Disentangling the Dynamical Origin of $P_{11}$ Nucleon Resonances

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We show that two almost degenerate poles near the $\pi\Delta$ threshold and the next higher mass pole in the $P_{11}$ partial wave of $\pi N$ scattering evolve from a single bare state through its coupling with $\pi N$, $\eta N$, and $\pi\pi N$ reaction channels. This finding provides new information on understanding the dynamical origins of the Roper $N^*$ (1440) and $N^*$ (1710) resonances listed by Particle Data Group. Our results for the resonance poles in other $\pi N$ partial waves are also presented.

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The excited nucleon states are unstable and couple strongly to the meson-baryon continuum states to form resonances in $\pi N$ and $\gamma N$ reactions. Therefore, the extraction of nucleon resonances (called collectively as $N^*$) from data has been a well-recognized important task in advancing our understanding of strong interactions. The $N^*$ parameters listed and periodically updated by Particle Data Group [1] (PDG) are commonly used in testing hadron structure calculations using QCD-based hadron models [2] and lattice QCD [3,4].

It is well known that resonances locate on the unphysical sheets of the complex energy plane and thus their properties can only be extracted from the empirical partial-wave amplitudes (PWA) by analytic continuation. In extracting resonances from $\pi N$ data up to invariant mass $W = 2$ GeV we face a multichannel complication, namely, that a resonance may appear as a pole on more than one of the unphysical Riemann sheets, as investigated previously by Eden and Taylor [5], Kato [6], and Morgan and Pennington [7]. It is custom to name the pole which is closest to physical region as the resonance pole, and others as shadow poles. In general, the observables are mainly determined by the resonance poles. However, under certain circumstances a shadow pole could lie close to the threshold of one of the channels and could therefore affect the physical observables, as discussed in Refs. [5,7]. A theoretical understanding of the dynamical origins of these poles and their interrelations is needed to interpret the resonance parameters. In this Letter, we report progress in this direction for the $N^*$ in the $P_{11}$ partial wave of $\pi N$ scattering. Our results for other partial waves will also be presented.

The determination of resonance poles in the $P_{11}$ partial wave has been difficult since the discovery [8] of the Roper, $N^*$ (1440), resonance in 1964. It was first found by Arndt, Ford, and Roper [9] that this partial wave has two almost degenerate poles near the $\pi\Delta$ threshold. This was confirmed and investigated in more detail by Cutkosky and Wang [10]. This two pole structure has also been obtained in the recent analysis by the GWU/VPI [11] and Jülich [12] groups. In this Letter, we demonstrate that these two poles near the $\pi\Delta$ threshold ($\sim 1360$ MeV) and a pole at about 1800 MeV correspond to a single bare state within a dynamical coupled-channels model (JLMS) developed in Ref. [13]. Thus they have the resonance pole-shadow pole relation as discussed in Refs. [5–7]. Our result suggests that the $N^*$ (1440) and $N^*$ (1710) listed by PDG originate from the same excited nucleon state modeled as a bare particle within the JLMS model.

The JLMS model is defined within a Hamiltonian formulation [14] of multichannel reactions. It describes meson-baryon (MB) reactions involving the following channels: $\pi N$, $\eta N$, and $\pi\pi N$ which has $\pi\Delta$, $\rho N$, and $\sigma N$ resonant components. The excitation of the internal structure of a baryon ($B$) by a meson ($M$) to a bare $N^*$ state is modeled by a vertex interaction $\Gamma_{MB-N^*}$. The Hamiltonian also has energy independent interactions $v_{MB-M'B'}$ which describe the meson-exchange mechanisms deduced from phenomenological Lagrangians. Nucleon resonances can be due to the $MB \rightarrow N^* \rightarrow M'B'$ transitions induced by the vertex interaction $\Gamma_{MB-N^*}$ in this formulation. But they can also be due to the attractive forces of $v_{MB-M'B'}$ and channel coupling effects. For investigating the $N^*$ structure, the second type of resonances, called molecular-type resonances in the literature, must also be identified in the analysis. For the same consideration, the parameterization of $v_{MB-M'B'}$, in particular, their phenomenological form factors, must be carefully constrained by the data. This had been achieved by performing rather complex $\chi^2$ fits to the $\pi N$ scattering data, as detailed in Ref. [13]. Briefly, the JLMS model is able to describe the data of $\pi N$ elastic scattering up to invariant mass $W = 2$ GeV. The resulting $\pi N$ scattering amplitudes and total cross sections are in good agreement with those from
Within the JLM model, it is convenient to cast the partial-wave amplitude of the \( M(\vec{k}) + B(-\vec{k}) \to M'(\vec{k}') + B'(-\vec{k}') \) reaction into the following form (suppressing the angular momentum and isospin indices):

