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Emergence of geometry: A two-dimensional toy model

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We review the similarities between the effective chiral Lagrangrian, relevant for low-energy strong interactions, and the Einstein-Hilbert action. We use these analogies to suggest a specific mechanism whereby gravitons would emerge as Goldstone bosons of a global $SO(D) \times GL(D)$ symmetry broken down to $SO(D)$ by fermion condensation. We propose a two-dimensional toy model where a dynamical zweibein is generated from a topological theory without any preexisting metric structure, the space being endowed only with an affine connection. A metric appears only after the symmetry breaking; thus the notion of distance is an induced effective one. In spite of several nonstandard features this simple toy model appears to be renormalizable and at long distances is described by an effective Lagrangian that corresponds to that of two-dimensional gravity (Liouville theory). The induced cosmological constant is related to the dynamical mass M acquired by the fermion fields in the breaking, which also acts as an infrared regulator. The low-energy expansion is valid for momenta $k > M$, i.e. for supra-horizon scales. We briefly discuss a possible implementation of a similar mechanism in four dimensions.

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I. INTRODUCTION

Einstein formulated general relativity in 1915 and 95 years later we still have little or no clue as to the true quantum nature of this theory.

It is surely correct to say that string theory is able to provide a consistent perturbative quantum theory of gravitation, at the price of a rather radical modification of quantum field theory, including the acceptance that our world has more than four dimensions. Unfortunately string theory is not able to select a unique vacuum, in particular, it does not shed light at present on the fact that we live in a world where $\langle g_{\mu\nu} \rangle \neq 0$. Other modifications of gravity that include extra dimensions, although extremely interesting from a conceptual and phenomenological point of view, typically lack an ultraviolet completion and therefore should probably find their ultimate justification in specific compactifications of string theory (where again the choice of vacuum appears itself).

Less popular alternatives, but of considerable interest nonetheless, are the search for nontrivial ultraviolet fixed points in gravity (asymptotic safety [1]) and the notion of induced gravity [2]. The former approach is the one pursued by exact renormalization-group (RG) practitioners [3] and by lattice and numerical techniques such as Lorentzian triangulation analysis [4]. Induced gravity advocates that a possible explanation of the relative weakness of gravity as compared to other interactions is that it is a residual or induced force, a subproduct of all the rest of matter and interaction fields. With the exception of lattice studies, all these approaches also rely on the introduction of a metric from the very beginning. On the contrary, lattice analysis only requires some premetric input, in particular, a notion of causality (hence transport of a timelike vector).

It has been pointed out several times in the literature (see e.g. [5]) that gravitons should perhaps be considered as Goldstone bosons of some broken symmetry. This is exactly the point of view that we adopt in this paper. This idea goes back probably to early papers by Salam and coworkers [6], and Ogievetsky and co-workers [7], if not earlier[1] [8], but a concrete proposal has been lacking so far (see however [9]). By concrete proposal we mean some field theory that does not contain the graviton field as an elementary degree of freedom. Ideally it should not even contain the tensor $\eta_{\mu\nu}$ as this already implies the use of some background metric. Indeed we would like to see the metric degrees of freedom emerging dynamically, like the pions appear dynamically after chiral symmetry breaking in QCD. Furthermore, if possible, we would like the underlying theory to be in some sense "simpler" than gravity, in

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[1] We thank L. Alvarez-Gaumé and B. McElrath for pointing out to us some of the earlier work on this subject.

particular, it should be renormalizable. One could then pose questions that are left unanswered in gravity, such as the fate of black hole singularities and the counting of degrees of freedom.

II. THE LOW-ENERGY EFFECTIVE ACTION OF QCD

The four-dimensional chiral Lagrangian is a nonrenormalizable theory describing accurately pion physics at low energies. It has a long history, with the first formal studies concerning renormalizability being due mostly to Weinberg [10] and later considerably extended by Gasser and Leutwyler [11]. The chiral Lagrangian contains a (infinite) number of operators

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + \alpha_1 \mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger + \alpha_2 \mathrm{Tr}\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger + \cdots,$$
$$U \equiv \exp i\tilde{\pi}/f_\pi, \qquad \tilde{\pi} \equiv \pi^a \tau^a/2, \tag{1}$$

organized according to the number of derivatives

$$\mathcal{L} = \mathcal{O}(p^2) + \mathcal{O}(p^4) + \mathcal{O}(p^6) + \cdots. \tag{2}$$

Pions are the Goldstone bosons associated with the (global) symmetry breaking pattern of QCD

$$SU(2)_L \times SU(2)_R \to SU(2)_V. \tag{3}$$

The above Lagrangian is the most general one compatible with the symmetries of QCD and their breaking. Locality, symmetry, and relevance (in the renormalization-group sense) are the guiding principles to construct $\mathcal{L}$. Renormalizability is not; in fact if we cut off the derivative expansion at a given order the theory requires counterterms beyond that order no matter how large the order is. Note that, although the symmetry has been spontaneously broken, the effective Lagrangian still has the full symmetry $U \to LUR^\dagger$ with L and R being $SU(2)$ matrices belonging to the left and right groups, respectively.

The lowest-order, tree level contribution to pion-pion scattering derived from the previous Lagrangian is $\sim p^2/f_\pi^2$. Simple counting arguments show that the one-loop chiral corrections are $\sim p^4/(16\pi^2 f_\pi^4)$. Thus the counting parameter in the loop (chiral) expansion in 4D is

$$\frac{p^2}{16\pi^2 f_\pi^2}. \tag{4}$$

Each chiral loop gives an additional power of p^2.

At each order in perturbation theory the divergences that arise can be eliminated by redefining the coefficients in the higher order operators

$$\alpha_i \to \alpha_i + \frac{c_i}{\epsilon}. \tag{5}$$

In addition to the pure pole in ϵ, logarithmic nonlocal terms necessarily appear. For instance in a two-point function they appear in the combination

$$\frac{1}{\epsilon} + \log \frac{-p^2}{\mu^2}, \tag{6}$$

with p being the external momentum. Note that the cut provided by the log is actually absolutely required by unitarity. All coefficients in the chiral Lagrangian are nominally of $\mathcal{O}(N_c)$. Loops are automatically suppressed by powers of N_c, because $f_\pi^2 \sim N_c$ appears in the denominator, but they are enhanced by logs at low momenta.

We have also acquired experience from chiral Lagrangians in the use of the equations of motion in an effective theory: at any order in the chiral expansion we can use the equations of motion derived from previous orders. For instance, using that at the lowest order $U\Box U^\dagger - (\Box U)U^\dagger = 0$ [from the $\mathcal{O}(p^2)$ Lagrangian], one can reduce the number of operators at $\mathcal{O}(p^4)$.

