Contribution of demography to the Spanish economic growth from 1850 to 2000*

Miguel Sanchez-Romero[1], Gemma Abio[2], Concepció Patxot[2], and Guadalupe Souto[3]

[1]Wittgenstein Centre for Demography and Global Human Capital (IIASA,VID/ÖAW, WU)
[2]Universitat de Barcelona (UB)
[3]Universitat Autonoma de Barcelona (UAB)

April 5, 2016

Abstract

In the span of 150 years, the life expectancy at birth in Spain rose from 30 to 80 years and the average number of children per woman fell from 4-5 children to slightly more than one. To gauge the economic consequences of these demographic trends, we implement an OLG model with heterogeneity by level of education in which individuals optimally decide their consumption and time spent on market goods and home-produced goods. The economic model is supplemented with a reconstruction of the Spanish population by single years of age based on historical demographic records. We find that 23.1% of the observed increase in per capita income growth from 1850 to 2000 is due to the demographic transition. Moreover, we show that the model is consistent with other historical trends such as the evolution of the number of hours worked in the market and the total number of employed workers.

*We thank Meritxell Sole Joves and Bernhard Hammer for their research assistance and fruitful discussions. We also thank David Reher and Diego Ramiro for providing historical demographic data and references. This project has received funding from the European Unions Seventh Framework Programme for research, technological development and demonstration under grant agreement no 613247.

1 Introduction

Inconsistent results on the correlation between population growth and economic growth led for a long time to neglect the importance of the demographic transition on per capita income growth (Kelley, 1998).[1] It is not until the 1990s, using empirical convergence models à la Barro (1991, 1997), that several scholars were able to better isolate the effect of demography on economic growth (Kelley and Schmidt, 1995; Bloom and Williamson, 1997, 1998). Their main finding was that demography has a strong and positive effect on economic growth when the working-age population grows faster than the dependent population, known as *first demographic dividend*. Later on, Kelley and Schmidt (2005) did an important contribution by considering in the convergence model that changes in the age distribution of the population (*translation component*) were likely to affect the productivity of workers (*productivity component*). By doing so, Kelley and Schmidt (2005) estimates that demography accounted worldwide for twenty percent of per capita income growth between 1965 and 1990.

Despite all the recent findings there are still many unanswered questions (Williamson, 2013). For instance, to what extent does demography influence economic growth over a longer time span? Is the twenty percent of per capita income growth also explained by demographic changes historically? The demographic dividend literature has extensively used cross-country panel data for the period 1950-2010, which coincides historically with the most rapid growth of the *Homo sapiens*. However, as far as we know, there is no study of the first demographic dividend starting in the nineteenth century, exactly at the onset of the demographic transition in Europe (Livi-Bacci, 2000; Lee, 2003).

The aim of this paper is to assess the impact of the Spanish demographic transition to per capita income growth along the period 1850-2000. Spain is of great interest to economists, demographers, and historians because of the good quality of data and the similarities with the East Asian "tigers" in the second half of the twentieth century (Prados de la Escosura and Rosés, 2010). Spain started the demographic transition later than northern European countries (Livi-Bacci, 2000). In 1850 the Spanish population size was around 15 million inhabitants, the average woman expected to have between four and five children, and life expectancy at birth was close to 30 years –due to an extremely high infant mortality– (Ramiro Fariñas and Sanz Gimeno, 2000). In 2000, the Spanish population was over 40 million people, the average woman

[1] We use interchangeable per capita income growth and economic growth in the article. The demographic transition refers to the transition from high birth and death rates to low birth and death rates.

expected to have 1.23 children, and life expectancy at birth was close to 80 years. The Spanish population also witnessed an economic revolution along this period. According to Prados de la Escosura and Rosés (2009) the average labor income per worker from 3.000-3.500 euros per year in 1850 to over 33.500 euros in 2000.[2] Moreover, the average number of hours worked declined by thirty-six percentage points and the entrance in the labor market was delayed due to the educational expansion. Indeed, sixty-four percent of the cohort born in 1850 was illiterate and thirty-four percent had only primary education (Nuñez, 2005). In 2000, the average number of years of schooling was 8.4 for adults (Barro and Lee, 2013). Thus, the increase is even more remarkable if we focus on the wage rate per hour worked that rose from 1.2 euros in 1850 to 19.2 in 2000.

