Implications for primordial non-Gaussianity ($f_{\rm NL}$) from weak lensing masses of high-z galaxy clusters

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The recent weak-lensing measurement of the dark matter mass of the high-redshift galaxy cluster XMMUJ2235.3-2557 of $(8.5 \pm 1.7) \times 10^{14} M_{\odot}$ at $z = 1.4$, indicates that, if the cluster is assumed to be the result of the collapse of dark matter in a primordial Gaussian field in the standard lambda cold dark matter model, then its abundance should be $<2 \times 10^{-3}$ clusters in the observed area. Here we investigate how to boost the probability of XMMUJ2235.3-2557, in particular, resorting to deviations from Gaussian initial conditions. We show that this abundance can be boosted by factors >3–10 if the non-Gaussianity parameter $f_{\rm NL}^{\rm local}$ is in the range 150–200. This value is comparable to the limit for $f_{\rm NL}$ obtained by current constraints from the cosmic microwave background. We conclude that mass determination of high-redshift, massive clusters can offer a complementary probe of primordial non-Gaussianity.

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I. INTRODUCTION

It has been recognized for almost a decade that the abundance of the most massive and/or high-redshift collapsed objects could be used to constraint the nature of the primordial fluctuation field [1–4]. The subject has recently received renewed attention [5–9] possibly sparked by a claimed detection of deviations from Gaussianity on cosmic microwave background (CMB) maps [10]. Depending on the sign of the non-Gaussian perturbation, the abundance of rare objects will be enhanced or depleted. In [1] we developed the necessary theoretical tools to interpret any enhancement (depletion) in the abundance of rare-peak objects over the Gaussian initial conditions case. Working with ratios of non-Gausian over the Gaussian case makes the theoretical predictions more robust. Later on, Ref. [5] generalized the procedure to more modern mass functions and type of non-Gaussianity including scale dependence. The validity of the analytical formulas developed in [1] has been recently confirmed by detailed N-body numerical simulations with non-Gaussian initial conditions [7]. These authors have shown that the analytical findings in [1] provide an excellent fit to the non-Gaussian mass function found in N-body simulations with a simple "calibration" procedure.

The authors of Reference [11] have recently reported a weak-lensing analysis of the $z = 1.4$ galaxy cluster XMMU J2235.3-2557 based in Hubble Space Telescope (advance camera survey) images. Assuming a Navarro-Frenk-White [12] dark matter profile for the cluster, they estimate a projected mass within 1 Mpc of $(8.5 \pm 1.7) \times 10^{14} M_{\odot}$, therefore the total mass will be larger. Adopting a lambda cold dark matter cosmology with cosmological parameters given by WMAP 5 yr data ([13]) and assuming Gaussian initial conditions, they estimate that in the surveyed 11 sq. deg. there should be 0.005 clusters above that mass. Therefore, the observed cluster is unlikely at the 3σ level. In this paper we explore which level of non-Gaussianity is required to boost this abundance by a factor ~ 10 and how this relates to the available constraints obtained from the CMB. We show that with $f_{\rm NL}^{\rm local}$ in the range 150–200 it is possible to significantly enhance the abundance expected for such a massive cluster. This value of $f_{\rm NL}$ is comparable with current limits from the CMB [13], [10].

II. HIGH REDSHIFT AND/OR MASSIVE OBJECTS

While there are in principle infinite types of possible deviations from Gaussianity, it is common to parameterize these deviations in terms of the dimensionless parameter $f_{\rm NL}$ (e.g., [1,14–16]).

$$\Phi = \phi + f_{\rm NL}^{\rm local}(\phi^2 - \langle \phi^2 \rangle), \tag{1}$$

where Φ denotes the *primordial* Bardeen potential [17] and ϕ denotes a Gaussian random field. With this convention a positive value of $f_{\rm NL}^{\rm local}$ will yield to a positive skewness in the density field and an enhancement in the number of rare, collapsed objects.

Although not fully general, this model (called *local* type) may be considered as the lowest-order terms in Taylor expansions of more general fields. Local non-Gaussianity arises in standard slow roll inflation (although in this case $f_{\rm NL}^{\rm local}$ is unnmeasurably small), and in multifield models (e.g., [18–21]). For other types of non-Gaussianity (as we will see below) an "effective" $f_{\rm NL}$ can be defined and related to this model.