III. IS GRAVITY A GOLDSTONE PHENOMENON?

The 4D Einstein-Hilbert action shares several remarkable aspects with the pion chiral Lagrangian. It is a nonrenormalizable theory as well as it is also described, considering the most relevant operator (we ignore here for a moment the cosmological constant), by a dimension two operator containing in both cases two derivatives of the dynamical variable. Both Lagrangians contain necessarily a dimensionful constant in four dimensions: M_P, the Planck mass, is the counterpart of the constant f_π in the pion Lagrangian (of course the value of both constants is radically different). Both theories are nonlinear and, finally, both describe the interactions of massless quanta. The Einstein-Hilbert action is

$$\mathcal{L} = M_P^2 \sqrt{-g}\mathcal{R} + \mathcal{L}_{\mathrm{matter}}, \tag{7}$$

where as just mentioned $\mathcal{R}$ contains two derivatives of the dynamical variable which is the metric $g_{\mu\nu}$

$$\mathcal{R}_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{\mu\nu} - \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha}, \tag{8}$$

$$\Gamma^\gamma_{\alpha\beta} = \tfrac{1}{2} g^{\gamma\rho}(\partial_\beta g_{\rho\alpha} + \partial_\alpha g_{\rho\beta} - \partial_\rho g_{\alpha\beta}). \tag{9}$$

In the chiral language, the Einstein-Hilbert action would be $\mathcal{O}(p^2)$, i.e., most relevant, if we omit the presence of the cosmological constant which accompanies the identity operator. Arguably, locality, symmetry, and relevance in the RG sense (and not renormalizability) are the ones that single out Einstein-Hilbert action in front of e.g. $\mathcal{R}^2$.

Unlike the chiral Lagrangian, the Einstein-Hilbert Lagrangian, or extensions that include higher derivative terms, has a local gauge symmetry. Indeed, gravity can be (somewhat loosely) described as the result of promoting a global symmetry (Lorentz) to a local one (for a detailed discussion on the gauge structure of gravity see e.g. [12]). This means that the gauge symmetry that is present in gravity will in practice reduce the number of degrees of freedom that are physically relevant.

Exactly like the chiral Lagrangian, the Einstein-Hilbert action requires an infinite number of counterterms

$$\mathcal{L} = M_P^2\sqrt{-g}\mathcal{R} + \alpha_1\sqrt{-g}\mathcal{R}^2 + \alpha_2\sqrt{-g}(\mathcal{R}_{\mu\nu})^2 + \alpha_3\sqrt{-g}(\mathcal{R}_{\mu\nu\alpha\beta})^2 + \cdots. \quad (10)$$

The divergences can be absorbed order by order by redefining the coefficients α_i just as done in the previous section for the pion effective Lagrangian. Power counting in gravity appears, at least superficially, quite similar to the one that can be implemented in pion physics. Of course, the natural expansion parameter is a tiny number in normal circumstances, namely,

$$p^2/16\pi^2M_P^2 \quad \text{or} \quad \nabla^2/16\pi^2M_P^2, \qquad \mathcal{R}/16\pi^2M_P^2, \quad (11)$$

making quantum effects usually quite negligible. There are some subtleties when matter fields are included (see [13] for a discussion).

Like in the pion chiral Lagrangian, nonlocal logarithmic pieces accompany the divergences. In position space they look like

$$\frac{1}{\epsilon} + \log\frac{\nabla^2}{\mu^2}, \quad (12)$$

where ∇ is the covariant derivative on symmetry grounds, ∇^2 reducing to $-p^2$ in flat space-time. These nonlocalities are due to the propagation of strictly massless nonconformal modes, such as the graviton itself. Therefore they are unavoidable in quantum gravity. Notice that the coefficients of these nonlocal terms are entirely predictable from the infrared properties of gravity.

Let us use "chiral counting" arguments to derive the relevant quantum corrections to Newton's law (up to a constant). The propagator at tree level gets modified by one-loop "chiral-like" corrections

$$\frac{1}{p^2} \to \frac{1}{p^2}\left(1 + A\frac{p^2}{M_P^2} + B\frac{p^2}{M_P^2}\log p^2\right). \quad (13)$$

Consider now the interaction of a pointlike particle with a static source ($p^0 = 0$) and let us Fourier transform the previous expression for the loop-corrected propagator in order to get the potential in the nonrelativistic limit. We recall that

$$\int d^3x \exp(i\vec{p}\,\vec{x})\frac{1}{p^2} \sim \frac{1}{r}, \qquad \int d^3x \exp(i\vec{p}\,\vec{x})1 \sim \delta(\vec{x}),$$
$$\int d^3x \exp(i\vec{p}\,\vec{x})\log p^2 \sim \frac{1}{r^3}, \quad (14)$$

with $r = |\vec{x}|$. Thus quantum corrections to Newton's law are of the form

$$\frac{GMm}{r}\left(1 + K\delta(\vec{x}) + C\frac{G\hbar}{c^3}\frac{1}{r^2} + \cdots\right). \quad (15)$$

We have restored for a moment $\hbar$ and c to make evident that C is a pure number. The contribution proportional to $\delta(\vec{x})$ is of course nonobservable, even as a matter of principle. It comes from the contact divergent term (that may eventually collect contributions from arbitrarily high frequency modes). C, however, is calculable. It depends only on the infrared properties of the theory.

A long controversy regarding the value of C exists in the literature [14–16]. The result now accepted as the correct one, $C = 41/10\pi$ [17], is obtained by considering the inclusion of quantum matter fields and considering the on-shell scattering matrix. Note that quantum corrections make gravity more attractive (by a really tiny amount) at long distances than predicted by Newton's law. In addition to quantum corrections there are post-Newtonian classical corrections that are not discussed here (see [13]).

There are in the literature definitions of an "effective" or "running" Newton constant [18]. A class of diagrams is identified that dresses up G and turns it into a distance (or energy)-dependent constant $G(r)$. Unfortunately it is not clear that these definitions are gauge invariant; only physical observables (such as a scattering matrix) are guaranteed to be. Nevertheless the renormalization-group analysis derived from this running coupling constant is of course very interesting and may bear relevance to the issue of asymptotic safety mentioned in the Introduction.

IV. A TWO-DIMENSIONAL TOY MODEL

In the previous sections we have given arguments why the Einstein-Hilbert action could be viewed as the most relevant term, in the sense of the renormalization group, of an effective theory describing the long distance behavior of some underlying dynamics.

Here we want to pursue this line of thought further. As a logical possibility, without making any particularly strong claim of physical relevance, we shall investigate a formulation inspired as much as possible in the chiral symmetry breaking of QCD. It should have the following characteristics:

(1) No *a priori* notion of metric should exist, only an affine connection defining the parallel transport of tangent vectors v^a on a manifold.
(2) The Lagrangian should be manifestly independent of the field $g_{\mu\nu}(x)$.
(3) The graviton field should appear as the Goldstone boson of a suitably broken global symmetry.
(4) The breaking should be triggered by a fermion condensate.

