Euro-dollar real exchange rate misalignments: Is the euro overvalued?*

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Abstract: This paper estimates both short term and long run relationship between the real bilateral EUR-USD exchange rate and its real determinants. In the long run, it finds that the non-stationary real exchange rate in levels is linearly co-integrated with real variables. Using an ordinary least squares method with error correction mechanism, it investigates real EUR-USD exchange rate misalignment in the short term. By analysing real variables and their influence on international trade and capital movements, potential economic policies capable of maintaining equilibrium in the balance of payments and avoiding currency overvaluation are considered.

Key words: real exchange rate misalignment; purchasing power parity; co-integration; unit roots test; the Ricardo-Samuelson-Balassa effect

1. Introduction

Unites States dollar exchange rate fluctuations have allowed the growing relevance of substitute currencies and other payment forms in the international markets to be observed. In both academic and business environments, research related with exchange rates are pertinent not only because it can explain balance of payment crises, but also because it is connected with matters such as competitiveness and economic growth, exchange rate structural changes, and generally most of the macroeconomics time series structural changes, i.e., changes in terms of trade, public expenditure, net foreign assets, balance of trade or productivity, affect monetary and capital markets equilibriums.

Real variable behaviour explains real exchange rate in equilibrium (RERe) movements between two different currencies and affect the real exchange rate (RER). The percental difference between the RERe and the RER represents the level of overvaluation.

This literature has been debated. Dornbusch (1976) analyses capital mobility in the Mundell (1964) and Fleming (1962) model with an open economy. His research focus on exchange rates determinants in the short term and international transmissions of monetary perturbations in the long run. Edwards (1988), based on Balassa (1964) and Samuelson (1964), finds other real determinants of the RER, and redefines the theory of the international economy equilibrium. Meese and Rogoff (1988), Froot and Rogoff (1994) and Clarida and Gali (1994) empirical evidence shows how significant monetary variables are in explaining exchange rates policies, but they do not agree about the levels or the sings of these changes affecting the RER behaviour.

This paper looks again at the co-integration theory to estimate the euro-dollar RERe. Through real variables,

* This research was presented at the Universitat de Barcelona in 2008 through the DEA's dissertation. Thank to seminar participants at the DIW macro econometrics workshop for comments and discussions.

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a short term econometric model and the RERe time series can be built. The real exchange misalignment is calculated by comparing the adjusted results with the RER data in levels. Cointegration techniques permit the establishment of long run relationships between the RER and its determinants. Following this, purchasing power parity is contrasted with the ordinary least squares (OLS) model using the error correction mechanism (ECM). The subsequent chapters follow an examination of the theoretical framework and the RER definition. Secondly, the implemented methodology contrasting the empirical evidence is specified. Finally, the results and conclusions are presented.

2. Theoretical framework

2.1 Mundell-Fleming model and the RERe with real determinants

There are two main approaches which explain RERe. The first approach is based on purchasing power parity (PPP) theory and the Mundell-Fleming model analysed by Dornbusch (1976). This theory extends the IS-LM model with free capital mobility and flexible prices in a flexible exchange rate framework. Under these monetary assumptions, changes in the nominal exchange rate are diminished by the domestic-foreign price relationship between countries. Monetary variations change prices, however, the nominal exchange rate reflects and opposes international price changes in a way that ensures that the PPP or the RER are constant over time.

In the long run, the Mundell-Fleming model with flexible prices in a flexible exchange rate framework predicts that monetary expansion increases money supply, prices and nominal exchange rate, but it does not affect real variables such as RER. Nominal exchange rate depreciation keeps the purchasing power of domestic goods, with respect foreign goods, in between the initial and the final equilibrium points. This fact implies international prices level equivalence when it is measured as a function of only one currency, value of money international equivalence, equilibrium stability in the PPP, neutrality of the long run RERe changes and the money causality function.

2.1.1 Mundell-Fleming model and the exchange rate overreaction

Assuming perfect capital mobility, the balance of payments is in equilibrium ($BP=0$) when the domestic interest rate is equal to the foreign interest rate ($i=i^*$).

(1) Mundell-Fleming simple model

This model explains balance of payments disequilibria and the monetary policy maker role within a fixed exchange rate system and perfect mobility of capital. IS-LM model with an open economy will be shown graphically. The macroeconomic variable effects are the following: a. Monetary contraction →b. Interest rate increases →c. Capital flow from foreign to the domestic economy and balance of payment surplus →d. Pressure on currency appreciation →e. Monetary authority intervention selling national currency and buying foreign currency →f. Money supply increases by the monetary authority, reduce the interest rate →g. Interest rates, money supply and balance of payments return to equilibrium.

In the case of money supply expansion (see Fig. 1), if monetary supply increases, the *LM* curve moves to *LM'* and the interest rate decreases. Because the domestic interest rate is lower than the foreign interest rate, capital flows to the foreign economy (*E'*), producing both pressures over the exchange rate and balance of payments deficit. Monetary authorities sell foreign currencies and buy domestic currency until money supply decreases and the *LM* curve returns to the initial equilibrium.

(2) Mundell-Fleming model adding flexible exchange rate

This model explains balance of payments disequilibria and the monetary policy maker role with a flexible exchange rate system and perfect mobility of capital.

In the case of money supply expansion (see Fig. 2), when money supply increases, the *LM* curve moves to *LM'*. Goods and money markets are in a new equilibrium at the point *E'*. Domestic interest rates decrease, and capital flows to foreign economies. This produces a balance of payments deficit, and leads to exchange rate depreciation. National currency depreciation improves relative domestic competitiveness. Exports expands and the *IS* curve moves to the right. The inner process is repeated until the *IS* curve reaches *E"* point. The domestic interest rate equals the foreign interest rate and currency depreciation augments the level of income. Expansive monetary policies expand exports.

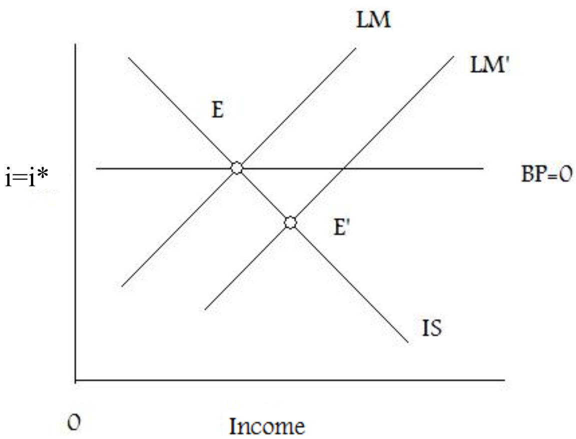


Fig. 1 Mundell-Fleming simple model

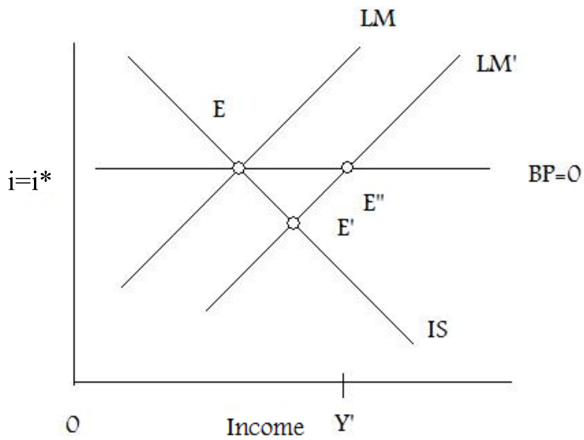


Fig. 2 Mundell-Fleming model adding flexible exchange rate

(3) Mundell-Fleming model adding flexible exchange rate and flexible prices

This model explains balance of payments disequilibria and the monetary policy maker role within a flexible exchange rate system, perfect mobility of capital and flexible prices.

In the case of money supply expansion (see Fig. 3), point *E'* is the initial equilibrium and the supply expansion moves *LM* curve to *LM'*. The new goods and money markets equilibrium *E'* implies an interest rate which is lower than the foreign interest rate. Capital flows to the foreign economy produce exchange rate depreciations. The *IS* curve moves to *IS'* and the domestic economy goes from point *E'* to point *E"*. At this point, higher levels of income increase prices if the labour demand is lower than the labour supply. Price increases reduce real variables and moves the *LM* curve back to the initial equilibrium *E*. Changes in the real variables increase the interest rate, with capital flowing to the domestic economy and appreciating the national currency. The *IS'* moves to the initial equilibrium *E*. In the long run, production comes back to initial levels and money, prices and exchange rate increases in the same proportion.

(4) Exchange rate overreaction

This model explains exchange rate and price adjustments, with a flexible exchange rate system, perfect mobility of capital and flexible prices.

In the case of monetary supply expansion (see Fig. 4), initially, the domestic economy is in equilibrium and all the indexes are equal to 100. Money supply increases permanently by 50% at *To*. Exchange rate depreciation from *A* to *A'* is higher than money supply increase. Prices adjust slowly. In the short term, prices of imports increase and relative domestic prices decrease. An improvement in relative domestic competitiveness expands transitorily domestic income. Commercial trade increases domestic prices and leads to an exchange rate appreciation, neutralizing the initial overreaction. In the long run, nominal money, exchange rates and prices increase by the same proportion (from 100 to 150), keeping real variables and relative prices constant.