For the purpose of this paper, the literature has frequently used convergence models. However, to calculate the demographic dividend starting in the nineteen century, convergence models face two important challenges: i) the lack of data necessary to guarantee robustness of the results and ii) the endogeneity problem in the accumulation of input factors (Feyrer, 2007). An alternative to convergence models for calculating the demographic dividend is the overlapping generation models (OLG) (Sánchez-Romero, 2013), which are less data demanding than convergence models, do not suffer from endogeneity problems, and under the correct experiments give similar results. For this reason, to measure the Spanish demographic dividend from 1850 to 2000, we follow Sánchez-Romero (2013) by implementing an OLG model calibrated to the Spanish economy. Then, we compare the baseline to a set of alternative demographic scenarios in which the vital rates are fixed at least one generation before the period analyzed. Our paper, however, differs from Sánchez-Romero (2013) in several aspects. First, the model is calibrated to the Spanish economy rather than to the Taiwanese economy. Second, the period of analysis increases from 40 years to 150 years. And third, we extend the model by introducing household production. We follow the works of Greenwood, Seshadri, and Yorukoglu (2005) and Ramey (2009) in order to account for the impact of technological progress on the value of time, especially on the time of rearing children. The costs of rearing children in terms of money and time to household heads are introduced in the utility function through the family size (Browning and Ejrnæs, 2009). Given that we have detailed demographic information for the period 1850-2000, our family size not only changes over time but also by age of the household head. The units of equivalent adult consumption of market and home-produced goods rely on the AGENTA database (Vargha,

[2] All figures are measured at constant prices of 2010.

Gál, and Crosby-Nagy, 2015; Renteria, Scandurra, Souto, and Patxot, 2015). In addition, to control for the educational dividend (Crespo Cuaresma, Lutz, and Sanderson, 2014), we model heterogenous agents by educational attainment using data from Wittgenstein Centre for Demography and Global Human Capital (2015).

We find that the changes in the age structure of the population accounts for 23.1% of the Spanish per capita income growth for the period 1850-2000. This result is in line with the literature (Kelley and Schmidt, 2005; Sánchez-Romero, 2013). Additional counterfactual experiments show that fertility explains around 15% of the impact of demographic changes in per capita income growth, while mortality explains 13% of the impact of demographic changes in per capita income growth. We obtain a greater effect of mortality on income growth, relative to that in the literature, due to the longer period analyzed. Moreover, we have further decomposed the contribution in the *translation component* and the *productivity component*. Our results suggest that for the one-and-a-half centuries, the translation component accounts for 40% of the total income growth, while the productivity component accounts for 60%.

The paper is organized as follows. Section 2 details the theoretical model and its main theoretical implications. Section 3 presents the Spanish demographic transition, the economic data, and the model calibration. The contribution of the Spanish demographic transition on the per capita income growth rate is presented in Section 4 and the impact of different model assumptions on our results are discussed in Section 5. Section 6 concludes.

2 Theoretical model

We implement a large-scale OLG model à la Auerbach and Kotlikoff (1987) in which heterogeneous household heads by level of education endogenously choose the consumption of the household and the time spent in the market and in home production. Demographics and technological progress are exogenous. Firms are assumed to operate in perfectly competitive markets and produce under constant returns to scale. To account for the full effect of demography on the economy, we assume Spain is a closed economy. Hence, changes in the population structure might have an impact on input prices. Moreover, to better capture the accumulation of capital over time, we consider the historical evolution of the pension system through the average replacement rate. Thus, our individuals contribute a fraction of their wages to the pension system, and retirees receive a fraction of their average past earnings as pension benefits.

