^{+0.007}, \\
R_{\rho} &= \frac{\mathcal{B}(B \rightarrow \rho \gamma)}{\mathcal{B}(B \rightarrow K^* \gamma)} = 0.042 \pm 0.009, \\
R_{\rho/\omega} &= \frac{\mathcal{B}[B \rightarrow (\rho/\omega) \gamma]}{\mathcal{B}(B \rightarrow K^* \gamma)} = 0.039 \pm 0.008.
\end{aligned}$$

These ratios of branching fractions can be used to calculate  $|V_{td}/V_{ts}|$  [3,5,17]. Following Eq. (1) and using  $1/\zeta_{\rho} = 1.17 \pm 0.09$ ,  $1/\zeta_{\omega} = 1.30 \pm 0.10$  [3],  $\Delta R_{\rho^+} = 0.057_{-0.055}^{+0.057}$ ,  $\Delta R_{\rho^0} = 0.006_{-0.043}^{+0.046}$ , and  $\Delta R_{\omega} = -0.002_{-0.043}^{+0.046}$  [1], we obtain

$$\begin{aligned}
|V_{td}/V_{ts}|_{\rho^+} &= 0.198_{-0.035}^{+0.039} \pm 0.016, \\
|V_{td}/V_{ts}|_{\rho^0} &= 0.254_{-0.031}^{+0.033} \pm 0.021, \\
|V_{td}/V_{ts}|_{\omega} &= 0.202_{-0.050}^{+0.058} \pm 0.016,
\end{aligned}$$

where the first error is experimental and the second is theoretical. Using the average branching fractions and following Ref. [1], we obtain

$$\begin{aligned}
|V_{td}/V_{ts}|_{\rho} &= 0.235_{-0.025}^{+0.026} \pm 0.020, \\
|V_{td}/V_{ts}|_{\rho/\omega} &= 0.233_{-0.024}^{+0.025+0.022}.
\end{aligned}$$

Similar values are found following Ref. [3]. These results are consistent with the value of this ratio,  $0.208 \pm 0.002(\text{exp})_{-0.006}^{+0.008}(\text{theory})$  [12], obtained from the studies of  $B_d$  and  $B_s$  mixing by the CDF and D0 Collaborations.

## VII. SUMMARY

We report the updated measurements of the branching fractions for the radiative decays  $B^+ \rightarrow \rho^+ \gamma$ ,  $B^0 \rightarrow \rho^0 \gamma$ , and  $B^0 \rightarrow \omega \gamma$ ,

$$\begin{aligned}
\mathcal{B}(B^+ \rightarrow \rho^+ \gamma) &= (1.20_{-0.37}^{+0.42} \pm 0.20) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow \rho^0 \gamma) &= (0.97_{-0.22}^{+0.24} \pm 0.06) \times 10^{-6}, \\
\mathcal{B}(B^0 \rightarrow \omega \gamma) &< 0.9 \times 10^{-6} (90\% \text{ C.L.}).
\end{aligned}$$

We test the hypothesis of isospin symmetry by measuring the quantity  $\Delta_{\rho} = -0.43_{-0.22}^{+0.25} \pm 0.10$ . We also measure the averaged branching fractions  $\mathcal{B}(B \rightarrow \rho \gamma) = (1.73_{-0.32}^{+0.34} \pm 0.17) \times 10^{-6}$  and  $\mathcal{B}[B \rightarrow (\rho/\omega) \gamma] = (1.63_{-0.28}^{+0.30} \pm 0.16) \times 10^{-6}$ . These results are in good agreement with, and supersede, the previous published *BABAR* measurement [6], which uses a subsample of the data used for this analysis. These results are also consistent with the measurements from Belle [18]. These branching fraction measurements are used to extract  $|V_{td}/V_{ts}|$  in a way that is complementary to the approach using  $B$  mixing [4].

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