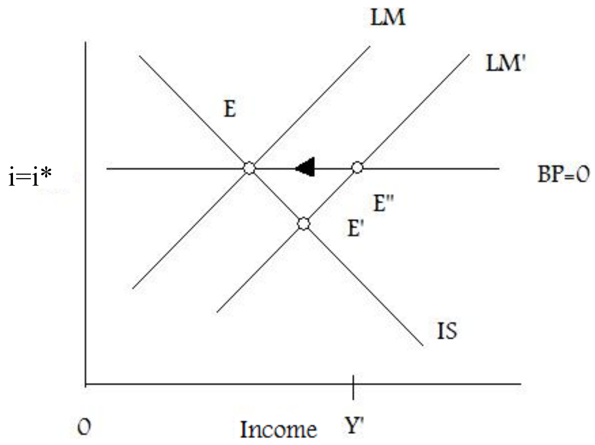


Fig. 3 Mundell-Fleming model adding flexible exchange rate and flexible prices

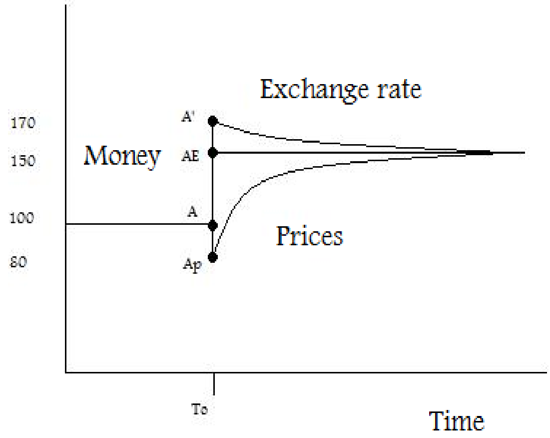


Fig. 4 Exchange rate overreaction

Domestic or foreign price changes are compensated by nominal exchange rate variations, so that *RER* is constant in the long run. The model is the following:

$$RER = c_1 * [(NER * PE)/PD] \quad (1)$$

where *NER* is the nominal exchange rate, *PE* is external price for goods and services and *PD* is the domestic price for goods and services. Assuming *PPP*:

$$NER = c_2 * (PD/PE) \quad (2)$$

Substituting equation (2) in equation (1), obtains:

$$RERe = c_1 * c_2 = c_3 \quad (3)$$

Any condition misalignment equation (3) is temporary and is associated with transitory and speculative deviations.

On the other hand, a second approach supported by Edwards (1988) shows that the RERe behaviour is not explained only by monetary variables, but also by real variables. *RER* changes are not transitory. Fundamental real determinants cause permanent misalignments in the RERe. “The equilibrium real exchange rate is relative price of tradables to non-tradables that, for given sustainable (equilibrium) values of other relevant variables such as taxes, international prices and technology, results in the simultaneous attainment of internal and external equilibrium. Internal equilibrium means that the non-tradable goods market clears in the current period, and is expected to be in equilibrium in future periods. In this definition of equilibrium RER, it implies the idea that this equilibrium takes place with unemployment at the “natural” level. External equilibrium, on the other hand, is attained when the inter temporal budget constraint that states that the discounted sum of a country’s current account has to be equal to zero, is satisfied. In other words, external equilibrium means that the current account balances (current and future) are compatible with long run sustainable capital flows” (Edwards, 1988).

The general inter temporal subjacent equilibrium theory, with real determinants, contradicts the PPP theory because of the following reasons: a. contrary to the assumption of the law of one price, transport costs and foreign trade restrictions exist and impede the free trade of good and services, b. monopolistic and oligopolistic practices impede in many cases, price equalisation between 2 countries, c. the basket of goods and services used to measure the consumer price index vary across countries, d. PPP do not take into account tradable goods, differences in productivity levels and preference changes between economic agents and, e. RERe does not necessarily have to be constant in the long run.

In short, the theory maintains that nominal exchange rates changes do not necessarily oppose international prices ratio to maintain RER constant over time. The core proposes a relationship between RERe and the following real variables:

$$RERe = c_4TT + c_5PE + c_6NFA + c_7BT + c_8PR + c_9CR \quad (4)$$

where TT is terms of trade, PE is public expenditure, NFA is net foreign assets, BT is balance of trade or openness level and PR is productivity. There is a monetary variable in the lineal model to test the interest rate differential or country risk influence over the RERe.

2.1.2 Balassa-Samuelson theory and RERe determinant signs

The Balassa-Samuelson theory is usually related to two assumptions: (1) Non-tradable goods prices grow faster than tradable goods prices and, (2) the productivity growth rate of tradable goods relative to the non-tradable goods productivity is higher in countries which are tradable goods intensive. According to this theory, the prices of non-tradable goods grow higher than tradable goods price growth because the productivity growth rate of tradable goods is higher than the productivity growth rate of tradable goods. If the growth rate of the productivity tradables-non-tradables ratio is higher in domestic economies than in foreign economies, the domestic economy RER decreases or has an appreciation.

In this model, there are two countries with tradable and non-tradable goods with competitive labour markets for each country. The tradable goods sector presents higher relative productivity, and workers mobility in both productive tradable and non-tradable sectors is perfect. PPP is valid only for tradable goods but non-tradable goods prices are different across countries. There is perfect mobility of capital. Tradable and non-tradable production functions YT = ATF (KT, LT) and YNT = ANTF (KNT, LNT), satisfying the following conditions:

(1) Constant returns to scale in $F(.)$

Multiplying each input K and L by λ, obtains: $AF(\lambda K, \lambda L) \rightarrow \lambda AF(K, L)$ for all $\lambda > 0$. Where K is capital, L is labour, A is technology and λ is a constant.

(2) Positive and diminishing returns to private inputs.

Calculating derivatives of $F(.)$ with respect to each input:

$$\partial F/\partial K = r > 0,\ \partial^2 F/\partial K^2 < 0$$
$$\partial F/\partial L = w > 0,\ \partial^2 F/\partial L^2 < 0$$

where r is the marginal product of capital and w is the marginal product of labour.

(3) Inada condition

In the limit, the first derivatives of $F(.)$ with respect to each input satisfying the following conditions:

$$Lim_{K\rightarrow 0}(\partial F/\partial K) = lim_{L\rightarrow 0}(\partial F/\partial L) = \infty$$
$$Lim_{K\rightarrow \infty}(\partial F/\partial K) = lim_{L\rightarrow \infty}(\partial F/\partial L) = 0$$

Note that the marginal product of each input depends on the capital-labour ratio $k=K/L$.

Moreover, $Y = AF(K, L) \rightarrow Y = ALF(K/L, L/L) \rightarrow Y = ALF(K/L, 1) \rightarrow Y = ALF(k,1) \rightarrow Y = ALf(k) \rightarrow Y = ALf(K/L)$.

$$\partial Y/\partial K = \partial ALf(K/L)/\partial K = A[Lf'(K/L) * (1/L)] = A(L/L)f'(K/L) = Af'(K/L) = Af'(k) \quad (5)$$
$$\partial Y/\partial L = \partial ALf(K/L)/\partial L = A[(1 * f(K/L)) + Lf'(K/L) * ((0*L-K*1)/L^2) = A[f(K/L)$$

$$+ (L/L)f'(K/L) * (-K/L)] = A[f(K/L)-f'(K/L) (K/L)] = A[f(k)-f'(k)k] \quad (6)$$

The firm maximization problem is the following:

Maximize profit $(\pi) = \Sigma_t^{\infty} (1 / (1+z))^t [P * AF(K,L) - wL - rK]$, such that conditions *A, B* and *C* are satisfied. Where z is the discount factor, P is the goods and services prices, w are the wages to workers, r is the capital price and, it is assumed for simplicity, that capital depreciation is equal to zero.

Rewriting equation (5) and equation (6), first order conditions are the following:

$$\partial\pi/\partial K = 0 \rightarrow P * Af'(k)-r = 0 \rightarrow r = P * Af'(k) \quad (5')$$

$$\partial\pi/\partial L = 0 \rightarrow P * A[f(k)-f'(k)k]-w = 0 \rightarrow w = P * A[f(k)-f'(k)k] \quad (6')$$

Tradable goods sector:

$$r = P_T * A_T f'(k)$$
$$w = P_T * A_T[f(k)-f'(k)k]$$

Non-tradable goods sector:

$$r = P_{NT} * A_{NT} f'(k)$$
$$w = P_{NT} * A_{NT}[f(k)-f'(k)k]$$

where T is tradable goods and NT are non-tradable goods. It is assumed that the level of prices is defined in geometric averages with weights equal to γ and $1-\gamma$ for tradable goods prices and non-tradable goods prices respectively.

$$PD = PD_T^{\gamma} * PD_{NT}^{1-\gamma} \quad (7)$$

$$PE = PE_T^{\gamma} * PE'^{1-\gamma}_{NT} \quad (8)$$

where PD is goods and services at domestic prices and PE is goods and services at foreign prices.

