Mild Quasilocal Non-Gaussianity as a Signature of Modified Gravity During Inflation

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(Received 24 July 2014; published 17 October 2014)

We show that modifications of Einstein gravity during inflation could leave potentially measurable imprints on cosmological observables in the form of non-Gaussian perturbations. This is due to the fact that these modifications appear in the form of an extra field that could have nontrivial interactions with the inflaton. We show it explicitly for the case $R + \alpha R^2$, where nearly scale-invariant non-Gaussianity at the level of $f_{\mathrm{NL}} \approx -(1 \text{ to } 30)$ can be obtained, in a quasilocal configuration.

DOI: 10.1103/PhysRevLett.113.161303 PACS numbers: 98.80.-k

The current inflationary paradigm [1–7] is the most economical at successfully describing many observed features in the Universe, from its homogeneity, flatness and size, to the origin of the structure in the Universe as quantum fluctuations, e.g., [8,9]. In the vast majority of inflationary models, Einstein gravity is assumed as the correct description of gravity. However, it might be that Einstein gravity is not the correct description of gravity at very high energies either via a true modification of general relativity or because quantum effects become relevant. Departures from Einstein gravity during inflation have been considered in the first inflationary model proposed [10], in Refs. [11–17], and most recently in Ref. [18]. In Ref. [15] (see also [16,17]) graviton non-Gaussianities are considered beyond ordinary Einstein gravity. However, such non-Gaussianities are well below the sensitivity of future measurements and in fact well below the cosmic variance limit for the full sky. In this Letter we investigate if deviations from general relativity (GR) could be observable and measurable in the sky through the enhancement of non-Gaussianity (NG) of curvature perturbations. In the simplest models of inflation with standard gravity (or inflation models within modified gravity, which can be described as GR plus single-field slow-roll inflation), the amount of primordial non-Gaussianity (NG) is too small to be measurable, the NG parameter f_{NL} being $\sim\mathcal{O}(\epsilon)$ [19–21].

NG has been recognized as a powerful tool to learn about fundamental physics at play during inflation, being a probe of the interactions of the field(s) driving inflation. Other statistics, such as the power spectrum, do not carry as *specific* signatures as NG does. For this reason we expect that the effect of modifying gravity will leave specific signatures on the departures from Gaussianity. We find that this is the case, in particular, we show that modifications of Einstein gravity, if already relevant during the epoch of inflation, could lead to a measurable non-Gaussian signature in the cosmological fluctuation field. Such non-Gaussian signatures would be the imprints of departures from GR that, on the other hand, might be much harder to probe in the power spectrum of scalar perturbations. Also, we will show that, for a large part of the parameter space, the generated non-Gaussianities have a quasilocal shape. This is observationally promising given that future LSS surveys can be sensitive to values of local NG $f_{\mathrm{NL}} \sim \mathcal{O}(1)$ or even smaller (see, e.g., [22–25]).

Let us start from a Lagrangian that contains all generally covariant terms up to two derivatives built with the metric and one scalar field, which we will assume to drive inflation [26]:

$$\begin{aligned} L = \sqrt{-g}\bigg[&\frac{1}{2}M_{Pl}^2\Omega(\psi)^2 R - \frac{1}{2}h(\psi)g^{\mu\nu}\partial_\mu\psi\partial^\mu\psi - U(\psi) \\ &+ f_1(\psi)(g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi)^2 + f_2(\psi)g^{\rho\sigma}\partial_\rho\psi\partial_\sigma\psi\Box\psi \\ &+ f_3(\psi)(\Box\psi)^2 + f_4(\psi)R^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + f_5(\psi)Rg^{\mu\nu}\partial_\mu\psi\partial_\nu\psi \\ &+ f_6(\psi)R\Box\psi + f_7(\psi)R^2 + f_8(\psi)R^{\mu\nu}R_{\mu\nu} \\ &+ f_9(\psi)C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}\bigg] + f_{10}(\psi)\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda}. \end{aligned} \quad (1)$$