2.1 Household preferences

For expositional purposes, let us consider in this subsection a static framework. Household heads may belong to any of the three possible levels of education that we denote by $e \in \mathbf{E}$={*Primary or less; Secondary; Tertiary*}. Households derive utility from consumption of market goods c^m, home goods and services c^h, and leisure z. The period utility of a household, whose head has a level of education $e \in \mathbf{E}$, is given by

$$u_e^h(c^m, c^h, z) = (n^m)^{\eta^m} \ln\left(\frac{c^m}{n^m} - \bar{c}\right) + \beta_e (n^h)^{\eta^h} \ln\left(\frac{c^h}{n^h}\right) + \zeta_e \frac{z^{1-\frac{1}{\sigma_e}} - 1}{1 - \frac{1}{\sigma_e}}, \tag{2.1.1}$$

where n^m and n^h denote the number of equivalent adult consumers of market goods and home production, respectively. We explicitly distinguish between n^m and n^h to account for the fact that home production mainly accounts for services demanded within the household, such as child-caring, cleaning, cooking, etcetera, which are time demanding –especially from children–, while the consumption of market goods at young ages is smaller than that at adult ages (Vargha, Gál, and Crosby-Nagy, 2015); $\bar{c} > 0$ is the subsistence level of consumption; η^m and η^h represent the degree of economies of scale in market goods and in home-production; $\beta_e > 0$ and $\zeta_e > 0$ are relative utility weights on home production and leisure, respectively, and $\sigma_e > 0$ is the elasticity of substitution on labor supply. The set of parameters $\{\beta_e, \zeta_e, \sigma_e\}$ depends on 'e' in order to better account for differences in the labor supply by level of education.

We assume home production require intermediate goods and labor

$$f^h(c^x, h) = \left[\theta\,(c^x)^{\frac{\rho-1}{\rho}} + (1-\theta)\,(h)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \text{ with } \rho \geq 0, \tag{2.1.2}$$

where c^x stands for intermediate goods used for home production, h is the time spent on home production, and θ is assumed to be positive and between zero and one. Therefore, according to Eq. (2.1.2) technological progress has an impact on home production through intermediate goods. Moreover, we implicitly assume that home production is always consumed in the household and not sold in the market; i.e. $c^h = f^h(c^x, h)$.

2.2 Households' problem

In each year t household heads are heterogenous by their educational attainment (e), age (j), year of birth ($t - j$), assets accumulated (a), their pension base (q), and their household size of market goods and home production

(n^m, n^h). We denote by $\boldsymbol{x}_{e,j,t} = \{e, j, t, a_{e,j,t}, q_{e,j,t}, n^m_{j,t}, n^h_{j,t}\}$ the state variables of a household head with a level of education e at age j in year t. Each household head at age j in year t faces a probability to surviving to the next age of $\pi_{j+1,t+1}$, which is independent of other household characteristics.[3] Individuals can live to a maximum of 100 years. For simplicity, we assume surviving children between age 0 and 16 are dependents and do not work either in the market or at home.[4] At the age of 17 (J_0) individuals start making decisions, leave their parents home, and establish their own household. After age 65 (J_R) all individuals retire. Adults are endowed with one unit of time that they distribute between market work, home work, and leisure. Annuity markets are absent and accidental bequests are distributed by the government to all households in the economy.

Household heads maximize their lifetime utility (V) by optimally choosing their consumption of market goods, home production, intermediary goods, leisure time, and time spent on home-production. Let the control variables of a household head with a level of education e at age j in year t be $\boldsymbol{c}_{e,j,t} = \{c^m_{e,j,t}, c^h_{e,j,t}, c^x_{e,j,t}, h_{e,j,t}, z_{e,j,t}\} \in \mathbf{C}$. Thus, household heads solve the following problem

$$V_{j,t}(\boldsymbol{x}_{e,j,t}) = \max_{\boldsymbol{c}_{e,j,t} \in \mathbf{C}} \left\{ u^h_e(\boldsymbol{c}_{e,j,t}) + \pi_{j+1,t+1} V_{j+1,t+1}(\boldsymbol{x}_{e,j+1,t+1}) \right\} \tag{2.2.1}$$

subject to the time constraint $\ell_{e,j,t} + h_{e,j,t} + z_{e,j,t} = 1$, the budget constraint