Taking into account the perfect mobility of labour in between both tradable and non-tradable productive sectors, the following is obtained for each country:

$$PD_T * AD_T[d(k)-d'(k)k] = w = PD_{NT} * AD_{NT}[d(k)-d'(k)k] \quad (9)$$

$$PE_T * AE_T[g(k)-g'(k)k] = w = PE_{NT} * AE_{NT}[g(k)-g'(k)k] \quad (10)$$

where D is the domestic country and E is the foreign country. Without losing generalization, tradable goods prices can be equal to the numeraire $(P_T^{\gamma} = P'^{\gamma}_T = 1)$. Rewriting equation (9) and equation (10):

$$PD_{NT} = AD_T[d(k)-d'(k)k]/AD_{NT}[d(k) - d'(k)k] \quad (9')$$

$$PE_{NT} = AE_T[g(k)-g'(k)k]/AE_{NT}[g(k) - g'(k)k] \quad (10')$$

Similarly, rewriting equation (7) and equation (8):

$$PD = (1)^{\gamma} * PD_{NT}^{1-\gamma} = PD_{NT}^{1-\gamma} \quad (7')$$

$$PE = (1)^{\gamma} * PE_{NT}^{1-\gamma} = PE_{NT}^{1-\gamma} \quad (8')$$

Real exchange rate is defined as: $RER = c_1 * [(NER * PE) / PD]$. Using the PPP assumption in the tradable goods market and substituting the numeraire:

$$P_T = NER * P'_T \rightarrow 1 = NER * 1 \rightarrow NER = 1. \textit{ Finally, } RER = c_1 * [PE/PD] \quad (11)$$

Substituting equation (9') and equation (10') on equation (7'), equation (8') and equation (11) the Balassa-Samuelson effect can be obtained:

$$RER = c_1 * \left[\frac{AE_T[g(k)-g'(k)k]/AE_{NT}[g(k)-g'(k)k]}{AD_T[d(k)-d'(k)k]/AD_{NT}[d(k)-d'(k)k]} \right]^{1-\gamma}$$

If an increase in the tradable goods productivity, relative to non-tradable goods productivity, is higher in the domestic economy than in the foreign economy, the RER decreases and has an appreciation.

The perfect mobility of capital assumption implies that non-tradable goods prices change as a result of wage changes. Customers' decisions between saving and consumption are given and stay constant because the capital price is in its equilibrium level. The Balassa-Samuelson is robust including when there are international differences between consumer preferences. Nominal exchange rates oppose only changes in tradable goods prices. Assuming a non-tradable sector intensive in labour and a tradable sector intensive in capital, the Balassa-Samuelson theory explains that domestic economic growth increases technological progress and improves tradable goods productivity levels relative to non-tradable goods productivity. This productivity improvement in the domestic economy, relative to the foreign economy, decreases the RER. Moreover, if the non-tradable goods prices increase in the domestic economy, the RER decreases.

Example of the Balassa-Samuelson effect:

$$\uparrow AE_T \text{ or } \downarrow AE_{NT} \rightarrow \uparrow PE_{NT}^{1-\gamma} \rightarrow \uparrow TCR \text{ (depreciation in the domestic currency)}$$

$$\uparrow AD_T \text{ or } \downarrow AD_{NT} \rightarrow \uparrow PD_{NT}^{1-\gamma} \rightarrow \downarrow TCR \text{ (appreciation in the domestic currency)}$$

In other words:

$$\partial TCR/\partial AE_T > 0 = \partial TCR/AE_{NT} < 0,\ \partial TCR/\partial AD_T < 0 = \partial TCR/\partial AD_{NT} > 0,$$

RERe real determinant signs are the following:

a. Terms of trade (TT): An increase in the international relative price of imports, as a proxy of TT, implies the following effects over RER:

- Income effect: An increase in export prices (or a decrease in import prices) augments TT and improves wages, encouraging goods and services consumption. This productivity growth in tradable goods or an increase in nontradable goods prices decreases or appreciates RER (negative relationship).
- Substitutive effect: A decrease in export prices (or an increase in the import prices) decreases TT and increases substitution of foreign goods by domestic goods. This productivity growth in tradable goods or the increase in nontradable goods prices decreases or appreciates RER (positive relationship).

The relationship TT and RER depends on the dominant effect:

An example of when the income effect dominates: ↑ Export prices (or ↓ Import prices)→↑ TT → ↑ Wages and domestic goods and services consumption→↑ AD_T (relative to ↓ AD_{NT}) ↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (negative relationship).

An example of when the substitutive effect dominates: ↓ Export prices (or ↑ Import prices) → ↓ TT → ↑ Substitution of foreign goods by domestic goods → ↑ AD_T (relative to ↓ AD_{NT})→↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (positive relationship).

b. Public expenditure (PE): The PE effect over the RER behaviour depends on the expenditure composition in tradable or non-tradable goods. This effect also depends on how governmental expenditure is financed as levels of investment, consumption and resources can be modified in the private sector. An increase in public expenditure implies the following effects over RER:

- Direct effect: An increase in government demand of domestic goods and services incentives domestic production to grow. This productivity growth in tradable goods or an increase in nontradable goods prices, decreases or appreciates RER (negative relationship).
- Indirect effect: If the increase in government demand of domestic goods and services overshoots private consumption, the productivity decrease in tradable goods or the decrease in nontradable goods prices increases or depreciates RER (positive relationship).

The relationship between PE and RER depends on the difference between both the marginal domestic propensity to consume in the public and private sectors:

An example when direct effect dominates: ↑ PE in domestic consumption → ↑ AD_T (relative to ↓ AD_{NT}) → ↑ $PD_{NT}^{1-\gamma}$→↓ RER (negative relationship).

An example when indirect effect dominates: ↑ PE in domestic consumption → ↓ Private domestic consumption → ↓ AD_T (relative to ↑ AD_{NT}) ↓ $PD_{NT}^{1-\gamma}$ → ↑ RER (positive relationship).

c. Net foreign assets (NFA): This variable is a measurement of wealth of the national agents in a foreign currency. There are two transmission mechanisms to the *RER*:

- Income effect: An increase in net foreign assets increases wealth and domestic consumption levels. This growth in productivity of tradable goods or an increase in non-tradable goods prices decreases or appreciates *RER* (negative relationship).
- Substitutive effect: A fall in net foreign assets decreases the savings and investment levels and increases domestic consumption. The productivity growth in tradable goods or the increase in non-tradable goods prices decreases or appreciates RER (positive relationship).

The relationship between NFA and RER depends on the dominant effect:

Example when income effect dominates: ↑ NFA → ↑ Domestic goods and services consumption → ↑ AD_T (relative to ↓ AD_{NT}) ↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (negative relationship).

Example when substitutive effect dominates: ↓ NFA→ ↑ Domestic goods and services consumption → ↑ AD_T (relative to ↓ AD_{NT}) → ↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (positive relationship).

d. Balance of trade (BT): This variable represents the level of trade openness. It is a measure of wealth of national agents in a foreign currency. There are two transmission mechanisms to the RER:

- Income effect: An increase in the level of trade openness (through diminishing tariffs or improving international trade bureaucratic procedures) decreases international trade discretional distortions and augments wealth, increasing private domestic consumption. This productivity growth in tradable goods or the increase in non-tradable goods prices decreases or appreciates RER (negative relationship).
- Substitutive effect: A decrease in the level of trade openness (through diminishing tariffs or improving international trade bureaucratic procedures) decreases international trade discretional distortions and augments wealth, increasing private foreign consumption. This productivity growth in foreign tradable goods or increase in foreign non-tradable goods prices increases or depreciates RER (positive relationship).

The relationship between *BT* and *RER* depends on the dominant effect:

Example of when income effect dominates: ↑ BT → ↑ Domestic goods and services consumption → ↑ AD_T (relative to ↓ AD_{NT}) ↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (negative relationship).

Example of when substitutive effect dominates: ↑ BT → ↑ Foreign goods and services consumption → ↑ AE_T (relative to ↓ AE_{NT}) → ↑ $PE_{NT}^{1-\gamma}$ → ↑ RER (positive relationship).

e. Productivity (PR): An increase in productivity implies an improvement in the production capacity in the economy and allows for an increase in its level of economic activity. An increase in the productivity of tradable goods or an increase in the price of non-tradable goods diminishes or appreciates the RER (negative relationship).

Example: ↑ PR → ↑ AD_T (relative to ↓ AD_{NT}) → ↑ $PD_{NT}^{1-\gamma}$ → ↓ RER (negative relationship).

f. Interest rate differential or country risk (CR): This variable represents the short term interest rate differential across countries. There are two transmission mechanisms to the RER:

- Short term effect: An increase in the domestic interest rate (or a decrease in the foreign interest rate) increases the domestic end foreign incentives to save in the domestic economy, promoting capital flow from the foreign economy to the domestic economy and improving the balance of payments. A decrease in the foreign currency demand decreases or appreciates RER (negative depreciation).

- Lung run effect: A convergence in international real interest rates (in between domestic and foreign interest rates) reduces international monetary distortions and modifies customer preferences, changing future consumption for present consumption. This growth in the productivity of tradable goods or the increase in domestic non-tradable goods prices decreases or appreciates RER. If CR converges to 1, the RER decreases and when CR tends to disperse from 1, the RER increases.

