Supertubes

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It is shown that a IIA superstring carrying D0-brane charge can be "blown up," in a *Minkowski vacuum* background, to a (1/4)-supersymmetric tubular D2-brane, supported against collapse by the angular momentum generated by crossed electric and magnetic Born-Infeld fields. This "supertube" can be viewed as a world-volume realization of the sigma-model Q lump.

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(I) *Introduction.*—*M* theory can be viewed as an extension of string theory to a "democratic" theory of branes, with dualities that allow the whole theory to be constructed from any brane. One aspect of this is that a given brane may often be considered as a partially collapsed version of another, higher-dimensional, brane. Conversely, there may be circumstances in which the lower-dimensional brane is "blown up" into the higher-dimensional brane. One example of this is the observation that a IIA superstring can be blown up to a tubular D2-brane by placing it in an appropriate (nonsupersymmetric) background [1]; the background fields impose an external force that prevents the collapse of the D2-brane. Another example is the observation that a similar nontrivial background can blow up a collection of D0-branes into a fuzzy 2-sphere [2]; this can again be considered as a D2-brane prevented from collapse by an external force [3]. Another way to support a brane against collapse to a lower-dimensional one is to allow it to carry angular momentum. The case of branes on spheres, first analyzed in [4], has recently found an application to "giant gravitons" [5]; these partially preserve the supersymmetry of an $\mathrm{AdS}_n \times S^m$ supergravity background [6], a fact that is presumably related to the appearance of angular momentum in the anticommutator of anti–de Sitter (AdS) supersymmetry charges.

Here we shall show that a cylindrical, or tubular, D2-brane in a *Minkowski vacuum* spacetime can be supported against collapse by the angular momentum generated by crossed electric and magnetic Born-Infeld (BI) fields. The construction involves a uniform electric field along the tube, and a constant magnetic flux. The electric field can be interpreted as a "dissolved" IIA superstring, so the tube is a IIA superstring that has been blown up to a tubular D2-brane. The magnetic flux can be interpreted as a dissolved D0-brane charge (per unit length). There are thus similarities to some of the previously proposed stabilization mechanisms but also some differences. First, the background in our case is trivial; there is *no external force*. Second, the tubular D2-brane configuration presented here is *supersymmetric*; it preserves one-fourth of the supersymmetry of the IIA Minkowski vacuum, hence the term "supertube."

At first sight it would appear unlikely that a brane configuration stabilized by angular momentum could be supersymmetric. For a start, supersymmetry requires a time-independent energy density, which is certainly nongeneric for configurations with nonzero angular momentum. Even if this can be arranged, there is still the fact that a supersymmetric configuration in Minkowski space minimizes the energy subject to fixed values of the central charges appearing in the supersymmetry algebra, but *angular momentum is not one of these charges*. However, although these considerations may make supersymmetric stabilization by angular momentum unlikely, they do not make it impossible. Indeed, in the somewhat different context of field theory solitons there are *known* examples of supersymmetric solitons that are stabilized by angular momentum. One such example is the Q lump of certain massive $D = 3$ supersymmetric sigma models [7], and the D2-brane supertube is essentially its *world-volume* realization.

The Q lump saturates an energy bound of the form $M = |L| + |Q|$, where L is a topological (lump) charge and Q is a Noether charge [8]. When viewed as a stringlike solution of the maximally supersymmetric $D = 4$ sigma model, it preserves one-fourth of the supersymmetry of the sigma-model vacuum [9]. The simplest Q lump string has cylindrical symmetry [10], but for large Q its energy density is *not* concentrated on the axis of symmetry but rather in a hollow tube of radius Q. Therefore the Q lump-string tube can be interpreted, for large Q, as a lump string that has been blown up into a cylindrical tube of kink-2-brane [11]. This tube is prevented from collapse by a centrifugal force, generated by an angular momentum proportional to the charge Q. Because the effective action for the kink-2-brane is a Dirac-Born-Infeld (DBI) action [9], there should be a *world-volume* description of the blown-up sigma-model lump string as a (1/4)-supersymmetric solution of the $D = 4$ DBI equations. Any such solution would also solve the DBI equations of the $D = 10$ IIA D2-brane (in a IIA Minkowski vacuum) and must preserve at least two supersymmetries (this being one-fourth of the eight supersymmetries of the sigma-model vacuum). This solution is precisely the D2-brane

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supertube to be discussed below; it actually preserves eight of the thirty-two supersymmetries of the IIA vacuum and is therefore still one-fourth supersymmetric.