A model along these lines was considered some time ago by Russo and others [9]. Our proposal appears to be perturbatively renormalizable and leads to finite calculable predictions, unlike the one in [9].

As announced we seek inspiration in the effective Lagrangians of QCD at long distances. A successful model for QCD is the so-called chiral quark model [19]. Consider the matter part Lagrangian of QCD with massless quarks (2 flavors)

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi = i\bar{\psi}_L\partial\!\!\!/\psi_L + i\bar{\psi}_R\partial\!\!\!/\psi_R. \tag{16}$$

This theory has a global $SU(2) \times SU(2)$ symmetry that forbids a mass term M. However after chiral symmetry breaking pions appear and they must be included in the effective theory. Then it is possible to add the following term:

$$-M\bar{\psi}_L U\psi_R - M\bar{\psi}_R U^\dagger \psi_L \tag{17}$$

that is invariant under the full global symmetry $\psi_L \to L\psi_L$, $\psi_R \to R\psi_R$, $U \to LUR^\dagger$.

Chiral symmetry breaking is triggered by a nonzero fermion condensate $\langle\bar{\psi}\psi\rangle \neq 0$. In order to determine the value of this condensate, and, in particular, whether it is zero or not, one is to solve a ‘‘gaplike’’ equation in some modelization of QCD, or on the lattice. The final step is to integrate out the fermions using the self-generated effective mass as an infrared regulator. This reproduces the chiral effective Lagrangian discussed in the first section, although the low-energy constants α_i obtained in this way are not necessarily the real ones, as the chiral quark model is only a modelization of QCD.

We shall use the Euclidean conventions. Our idea is to find out a two-dimensional field theory with the characteristics outlined above. There is only one possible ‘‘kinetic’’ term bilinear in fermions that is invariant under Lorentz $\times$ $Diff$ [actually $SO(D)$ rather than Lorentz] and it is local and Hermitian.[2] It is

$$i\bar{\psi}_a\gamma^a\nabla_\mu\psi^\mu + i\bar{\psi}^\mu\gamma^a\nabla_\mu\psi_a. \tag{18}$$

To define ∇_μ we only need an affine connection

$$\nabla_\mu\psi^\mu = \partial_\mu\psi^\mu + \omega_\mu^{ab}\sigma_{ab}\psi^\mu + \Gamma^\nu_{\mu\nu}\psi^\mu. \tag{19}$$

Here $a, b, \ldots$ are tangent space indices, while $\mu, \nu, \ldots$ are world indices. The coordinates on the manifold shall be denoted by x^μ and of course there is no way of raising or lowering indices because there is no metric. Only δ_{ab} as invariant tensor of the tangent space is admissible. ψ_a and ψ^μ are independent spinor fields. The field ψ^μ is expected to have a spin $1/2$ as well as a $3/2$ component. No attempt has been made to project out the $1/2$ component.

Note that no metric is needed at all to define the action if we assume that ψ^μ behaves as a contravariant spinorial vector density under *Diff*. Then, $\Gamma^\mu_{\nu\rho}$ does not enter in the covariant derivative, only the spin connection ω_μ^{ab}. If we keep this spin connection fixed, i.e. we do not consider it to be a dynamical field for the time being, there is no invariance under general coordinate transformations, but only under the global group $G = SO(D) \times GL(D)$. Notice once more that the spin connection is the only geometrical quantity introduced.

We would like to find a nonzero value for the fermion condensate

$$\langle\bar{\psi}_a\psi^\mu + \bar{\psi}^\mu\psi_a\rangle \sim A_a^\mu \neq 0. \tag{20}$$

Because the broken theory has still the full symmetry it is of course irrelevant in which direction the condensate points; all the vacua will be equivalent. We can choose $A_a^\mu = \delta_a^\mu$ without loss of generality.

Along with the breaking a large number of Goldstone bosons are produced. The original symmetry group $G = SO(D) \times GL(D)$ has $\frac{D(D-1)}{2} + D^2$ generators. After the breaking $G \to H$, with $H = SO(D)$ there are D^2 broken generators, as expected. It remains to be seen how many of those actually couple to physical states.

In order to trigger the appearance of a vacuum expectation value we have to include some dynamics to induce the symmetry breaking. The model we propose is to add the interaction piece

$$S_I = \int d^4x(iB_\mu^a(\bar{\psi}_a\psi^\mu + \bar{\psi}^\mu\psi_a) + c\det(B_\mu^a)). \tag{21}$$

Note that the interaction term also behaves as a density thanks to the covariant Levi-Civita symbol hidden in the determinant of B_μ^a so no metric is needed. Note that (21) is non-Hermitian, but the continuation to Minkowski is: B_μ^a upon continuation changes like a Euclidean mass does $B_\mu^a \to iB_\mu^a$. Since the field B_μ^a is auxiliary, it is clear that we are dealing with a four-fermion interaction; fermions are the only dynamical fields.

A. Equations of motion

If we consider the equation of motion for the auxiliary field B_μ^a we get

$$\langle\bar{\psi}_a\psi^\mu + \text{H.c.}\rangle = -ic\epsilon^{\mu\nu}\epsilon_{ab}B_\nu^b. \tag{22}$$

We conjecture the field B_μ^a to correspond to the zweibein, e_μ^a, up to a (dimensionful) constant.

Making use of this equation of motion, the interaction term in 2D corresponds to a four-fermion interaction.[3]

$$\epsilon_{\mu\nu}\epsilon^{ab}(\bar{\psi}_a\psi^\mu + \bar{\psi}^\mu\psi_a)(\bar{\psi}_b\psi^\nu + \bar{\psi}^\nu\psi_b). \tag{23}$$

This can be integrated over the manifold without having to appeal to a measure if we assume that ψ^μ is a spinorial density. Note that if $\langle\bar{\psi}_a\psi^\mu + \bar{\psi}^\mu\psi_a\rangle$ acquires a vacuum expectation value (VEV) it is possible to write new operators.

[2] Actually what we really should require is that the continuation to Minkowski space is Hermitian.

[3] Although this would take us too far away, note that this is reminiscent of an instanton-generated interaction. We are grateful to C. Gómez for a discussion on this subject.

The equations of motion for the fermion fields are

$$\gamma^a \nabla_\mu \psi^\mu + B^a_\mu \psi^\mu = 0, \tag{24}$$

and

$$\gamma^a \nabla_\mu \psi_a + B^a_\mu \psi_a = 0. \tag{25}$$

Note that after use of the equations of motion the Lagrangian itself reduces to the term $c\,\det(B^a_\mu)$.