Example when short term effect dominates: ↑ Domestic interest rate → ↑ Foreign capital flows to domestic economy → ↓ NER → ↓ RER (negative relationship).

3. Methodology

Basically, there are three econometrics techniques used as a methodology. The first procedure is the augmented Dickey-Fuller test to find unit roots. The optimum lags order is calculated running the Schwarz information criterion, and the critical values are based in MacKinnon to 1%, 5% and 10%. The second procedure is the Johansen method to test the number of co-integration vectors under unrestricted intercepts and restricted tends assumptions. Finally, the third procedure is a logarithm to minimize the sum of squared residuals of a lineal regression under estimator efficient properties. The ordinary least squares (OLS) model includes the stationary vectors found through using the cointegration procedure. The adjustment of the model to the data taking into account the R^2 is consider, as it is the individual significance of the estimators, the autocorrelation, the heteroskedasticity and the normality of the errors.

3.1 Unit roots test

The augmented Dickey-Fuller (1979) unit roots test presents one unit root ($I(1)$) as a null hypothesis and MacKinnon (1991) critical values to 1%, 5% and 10%.

Consider a sample regression function represented by a first order autoregressive process or:

$$AR(1):\ y_t = \rho y_{t-1} + f(t) + \varepsilon_t \tag{12}$$

where $f(t)$ can be zero (0), constant (δ) or constant and tend ($\delta + \beta_0 t$) and where $\varepsilon_t \sim iid\ (0,\sigma_\varepsilon^2)$. Subtracting y_{t-1} in both sides of the equation (12):

$$\Delta y_t = \alpha y_{t-1} + f(t) + \varepsilon_t \tag{13}$$

where $\alpha = \rho\text{-}1$. The statistical hypothesis are the following: H_0: $|\alpha| = 0 \equiv y_t \sim I(1)$ y H_1: $|\alpha| < 0 \equiv y_t \sim I(0)$. One of the two Dickey-Fuller (1979) statistics test proposes to evaluate the integration order of the variable y_t, the pseudo t-ratio:

$$t_{\alpha^\wedge} = \alpha^\wedge \ / \ (se(\alpha^\wedge)) \tag{14}$$

where $\alpha^\wedge$ is the OLS estimation of α in the model (13) and $se(\alpha^\wedge)$ is the standard error coefficient of $\alpha^\wedge$. Dickey-Fuller shows that under one unit root null hypothesis, the pseudo t-ratio does not converge to the t-student distribution and they calculate critical values for different sample sizes with results which derivate asintotically. MacKinnon (1991), running a higher number of simulations, calculates Dickey-Fuller critical values for arbitrary sample sizes. The augmented Dickey-Fuller (1981) test builds a parametric correction for autocorrelations with order higher than 1 and assumes that y_t follow a process $AR(p)$:

$$\Delta y_t = \alpha y_{t-1} + f(t) + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \ldots + \beta_p \Delta y_{t-p} + \upsilon_t \tag{15}$$

This augmented equation evaluates the statistical hypothesis through the pseudo t-ratio statistic equation (14).

The asintotic distribution of the pseudo t-ratio equation (14) for α is independent of the number of lags in first differences in equation (15).

The optimum lags order is calculated running the Schwarz information criterion (SIC). This criterion is based on an asintotic result derived from the exponential distribution of the data assumption. The formula is the following:

$$SIC = -2*lnL + k*ln(n) \tag{16}$$

where n is the number of observations in the sample, k is the number of repressors (including the constant) and L is the maximum value in the maximum likelihood function of the estimated model. Under the assumption of normality in the perturbation distribution in equation (16), one can derive: $SIC = n*ln((RSS)/n) + k*ln(n)$; where RSS is the square sum error of the estimated model. Given two estimated models, the Schwarz criterion chooses the model with the minimum SIC value.

3.2 Cointegration test

An autorregresive vector (VAR) evaluates the number of co-integration vectors defined by Engle and Granger (1987) with Johansen (1988, 1991) methodology, under unrestricted intercepts and restricted tends assumptions by Pesaran, Shin and Smith (2000).

Consider a sample regression function represented by $y_t = X_t + f(t) + u_t$; where $f(t)$ can be zero (0), constant (δ) or constant and trend ($\delta + \beta_0 t$), y_t is the endogenous variable, X_t is the exogenous variables vector and u_t are the regression errors. Engle and Granger (1987) propose that if there is not an unit root in $u_t = y_t - X_t + f(t)$ and the variables of the model show the same integration order process, the variables could be co-integrated.

In a $VAR\ (p)$ model framework:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + B(t) + \varepsilon_t \tag{12'}$$

where Y_t is a vector with dimension $(k*1)$ and variables $I(1)$, $B(t)$ is a d-vector of deterministic variables and where ε_t are regression errors. Rewriting model (12'):

$$\Delta Y_t = \Pi Y_{t-1} + \Sigma^{p-1}_{i=1} \Gamma_i \Delta Y_{t-i} + B(t) + \varepsilon_t \tag{13'}$$

where $\Pi = - I + A_1 + A_2 + \ldots + A_p\ y\ \Gamma_i = - A_1 + A_2 + \ldots + A_i$ *for all* $i = 1, 2, \ldots, p-1$.

If the coefficient Π_{kxk} has rank $0 < r < k$, there will be $k*r$ matrices $\theta_{kxr}\ y\ \lambda_{rxk}$ such that $\Pi = \theta*\lambda'\ y\ \lambda'*Y_t$ is $I(0)$. Where k is the number of endogenous variables, θ is the error correction velocity, r is the number of co-integration relationships and each column of the matrix λ are the co-integration vectors.

Considering an unrestricted VAR and five different types of trends as determinist variables, Johansen (1988, 1991) estimates the matrix Π_{kxk} and determines the number of variables lineal independent r, proceeding in sequences from $r = 0$ to $r = k-1$. The trace statistic evaluates null hypothesis Hr: rank $[\Pi] = r$ *and* $r = 0, 1, \ldots, k$ co-integration relationships with the following equation:

$$LR_{tr}\ (r/k) = -T\ \Sigma^{k}_{i=r+1}\ log\ (1-\tau_i)\ para\ r = 0, 1, \ldots, k-1 \tag{17}$$

where $LR_{tr}\ (r/k)$ is the maximum likelihood ratio $ln[Lmax(r)/Lmax(k)],\ t = -k+1, \ldots, T,\ \tau_i$ is the *i-ésimo* higher value or eigenvalue of the matrix Π_{kxk}. If the statistic is higher than the critical value calculated by Johansen as a nonstandard asintotic distribution, it cannot be rejected the null hypothesis of r co-integration relationships.

Pesaran, Shin and Smith (2000) propose five different cases to test restricted VAR. Case 4 presents an unrestricted intercept and a restricted tend modifying equation (13') in the following way:

$$\Delta Y_t = \Pi Y_{t-1} + \Sigma^{p-1}_{i=1} \Gamma_i \Delta Y_{t-i} + c_0 + (-\Pi\gamma)t + \varepsilon_t \tag{13''}$$

where $\Pi = - I + A_1 + A_2 + \ldots + A_p$, $\Gamma_i = - A_1 + A_2 + \ldots + A_i$ *for all* $i = 1, 2, \ldots, p-1$, $c_0 \neq 0$ and γ is an unknown parameter. Equation (17) tests the number of co-integration vectors comparing the statistics with the critical values calculated by Pesaran, Shin and Smith (2000) in equation (13'').

3.3 Ordinary least squares (OLS) model with error correction mechanism (ECM)

A logarithm minimizes the sum of squared residuals of a lineal regression under estimator efficient properties. The stationary vector found with the cointegration test is also included.

Consider a sample regression function represented by

$$y_t = X_t + f(t) + ECM + u_t \quad (18)$$

where $f(t)$ can be zero (0), constant (δ) or constant and trend ($\delta + \beta_0 t$), y_t is the endogenous variable, X_t is the exogenous variables vector, ECM is the stationary cointegration vector and u_t are the regression errors. The OLS model calculates the regression estimators minimizing the following equation $\Sigma\, \varepsilon_t^2 = \Sigma\, (y_t - y_t\hat{})^2$. Where $y_t - y_t\hat{}$ is the difference between observed values and estimated values of the endogenous variable and ε_t^2 are the errors of the sample.

Classical OLS properties for regression analysis are the following:

- The regression model is linear in the parameters.
- The X_t values are fixed in repeated sampling or X_t is non-stochastic.
- Given X_t, there is zero mean value of disturbance u_t.
- Given X_t, the disturbance u_t variance is constant or there is homoskedasticity $[Var(u_t \mid X_t) = \sigma 2]$.
- Given X_t, there is not autocorrelation or correlation between u_i and u_j $(i \neq j)$ is zero.
- If X_t is stochastic, u_t and X_t are independent or zero covariance between u_i and X_i $[E(u_i X_i)=0]$.
- The number of observations must be greater than the number of parameters X_t.
- The regression model is correctly specified or the specification is not biased.
- There is not perfect multi-collinearity or there are not perfect linear relationships in between the repressors X_t.
- Disturbance u_t is normally distributed.