(II) *Energetics of the supertube.*—The D2-brane Lagrangian, for unit surface tension, is

$$\mathcal{L} \equiv -\Delta = -\sqrt{\det(g + F)}, \tag{1}$$

where g is the induced world-volume 3-metric and F is the BI 2-form field strength. We shall choose spacetime coordinates such that the $D = 10$ Minkowski metric is

$$ds^2 = -dT^2 + dX^2 + dR^2 + R^2 d\Phi^2 + ds^2(\mathbb{E}^6), \tag{2}$$

with $\Phi \sim \Phi + 1$. If we take the world-volume coordinates to be (t, x, φ) with $\varphi \sim \varphi + 1$, then we may fix the world-volume diffeomorphisms for a D2-brane of cylindrical topology by the "physical" gauge choice

$$T = t, \qquad X = x, \qquad \Phi = \varphi. \tag{3}$$

For a cylindrical D2-brane of (possibly varying, but time-independent) radius $R(x, \varphi)$ at a fixed position in $\mathbb{E}^6$ (and with the X axis as the axis of symmetry), the induced metric is

$$ds^2(g) = -dt^2 + dx^2 + R^2 d\varphi^2 + (R_x dx + R_\varphi d\varphi)^2, \tag{4}$$

where $R_x \equiv \partial_x R$ and $R_\varphi \equiv \partial_\varphi R$. We will allow for a time-independent electric field E in the x direction, and a time-independent magnetic field B, so that the BI 2-form field strength is

$$F = E dt \wedge dx + B dx \wedge d\varphi . \tag{5}$$

Under these conditions the Lagrangian becomes

$$\mathcal{L} = -\sqrt{(R^2 + R_\varphi^2)(1 - E^2) + B^2 + R^2 R_x^2}. \tag{6}$$

The corresponding Hamiltonian density is defined as

$$\mathcal{H} \equiv \Pi E - \mathcal{L}, \tag{7}$$

where $\Pi \equiv \partial \mathcal{L} / \partial E$ is the "electric displacement" subject to the Gauss law constraint $\partial_x \Pi = 0$.

Let us now focus on the D2-brane supertube, for which R is constant. In this case, the relation between the electric field and the electric displacement takes the form

$$E = \frac{\Pi}{R}\sqrt{\frac{B^2 + R^2}{\Pi^2 + R^2}}, \tag{8}$$

so the Hamiltonian density becomes

$$\mathcal{H} = R^{-1}\sqrt{(\Pi^2 + R^2)(B^2 + R^2)}, \tag{9}$$

where it should be noted that $B = \boldsymbol{\nabla} \times \boldsymbol{A}$ for BI 2-vector potential $\boldsymbol{A}$. The Gauss law constraint implies that Π is x independent. In addition, the equation of motion for $\boldsymbol{A}$ forces B to be x independent when R is constant, as we are now assuming. Under these conditions, $\mathcal{H}$ is x independent, and its integral over the circle parametrized by φ yields a constant energy per unit length, namely, the tube tension

$$\tau = \oint d\varphi\, \mathcal{H}. \tag{10}$$

This is a function of R and a functional of $\Pi(\varphi)$ and $B(\varphi)$. For an appropriate choice of units, the integrals

$$q_s \equiv \oint d\varphi\, \Pi \quad \text{and} \quad q_0 \equiv \oint d\varphi\, B \tag{11}$$

are, respectively, the conserved IIA string charge and the D0-brane charge per unit length carried by the tube. The total D0-brane charge is also conserved, so imposing periodic boundary conditions on the tube, with period L, implies conservation of q_0, and also that q_0 is quantized in multiples of some unit proportional to L^{-1}. For fixed values of these charges, the tube tension is bounded from below:

$$\tau \geq |q_s| + |q_0|, \tag{12}$$

with equality if, and only if,

$$\Pi = q_s, \qquad B = q_0, \qquad R = \sqrt{|q_s q_0|}. \tag{13}$$

The crossed electric and magnetic fields generate a Poynting 2-vector density with

$$\mathcal{P}_\varphi = \Pi B \tag{14}$$

as its only nonzero component. The integral of $\mathcal{P}_\varphi$ over φ yields an angular momentum per unit length $J = \Pi B = q_s q_0$ along the axis of the cylinder. It is this angular momentum that supports the tube at the constant radius $\sqrt{|q_s q_0|}$. By substituting (13) into (8) we see that

$$E = \mathrm{sgn}(\Pi) = \pm 1. \tag{15}$$

This would be the "critical" electric field if the magnetic field were absent, as is shown by the fact that $\Delta = B$ when $|E| = 1$.

(III) *Supersymmetry.*—We now aim to show that the tubular D2-brane configuration just described is one-fourth supersymmetric, but as we also aim to relate it to some previously discussed D2-brane configurations we shall now drop the assumption that the world-volume fields are independent of x and φ. The number of supersymmetries preserved by any D2-brane configuration is the number of independent Killing spinors ϵ of the background for which

$$\Gamma \epsilon = \epsilon, \tag{16}$$

where Γ is the matrix appearing in the "κ-symmetry" transformation of the world-volume spinors [12]. By introducing $\Gamma_\natural$ as the constant matrix, with unit square which anticommutes with all ten spacetime Dirac matrices, and $(\gamma_t, \gamma_x, \gamma_\varphi)$ as the induced world-volume Dirac matrices, we have [13]

$$\Gamma = \Delta^{-1}(\gamma_{tx\varphi} + E\gamma_\varphi \Gamma_\natural + B\gamma_t \Gamma_\natural). \tag{17}$$

For the D2-brane configuration of interest here

$$\gamma_t = \Gamma_{\underline{T}}, \qquad \gamma_x = \Gamma_{\underline{X}} + R_x \Gamma_{\underline{R}}, \qquad \gamma_\varphi = R\Gamma_{\underline{\Phi}} + R_\varphi \Gamma_{\underline{R}}, \tag{18}$$

where $\Gamma_{\underline{T}}$, $\Gamma_{\underline{X}}$, $\Gamma_{\underline{R}}$, and $\Gamma_{\underline{\Phi}}$ are the constant Minkowski spacetime Dirac matrices (with $\Gamma_{\underline{\Phi}}^2 = 1$). For the spacetime coordinates that we have chosen, any Killing spinor ϵ can be written as

$$\epsilon = M_+ \epsilon_0, \qquad M_\pm \equiv \exp(\pm \tfrac{1}{2}\Phi \Gamma_{\underline{R\Phi}}), \tag{19}$$

where ϵ_0 is a constant 32-component spinor of *spin*(1, 9). The condition for preservation of supersymmetry can now be written as

$$\begin{aligned} 0 = {} & M_+(RR_x \Gamma_{\underline{TR\Phi}} + B\Gamma_{\underline{T}}\Gamma_\natural + R_x R_\varphi \Gamma_{\underline{T}} - \Delta)\epsilon_0 \\ & + M_- \gamma_\varphi \Gamma_\natural (\Gamma_{\underline{TX}}\Gamma_\natural + E)\epsilon_0 . \end{aligned} \tag{20}$$

It is clear that in order to satisfy this equation for all values of φ (equal to Φ for our gauge choice) both terms on the right-hand side must vanish independently. From the vanishing of the second term we recover the condition that $E = \pm 1$, and we further deduce that ϵ_0 must satisfy

$$\Gamma_{\underline{TX}}\Gamma_\natural \epsilon_0 = -\text{sgn}(E)\epsilon_0 . \tag{21}$$

The vanishing of the first term leads to $R_\varphi = 0$ and

$$(RR_x \Gamma_{\underline{TR\Phi}} + B\Gamma_{\underline{T}}\Gamma_\natural)\epsilon_0 = \sqrt{R^2 R_x^2 + B^2}\, \epsilon_0. \tag{22}$$