If the inflaton ψ is slowly rolling, then the functions $\Omega(\psi)$, $h(\psi)$ and $f_i(\psi)$ are varying slowly and can be simply treated as constants up to slow-roll corrections, which we will neglect. In this case, the Weyl-squared terms can be recast as a surface term (the Gauss-Bonnet term) plus R^2 and $R_{\mu\nu}R^{\mu\nu}$, which can then be reabsorbed. Moreover, in order to avoid ghosts, the terms proportional to f_2, f_3, f_6, and f_8 will be set to zero, as well as f_{10} as we are not interested in parity violating signatures. We are interested only in the terms that could give rise to a possibly enhanced local (or quasilocal) NG in the squeezed limit, different from the well-known result $f_{\rm NL} \sim \mathcal{O}(\epsilon)$ that is valid in standard gravity [19–21]. Therefore, we will not consider inflaton derivative self-interactions, which are known to generate NG mainly in the equilateral configuration. This is valid also for the ghost-free combination that can be built with the operators proportional to f_4 and f_5 [27], which would not generate significant NG in the local configuration. The only term left to consider is therefore the term R^2, which is nothing else than the first term in an expansion in powers of the Ricci scalar of a more general $f(R)$ theory:

$$\mathcal{L} = \sqrt{-g}\left[f(R) - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - U(\psi)\right]. \qquad (2)$$

This action describes one more degree of freedom associated to the $f(R)$ term. Through a standard procedure we use an auxiliary field $f'(\chi) = M_{\rm Pl}^2\phi/2$ to recast the action in the form

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2\phi R + \Lambda(\phi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - U(\psi)\right], \quad (3)$$

where $\Lambda(\phi) = f[\chi(\phi)] - M_{\rm Pl}^2\phi\chi/2$.

By performing a Weyl transformation $g_{\mu\nu} \to e^{-2\omega}g_{\mu\nu}$, with $e^{2\omega} = \phi$, to go to the Einstein frame, the action appears as a two-field interacting model:

$$\begin{aligned}\tilde{\mathcal{L}} \ &= \sqrt{-g}\bigg[\tfrac{1}{2}M_{\rm Pl}^2 R - \tfrac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\varphi^a\partial_\nu\varphi^b \\ &\quad -U_1(\varphi_1) - e^{-4\varphi_1/\sqrt{6}M_{\rm Pl}}U(\varphi_2)\bigg], \end{aligned} \qquad (4)$$

where $a, b = 1, 2$ we have normalized the fields as

$$\sqrt{6}M_{\rm Pl}\omega = \varphi_1, \qquad \psi = \varphi_2, \qquad (5)$$

defined U_1 as

$$U_1(\varphi_1) = -e^{-4\varphi_1/\sqrt{6}M_{\rm Pl}}\Lambda(\phi[\omega(\varphi_1)]), \qquad (6)$$

and defined the field metric

$$\gamma_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-2\varphi_1/\sqrt{6}M_{\rm Pl}} \end{pmatrix}. \qquad (7)$$

As expected, there is an equivalence between "$f(R)$ + scalar" and a two-field model with a specific field metric, a generic potential for φ_1 and a "conformally stretched" potential for φ_2. Then it is conceivable that the interactions between the two fields could induce some observable effects, possibly enhancing also local NG to an observable level. It is important to note here that if both fields contribute to the dynamics of the background, we should rigorously impose slow-roll conditions on both of them. However, if the field associated to the R^2 terms is subdominant, then this condition could be relaxed and its possible NG could be transferred to the inflaton field. In the Einstein frame this is equivalent to a transfer of non-Gaussian isocurvature perturbations to the adiabatic perturbation mode [28]. To study this effect, we will consider $f(R) = \frac{1}{2}M_{\rm Pl}^2 R + R^2/12M^2$.

This choice is motivated by the fact that it corresponds to the leading-order term in an expansion of a generic $f(R)$ in powers of R (or, equivalently, in derivatives of the metric). In this case, we obtain a complete potential $V(\varphi_1, \varphi_2)$ given by

$$\begin{aligned} V(\varphi_1, \varphi_2) = &\frac{3}{4}M^2M_{\rm Pl}^4(1 - e^{-2\varphi_1/\sqrt{6}M_{\rm Pl}})^2 \\ &+ e^{-4\varphi_1/\sqrt{6}M_{\rm Pl}}U(\varphi_2). \end{aligned} \qquad (8)$$

It is clear that if the field φ_1 is very heavy and the scale of the new physics induced by the R^2 term is much higher than the energy scale of the inflaton φ_2, then its effect should be vanishingly small. Indeed, if φ_1 is heavy enough, it could not be excited during inflation and its kinetic energy would be completely negligible. Therefore, we could integrate it out of the action (4), coming back to a standard effective single-field scenario. This would correspond to a value of $M \sim 1$ or higher, which implies that the new physics simply enters at the Planck scale or beyond. On the other hand, lowering the scale $M \lesssim 1$, the first regime we encounter is the quasisingle field regime [29]. Progressively reducing the value of M, other regimes are possible: first the multifield inflation where both scalar fields are actively at play and then, when the field φ_1 dominates the dynamics, single-field Starobinsky inflation [10]. Hereafter, we adopt a monomial potential $U(\varphi_2) = m^{4-\beta}\varphi_2^\beta$, with $\beta < 2$ (motivated by current Planck-satellite constraints). Our results are insensitive to the choice of β. We have also explored other nonmonomial potential, like cosine potentials $[U[\varphi_2] = m^4(1 - \cos[\varphi_2/f])$; with f a free parameter], and found our results do not depend on the particular choice of $U(\varphi_2)$.