To evaluate properties in the OLS model, this paper finds individual significance of the estimators observes the model adjustment to the data, tests autocorrelation and heteroskedasticity and verifies the normality of the errors.

(1) "*t*-student" or t-ratio test

The *t*-student evaluates if the estimator b_k of a parameter β_k is statistically different from 0. If $t = b / s_b$. *Si* $[b_k - \beta_k] / s_b > t_{\lambda/2}$, with the null hypothesis $b_k = 0$ being rejected. Where k is the number of variables X in the equation (18), β_k are the parameters of the vector X, $b_k = (X'X)^{-1}X'(X\beta + \varepsilon_t)$ is the OLS estimator of β_k, s_b is the sample variance of b_k and $t_{\lambda/2}$ is the $100(1-\lambda/2)$ percental critical value with a t distribution and $(n - k)$ degrees of freedom.

(2) Coefficient of determination R^2

The autocorrelation and heteroskedasticity properties in the errors of the model can be tested calculating the model adjustment level to the data or R^2. The formula is the following:

$$R^2 = [\Sigma_i\, (y_i - y^*)\, (y_i\hat{} - y\hat{}^*)]^2 / [\Sigma_i\, (y_i - y^*)^2]\, [\Sigma_i\, (y_i\hat{} - y\hat{}^*)^2]$$

where y_i is the observed endogenous variable values in equation (18), y^* is the average of y_i, $y_i\hat{}$ is the OLS estimated values of y_i and $y\hat{}^*$ is the average of $y_i\hat{}$.

(3) Breush-Godfrey (BG) autocorrelation test of order (p)

The auxiliar regression in the errors of the sample regression function in equation (18), to find autocorrelation of order p, is the following equation: $\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + \ldots + \rho_p\varepsilon_{t-p} + a_t$, where ε_t are the sample errors in equation (18) and $a_t \sim iid\ (0,\sigma_\varepsilon^2)$. The associated statistic is $n{*}R^2{\sim}\chi^2$. Where n is the number of observation, R^2 is the model adjustment level to the data and χ^2 is the chi-square distribution.

If $t = b/s_b$. *Si* $[b_k - \beta_k]/s_b > t_{\lambda/2}$, the null hypothesis $b_k = 0$ is rejected. Where k is the number of variables X in the equation (18), β_k are the parameters of the vector X, $b_k = (X'X)^{-1}X'(X\beta + \varepsilon_t)$ is the OLS estimator of β_k, s_b is the sample variance of b_k and $t_{\lambda/2}$ is the $100(1-\lambda/2)$ percental critical value with a t distribution and $(n - k)$ degrees of freedom.

If the statistical value is higher than the chi-square value at a specific level of significance, the null hypothesis of no serial autocorrelation of order p is rejected.

(4) White heteroskedasticity test

The auxiliar regression in the errors of the sample regression function in equation (18), to find heteroskedasticity, is the following equation: $\varepsilon_t^2 = \pi_o + \pi_1X_t + \pi_2X_j + \pi_3X_t^2 + \pi_4X_j^2 + \pi_5X_tX_j + b_t$, where ε_t are the sample errors in equation (18), X_t and X_t are the exogenous variables vectors and $b_t \sim iid\ (0,\sigma_\varepsilon^2)$. The associated statistic is $(n){*}R^2 \sim\chi^2$. Where n is the number of observation, R^2 is the model adjustment level to the data and χ^2 is the chi-square distribution.

If the statistic value is higher than the chi-square value at a specific level of significance, the null hypothesis of no heteroskedasticity is rejected.

(5) Jarque-Bera errors normality test

The Jarque-Bera goodness of fit measure evaluates the statistical difference in between the kurtosis and asymmetry in a normal distribution and the kurtosis and asymmetry in the errors distribution function of the sample regression in equation (18). The formula is the following: $Jarque\text{-}Bera = (n-k)/\sigma * [S^2 + ((K-3)^2 / \varphi)]{\sim}\chi^2$, where n is the number of observations, S is the asymmetry, K is the kurtosis, k, σ and φ are the estimated coefficients and χ^2 is the chi-square distribution. If the statistical value is higher than the chi-square value at a specific level of significance, the null hypothesis of normal distribution is rejected.

4. Empirical evidence

The theoretical model is applied to the euro-dollar relationship because there are important commercial transactions and capital flows between both regions. In the next sections, the data is defined and calculated in the first instance, the time series with unit roots tests is evaluated and contrasted with the purchasing power parity theory; Secondly, the long run relationship between the cointegrated variables is found, and finally, the short term model with the ECM to built to estimate the ERE overvaluation.

4.1 Data

The euro replaced the ECU (European currency unit) on January 1st, 1999 as the official currency of the European Union country members. Conversion factors of the exchange rate mechanism adopted by 11 countries of the euro area were the following: Austria (S. 13.7603), Belgium (BF. 40.3399), Finland (Fmk. 5.94573), France (F. 6.55957), Germany (DM. 1.95583), Ireland (IR£. 0.787564), Italy (Lit. 1936.27), Luxembourg (Lux F. 40.3399), Holland (f. 2.20371), Portugal (Esc. 200.482) and Spain (Pta. 166.386). At the same time, an exchange rate agreement in Greece came into being (d. ± 15% surrounding the euro central exchange rate). On January 1st, 2001, the euro was adopted in Greece (d. 340.750) and on January 1st, 2007 in Slovenia (t. 239.640). This paper analyzes the quarterly data of 13 country members of the euro zone (euro 13) from January 1999 to

April 2007 (see Table 1) to find euro-dollar RER real determinants. The base year data is 2000 and the primary sources are the following:

(1) The international finances statistics from the International Monetary Fund (IFS/IMF);

(2) The macroeconomics data from the Statistical Office of the European Communities (Eurostat-European Commission) and from the European Central Bank (ECB);

(3) The inflation data from the Bureau of Labor Statistics of United States (BLS).

Table 1 Data RER euro (Index 2000=100) levels

Year	RER NER*MUICP/ CPIUSA Index	TT (UVX/UVI)/ GDPIndex	PE ECBPE/MUICP Millions of Euros	NFA IR+Gold/CPIUSA Millions of Euros	BT (M+X)/ GDPIndex	CR Eur.(3-m)/ Tr.USA(3-m) Ratio	PR (GDP/Nº of Em.)Index
1999Q1	0.93658	0.01192	6928.5923	3911.26492	1.7515	0.68415	0.92725
1999Q2	0.97886	0.01145	7059.6842	3714.67023	1.7960	0.57512	0.92095
1999Q3	0.96038	0.01080	7026.8355	3869.56369	1.7692	0.56473	0.91998
1999Q4	1.00081	0.01043	7854.9065	3827.83316	1.9192	0.66013	0.92810
2000Q1	1.03834	0.00978	7001.1621	3703.20638	1.8762	0.62256	0.94443
2000Q2	1.05365	0.00957	7167.6418	3691.00000	1.8892	0.72332	0.93696
2000Q3	1.14585	0.00941	7094.9263	3550.04055	1.9112	0.76433	0.92982
2000Q4	1.11823	0.00921	7984.2767	3487.43402	1.9133	0.80802	0.93292
2001Q1	1.09349	0.00959	7088.6651	3399.41099	1.9102	0.95420	0.94872
2001Q2	1.16744	0.00933	7283.2279	3367.95991	1.9199	1.22638	0.93889
2001Q3	1.09154	0.00933	7178.3570	3480.33346	1.9035	1.31762	0.93095
2001Q4	1.12950	0.00922	8204.2410	3376.28862	1.8790	1.76785	0.93248
2002Q1	1.14665	0.00958	7199.3572	3439.11002	1.8980	1.91980	0.94224
2002Q2	1.05160	0.00945	7473.3776	3507.92314	1.8957	1.97515	0.93725
2002Q3	1.01966	0.00937	7444.1271	3576.89996	1.8866	2.00526	0.93497
2002Q4	0.98908	0.00921	8335.6561	3658.93030	1.8793	2.28234	0.93801
2003Q1	0.92435	0.00941	7354.3218	3459.54949	1.9020	2.29007	0.95050
2003Q2	0.85943	0.00934	7620.4180	3496.92331	1.8866	2.25448	0.93880
2003Q3	0.89016	0.00935	7532.5624	3610.88635	1.8701	2.28185	0.93769
2003Q4	0.82011	0.00919	8433.4693	3623.40273	1.8594	2.31964	0.94160
2004Q1	0.81399	0.00931	7492.5411	3473.30803	1.8653	2.21224	0.96068
2004Q2	0.81863	0.00915	7699.5027	3335.47197	1.8820	1.92103	0.95176
2004Q3	0.81385	0.00905	7528.1787	3365.73097	1.8994	1.40462	0.94820
2004Q4	0.74690	0.00891	8448.1989	3467.84004	1.8974	1.06398	0.94905
2005Q1	0.74893	0.00909	7573.9508	3294.57916	1.9228	0.82665	0.96444
2005Q2	0.81317	0.00899	7795.1882	3235.78402	1.9195	0.72931	0.95703
2005Q3	0.79512	0.00899	7543.8588	3249.36710	1.9257	0.62076	0.95463
2005Q4	0.83301	0.00882	8592.1603	3307.22419	1.9428	0.59828	0.95579
2006Q1	0.81275	0.00888	7637.0987	3415.91876	1.9655	0.58031	0.97298
2006Q2	0.76822	0.00868	7856.8206	3498.08862	1.9568	0.59942	0.96695
2006Q3	0.76320	0.00857	7682.2845	3496.34677	1.9653	0.64033	0.96509
2006Q4	0.74292	0.00837	8725.6060	3666.46446	1.9499	0.71359	0.97077
2007Q1	0.73173	0.00865	7707.0811	3708.04568	1.9753	0.74720	0.98393
2007Q2	0.71857	0.00836	7920.4142	3635.06069	1.9851	0.83535	0.97190
2007Q3	0.69490	0.00828	7754.7759	3992.33880	1.9838	1.02927	0.97209
2007Q4	0.66673	0.00820	7690.5583	4195.56244	1.9896	1.36660	0.97453

Data source: IFS/IMF-ECB-BLS.