The abundance of *rare events* (high redshift and/or massive objects) is determined by the form of the high-

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density tail of the primordial density distribution function. A probability distribution function (PDF) that produces a larger number of $>3\sigma$ peaks than a Gaussian distribution will lead to a larger abundance of rare events. Since small deviations from Gaussianity have deep impact on those statistics that probe the tail of the distribution (e.g. [1,22]), rare events should be powerful probes of primordial non-Gaussianity. The non-Gaussianity parameter $f_{\rm NL}$ is effectively a "tail enhancement" parameter (cf., [1]).

In addition, high-redshift clusters are sensitive to the primordial skewness, and not too sensitive to the shape of non-Gaussianity as long as it yields the same skewness. CMB bispectrum is directly sensitive to the shapes of non-Gaussianity (thus has widely different error bars depending on the non-Gaussian shape), while halo bias is totally blind to some shapes and exponentially sensitive to others. Determining the shapes of non-Gaussianity would determine the physics behind deviations from the simplest single field, Bunch-Davies vacuum, slow roll inflation. When deviating from a simple local non-Gaussian model, the combination of different observables will be crucial in determining the non-Gaussian shapes.

As shown in [4,5,7] when using an analytical approach to compute the mass function a robust quantity to use is the fractional non-Gaussian correction to the Gaussian mass function $\mathcal{R}_{\rm NG}(M, z)$. This quantity was calibrated on non-Gaussian N-body simulations in [7]. For our purpose here we want to compute a closely related quantity: the *non-Gaussianity enhancement*, i.e. ratio of the non-Gaussian to Gaussian abundance of halos above a mass threshold [4]. As the mass function is exponentially steep for rare events here we can safely make the identification of the non-Gaussianity enhancement with $\mathcal{R}_{\rm NG}$.

To understand the effect of non-Gaussianity on halo abundance let us recall that to first order the *non-Gaussianity enhancement* is given by [5,7]

$$\mathcal{R}_{\rm NG}(M, z) \sim 1 + S_{3,M}\frac{\delta_c'(z)^3}{6\sigma_M^2}, \quad (2)$$

where $S_{3,M}$ denotes the skewness of the density field linearly extrapolated at $z = 0$ and smoothed on a scale R corresponding to the comoving Lagrangian radius of the halo of mass M, σ_M denotes the *rms* if the –linearly extrapolated at $z = 0$– density field also smoothed on the same scale R; $\delta_c'(z_f) = \sqrt{q}\delta_c(z_f)$ and $\delta_c(z_f)$ denotes critical collapse density at the formation redshift of the cluster z_f. Note that $\delta_c(z) = \Delta_c D(z = 0)/D(z)$ with $D(z)$ denoting the linear growth factor and Δ_c is a quantity slightly dependent on redshift and on cosmology, which only for an Einstein-de Sitter universe is constant $\Delta_c = 1.68$. The constant $q \simeq 0.75$ (which we will call "barrier factor") can be physically understood as the effect of nonspherical collapse [23,24] lowering the critical collapse threshold of a diffusing barrier [25], see also [26], and has been calibrated on N-body simulations in Ref. [7]. The full expression for $\mathcal{R}_{\rm NG}$ is (cf. Eqs. 6 and 7 in Ref. [7])

$$\mathcal{R}_{\rm NG}(M, z) = \exp\left[(\delta_c')^3\frac{S_{3,M}}{6\sigma_M^2}\right] \times \left|\frac{1}{6}\frac{\delta_{ec}^2}{\sqrt{1-\delta_c' S_{3,M}/3}}\frac{dS_{3,M}}{d\ln\sigma_M} + \frac{\delta_c'\sqrt{1-\delta_c' S_{3,M}/3}}{\delta_c'}\right|. \quad (3)$$

Let us reiterate that in principle the enhancement factor should be computed by integrating the mass function $n(M, z, f_{\rm NL})$ between the minimum and the maximum mass and for redshifts above the observed one [4]:

$$\hat{\mathcal{R}}_{\rm NG} = \frac{\int n(M, z, f_{\rm NL})dMdz}{\int n(M, z, f_{\rm NL} \equiv 0)dMdz} \quad (4)$$

but since the mass function, in the regime we are interested in, is exponentially steep, we can identify $\hat{\mathcal{R}}_{\rm NG} = \mathcal{R}_{\rm NG}$. The mass function of Ref. [5] applies to lower δ/σ than the regime we explore here. For high δ/σ the Matarrese-Verde-Jimenez mass function is much better suited. This consideration based on the approximations made in deriving the mass function are supported by N-body simulation results [7].