We are interested in the quasi single-field regime, as observables do not depend on the particular choice of the initial conditions. In this sense we look for *generic* predictions. In this case, assuming that the adiabatic direction is given by $\varphi_2 \equiv \varphi_I$, we obtain nontrivial effects from the coupling with the isocurvature field $\varphi_1 \equiv \varphi_G$. (Here, by using the subscripts I and G we have made explicit that the field φ_I is the inflation and φ_G describes the modifications of gravity). To make an estimate of the magnitude of the effect, we can expand the action Eq. (4) in the flat gauge and ignore metric perturbations for simplicity. At second order, we find the leading transfer vertex

$$\delta\mathcal{L}_2 = \frac{2}{\sqrt{6}M_{\rm Pl}}e^{(-2\bar{\varphi}_G/\sqrt{6}M_{\rm Pl})}\dot{\bar{\varphi}}_I\delta\varphi_G\delta\dot{\varphi}_I, \qquad (9)$$

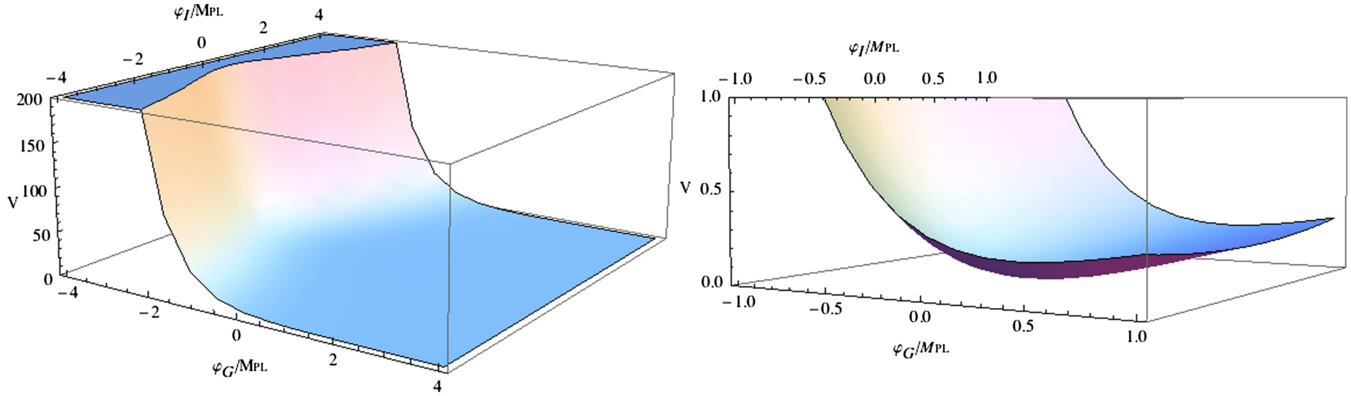

FIG. 1 (color online). Potential as a function of the two scalar fields. φ_G describes the "scalaron" field that accounts for modifications of Einstein gravity while φ_I is the one driving inflation. Significant non-Gaussianities ($|f_{\rm NL}| \approx 1$–30) are generated for generic initial field values, provided $\varphi_G > -3$. Parameters are chosen for illustration purposes. In particular, we chose a quadratic potential [30] for the inflaton field φ_I. The right panel shows the potential around the minimum.

where the bar refers to homogeneous quantities computed on the background. At third order, as the isocurvature potential U_1''' is not subject to slow-roll conditions, the leading vertex is

$$\delta\mathcal{L}_3 = -\frac{1}{6}U_1'''(\bar{\varphi}_I)\delta\varphi_G^3. \qquad (10)$$