For the supertube configuration $R_x = 0$, and B is constant, so the constraint above becomes simply

$$\Gamma_{\underline{T}}\Gamma_\natural \epsilon_0 = \text{sgn}(B)\epsilon_0 . \tag{23}$$

The two conditions (21) and (23) are compatible and imply preservation of one-fourth supersymmetry; the minimal energy tubular D2-brane configuration is a *supertube*. Note that the constraints (21) and (23) are those associated with, respectively, a IIA superstring (in the X direction) and D0-brane charge; in particular, there is no trace of the D2-brane in these conditions. The physical reason for this is that a cylindrical D2-brane carries no net D2-brane charge.

When $R_x \neq 0$, the vanishing of the first term on the right-hand side of (20) implies that

$$B = B_0 R R_x \tag{24}$$

for some constant B_0, and the constraint on the spinor becomes

$$(\Gamma_{\underline{TR\Phi}} + B_0 \Gamma_{\underline{T}}\Gamma_\natural)\epsilon_0 = \sqrt{1 + B_0^2}\, \epsilon_0 . \tag{25}$$

The Gauss law now implies that

$$R(x) = Ce^{x/E_0} \tag{26}$$

for some constants C and E_0. This is just the "BIon in a magnetic background" solution of [9] representing a IIA string ending on a bound state of D2-branes and D0-branes. To see this more clearly, first note that the constraint (25) indeed corresponds to that of a D2-D0 bound state in the (R, Φ) plane. Second, we can invert (26) to find

$$x = E_0 \ln(R/C), \tag{27}$$

which shows that x is a harmonic function on the D2-brane two-dimensional world space, as expected for the BIon [14,15]. Finally, by using (27) we can rewrite the BI field strength (5) as

$$F = \frac{E_0}{R} dt \wedge dR + B_0 R dR \wedge d\varphi . \tag{28}$$

This corresponds to a radial Coulomb-like electric field on the D2-brane world space, as expected from the charge at the end point of the string, and to a constant density of D0-brane charge per unit world space area, precisely as in the solution of [9].

(IV) *Discussion.*—By definition, a Dp-brane is a sink for IIA string charge. However, the D0-brane, being point-like, is special because the IIA charge has nowhere to go and must exit on another IIA superstring. Thus D0-branes can appear as "beads" on a IIA superstring, breaking the one-half supersymmetry of the string to one-fourth supersymmetry. Of course, quantum mechanics will ensure that the ground state of such a superstring is one for which the D0-brane charge is uniformly distributed along the string. This "charged" IIA superstring will have a tension exactly equal to the D2-brane supertube discussed above, $\tau = |q_s| + |q_0|$. What distinguishes it is the angular momentum; the charged superstring has zero angular momentum while the supertube has angular momentum per unit length J equal to $q_s q_0$. In principle, J can be specified *independently* of the string and D0-brane charges, so, given q_s and q_0, we might expect there to be some supersymmetric string/tube configuration with arbitrary J. As long as $|J| \leq |q_s q_0|$ it is not difficult to see what this will be: a supertube with angular momentum per unit length J, together with a charged superstring along its central axis. Because J is quantized in the same units as B, it is always possible for excess string and D0-charge to "condense" out of a tubular D2-brane to leave behind a supertube supported from collapse by an given $|J|$ less than $|q_s q_0|$. On the other hand, it is unclear what the ground state could be when $|J| > |q_s q_0|$. It is conceivable that $|q_s q_0|$ is an upper bound on the angular momentum of a *supersymmetric* IIA superstring configuration with charges q_s and q_0, and that supersymmetry is spontaneously broken when $|J| > |q_s q_0|$.

There is some similarity here to the status of angular momentum in the context of black holes of $D = 5$ supergravity [16–18]. The black hole mass is determined entirely by its charge, if it is supersymmetric. For a given charge there is a one-parameter family of supersymmetric black hole spacetimes, parametrized by an angular momentum, but there is a critical value of the angular momentum beyond which the physics is qualitatively different. There is also a similarity to the suggestion [6] that the "giant graviton" ground state is not supersymmetric above a critical value of the angular momentum, although the supersymmetry of relevance in that case is AdS rather than Poincaré.