Small deviations from Gaussian initial conditions will lead to a nonzero skewness and, in particular, for local non-Gaussianity $S_{3,M} = f_{\rm NL}^{\rm local} S_{3,M}^1$, where $S_{3,M}^1$ denotes the skewness produced by $f_{\rm NL}^{\rm local} = 1$. Since non-Gaussianity comes in the expression for $\mathcal{R}_{\rm NG}$ only through the skewness, the same expression can be used for other types of non-Gaussianity such as the *equilateral* type (see e.g. Refs. [5,6] for an example of applications). For example, at the scales of interest $R = 13$ Mpc/h, $S_{3,R}^{1,\rm local} = 3.4 S_{3,R}^{1,\rm equil}$ thus when working on these scales to obtain the same non-Gaussian enhancement as a local model, an equilateral model needs a higher effective $f_{\rm NL}$: we can make the identification $f_{\rm NL}^{\rm equil} = 3.4 f_{\rm NL}^{\rm local}$.

Here, we will use the full [1] expression, corrected for the "barrier factor," for the non-Gaussian mass function to compute the *non-Gaussianity enhancement*. Note that the estimated mass and redshift of XMMUJ2235.3-2557, places it just outside the range where the mass function expressions of [1,5] have been directly reliably tested with non-Gaussian N-body simulations. Simulations seem to indicate that the [1] expression is a better fit than [5] at high masses/redshift and large $f_{\rm NL}$; this is also supported by theoretical considerations [7].

III. RESULTS

Figure 1 shows the enhancement factor $\mathcal{R}_{\rm NG}$ as a function of the mass of the galaxy cluster for different values of $f_{\rm NL}^{\rm local}$ and the redshift of collapse. The shaded

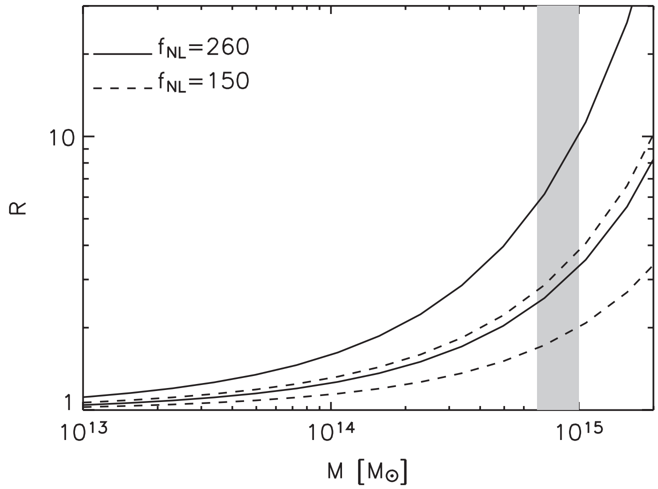

FIG. 1. Enhancement factor $\mathcal{R}_{NG}$ of the number of rare objects for different values of the dark matter mass of the galaxy cluster. The lines correspond to different values of f_{NL}. The upper lines are for a collapse redshift of $z_f = 2$ and the lower lines for $z_f = 1.4$. The shaded area is the range for the weak-lensing mass estimate of the clusters XMMUJ2235.3-2557. Note that for the quoted values of f_{NL}^{local} it is possible to obtain enhancements of order 10 in the cluster number abundance. This enhancement brings the expected abundance of such massive clusters in better agreement with the observations. Note that for masses above the estimated central value ($8.5 \times 10^{14} M_\odot$) one expects to find *zero* such objects in the *whole* sky (one expects 7 objects in the whole sky at the lowest value of the mass estimate) which emphasizes the need of an enhancement as the one provided by primordial non-Gaussianity studied here.