B. Energy-momentum tensor and symmetries

Altough the above theory is ‘‘topological’’ inasmuch as it is described by an action that does not contain a metric (albeit it depends on a connection), the energy-momentum tensor understood as the Noether currents of translation invariance is nonvanishing

$$T^\mu_\nu = i\bar{\psi}^\mu \gamma^a \partial_\nu \psi_a + i\bar{\psi}_a \gamma^a \partial_\nu \psi^\mu - \delta^\mu_\nu L. \tag{26}$$

Note that no metric is needed to define T^μ_ν. In the absence of the external connection T^μ_ν is traceless as expected given that the theory is formally conformal, but we will see later that it will not remain so at the quantum level as anomalous dimensions develop.

The free action (18), without considering the interaction term, is invariant under the symmetry

$$\psi_a \to \psi'_a = \left(\delta^b_a - \frac{1}{D}\gamma_a \gamma^b\right)\psi_b. \tag{27}$$

Another invariance of the free action is provided by redefining, in Fourier space,

$$\psi^\mu(k) \to \psi'^\mu = P^\mu_\nu \psi^\nu(k), \tag{28}$$

where $k_\mu P^\mu_\nu = 0$. These two invariances make it considerably difficult for the heat-kernel derivation of an effective action for the field B^a_μ that will be discussed below.

C. Free propagator and renormalizability

Note the peculiar ‘‘free’’ kinetic term $\gamma^a \otimes k_\mu$. It is of course reminiscent of the Dirac equation, but it is not quite identical (the Dirac equation needs a metric or an n-bein to be defined). In the next section we will see that after the introduction of the interaction term $\sim \det B$, the field B^a_μ will indeed develop a VEV that we conventionally take to be

$$\langle B^a_\mu \rangle = M\delta^a_\mu. \tag{29}$$

Any other direction would be equivalent. The only substantial fact is whether M is zero or not. Via (22) this VEV for B^a_μ translates into a VEV for $\bar{\psi}_a \psi^\mu + \bar{\psi}^\mu \psi_a$. From (21) we see that the scale M plays the role of a dynamically generated mass for the fermions (not unlikely the ‘‘constituent mass’’ in chiral dynamics, except that here it will be possible, as we will see, to determine exactly its relation to the fundamental parameters of the model).

Below we write explicitly in two dimensions the bilinear operator acting on the fermion fields. Considered as a matrix, we shall not distinguish at this point between tangent and world indices (we then use indices in the middle of the alphabet $i, j, k, \ldots$):

$$\Delta_{ij} = \begin{pmatrix} iB_{11} & k_1 & iB_{12} & k_2 \\ k_1 & iB_{11} & k_2 & iB_{12} \\ iB_{21} & -ik_1 & iB_{22} & -ik_2 \\ ik_1 & iB_{21} & ik_2 & iB_{22} \end{pmatrix}. \tag{30}$$

When B_{ij} develops a VEV, $B_{ij} = M\delta_{ij}$, this reads

$$\Delta(k)_{ij} = \begin{pmatrix} iM & k_1 & 0 & k_2 \\ k_1 & iM & k_2 & 0 \\ 0 & -ik_1 & iM & -ik_2 \\ ik_1 & 0 & ik_2 & iM \end{pmatrix}. \tag{31}$$

The inverse of this matrix will give the propagator of the fermion field. It can be written (in any number of dimensions) as

$$\Delta^{-1}(k)_{ij} = \frac{-i}{M}\left(\delta_{ij} - \frac{\gamma_i(\not{k} - iM)k_j}{k^2 + M^2}\right), \tag{32}$$

with $k^2 = \sum_i k_i^2$. The covariance of the results, not evident at all from these expressions, will be discussed in the next section.

This is an appropriate point to discuss the renormalizability of the model. Naively, because the coupling constant c is dimensionless in 2D, we would expect the model to be renormalizable. However, this expectation is jeopardized by the behavior of the propagator. Indeed the diagonalization of (31) gives as eigenvaules M (twice), $k + iM$, and $k - iM$. Therefore the propagator does not behave, in general, as $1/k$ and therefore the usual counting rules simply do not apply.

There is however a further twist to the issue of renormalizability. The model proposed does not contain a metric and therefore the number of counterterms that one can write is extremely limited. For instance, a mass term for the B field is impossible. Higher dimensional operators would require powers of $\sqrt{g}$ to preserve the *Diff* invariance that the model has (when w is a dynamical variable), but there is no metric. In fact the metric will be generated *after* the breaking, but the counterterms of a field theory do not depend on whether there is spontaneous breaking of a global symmetry or not. In summary, the lack of counterterms makes us believe that the theory is renormalizable after all. Indeed this expectation is supported by an explicit one-loop calculation (see Sec. VI), where the only divergence that appears can be absorbed by a redefinition of c. We find this quite remarkable.

D. Gap equation

If $w_\mu = 0$ then one can use homogeneity and isotropy arguments to look for constant solutions of the gap equation associated with the following effective potential:

$$V_{\text{eff}} = c\det(B^a_\mu) - 2\int\frac{d^Dk}{(2\pi)^D}\,\mathrm{tr}(\log(\gamma^a k_\mu + iB^a_\mu)). \tag{33}$$

The factor 2 is due to the fact that ψ^μ and ψ_a are independent degrees of freedom. By deriving with respect to B^a_μ, the extrema of V_{eff} are found from the equation

$$cn\epsilon_{aa_2\cdots a_n}\epsilon^{\mu\mu_2\cdots\mu_n}B^{a_2}_{\mu_2}\cdots B^{a_n}_{\mu_n} - 2i\,\mathrm{tr}\int\frac{d^Dk}{(2\pi)^D}(\gamma\otimes k + iB)^{-1}|^\mu_a = 0. \tag{34}$$

In 2D this equation is particularly simple

$$c\epsilon_{ab}\epsilon^{\mu\nu}B^b_\nu - i\,\mathrm{tr}\int\frac{d^Dk}{(2\pi)^D}(\gamma\otimes k + iB)^{-1}|^\mu_a = 0. \tag{35}$$

The "gap equation" to solve for constant values of B_{ij} is

$$cB_{ij} + \frac{1}{2\pi}B_{ij}\log\frac{\det B}{\mu^2} = 0. \tag{36}$$

A logarithmic divergence has been absorbed in c. Notice that the equations are invariant under the permutation

$$B_{ij} \to B_{\sigma(i)\sigma(j)},\, k_i \to k_{\sigma(i)}, \qquad \sigma\epsilon S_2. \tag{37}$$

This equation has a nontrivial solution that we can always choose, as indicated before, to be $B_{ij}\sim\delta_{ij}$. We thus see that the dynamical mass for the fermions is indeed generated hence justifying *a posteriori* the propagator introduced in the previous section. The solution for the dynamical mass is

$$M = \mu e^{-\pi c(\mu)}. \tag{38}$$

Plugging this back into the effective potential we obtain

$$V_{\text{eff}} = -\frac{\mu^2 e^{-2\pi c(\mu)}}{2\pi}. \tag{39}$$

Upon continuation to Minkowski space-time this term is to be identified with the cosmological constant, because when rotating to Minkowski space $V\to -V$, the cosmological constant is positive. At this level M is an observable and as such it should be a renormalization-group invariant. This is guaranteed if c runs according to the rather trivial beta function

$$\mu\frac{dc}{d\mu} = \frac{1}{\pi}. \tag{40}$$

Note that the coefficient of this term is related to the coefficient of the logarithmic divergence and hence it is universal.