Therefore, we expect a contribution to the bispectrum of size

$$\begin{aligned} f_{\rm NL} &\simeq \alpha(\nu)(\widehat{\delta\mathcal{L}_2})^3\widehat{\delta\mathcal{L}_3}\mathcal{P}_\zeta^{-1/2} \\ &= -\frac{4}{9\pi}\alpha(\nu)\frac{\mathcal{P}_\zeta^{-1}}{\sqrt{\epsilon}}M^2\left[\epsilon - 3\left(\frac{\dot{M}_{\rm Pl,eff}}{HM_{\rm Pl,eff}}\right)^2\right]^{3/2} \\ &\quad\times\left[\left(\frac{M_{\rm Pl,eff}}{M_{\rm Pl}}\right)^2 - 4\right]\left(\frac{M_{\rm Pl,eff}}{M_{\rm Pl}}\right)^{-7}, \qquad (11) \end{aligned}$$

where $\widehat{\delta\mathcal{L}_2}$ and $\widehat{\delta\mathcal{L}_3}$ are the vertices of the interaction terms, Eqs. (9)–(10), $\nu = \sqrt{9/4 - (M_{\rm eff}/H)^2}$, $M_{\rm eff}$ is the effective mass of the isocurvature mode, and ϵ the total slow-roll parameter. In Eq. (11) $M_{\rm Pl,eff} = M_{\rm Pl}e^{\varphi_G/\sqrt{6}M_{\rm Pl}}$ is the effective (reduced) Planck mass during inflation in the Jordan frame. The numerical factor $\alpha(\nu)$ can range from 0.2, for heavier isocurvatons, to approximately 300; however, in the perturbative regime, NG can gain at most an effective enhancement factor proportional to the number of e foldings; see [29].

The shape of the potential as a function of the two fields φ_I and φ_G is shown in Fig. 1. On the left panel one can appreciate that the φ_I direction is flat but there are values of φ_G where the potential is steep. On the right panel we show the region around the global minimum. Figure (2) shows the NG parameter $f_{\rm NL}$ as a function of e folds adopting $U(\varphi_I) = m^3\varphi$; our results are not sensitive to the specific value adopted for β. As an example, for $M = 10^{-3}$ and $m = 10^{-8/3}$, in Planck units, we obtain $f_{\rm NL} \sim \mathcal{O}(-3)$, for initial values of the field $\varphi_G = 3, \varphi_I = 12$. Note the nearly scale invariant dependence. A similar value and scale dependence was found when using the cosine potential for $U(\varphi_2)$ instead. For this particular example at 60 e folds the field abandons slow-roll and reheating starts. The characteristic shape of this kind of NG is intermediate between an equilateral shape, which is reached for small values of ν, i.e., towards a single-field regime, and a local shape, for $\nu \geq 1/2$, i.e., closer to a multifield scenario.

In this set up $f_{\rm NL}$ is generically negative. A quasilocal shape with $f_{\rm NL} \approx -1$ to -30 can thus be achieved without necessity of much fine tuning. The value of $f_{\rm NL}$ scales as a function of the masses of the two potentials,

$$f_{\rm NL} \propto -(MM_{\rm Pl}/m)^2\alpha(\nu). \qquad (12)$$

This makes it possible to test deviations from GR, including quantum corrections of Einstein gravity, a couple of orders

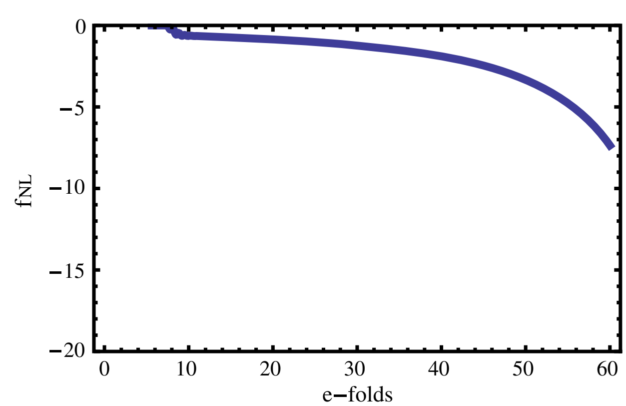

FIG. 2 (color online). The NG parameter $f_{\rm NL}$ as a function of number of e folds for $\alpha(\nu) = 1$, $M = 10^{-3}$, and $m = 10^{-8/3}$ in units of $M_{\rm Pl}$ to illustrate the scale dependence; $f_{\rm NL}$ can be smaller than -1 for fairly generic conditions.

of magnitude above the mass scale of the inflaton. Note that Eq. (11) gives a "consistency relation" between the amplitude of NG and its shape. In fact, f_{NL} measures departures from the effective gravitational constant G_{eff} during inflation as $G_{\mathrm{eff}}/G_{\mathrm{GR}} = \mathrm{e}^{-\varphi_G/\sqrt{6}M_{\mathrm{Pl}}}$.