area shows the error band for the mass determination of XMMUJ2235.3-2557 from Ref. [11] and the different lines have been computed using the [1] mass function, with the "barrier factor" correction. Reference [7] show that it fits very well the N-body numerical simulations for the case of rare peaks, which is the one we are concerned with. The solid lines correspond to $f_{NL} = 260$, the lower one is for a cluster collapse redshift of $z_f = 1.4$ (i.e. assuming that the cluster forms at the observed redshift) and the upper one for $z_f = 2$. The two dashed lines also depict the mentioned collapse redshifts but for $f_{NL} = 150$. An extended Press-Schecter approach shows that the likely redshift of formation for such an object is $z_f = 1.6$ with some distribution around it extending from a sharp lower limit of z_f equal to the observation redshift of 1.4 and extending up to $z_f = 2$. We see that the galaxy cluster abundance can be enhanced by a factor up to 10. In the mass range of interest, the same enhancement factor can be obtained for an equilateral-type non-Gaussianity for $f_{NL}^{equil} = 884$ and 510, respectively. The dependence of this result on the adopted value of σ_8 is small, from the range $0.77 < \sigma_8 < 0.84$, f_{NL} changes by less than 10%.

We should bear in mind that XMMUJ2235.3-2557 is an extremely rare object, sampling the tail of the mass function which may not be well known and may be strongly affected by cosmology. Using the [27] mass function we estimate that in the WMAP5 lambda cold dark matter model [28] one should find 7 galaxy clusters in the whole sky with mass greater or equal than the lower mass estimate of XMMUJ2235.3-2557 $M = 5 \times 10^{14} M_\odot$ and $z > 1.4$ corresponding to a probability of 0.002 for the 11 deg^2 of the survey. This should be compared with the reported number of 0.005 obtained by [11] for a different cosmology and different mass function. Thus the effects of cosmology and uncertainty in the mass function can account for a factor ~2 uncertainty in the predicted halo abundance.

Note that in all our calculations we have used a conservative lower limit for the mass of the cluster. If instead we use the central or upper value for the mass, using the WMAP5 cosmology and the [27] mass function we expect to find *zero* such clusters in the whole sky, which will make our conclusions even stronger.

The survey area used in Ref. [11] is 11 deg^2, but the XMM serendipitous survey in 2006 covered 168 deg^2, and today covers ~400 deg^2. Below, we report the Ref. [11] numbers and in parenthesis the numbers we obtain. The probability of finding XMMUJ2235.3-2557 is thus 0.005 (0.002) if using 11 deg^2; to avoid as much as possible biases due to *a posteriori* statistics one could use 168 deg^2 obtaining a probability of 0.07 (0.03), or, as a limiting case, even 0.17 (0.07) if using 400 deg^2. Note that it is likely that there are more clusters as massive in the survey area [29] and therefore these numbers are conservative. If we use from Fig. 1 the factor 3 to 10 enhancement, we find that it would help boost the probability to ~1 in the surveyed areas.

The latest WMAP compilation [13] reports $-9 < f_{NL}^{local} < 111$ and $-151 < f_{NL}^{equil} < 253$ at 95% confidence, [10] reports $27 < f_{NL}^{local} < 147$. The CMB however probes much larger scales ($R > 120$ Mpc/h) than those probed by clusters such as XMMUJ2235.3-2557 $R \sim 13$ Mpc/h: a scale-dependent f_{NL} with $k \sim -0.3$ can yield an effective f_{NL} on dependence XMMUJ2235.3-2557 scales that is larger than the CMB one by a factor of 3.

The f_{NL}^{local} values needed to accomodate the observed cluster at $z = 1.4$ is in the range 150 to 260. This is comparable to the limits quoted by Refs. [10,13], but slightly above Ref. [30], although we emphasize the fact that we are measuring f_{NL} at different scales than the CMB.

IV. CONCLUSIONS

Accurate masses of high-redshift clusters are now becoming available through weak-lensing analysis of deep images. As already discussed in previous papers [1,5], their abundance can be used to put constraints on primordial non-Gaussianity. f_{NL}^{local} in the range 150–260 can boost the expected number of massive ($> 5 \times 10^{14} M_\odot$) high-redshift ($z > 1.4$) clusters by factors of 3 to 10. Such large numbers would help make clusters like XMMUJ2235.3-2557 much more probable. The scales probed by clusters are smaller than the CMB scales, and in principle non-Gaussianity may be scale-dependent, making this a complementary approach, in fact some inflation models like Dirac-Born-Infeld predict scale-dependent $f_{\rm NL}$ [5].