The above effective potential and ensuing gap equation are exact in the limit where the number of fermions, N, is infinite. In fact we expect that it is exact *only* in this limit, as in 2D the phenomenon of spontaneous breaking of a continuous symmetry can take place only in the $N=\infty$ limit.

For a nonzero connection ($w_\mu\neq 0$) the gap equation is not applicable and one needs to derive the full effective action. Then one would minimize the fields B^a_μ as a function of w_μ. This is discussed in the next section.

V. DERIVATION OF THE EFFECTIVE ACTION

Let us now attempt to derive the effective action for the fields B^a_μ and the external affine connection w_μ that eventually we will allow to become a dynamical variable too. Hereafter we want to perform a double minimization with respect to these fields. This will be an exact procedure for $N=\infty$ and provide a guidance in the general case. Of course the really interesting question is what happens for $D>2$.

We would expect that this double minimization will provide us with two equations whose meaning would be schematically the following: One of them would provide a relation between the field B^a_μ (associated with the zweibein) and the affine connection w_μ. If the present model is to describe in its broken phase 2D gravity, this relation would be analogous to the relation of compatibility between the metric and the connection that appears when the Palatini formalism [20] is used in general relativity and the equations of motion for the connection w_μ are derived. The remaining equation, after imposition of the previous compatibility condition, should then be equivalent to Einstein's equations.

However, in 2D gravity it is rather peculiar and indeed the condition

$$w^{ab}_\mu = e^a_\nu\partial_\mu E^{\nu b} + e^a_\nu E^{\sigma b}\Gamma^\nu_{\sigma\mu}, \tag{41}$$

where E^μ_a is the inverse zweibein $E^\mu_a e^b_\mu = \delta^b_a$, holding in any number of dimensions, does not follow in 2D from any variational principle (see e.g. [21]). There are several ways to understand this fact, but perhaps the simplest one is to realize that the Einstein-Hilbert action in 2D depends on w_μ only through the two-form dw which is linear in the affine connection w_μ. In fact, the scalar curvature term $\sqrt{g}\mathcal{R}$ does not contain in 2D any coupling between $g_{\mu\nu}$ and w_μ. Adding higher derivatives does not really help as the Riemann tensor contains only an independent component that can be ultimately related to the scalar curvature. We shall see below that this peculiarity of two-dimensional gravity is faithfully reproduced in our proposal.

The starting point of the derivation of the effective action is the differential operator

$$D^a_\mu = \gamma^a(\partial_\mu + w_\mu\sigma_3) + B^a_\mu. \tag{42}$$

We consider the expansion around a fixed background preserving $SO(D)$ but not the full symmetry group G. We will take $B^a_\mu = M\delta^a_\mu$, where M will be determined via the gap equation discussed in the previous section, which corresponds to a solution of the equation of motion at the lowest order in a weak field and derivative expansion, in the spirit of effective Lagrangians. To go beyond this approximation we have to consider x-dependent fluctuations around this vacuum and include the external field w_μ. We shall decompose

$$B^a_\mu = \xi^a_{L\ b} \bar{B}^b_\nu \xi^{-1\nu}_{R\ \mu}, \tag{43}$$

where $\xi_L \in SO(D)$, $\xi_R \in GL(D)$, and $\bar{B}^a_\mu$ is a solution of the gap equation, $M\delta^a_\mu$ in our case. It is technically advantageous to absorb the matrices ξ_L and ξ_R in the fermion fields (in QCD this is the so-called "constituent" quark basis [19]). Then the differential operator to deal with will be

$$\mathcal{D}^b_\mu = \xi^{\dagger\ b}_{La} \gamma^a (\partial_\rho + w_\rho \sigma_3) \xi^\rho_{R\ \mu} + \bar{B}^b_\mu. \tag{44}$$

To evaluate the effective action generated by the integration of the fermion fields one possibility is to write the log of the fermion determinant as

$$W = -\frac{1}{2} \int_0^\infty \frac{dt}{t} \operatorname{tr}\langle x | e^{-tX} | x \rangle, \tag{45}$$

where

$$X_{\mu\nu} \equiv \mathcal{M}^\dagger \mathcal{M}, \tag{46}$$

with

$$\mathcal{M} = \mathcal{D}^b_\nu, \qquad \mathcal{M}^\dagger = -\mathcal{D}_{\mu b} \tag{47}$$

and

$$\begin{aligned} \mathcal{D}^b_\mu &= \xi^{\dagger\ b}_{La} \gamma^a (\partial_\rho + w_\rho \sigma_3) \xi^\rho_{R\ \mu} + \bar{B}^b_\mu, \\ \mathcal{D}_{\nu b} &= \xi^{\dagger\ \sigma}_{R\nu} (\partial_\sigma - w_\sigma \sigma_3) \gamma_a \xi^a_{L\ b} - \bar{B}_{\nu b}. \end{aligned} \tag{48}$$

$X_{\mu\nu}$ has both world and Dirac indices (the latter not explicitly written). Note that as previously discussed $\mathcal{M}$ is not Hermitian, but of course $X_{\mu\nu} = \mathcal{M}^\dagger \mathcal{M}$ is. We could have also considered the determinant of $\mathcal{M}\mathcal{M}^\dagger$ which is of course identical, but it is important to maintain a covariant appearance as long as possible (note that there is no metric so far and no way of lowering or raising indices). The final result has to be of course covariant, since our starting point is, but using, as we shall do, a plane basis to evaluate the traces in the heat-kernel expansion breaks in principle this covariance in intermediate steps.

Once $W(w, B)$ is known we can differentiate with respect to w_μ and obtain the relation between the zweibein and the spin connection using the logic behind the Palatini formalism.