The proxy of the variables is the following:

- Variable RER: The real exchange rate is calculated by multiplying the euro-dollar nominal exchange rate by the Monetary Union index of consumer prices over the consumer price index of United States. *RER= NER*MUICP/CPIUSA* (Sources: IFS/IMF-ECB-BLS).
- Variable TT: The euro zone terms of trade are obtained from the ratio unit value exports over unit value imports divided by the euro area gross domestic product. *TT = (UVX/UVI)/GDP* (Source: IFS/IMF).
- Variable PE: The proxy of public expenditure is the euro area total current government expenditure deflected by the Monetary Union index of consumer prices. *PE = ECBPE/MUICP* (Source: ECB).
- Variable NFA: The net foreign assets are calculated dividing the euro area total international reserves plus gold, deflected by the consumer price index of United States. *NFA = IR+Gold/CPIUSA* (Sources: IFS/IMF-BLS).
- Variable BT: The balance of trade is the sum of the volume of exports plus the volume of imports divided by the euro area gross domestic product. *BT = (M+X)/GDP* (Source: IFS/IMF).
- Variable CR: The country risk is calculated dividing the 3-month interbank Europe rate by the 3-month United States Treasury bill rate. *CR = Eurepo(3-m)/Treas. USA(3-m)* (Source: IFS/IMF).
- Variable PR: Productivity is obtained by dividing the euro area gross domestic product by the number of full time employees in the euro area. *PR = (GDP/Nº of Em.)* (Sources: IFS/IMF-ECB).

Rewriting equation (18) as the equation (4) in the theoretical framework:

$$RERe = f(t) + c_4TT + c_5PE + c_6NFA + c_7BT + c_8PR + c_9CR + ECM + u_t \quad (19)$$

4.2 Unit roots test

The unit roots tests contrast the purchasing parity power, observing if the real exchange rate deviations with respect to its equilibrium are transitory or permanent. The augmented Dickey-Fuller test to 1%, 5% and 10% obtains unit roots results when the data is presented in levels and in first differences. There are three different cases to test unit roots: models with only intercepts, models with intercepts and trends and models without intercepts or trends (see Table 2). The Schwarz information criterion (SIC) finds the optimum number of lags. Because the data is measured quarterly, the maximum number of lags including testing unit roots, is 9. Analysing the RER variable in levels with the optimum number of lags equal to 0, the one unit root null hypothesis cannot be rejected and the integration order is found to equal to 1 or *I(1)*. Fig. 5 shows the RER time series from January 1999 to April 2007. The RER behaviour does not present stationarity in the variance.

Table 2 Testing unit roots (augmented Dickey-Fuller)

	No trends No intercepts	Intercepts	Trends Intercepts		No trends No intercepts	Intercepts	Trends Intercepts
RER in levels	-1.035	-0.093	-2.562	RER in 1st differences	-6.185	-6.370	-6.593
1% level	-2.633	-3.633	-4.244	1% level	-2.635	-3.639	-4.253
5% level	-1.951	-2.948	-3.544	5% level	-1.951	-2.951	-3.548
10% level	-1.611	-2.613	-3.205	10% level	-1.611	-2.614	-3.207
SIC (maxlag=9)	0	0	0	SIC (maxlag=9)	0	0	0

Note: MacKinnon (1996) one-sided *p*-values (variable in levels and in first differences).
Data source: Author's calculations.

The fact that the one unit root null hypothesis cannot be rejected in the RER time series justifies the empirical real determinants according to the equation (19). To test time series stationarity in the right side of the equation (19), the Schwarz information criterion shows the following optimum number of lags: 3 lags for PE, 0 lags for NFA, 0 lags for BT, 4 lags for PR and 1 lag for CR. The variable TT presents 4 optimum lags in the cases where there are not intercepts and trends or with intercepts and trends, and 5 lags where there are only intercepts. Variables TT, PE, NFA and BT are *I(1)* and variables PR and CR are *I(2)*. The BT data in levels are weakly stationary to 10% when there are only intercepts or when there are intercepts and trends in the regression. When the model does not include intercepts and trends, the one unit root null hypothesis is rejected, testing PR and CR in the first differences with levels of significance equal to 5% and 10% (see Table 3).

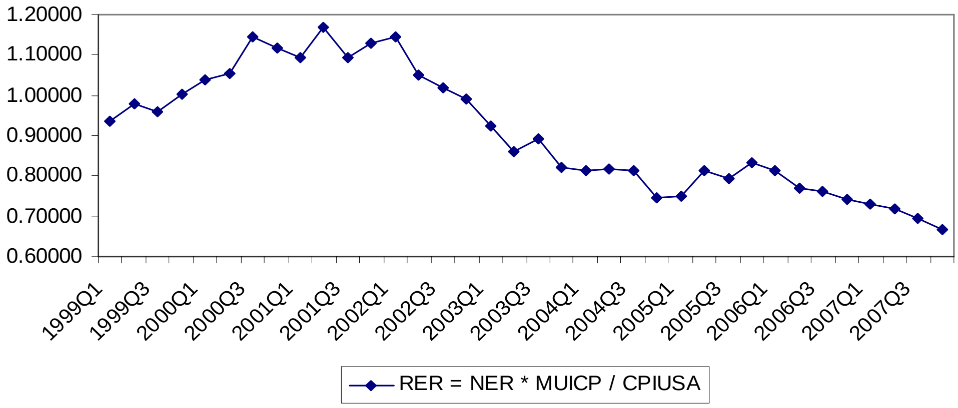


Fig. 5 Real exchange rate

Table 3 Testing unit roots (augmented Dickey-Fuller)

	No trends No intercepts	Intercepts	Trends Intercepts		No trends No intercepts	Intercepts	Trends Intercepts
TT in levels	-1.076	0.489	-3.416	TT in 1st differences	-4.522	-4.413	-4.480
1% level	-2.642	-3.670	-4.285	1% level	-2.642	-3.662	-4.297
5% level	-1.952	-2.964	-3.563	5% level	-1.952	-2.960	-3.568
10% level	-1.610	-2.621	-3.215	10% level	-1.610	-2.619	-3.218
SIC (maxlag=9)	4	5	4	SIC (maxlag=9)	3	3	4
PE in levels	1.605	-1.734	1.002	PE in 1st differences	-18.016	-18.615	-19.590
1% level	-2.639	-3.654	-4.273	1% level	-2.639	-3.654	-4.273
5% level	-1.952	-2.957	-3.558	5% level	-1.952	-2.957	-3.558
10% level	-1.611	-2.617	-3.212	10% level	-1.611	-2.617	-3.212
SIC (maxlag=9)	3	3	3	SIC (maxlag=9)	2	2	2
NFA in levels	0.343	-0.855	-0.242	NFA in 1st differences	-5.283	-5.250	-5.927
1% level	-2.633	-3.633	-4.244	1% level	-2.635	-3.639	-4.253
5% level	-1.951	-2.948	-3.544	5% level	-1.951	-2.951	-3.548
10% level	-1.611	-2.613	-3.205	10% level	-1.611	-2.614	-3.207
SIC (maxlag=9)	0	0	0	SIC (maxlag=9)	0	0	0
BT in levels	1.238	-2.809	-3.542	BT in 1st differences	-8.680	-9.113	-9.045
1% level	-2.633	-3.633	-4.244	1% level	-2.635	-3.639	-4.253
5% level	-1.951	-2.948	-3.544	5% level	-1.951	-2.951	-3.548
10% level	-1.611	-2.613	-3.205	10% level	-1.611	-2.614	-3.207
SIC (maxlag=9)	0	0	0	SIC (maxlag=9)	0	0	0
PR in levels	1.656	0.701	-2.405	PR in 1st differences	-2.138	-2.652	-3.646
1% level	-2.642	-3.662	-4.285	1% level	-2.642	-3.662	-4.297
5% level	-1.952	-2.960	-3.563	5% level	-1.952	-2.960	-3.568
10% level	-1.610	-2.619	-3.215	10% level	-1.610	-2.619	-3.218
SIC (maxlag=9)	4	4	4	SIC (maxlag=9)	3	3	4

(to be continued)

				PR in 2nd differences	-27.521	-27.233	-26.950
				1% level	-2.642	-3.662	-4.285
				5% level	-1.952	-2.960	-3.563
				10% level	-1.610	-2.619	-3.215
				SIC (maxlag=9)	2	2	2
CR in levels	-0.363	-2.186	-2.158	CR in 1st differences	-2.128	-2.146	-2.071
1% level	-2.635	-3.639	-4.253	1% level	-2.635	-3.639	-4.253
5% level	-1.951	-2.951	-3.548	5% level	-1.951	-2.951	-3.548
10% level	-1.611	-2.614	-3.207	10% level	-1.611	-2.614	-3.207
SIC (maxlag=9)	1	1	1	SIC (maxlag=9)	0	0	0
				CR in 2nd differences	-7.210	-7.144	-7.088
				1% level	-2.637	-3.646	-4.263
				5% level	-1.951	-2.954	-3.553
				10% level	-1.611	-2.616	-3.210
				SIC (maxlag=9)	0	0	0

Note: MacKinnon (1996) one-sided *p*-values (variables in levels, in first differences and in second differences).
Data source: Author's calculations.