Finally, we wish to comment on some M-theory configurations dual to the D2-brane supertube. The first one involves the M2-brane: the D2-brane action is equivalent to the action for the $D = 11$ supermembrane, the equivalence involving an exchange of the BI 1-form potential for a periodically identified scalar field Ψ (with unit period) representing position in the 11th dimension [13]. Specifically, one has

$$\partial_i \Psi = \frac{g_{ij} \varepsilon^{jkl} F_{kl}}{2\sqrt{-\det(g + F)}} . \tag{29}$$

For the D2-brane supertube this yields

$$\Psi = \Pi \varphi - t . \tag{30}$$

First, this implies that the $M2$-brane is wound Π times around the 11th dimension, as expected given the identification of Π with IIA string charge. Second, it implies that Π must be an integer, obviously equal to the number of IIA strings dissolved in the original D2-brane. Third, it implies that there is a wave at the speed of light in the 11th dimension, as expected from the D0-brane charge; the momentum of this wave is proportional to B. Note that if the x dimension is periodically identified with period L then one can take $L \rightarrow 0$ to arrive at a new (1/4)-supersymmetric IIA configuration in which a helical string rotates, producing a net momentum along the axis of the helix [19].

Another dual M-theory configuration consists of an $M5$-brane with topology $\mathbb{R}^4 \times S^1$, carrying dissolved membrane charges oriented along orthogonal planes in $\mathbb{R}^4$. To see this, let the S^1 be parametrized by φ, and let the two planes span the directions 1–2 and 3–4. Reducing to the IIA theory by compactifying the 4-direction, and T-dualizing along 2 and 3, leads to the D2-brane supertube.

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[1] R. Emparan, Phys. Lett. B **423**, 71 (1988).
[2] R. C. Myers, J. High Energy Phys. **9912**, 022 (1999).
[3] J. Pawelczyk and S-J. Rey, Phys. Lett. B **493**, 395 (2000).
[4] J. Hoppe and H. Nicolai, Phys. Lett. B **196**, 451 (1987).
[5] J. McGreevy, L. Susskind, and N. Toumbas, J. High Energy Phys. **0006**, 008 (2000).
[6] M. T. Grisaru, R. C. Myers, and O. Tafjord, J. High Energy Phys. **008**, 040 (2000).
[7] R. Leese, Nucl. Phys. **B366**, 283 (1991).
[8] E. R. C. Abraham, Phys. Lett. B **278**, 291 (1992).
[9] J. P. Gauntlett, R. Portugues, D. Tong, and P. K. Townsend, Phys. Rev. D **63**, 085002 (2001).
[10] This is an oversimplification because the integral of the charge density of the cylindrically symmetric solution has an infrared logarithmic divergence; for this reason the charge Q mentioned below should be regarded only as a measure of scale.
[11] E. R. C. Abraham and P. K. Townsend, Phys. Lett. B **295**, 225 (1992).
[12] See, for example, E. Bergshoeff, R. Kallosh, T. Ortín, and G. Papadopoulos, Nucl. Phys. **B502**, 149 (1997).
[13] See, for example, E. Bergshoeff and P. K. Townsend, Nucl. Phys. **B490**, 145 (1997).
[14] C. G. Callan and J. M. Maldacena, Nucl. Phys. **B513**, 198 (1998).
[15] G. W. Gibbons, Nucl. Phys. **B514**, 603 (1998).
[16] J. C. Breckenridge, D. A. Lowe, R. C. Myers, A. W. Peet, A. Strominger, and C. Vafa, Phys. Lett. B **381**, 423 (1996).
[17] J. P. Gauntlett, R. C. Myers, and P. K. Townsend, Classical Quantum Gravity **16**, 1 (1999).
[18] G. W. Gibbons and C. A. R. Herdeiro, Classical Quantum Gravity **16**, 3619 (1999).
[19] We owe this observation to Barak Kol and Juan Maldacena; note that T duality along the axis of the helix interchanges the string with the wave, yielding a IIB version of the rotating helical string.