\[
t_{MB,M'B'}(k, k', E) = v_{MB,M'B'}(k, k') + \sum_{M'B'} \int_{C_M'N'} t_{MB,M'B'}(k, k', E) = v_{MB,M'B'}(k, k') + \sum_{M'B'} \int_{C_M'N'} t_{MB,M'B'}(k, k', E),
\]

where \( C_{MB} \) is the integration contour in the complex-\( q \) plane used for channel MB. The term associated with the bare \( N^* \) states in Eq. (1) is

\[
t_{N^*N^*}',(k, k', E) = \sum_{N^*_i, N^*_j} \tilde{\Gamma}_{N^*', N^*}(k, E) \times [D(E)]_{i,j} \tilde{\Gamma}_{N^*', N^*}(k', E),
\]

where \( \tilde{\Gamma}_{N^*', N^*}(k, E) \) is the dressed vertex function which is calculated from the bare vertex \( \Gamma_{N^*', N^*}(k, E) \) and convolutions over the meson-exchange amplitudes \( t_{MB,M'B'}(k, k', E) \). The inverse of the propagator of dressed \( N^* \) states in Eq. (3) is \([D^{-1}(E)]_{i,j} = (E - M_{N^*}^2) \delta_{i,j} - [M(E)]_{i,j}, \) where \( M_{N^*} \) is the bare mass of the \( i \)th \( N^* \) state, and the \( N^* \) self-energy is defined by

\[
[M(E)]_{i,j} = \sum_{MB} \int_{C_M} q^2 dq \tilde{\Gamma}_{N^*', N^*}(q, E)G_{MB}(q, E) \times \Gamma_{MB,N^*}(q, E).
\]

Equation (1) indicates that if no pole is found in the first term \( t_{\pi\pi,N^*}(k, k', E) \), then the poles of the total amplitude can be found from the second term \( t_{\pi\pi,N^*}(k, k', E) \). But if \( t_{\pi\pi,N^*}(k, k', E) \) has a pole, we need to check whether it will be canceled by the second term, as demonstrated in Ref. [12]. Thus our procedure is to first use the standard method to determine whether \( t_{\pi\pi,N^*}(k, k', E) \) has poles by examining the determinant of \([1 - vG]^{-1}\) of Eq. (2). It turns out that we do not find any pole from these meson-exchange amplitudes. Thus there is no molecular-type nucleon resonance within JLM model.

We thus can search for poles of the total amplitudes from finding the zeros of the determinant of \( D^{-1}(E) \). Here we use the well-established Newton iteration method. We have performed searches in the regions \((m_\pi + m_\pi) \leq \text{Re}(E) \leq 2000 \text{ MeV} \) and \(-\text{Im}(E) \leq 250 \text{ MeV} \) region within which PDG's 3- and 4-star resonances are listed. Poles with very large widths are more difficult to locate precisely with our numerical methods and hence will not be discussed here.

We now focus on our results in \( P_{11} \) partial wave. We find two poles near the PDG value \((\text{Re}M_R, -\text{Im}M_R) = (1350–1380, 80–110)\) of the Roper, \( N^*(1440) \), resonance. This finding is consistent with the results from the analysis by Cutkosky and Wang [10] (CMB), GWU/VPI [11] and Jülich [12] groups, as seen in Table I. In our analysis, we find that they are on different sheets: (1357, 76) and (1364, 105) are on the unphysical sheet of the \( \pi N \) channel, and could be on either unphysical \((u)\) or physical \((p)\) sheets of other channels considered in this analysis. We will indicate the sheets where the identified poles are located by \((s_{\pi N}, s_{\eta N}, s_{\pi\pi N}, s_\Delta, s_\rho N, s_\sigma N)\), where \( s_{MB} \) and \( s_{\pi\pi N} \) can be \( u \) or \( p \) or—denoting no coupling to this channel.