4.3 Cointegration test

A first order VAR with unrestricted intercepts and restricted trends finds the long run relationship between RER, TT, PE, NFA and BT. The likelihood ratio test based on trace lists 2 cointegrated vectors:

At the 5% of significance level, Table 4 shows that the maximum likelihood ratio is higher than the critical value for $r >= 1$ and $r >= 2$.

Table 4 Cointegration with unrestricted intercepts and restricted trends in the VAR cointegration LR test based on trace of the stochastic matrix

35 observations from 1999 Q2 to 2007 Q4. Order of VAR = 1.				
List of variables included in the cointegrating vector:				
RER TT	PE	NFA	BT	Trend
List of eigenvalues in descending order:				
0.84878 0.74164	0.43825	0.35617	0.071126	0.0000
Null	Alternative	Statistic	95% Critical value	90% Critical value
r = 0	r>= 1	151.6629	87.1700	82.8800
r<= 1	r>= 2	85.5464	63.0000	59.1600
r<= 2	r>= 3	38.1778	42.3400	39.3400
r<= 3	r>= 4	17.9934	25.7700	23.0800
r<= 4	r = 5	2.5824	12.3900	10.5500

Note: This table is used to determine *r* (the number of cointegrating vectors).

Table 5 presents VAR estimator values in both cointegrated vectors. Normalized estimators values are in between brackets. According to the economics theory, cointegration vector 1 satisfies the real determinant signs. Note that only the public expenditure estimator sign is different comparing vector 1 and vector 2. If the PE effect over the RER behaviour depends on the expenditure weight in tradable or non-tradable goods, an increase in government demand in domestic goods and services incentives domestic production to grow. This productivity growth in tradable goods or an increase in non-tradable goods prices decreases or appreciates RER (negative relationship).

Table 5 Estimated cointegrated vectors in Johansen estimation cointegration with unrestricted intercepts and restricted trends in the VAR

List of variables included in the cointegrating vector	Vector 1	Vector 2
RER	0.79247	-0.52991
	(-1.0000)	(-1.0000)
TT	-292.3190	211.4530
	(368.8714)	(399.0326)
PE	0.1526E-3	0.4267E-3
	(-0.1925E-3)	(0.8052E-3)
NFA	-0.4177E-4	0.3202E-4
	(0.5271E-4)	(0.6042E-4)
BT	-3.1863	2.8858
	(4.0208)	(5.4457)
Trend	0.0053433	-0.017867
	(-0.0067426)	(-0.033717)

Notes: 35 observations from 1999 Q2 to 2007 Q4. Order of $VAR = 1$, chosen $r = 2$.

4.4 Ordinary least squares (OLS) model with error correction mechanism (ECM)

The OLS with ECM model determines the short term relationship and the exchange rate overvaluation. Note that equation (19) includes the monetary variable RC to explain RERe. In the model, all time series are *I(0)* because RER, TT, PE, NFA and BT are transformed to first differences and PR and CR to second differences. To build the final model with 4 distributed lags in each exogenous variable, there is an interactive procedure where the less significant variables are excluded. The ECM is calculated with the cointegration vector 1 and included in the OLS model with 1 lag and negative sign (see Table 6).[1]

Table 6 Dependent variable: D (RER)

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	-5.062393	1.078408	-4.694321	0.0001
D (RER(-3))	0.297707	0.128300	2.320406	0.0310
D (TT(-4))	67.134480	38.807680	1.729928	0.0990
D (PE(-2))	-5.00E-05	1.78E-05	-2.810193	0.0108
D (PE(-3))	-9.13E-05	2.78E-05	-3.283964	0.0037
D (PE(-4))	-0.000113	2.84E-05	-3.972838	0.0007
D (NFA(-1))	-0.000174	5.86E-05	-2.968203	0.0076
D (NFA(-2))	-0.000143	7.38E-05	-1.943140	0.0662
D (BT(-1))	-2.181635	0.485949	-4.489436	0.0002
D (BT(-2))	-1.072368	0.269786	-3.974889	0.0007
ECMFINAL	-0.583090	0.124415	-4.686660	0.0001
R-squared	0.702250	Mean dependent var	-0.011987	
Adejusted R-squared	0.553375	S.D. dependent var	0.044274	
S.E. of regression	0.029588	Akaike info criterion	-3.931475	
Sum squared resid	0.017509	Schwarz criterion	-3.422641	
Log likelihood	71.937860	*F*-statistic	4.717051	
Durbin-Watson stat	1.972143	Prob (*F*-statistic)	0.001562	

Notes: Method: Least Squares; Sample (adjusted): January 2000- April 2007; Included observations: 31 after adjusting endpoints.

[1] In the ordinary least squares model, the second error correction mechanism (calculated from the second cointegration vector) is not statistically significant to 1%, 5% and 10%. The individual associated *p*-value to the *t*-statistic parameter of the second mechanism is 0.5380.

Where *D* is first differences and the negative number between brackets is the number of lags. The ECM does not present unit roots to 5% and 10% with intercepts or with intercepts and trends (see Table 7). According to the economic theory, all the coefficients have right signs. Testing with the *t*-ratio, only *D* (TT (-4) and *D* (NFA (-2)) are significant to 10%. The rest of the coefficients are statistically significant to 5%. The regression includes 3 lags of the endogenous variable to adjust present RER values to past RER values speculative effects. After 4 periods, the substitutive effect is stronger than the income effect in between TT and RER. An increase in the *TT* index, as a result of a change in the relative import-export international price, is inversely proportional in the RER index. The *PE* shows the strongest global effect over the endogenous variable, increasing domestic productivity and decreasing the RER with 2, 3 and 4 lags. An increase in the public demand of domestic goods and services directly improves the domestic production without crowding out private consumption and decreasing RER. The relationship between variables NFA, BT and RER are explained by the income effect after 1 and 2 periods. An increase in net foreign assets or in trade openness levels similarly affects the wealth level and the domestic goods and services consumption level. If productivity in the domestic tradable sector or prices in the domestic non-tradable sector are higher, the RER decreases, appreciating the national currency. Trend, PR and CR coefficients are not statistically significant. The ECM coefficient of the first cointegration vector is negative, and as the econometric theory affirms, its values are between 0 and 1. Including 31 observations, the model adjusts acceptably to the statistic data with a R^2 value of 0.702250. In Fig. 6, the segmented line (representing *RER* variable in levels) and the continuous line (representing the adjusted *RER* variable) varies similarly. Before year 2002, changes in *RER and RERe* are a possible result of international speculative movements. After this year, there is less volatility in th*e RERe* behaviour and a better adjustment in the model.

Table 7 Testing unit roots (augmented Dickey-Fuller)

	No trends no intercepts	Intercepts	Trends intercepts
ECM	-0.182	-3.298	-3.893
1% level	-2.642	-3.662	-4.273
5% level	-1.952	-2.960	-3.558
10% level	-1.610	-2.619	-3.212
SIC (maxlag=9)	4	4	3
-ECM(-1)	-0.569	-4.021	-4.502
1% level	-2.644	-3.670	-4.297
5% level	-1.952	-2.964	-3.568
10% level	-1.610	-2.621	-3.218
SIC (maxlag=9)	4	4	4

Note: MacKinnon (1996) one-sided *p*-values (variable in levels and in first differences).

Data source: Author's calculations.

The Breush-Godfrey (*BG*) autocorrelation test of order (*p*) is in Table 8. Evaluating the chi-square probability associated with each statistic test for 1, 2, 3 and 4 lags, the null hypothesis of no autocorrelation in all the autorregresive coefficients of the auxiliar regression cannot be rejected.

The White heteroskedasticity test without cross terms shows a statistical test equal to 22.860 and an associated p-value to the statistic equal to 0.296. Because the *p*-value associated is higher than the significance level 1%, 5% and 10%, the null hypothesis of no heteroskedasticity cannot be rejected.