The starting point of the heat-kernel derivation is the evaluation of

$$\operatorname{tr}\langle x | e^{-tX_{\mu\nu}} | x \rangle = \frac{1}{t^{D/2}} \int \frac{d^D k}{(2\pi)^D} \operatorname{tr} e^{[-D\xi^{\top\ \sigma}_{R\mu} \xi^\rho_{R\nu} k_\sigma k_\rho + i\sqrt{t}\mathcal{D}_{\mu b} \xi^{-1b}_{L\ \ a} \gamma^a k_\rho \xi^\rho_{R\nu} + i\sqrt{t}\xi^{\top\ \sigma}_{R\mu} k_\sigma \gamma_a \xi^{-1a}_{L\ \ b} \mathcal{D}^b{}_\nu - tX_{\mu\nu})]}, \tag{49}$$

where for convenience we have rescaled k_μ and a plane wave basis resolution of the identity has been used. For simplicity let us call the exponent on the right-hand side of the previous equation $X(\sqrt{t})$. Then the way to proceed is to expand the exponential $e^{X(\sqrt{t})}$ in powers of $\sqrt{t}$. Only even powers of $\sqrt{t}$ (and thus of k) will contribute at the end to the series, so the first nontrivial term will be of order t. We define

$$F_n(X(0), \dot{X}(0), \ddot{X}(0)) \equiv \frac{d^{(n)}}{(d\sqrt{t})^n} e^{X(\sqrt{t})} |_{\sqrt{t}=0}, \tag{50}$$

and then

$$\begin{aligned} \operatorname{tr}\langle x | e^{-tX_{\mu\nu}} | x \rangle &\propto \operatorname{tr} \sum_n F_n \frac{(\sqrt{t})^n}{n!} \\ &= F_0 + \frac{t}{2} F_2 + \frac{t^2}{24} F_4 + \mathcal{O}(t^4). \end{aligned} \tag{51}$$

This expansion is quite tedious and to perform it we used repeatedly the well-known formula

$$\frac{d}{dt} e^{A(t)} = \int_0^1 da e^{(1-a)A(t)} \frac{dA(t)}{dt} e^{aA(t)}. \tag{52}$$

Note that the invariances discussed in the previous section introduce zero modes in the exponent and hence integrals that are not damped for large values of the momentum k. Of course they are not true zero modes of the full theory, just of the kinetic term, but the technical complications that they bring about are notable.

However, it is pleasant to see that a formally covariant result emerges. If we neglect w_μ and we take the matrices ξ to be constant it is not difficult to see that the lowest nontrivial order of the heat-kernel calculation gives

$$W = \frac{3M^2}{16\pi} \int d^2x \sqrt{\det[(\xi^\sigma_{R\mu} \xi^{\dagger\rho}_{R\mu})^{-1}]}, \tag{53}$$

where a summation over μ is to be understood and where M^2 is the dynamically generated mass. This is just a cosmological term with $g^{\sigma\rho} = \sum_\mu \xi^\sigma_{R\mu} \xi^{\dagger\rho}_{R\mu}$. One can likewise verify that other pieces in the effective action are covariant. The coefficient of the cosmological constant term obtained at the lowest order in the heat-kernel expansion does not agree with the one obtained through the gap equation. We shall see later why this is so.

Since the most general metric in two dimensions is conformally flat we can reconstruct the full covariant action from this particular choice. This notably simplifies the derivation of the effective action. We take $B^a{}_\mu(x) = \xi^a_{Lb}\bar{B}^b{}_\rho \xi_R^{-1\rho}{}_\mu(x) = M\phi^{-1}\delta^a_\mu$. The expressions that follow are specific to this gauge.

At second order in the heat-kernel expansion [order $(\sqrt{t})^2$] the corresponding piece of the effective action reads

$$W^{(2)} = \int d^2x\phi^{-2}\left[\frac{3M^2}{16\pi}\left(\frac{2}{\epsilon} - \gamma - \log\left(\frac{M^2}{\mu^2}\right) + \log(8\pi) + 4\right) + \frac{(\partial_\mu\phi)^2}{4\pi}\left(\frac{2}{\epsilon} - \gamma - \log\left(\frac{M^2}{\mu^2}\right) + \log(8\pi) - \frac{5}{3}\right) + \frac{w^2\phi^2}{4\pi}\left(\frac{2}{\epsilon} - \gamma - \log\left(\frac{M^2}{\mu^2}\right) + \log(8\pi)\right)\right], \tag{54}$$

where $D = 2 - \epsilon$. We can take one step further and calculate the contribution to order t^2

$$W^{(4)} = \int d^2x\phi^{-2}\left[-\frac{3M^2}{32\pi}\left(\frac{2}{\epsilon} + \log(8\pi) - \log\left(\frac{M^2}{\mu^2}\right) - \gamma + \frac{5}{18}\right) - \frac{\phi^4(\partial_\mu w_\mu)^2}{4\pi M^2} + \frac{\phi^3(2w_\nu\partial_\mu\phi\partial_\mu w_\nu - 3w_\mu\partial_\mu\phi\partial_\nu w_\nu)}{3\pi M^2}\frac{\phi^2(\partial_\mu\phi)^2 w^2}{6\pi M^2} - \frac{w_\mu w_\nu\partial_\mu\phi\partial_\nu\phi}{\pi M^2} + \frac{\phi^2 w^2}{8\pi} - \frac{\phi^4 w^4}{4\pi M^2} - \frac{5(\partial_\mu\phi\partial_\mu\phi)}{48\pi} + \frac{\phi}{M^2}\frac{\partial_\mu\phi\partial_\nu\phi\partial_\mu\partial_\nu\phi}{3\pi} + \frac{\phi}{M^2}\frac{\partial_\mu\phi\partial_\mu\phi\partial_\nu\partial_\nu\phi}{15\pi} - \frac{7\phi^3}{M^2}\frac{\partial_\mu\partial_\mu\partial_\nu\partial_\nu\phi}{60\pi} - \frac{\partial_\mu\phi\partial_\mu\phi\partial_\nu\phi\partial_\nu\phi}{5\pi M^2}\right]. \tag{55}$$

The calculation of the fourth-order coefficients in the heat-kernel expansion just shown is already a formidable task and we will not attempt to go beyond.

If we look at the results of the expansion at second order it is interesting to see that the terms that are generated are the ones expected from the point of view of general relativity. There is a cosmological term (proportional to ϕ^{-2}, which in covariant form corresponds to $\sqrt{g}$), and a Liouville term [proportional to $\phi^{-2}(\partial_\mu\phi)^2$, which in covariant form is nonlocal: $\sqrt{g}\mathcal{R}\nabla^{-2}\sqrt{g}\mathcal{R}$]. In addition there is a term proportional to w^2 (which once written in a covariant form would be $\sqrt{g}g^{\mu\nu}w_\mu w_\nu$). Note that the Einstein term itself is topological in 2D and it is not expected to show up. However, in spite of these satisfactory results, we notice that the cosmological term does not quite coincide with the one previously derived, via the gap equation, and the Liouville term is apparently divergent casting doubts on the renormalizability of the model. We note that like in the chiral Lagrangian, the effective theory still possesses the full symmetry group G.