To summarize, we have explored whether signatures of modified gravity during the period of inflation can produce observable effects. To be used to gain insight into the physics at play during inflation, these effects should be specific and not easily mimicked by standard gravity, yet arising under fairly generic conditions. For this reason we concentrated on local (or quasilocal) NG: departures from Gaussianity are $\mathcal{O}(\epsilon)$ in standard gravity single-field inflation (and higher-derivative inflaton self-interactions generate equilateral-like NG, the same being true for various gravity theories with one scalar degree of freedom that can be described in terms of a Horndeski theory, such as Galileon models [31]—for a summary of predictions see, e.g., [32]). Large non-Gaussianities can arise in multifield inflation but also other observational signatures can be generated such as isocurvature modes and breaking the tensor consistency relation. We have found that it is possible, in a very generic setup, for modifications of gravity to generate deviations from Gaussian initial conditions where the NG is close to the local type and has values $f_{\mathrm{NL}} \approx -1$ to -30.

It is interesting to note that in the same way that gravity, via its relativistic corrections, enhances the level of NG to $f_{\mathrm{NL}} \sim \mathcal{O}(-1)$ right after inflation (as pioneered by [23,33]), a modification of GR *during* inflation will lead to an enhancement of similar magnitude.

For quasilocal shapes NG is near maximal in the squeezed limit and the squeezed limit is made observationally accessible in the so-called large-scale halo bias.

Thanks to the halo bias effect, a local NG of this amplitude is expected to be measurable in forthcoming and future LSS surveys (see, e.g., [22–24,34,35]) if systematic effects can be kept under control (e.g., [36]). On the other hand, the departures from exact single-field behavior leave some imprint on the shape of NG, and, in particular, on the squeezed-limit dependence of the bispectrum on the (small) momentum. In fact, since the shape of the effective potential, Eq. (8), is given, there is a consistency relation linking the amplitude of non-Gaussianity, f_{NL}, to its shape (i.e., the parameter ν). For large enough values of f_{NL} it is possible to constrain the scale dependence of the bispectrum in the squeezed limit and hence ν, from forthcoming surveys [37,38]. Thus, in case of a detection of NG, it may be possible to test the consistency relation between amplitude and shape. If such a consistency relation were found to be satisfied to sufficient precision, it would require a fine-tuning to be produced by any multi-or quasisingle field inflation. Conversely, it is a fairly generic prediction of GR modification effects at high energies.

Further, because the noninflating field is related to gravity, the ratio between r (the tensor-to-scalar ratio) and its power law slope (n_T) will be modified from the standard single field relation—with its counterpart in the two-field description in the Einstein frame [39,40]. A given form for $f(R)$ [corresponding to a given shape of $U_1(\varphi_G)$] will break the standard consistency relation in a specific way.

Notice also that a specific running of the NG parameter f_{NL} in Eq. (11) can be left imprinted by the dynamics of the scalaron field φ_G, and, interestingly, the NG running will be correlated with the running of the scalar spectral index [29]. Specific signatures in the trispectrum of curvature perturbations, similar to those featured in Eq. (11), arise as well.

There are several features found above that, if found experimentally, will point to a modification of gravity and not simply an arbitrary two-field inflation model: the consistency relation between amplitude and shape, the specific running of f_{NL}, and a breaking of the consistency relation for r; these are specific features driven by the modified gravity sector, i.e., the fact that to go in the two-field formalism we make a Weyl transformation that brings the various exponentials in our expressions. So, if in the future we test observationally some of the above 3 points, this would be an indication of modified gravity that is difficult to mimic by an arbitrary two-field model. The fact that this result is independent of the inflaton potential sector makes our predictions general.

To conclude, these findings, if supported by data, would yield clear insights into the physical mechanism behind inflation. Conversely, a null result would place limits on possible departures from GR at the energy scale of inflation, 20 orders of magnitude beyond what has been currently tested.

The work of N. B. and S. M. was partially supported by the ASI/INAF Agreement 2014-024-R.0 for the Planck LFI Activity of Phase E2. D. C. thanks the ICG Portsmouth for its warm hospitality during the last stages of this work. L. V. is supported by the European Research Council under the European Community's Seventh Framework Programme FP7-IDEAS-Phys.LSS 240117. L. V. and R. J. acknowledge Mineco Grant No. FPA2011-29678-C02-02. N. B. and S. M. thank Frederico Arroja and Maresuke Shiraishi for useful discussions.

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