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We also find one higher mass pole at (1820, 248) in \( P_{11} \) partial wave, which is close to the \( N^*(1710) \) state listed by PDG. Within the JLM model, we find that this pole and the two poles listed in Table II are related to one of the two bare states needed to obtain a good fit to the \( P_{11} \) amplitude up to \( W = 2 \text{ GeV} \), see [13]. To see how these poles evolve
dynamically through their coupling with reaction channels, we trace the zeros of \( \det[D^{-1}(E)] = \det[E - M_{MP}^0 - \sum_{MB}y_{MB}M_{MB}(E)] \) in the region \( 0 \leq y_{MB} \leq 1 \), where \( M_{MB}(E) \) is the contribution of channel MB to the self-energy defined by Eq. (4). Each \( y_{MB} \) is varied independently to find continuous evolution paths through the various Riemann sheets on which our analytic continuation method is valid.

We find that the three poles listed in Table I are associated to the bare state at 1736 MeV as shown in Fig. 1. The solid blue curve shows the evolution of this bare state to the position at \( C(1820, 248) \) on the unphysical sheet of the \( \pi \Delta \) and \( \eta N \) channels. The poles \( A(1357, 76) \) and \( B(1364, 105) \) evolve from the same bare state on the physical sheet of the \( \eta N \) channel. The dashed red curve indicates how the bare state evolves through varying all coupling strengths except keeping \( y_{\pi\Delta} = 0 \), to about \( \text{Re}(M_{\pi\Delta}) \sim 1400 \) MeV. By further varying \( y_{\pi\Delta} \) to 1 of the full JLMS model, it then splits into two trajectories; one moves to pole \( A(1357, 76) \) on the unphysical sheet and the other to \( B(1364, 105) \) on the physical sheet of \( \pi \Delta \) channel. Figure 1 clearly shows how the coupled-channels effects induces multipoles from a single bare state. The evolution of the second bare state at 2037 MeV [13] into a resonance at \( \pi \eta \eta \) can be similarly investigated, but will not be discussed here.

To explore this interesting result further and to examine the stability of the determined three \( P_{11} \) poles, we have performed several refits of the \( P_{11} \) amplitudes within the JLMS model. We are able to get new fits by varying solely the parameters associated with the bare \( N^* \) state at 1763 MeV while keeping its bare mass value varied within the range 1763 ± 100 MeV. The quality of these fits is comparable to that of the original JLMS model. The above described features remain unchanged; we find in all refitted results two poles close to the \( \pi \Delta \) threshold, within 1 MeV of the positions reported in Table II. The third higher mass pole is also found but its position varies up to 30 MeV from the value given in Table II. The trajectories similar to that shown in Fig. 1 are also obtained. This is the extent to which the stability of the resonance pole-shadow pole relation among the three \( P_{11} \) poles we can establish here.

A more detailed analysis of the model dependence of our results would involve extensive refits by varying the parameters associated with both the meson-exchange interaction \( v_{MB,MB^R} \) and bare \( N^* \) states in all partial waves and can not be addressed here.

[Table I and II are omitted for brevity.]

To further compare our \( P_{11} \) poles with the \( N^*(1440) \) and \( N^*(1710) \) listed by PDG, we have applied the method explained in Ref. [16] to extract the residues \( F = \text{Re}\phi \) which is related to \( S \) matrix by \( S(E) \sim 1 + 2iF/(E - M_R) \) as \( E \to M_R \). We obtain \( \{R[\text{MeV}], \theta[\text{degrees}]\} = (36, -111), (64, -99), (20, -168) \) for the \( P_{11} \) poles at \( \text{Re}(M_{\pi\eta}) \sim 1357, 1364, 105 \), and \( 1820, 248 \), respectively. The branching ratio of the \( N^* \) decay into \( \pi N \) channel can then be estimated by evaluating \( \eta_{\pi N} \sim R/(\text{Im}(M_{\pi\eta})) \). Our results for the \( P_{11} \) poles at \( 1357, 1364, 105 \) are 49% and 61%, respectively. These values are close to 60–70% of the \( N^*(1440) \) listed by PDG. Our result for the pole at \( 1820, 248 \) is 8% which is also close to 10%–20% of \( N^*(1710) \). We thus have firmer evidence showing that these two \( N^* \) states listed by PDG do evolve from the same bare state through their coupling with \( \pi N, \eta N \), and \( \pi\eta N \) reaction channels.