Testing residual normality with the Jarque-Bera test, the *p*-value observed is 0.920. The null hypothesis of normal distribution in the errors with level of significance of α = 0.01, 0.05 and 0.10 (see Fig. 7) cannot be rejected.

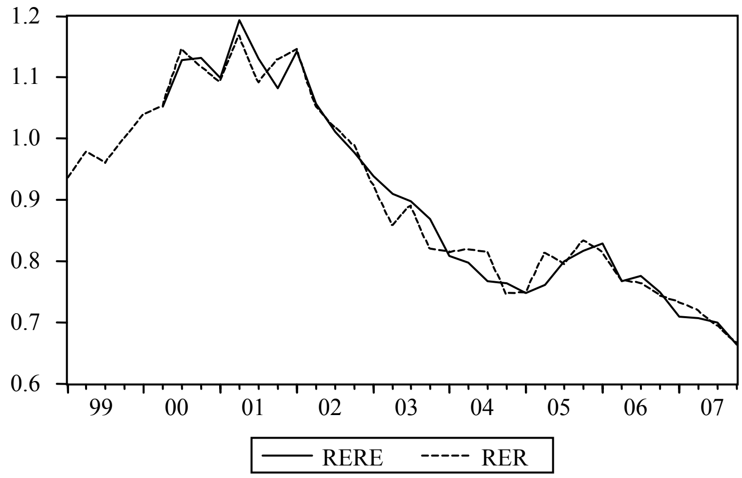
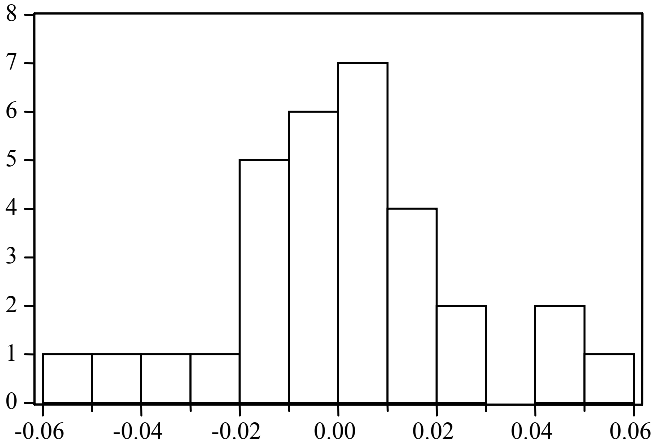


Fig. 6 Short term adjustment

Table 8 Breusch-Godfrey serial correlation LM test

Lag	1	2	3	4
Obs*R-squared	0.01	0.08	0.20	0.66
Prob. Chi-Square	0.93	0.96	0.98	0.96

Data source: Author's computations.



Series: Residuals	
Sample 2000Q2 2007Q4	
Observations 31	
Mean	1.35e-15
Median	0.001448
Maximum	0.051514
Minimum	-0.050641
Std. Dev.	0.024158
Skewness	0.090403
Kurtosis	3.311042
Jarque-Bera	0.167190
Probability	0.919804

Fig. 7 Error normality and other statistics

5. Results and conclusions

The unit roots test evaluates no stationarity of RER time series in levels and contrasts empirically the PPP theory. According to this test, variables TT, PE, NFA and BT are *I(1)* and PR and CR are *I(2)*. The cointegration test finds 2 cointegration vectors with statistically significant estimators and 1 cointegration vector with the economic right signs. After the long run relationship between RER and its real determinants TT, PE, NFA, and BT was found, the short term econometric model was estimated. The methodology OLS with ECM finds short term, statistically significant coefficients with economic right signs. The model adjustment is acceptable with a R^2 higher than 0.70. There is no residual autocorrelation or heteroskedasticity and the error distribution behaves normally. The strong relation between RER and PE is evident.

The RERe is found through transforming the short term econometric model to data in levels. The exchange rate overvaluation is obtained by calculating the percentage difference in between RER and RERe (see Table 9 and Fig. 8). RER fluctuating behaviour presents possibly speculative movements before the year 2002. The short term model is better suited to the statistic data after 2002. At the beginning of 2007, the euro overvaluation (subvaluation) was negative and equalled –3.06. This fact changed in line with the dollar nominal exchange rate depreciations. At the end of the year, the euro reached a value equal to –0.51 because the relationship between the *RER* and its equilibrium level is more stable.

Table 9 Exchange rate misalignment data RER and RERe (euro-dólar) (Index 2000=100)

	RER	DRERe	RERe	Overvaluation
1999Q1	0.93658	-	-	-
1999Q2	0.97886	-	-	-
1999Q3	0.96038	-	-	-
1999Q4	1.00081	-	-	-
2000Q1	1.03834	-	-	-
2000Q2	1.05365	0.01379	1.05213	-0.14
2000Q3	1.14585	0.07417	1.12782	-1.57
2000Q4	1.11823	-0.01441	1.13144	1.18
2001Q1	1.09349	-0.01934	1.09889	0.49
2001Q2	1.16744	0.10030	1.19379	2.26
2001Q3	1.09154	-0.03639	1.13105	3.62
2001Q4	1.12950	-0.00937	1.08217	-4.19
2002Q1	1.14665	0.01323	1.14273	-0.34
2002Q2	1.05160	-0.08931	1.05734	0.55
2002Q3	1.01966	-0.03995	1.01165	-0.79
2002Q4	0.98908	-0.04213	0.97753	-1.17
2003Q1	0.92435	-0.05063	0.93845	1.53
2003Q2	0.85943	-0.01428	0.91007	5.89
2003Q3	0.89016	0.03805	0.89748	0.82
2003Q4	0.82011	-0.02153	0.86863	5.92
2004Q1	0.81399	-0.01199	0.80812	-0.72
2004Q2	0.81863	-0.01719	0.79680	-2.67
2004Q3	0.81385	-0.05129	0.76734	-5.72
2004Q4	0.74690	-0.04988	0.76397	2.28
2005Q1	0.74893	0.00052	0.74742	-0.20
2005Q2	0.81317	0.01273	0.76166	-6.33
2005Q3	0.79512	-0.01297	0.80020	0.64
2005Q4	0.83301	0.02106	0.81618	-2.02
2006Q1	0.81275	-0.00385	0.82916	2.02
2006Q2	0.76822	-0.04598	0.76677	-0.19
2006Q3	0.76320	0.00768	0.77590	1.66
2006Q4	0.74292	-0.01372	0.74948	0.88
2007Q1	0.73173	-0.03361	0.70931	-3.06
2007Q2	0.71857	-0.02440	0.70733	-1.56
2007Q3	0.69490	-0.01933	0.69924	0.62
2007Q4	0.66673	-0.03156	0.66334	-0.51

Data source: EFI/FMI-BCE/EU-BLS-Author's calculations.

In order to analyse internal and external equilibrium behaviour in an economy, it is necessary to observe the

RER time series movements and estimate the RERe with its real determinants. The econometric estimators of the model parameters are not exact measures of economic stabilization, but they allow for the development of inferences over macroeconomics data and the most capable monetary and public policy to maintain sustainable balance of payments and real exchange rates in the short term and in the long run.

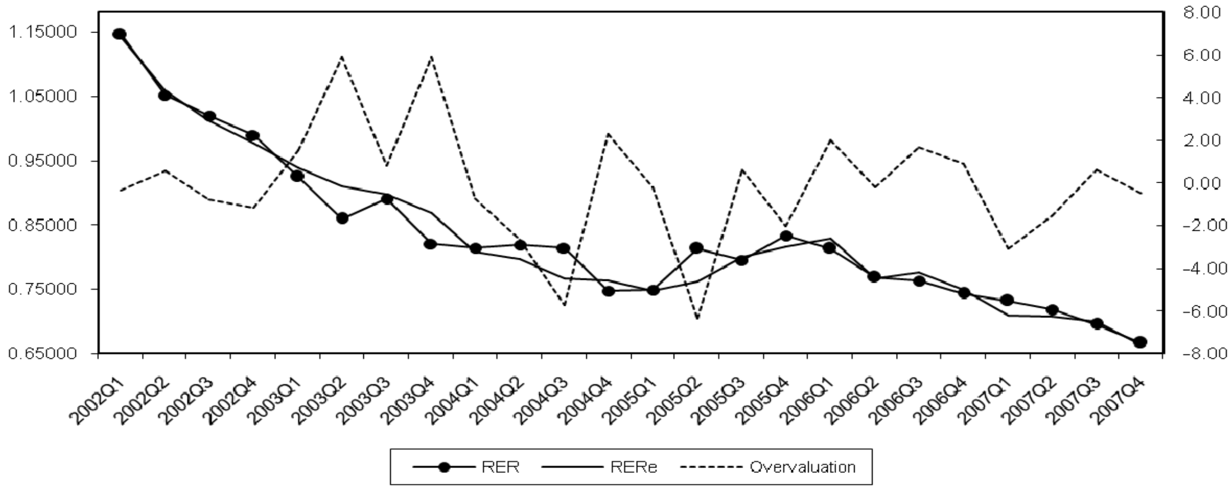


Fig. 8 Exchange rate misalignment

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(Edited by Ruby and Chris)