The adopted error range in the mass determination of XMMUJ2235.3-2557 is 100%; even with such a large mass uncertainty and considering the pessimistic estimate of 7 such objects expected in the entire sky with a Poisson error of ± 2.6, if the entire sky could be covered, $f_{\rm NL}^{\rm local} \sim 150$ could be detected at $>4\sigma$ level.

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[1] S. Matarrese, L. Verde, and R. Jimenez, Astrophys. J. **541**, 10 (2000).
[2] J. Robinson, E. Gawiser, and J. Silk, Astrophys. J. **532**, 1 (2000).
[3] J. A. Willick, Astrophys. J. **530**, 80 (2000).
[4] L. Verde, R. Jimenez, M. Kamionkowski, and S. Matarrese, Mon. Not. R. Astron. Soc. **325**, 412 (2001).
[5] M. Lo Verde, A. Miller, S. Shandera, and L. Verde, J. Cosmol. Astropart. Phys. 4 (2008) 014.
[6] M. Kamionkowski, L. Verde, and R. Jimenez, J. Cosmol. Astropart. Phys. 1 (2009) 010.
[7] M. Grossi, L. Verde, C. Carbone, K. Dolag, E. Branchini, F. Iannuzzi, S. Matarrese, and L. Moscardini, Mon. Not. R. Astron. Soc. **398**, 321 (2009).
[8] A. Pillepich, C. Porciani, and O. Hahn, arXiv:0811.4176.
[9] V. Desjacques, U. Seljak, and I. T. Iliev, Mon. Not. R. Astron. Soc. **396**, 85 (2009).
[10] A. P. S. Yadav and B. D. Wandelt, Phys. Rev. Lett. **100**, 181301 (2008).
[11] M. J. Jee *et al.*, Astrophys. J. **704**, 672 (2009).
[12] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1997).
[13] E. Komatsu *et al.*, Astrophys. J. Suppl. Ser. **180**, 330 (2009).
[14] P. Coles and J. D. Barrow, Mon. Not. R. Astron. Soc. **228**, 407 (1987).
[15] L. Verde, L. Wang, A. F. Heavens, and Kamionkowski, Mon. Not. R. Astron. Soc. **313**, 141 (2000).
[16] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001).
[17] Which, on sub-Hubble spaces reduces to the usual Newtonian potential with a negative sign.
[18] X. Luo, Astrophys. J. **427**, L71 (1994).
[19] T. Falk, R. Rangarajan, and M. Srednicki, Astrophys. J. **403**, L1 (1993).
[20] A. Gangui, F. Lucchin, S. Matarrese, and S. Mollerach, Astrophys. J. **430**, 447 (1994).
[21] Z. Fan and J. M. Bardeen, University of Washington, Report No. UW-PT-92-11, 1992.
[22] J. N. Fry, Astrophys. J. **308**, L71 (1986).
[23] J. Lee and S. F. Shandarin Astrophys. J. **500**, 14 (1998).
[24] R. Sheth and H. J. Mo and G. Tormen, arXiv:astro-ph/9907024.
[25] M. Maggiore and A. Riotto, arXiv:0903.1250.
[26] B. Robertson, A. Kravtsov, J. Tinker, and A. Zentner, Astrophys. J. **696**, 636 (2009).
[27] R. Sheth and G. Tormen, Mon. Not. R. Astron. Soc. **308**, 119 (1999).
[28] The cosmological parameter which uncertainty affect most the cluster abundance is σ_8, the value adopted is $\sigma_8 = 0.77$. However, if we were to adopt a value of $\sigma_8 = 0.84$, $f_{\rm NL}$ would be lower by only 10%.
[29] S. A. Stanford *et al.*, Astrophys. J. Lett. **646**, L13 (2006).
[30] A. Slosar, C. Hirata, U. Seljak, S. Ho, and N. Padmanabhan, J. Cosmol. Astropart. Phys. 8 (2008) 031.