Let us now turn to other partial waves. In Table II, the extracted resonance poles positions \( M_R \) are compared with the bare \( N^* \) masses \( M_{NP}^0 \) of the JLMS model and the 3- and 4-star nucleon resonances listed in the PDG [1]. With the exception of the \( P_{33}(1600), P_{13}, \) and \( P_{31} \) cases, all pole positions listed by the PDG are consistent with our results. One possible reason for not finding these poles is that their imaginary part may be beyond the \( -\text{Im}(M_{\pi\eta}) \approx 250 \) MeV region where our analytic continuation method is accurate and is covered in our searches. Another possibility is that these resonances, if indeed they exist, are perhaps due to the mechanisms which are beyond the JLMS model, but
are particularly sensitive to these partial waves. On the other hand, the possibility that these resonances do not exist cannot be excluded since the $\pi N$ data are not complete and all partial-wave analyses involve unavoidable theoretical assumptions. For the $F_{35}$ partial wave, we have also analyzed the evolution trajectories and found that the two poles listed in Table II correspond to the same bare state at 2162 MeV.

In summary, we have applied an analytic continuation method [16] to extract nucleon resonances from a dynamical coupled-channels model within which the bare $N^*$ states were determined from fitting the $\pi N$ scattering data up to $W = 2$ GeV [13]. Compared with all previous analysis, the new aspect of this work is to study the evolution of resonance pole parameters as a function of the coupling to continuum meson-baryon channels. Our most important finding is that the two lowest $P_{11}$ nucleon resonances, the Roper $N^*(1440)$ and $N^*(1710)$, originate from a single bare state. Our finding has an important implication in understanding how nucleon resonances arise in QCD. It implies that in some limits in which the coupling to the continuum is not fully implemented, for example, large $N_c$ QCD or quenched lattice QCD, there could be fewer nucleon resonances. Another possible implication is that the bare $N^*$ states, not the resonance poles, determined within our model could correspond to hadron structure calculations which exclude the coupling with meson-baryon continuum. Further investigations of these possibilities as well as related theoretical questions are needed to open a new direction towards understanding nucleon resonances and their connection to QCD. Finally, we mention that our results have confirmed most of the 3- and 4-star nucleon resonance poles listed by PDG but found no evidence of two four-star resonances, $P_{13}(1720)$, $P_{31}(1910)$, and one three-star one, $P_{33}(1600)$.

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FIG. 1 (color online). (left) Trajectories of the evolution of $P_{11}$ resonance poles $A(1357, 76), B(1364, 105),$ and $C(1820, 248)$ from a bare $N^*$ with 1763 MeV, as the couplings of the bare $N^*$ with the meson-baryon reaction channels are varied from zero to the full strengths of the JLMS model. See text for detailed explanations. Branch cuts for all channels are denoted as dashed lines. The branch points, $E_{b,p}$, for unstable channels are determined by $E_{b,p} - E_M(k) - E_B(k) - \Sigma_{MB}(k, E_{b,p}) = 0$ of the their propagators (described in the text) evaluated at the spectator momentum $k = 0$. With the parameters [14] used in JLMS model, we find that $E_{b,p}(\text{MeV}) = (1365.40, -32.46), (1704.08, -74.98), (1907.57, -323.62)$ for $\pi \Delta, \rho N,$ and $\sigma N,$ respectively. (right) Three-dimensional depiction of the behavior of $|\det[D(E)]|^2$ of the $P_{11} N^*$ propagator (in arbitrary units) as a function of complex $E$.