Yet it is easy to see that the above results are by necessity incomplete. For instance, the same operator ϕ^{-2} gets a contribution from the terms of order t and from t^2, ditto for Liouville. This comes from the fact that because the operator $X(\sqrt{t})$ contains terms linear in M and the heat-kernel expansion is effectively an expansion in inverse powers of M, a given order in t does not correspond to a given order in derivatives or external fields. Therefore although the heat-kernel calculation gives an interesting guidance to the form of the effective action and it shows the reappearance of covariance, the precise values of the coefficients of the different operators cannot be extracted from it. To solve this difficulty we turn to a diagrammatic calculation.

VI. DIAGRAMMATIC CALCULATION

Let us recapitulate. The heat-kernel calculation is plagued by two problems. The first one is related to the zero modes of the kinetic term, which increase considerably the difficulty of the calculations. The other one lies in the fact that the expansion is ill defined in the sense of relevance of the subsequent orders. In a way, the heat kernel fails to provide exact coefficients for the different operators but gives an accurate catalog of the possible terms one could expect.

In this section we derive the Feynman rules of our toy model and proceed to calculate the exact contributions of the zero-, one-, and two-point functions. As will be shown, we obtain finite contributions except for the cosmological term which nevertheless can be renormalized. The theory appears to be perfectly renormalizable in spite of the apparent bad power counting (due to the zero modes of the propagator).

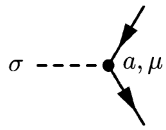
A. Feynman rules

We start by writing the generating functional of the theory in the conformal gauge in Euclidean space from which we can read off the Feynman rules for the one- and two-point functions we are interested in. We know that the diagrammatic expansion is not covariant, but once we have convinced ourselves that covariance is recovered, we can use this method for identifying specific coefficients. In this section it will be convenient to express the conformal gauge in the form

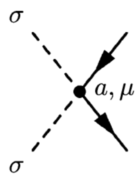
$$B^a_\mu(x) = Me^{-\sigma(x)/2}\delta^a_\mu. \tag{56}$$

The first term in the expansion of the exponential provides the dynamically generated mass for the fermions. Incidentally, this formalism is clearly quite reminiscent of chiral dynamics.

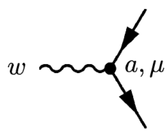
The interaction vertices are



$$i\frac{1}{2}M\delta^a_\mu\ ,$$



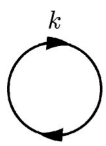
$$-i\frac{1}{8}M\delta^a_\mu\ , \tag{57}$$



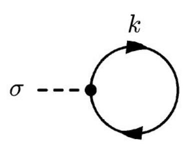
$$-i\gamma^a\sigma_3\ .$$

B. Zero-, one-, and two-point functions

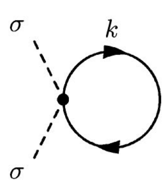
With the rules described in the previous section and the propagator derived in Sec. IV C we can calculate the exact contributions of the zero-, one-, and two-point functions of the theory. Since the theory is nonstandard, and it has a nonfamiliar set of Feynman rules, we will provide the diagrams, after transcribing the Feynman rules, and the final result. Note that because there are two species of fermions the result from the Feynman diagrams has to be multiplied by a factor 2. Let us first consider one-particle irreducible diagrams containing the σ field as the external one:



$$\begin{aligned}&= -\,\mathrm{Tr}\left[\int \frac{d^Dk}{(2\pi)^D}\Delta^{-1}(k)^{\mu}{}_{a}\delta^{a}{}_{\mu}\right]\\ &=\frac{iM}{2\pi}\left(\frac{2}{\epsilon}-\gamma-\log\left(\frac{M^2}{\mu^2}\right)+\log(4\pi)\right),\end{aligned}$$

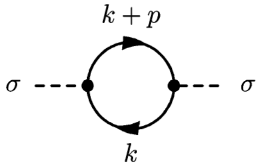


$$\begin{aligned}&= -\,\mathrm{Tr}\left[\int \frac{d^Dk}{(2\pi)^D}\Delta^{-1}(k)^{\mu}{}_{a}\frac{i}{2}M\delta^{a}{}_{\mu}\right]\\ &=-\frac{M^2}{2\pi}\frac{1}{2}\left(\frac{2}{\epsilon}-\gamma-\log\left(\frac{M^2}{\mu^2}\right)+\log(4\pi)\right),\end{aligned} \tag{58}$$



$$\begin{aligned}&= -\,\mathrm{Tr}\left[\int \frac{d^Dk}{(2\pi)^D}\Delta^{-1}(k)^{\mu}{}_{a}\frac{-i}{8}M\delta^{a}{}_{\mu}\right]\\ &=\frac{M^2}{2\pi}\frac{1}{8}\left(\frac{2}{\epsilon}-\gamma-\log\left(\frac{M^2}{\mu^2}\right)+\log(4\pi)\right).\end{aligned}$$

There is another diagram with two external scalar legs:



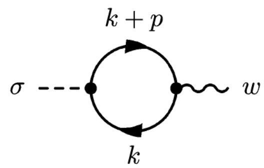
$$\begin{aligned}&= -\,\mathrm{Tr}\left[\int \frac{d^Dk}{(2\pi)^D}\frac{i}{2}M\delta^{a}{}_{\mu}\Delta^{-1}(k)^{\mu}{}_{b}\frac{i}{2}M\delta^{b}{}_{\nu}\Delta^{-1}(k+p)^{\nu}{}_{a}\right]\\ &=\frac{M^2}{4\pi}\frac{1}{4}\left[\frac{2}{\epsilon}-\gamma-\log\left(\frac{M^2}{\mu^2}\right)+\log(4\pi)-2\right]-\frac{1}{48\pi}p^2\ .\end{aligned} \tag{59}$$

From the M^2 terms in (58) and (59) we can already infer the total contribution to the cosmological term

$$\frac{M^2 e^{-\sigma}}{4\pi}\left(\frac{2}{\epsilon} - \gamma - \log\left(\frac{M^2 e^{-\sigma}}{\mu^2}\right) + \log(4\pi) + 1\right). \tag{60}$$

The divergence can be absorbed in the redefinition of the coupling constant, c. This result fully agrees with the one derived via the gap equation previously. In addition we observe that the p^2 piece in the last diagram will correspond in position space to the Liouville term. As it can be seen it is finite.

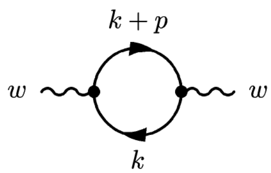
Next we look at the two-point function that mixes a σ field with a w field. This could yield a $\mathcal{R}$-type term but since in two dimensions gravity is topological we do not expect to see such a term. Indeed, the diagram gives zero



$$= -\,\mathrm{Tr}\left[\int \frac{d^D k}{(2\pi)^D}\frac{i}{2}M\delta^a{}_\mu \Delta^{-1}(k)^\mu{}_b(-i\gamma^b\sigma_3)\Delta^{-1}(k+p)^\nu{}_a\right]$$

$$= 0. \tag{61}$$

Finally we calculate the last of the two-point functions possible. Again we obtain a finite result



$$= -\,\mathrm{Tr}\left[\int \frac{d^D k}{(2\pi)^D}(-i\gamma^a\sigma_3)\Delta^{-1}(k)^\mu{}_b(-i\gamma^b\sigma_3)\Delta^{-1}(k+p)^\nu{}_a\right]$$

$$= \frac{\delta^{\mu\nu}}{2\pi} - \frac{p^\mu p^\nu}{6\pi M^2}\,. \tag{62}$$

We see with relief that even if the ultraviolet behavior of each and every one of the integrals is very bad, the final result hints of the renormalizability of the theory. After renormalizing the only coupling constant c of the theory the final result is perfectly finite.

C. Effective action

Let us now put all the pieces together and use the lowest-order equations of motion for the field B^a_μ, or what is tantamount, for the dynamically generated mass M, to write the effective action. The result is

$$S_{\mathrm{eff}} = \int d^2x\left[-\frac{M^2}{2\pi}e^{-\sigma} + \frac{1}{24\pi}\partial_\mu\sigma\partial_\mu\sigma + \frac{(\partial_\mu w_\mu)^2}{3\pi M^2} - \frac{w^2}{\pi} + \cdots\right], \tag{63}$$

with M is given by (38). This is our final result.

Several comments are in order. First we recall that the effective action is written in the conformal gauge for the metric, but it is trivial to recover a full covariant form. Secondly, we note that there is no coupling between metric and connection, as befits the Palatini formalism in two dimensions where, exceptionally, metric and connection are unrelated. One can apply a variational principle to the affine connection w_μ in the above effective action, obtaining some equations of motion at $\mathcal{O}(p^2)$, but in 2D they do not provide any information on the conformal factor σ.

One is then left with a cosmological and a Liouville term, as corresponds to two-dimensional gravity [22]. The dots correspond to higher curvatures that we have not attempted to compute. In general they will be nonzero. Notice that the expansion is valid as long as the characteristic momenta fulfill $k > M$. Since M is the mass scale related to the two-dimensional cosmological constant, this would correspond to scales larger than the horizon.

VII. FOUR DIMENSIONS

It is almost compulsory to discuss the possible extension of these results to four dimensions.

There is apparently a fundamental problem in considering four-dimensional theories where the graviton is generated dynamically. If we refer to the original paper by Weinberg and Witten [23] (see also [24]), the apparent pathology of these theories lies in the fact that the energy-momentum tensor has to be identically zero if particles with spin higher than 1 appear and we insist on the energy-momentum tensor being Lorentz covariant. This result does not hold in two dimensions as it relies on angular momentum considerations that do not apply. Of course there is no true spontaneous breaking of continuous symmetries in two dimensions either, but this is circumvented by appealing to the large N limit.

However, very preliminary results indicate that one gets from the simple toy model proposed here a result that looks close to what one expects in four-dimensional gravity, so it is legitimate to ask whether the Weinberg and Witten result applies. We note something peculiar in our model, namely, the energy-momentum tensor derived in Sec. IV does not have tangent (Lorentz) indices. In fact, Lorentz indices are of an internal nature in the present approach; the connection between Lorentz and space-time indices appears only after a n-bein is dynamically generated. But then one is exactly in the same situation as general relativity where the applicability of [23] is excluded. Thus it is not clear to us to what extent the conditions assumed by Weinberg and Witten apply to the present model.

Of course in four dimensions we would possibly generate a graviton field, hopefully with the same couplings dictated by general relativity, but with all the degrees of freedom thrown in. To see that only two of them are physical one has to go through the usual procedure of gauge fixing.

It is worth noticing that the loop integrals of the present model show an even worse ultraviolet behavior in four dimensions. However, exactly as in two dimensions, the absence of a metric in the fundamental theory seriously limits the number of possible counterterms.

Then while the previous two-dimensional example is all too trivial it shows perfectly the general ideas. It seems conceivable to entertain the idea that a mechanism analogous to chiral symmetry breaking may trigger the dynamical appearance of some degrees of freedom that, at the very least, formally reproduce the Einstein-Hilbert action.

In four dimensions there are two independent dimensionful parameters in gravity: the scale associated with the cosmological constant $\Lambda^{1/4}$ and the Planck mass M_P, which is absent in 2D. In a simple model (such as the one proposed here) it is to be expected that the two scales are related (this may not necessarily be so if the 4D model requires additional subtractions). This is of course the old problem of fine-tuning associated with the cosmological constant reemerging, the novel aspect here being that in the present microscopic model of gravity this can be made quantitative. Solving this fine-tuning problem was not however our primary motivation, but rather trying to circumvent the difficulties of the Einstein-Hilbert Lagrangian at the quantum level by considering it as an effective theory and proposing a microscopic toy model.

VIII. SUMMARY

In this work we have proposed a model where two-dimensional gravity emerges from a theory without any predefined metric. The minimal input is provided by assuming a differential manifold structure endowed with an affine connection.

We have made an allusion in the title of this article to the emergence of geometry and this is really what happens in the model proposed. Gravity and distance are induced rather than fundamental concepts. At sufficiently short scales, when the effective action does not make sense anymore, the physical degrees of freedom are fermionic. Below that scale there is not even the notion of a smaller scale: in a sense that is the shortest scale that can exist.

A very important aspect of the model is that it appears to be renormalizable. All divergences can be absorbed in the redefinition of a unique coupling constant. With the appropriate running, dictated by the corresponding beta function, the cosmological constant becomes a renormalization-group invariant. Everything else is finite. Of course at long distances the conformal mode is the relevant degree of freedom and this induces new divergences which are, nevertheless, the ones characteristic of two-dimensional gravity. They have been discussed in Sec. III. The renormalizability aspect of this model (and its relative technical simplicity) is its main virtue when compared with previous proposals [9], where even semiquantitative discussions appear impossible. Here one is able to derive in all detail the effective action. The derivation remains valid for suprahorizon momenta, where the theory is governed by the induced metric modes, while at subhorizon energy scales everything seems to indicate that the fundamental fermionic degrees of freedom are the relevant ones.

The renormalizability of the model can be traced back eventually to the absence of a metric. There are no obvious counterterms to be written, a behavior that could possibly persist in four dimensions.

ACKNOWLEDGMENTS

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