$$a_{e,j+1,t+1} = \begin{cases} R_t a_{e,j,t} + tr_{j,t} + (1-\tau_t)\hat{w}_{e,j,t}\ell_{e,j,t} - c^x_{e,j,t} - c^m_{e,j,t} & \text{for } J_0 \leq j \leq J, \\ R_t a_{e,j,t} + tr_{j,t} + \psi_{t-j} q_{e,j,t} - c^x_{e,j,t} - c^m_{e,j,t} & \text{for } j > J_R, \end{cases} \tag{2.2.2}$$

and the benefits base, or pension rights, which is calculated according to

$$q_{e,j+1,t+1} = \begin{cases} q_{e,j,t} + \varrho \hat{w}_{e,j,t}\ell_{e,j,t} & \text{for } J_0 \leq j \leq J_R, \\ q_{e,j,t} & \text{for } j > J_R, \end{cases} \tag{2.2.3}$$

and the standard boundary conditions $a_{e,J_0,\cdot} = 0$, $q_{e,J_0,\cdot} = 0$, and $a_{e,100,\cdot} \geq 0$, where ℓ denotes the hours worked (or intensive labor supply), R is the capitalization factor, tr is the accidental bequests distributed by the government

[3] Although the literature show that there exists a positive correlation between educational attainment and longevity, we do not have information on death rates by educational attainment for the period analyzed (Lleras-Muney, 2005).

[4] Existing estimates suggest children between ages 0-17 supply on average four hours per week doing homework, which represents one-sixth of the total average time devoted to homework by a prime-age adult (Ramey, 2009).

to the household head, τ is the social contribution rate to the pension system, $\hat{w}_{e,j,t} = r_t^H \epsilon_{e,j}$ is the wage rate per hour worked of an individual with education e at age j in year t, which depends on the wage rate per efficient unit of labor (r_t^H) and the age-specific productivity by educational attainment ($\epsilon_{e,j}$), ψ_{t-j} is the replacement rate for the cohort born in year $t-j$, and ϱ is the weight of current labor earnings on the benefits base.

After manipulating the first-order condition (A.1) and the envelope condition (A.6), the optimal consumption of market goods can be characterized through the Euler condition augmented with household size and subsistence level

$$\frac{\tilde{C}^m_{e,j+1,t+1}}{(n^m_{j+1,t+1})^{\eta^m}} \Bigg/ \frac{\tilde{C}^m_{e,j,t}}{(n^m_{j,t})^{\eta^m}} = \pi_{j+1,t+1} R_{t+1} \text{ with } \tilde{C}^m_{e,j,t} = c^m_{e,j,t} - \bar{c} n^m_{j,t}. \tag{2.2.4}$$

Assuming for expositional simplicity that $\bar{c}/A_t$ is sufficiently small ($\tilde{C}^m_{e,j,t} \approx c^m_{e,j,t}$), Eq. (2.2.4) indicates that the path of household consumption of market goods is more responsive to changes in household size as η^m approaches to one. In the extreme case that $\eta^m = 1$, consumption smoothing occurs at the per capita consumption level, and hence the household consumption changes at the same rate as the number of equivalent adult consumers. Recent large scale OLG models stress the importance of $\eta^m > 0$ as a channel for demographics to affect household saving (Braun, Ikeda, and Joines, 2009; Sánchez-Romero, 2013; Curtis, Lugauer, and Mark, 2015). Also, a value of $\eta^m > 0$ is supported by a large econometric evidence (Browning and Lusardi, 1996; Browning and Ejrnæs, 2009; Attanasio and Weber, 2010). The introduction of a subsistence level of consumption $\bar{c}$ is key for explaining a decline in the number of hours worked, since it allows the income effect to dominate over the substitution effect when productivity increases (Restuccia and Vandenbroucke, 2013).

Combining (A.1), (A.4)-(A.5), and using (2.1.1) the optimal hours worked (or intensive labor supply) is

$$\ell_{e,j,t} = 1 - \left(\zeta_e \frac{\tilde{C}^m_{e,j,t}/(n^m_{j,t})^{\eta^m}}{\hat{\omega}_{e,j,t}} \right)^{\sigma_e} - \beta_e (n^h_{j,t})^{\eta^h} \frac{\tilde{C}^m_{e,j,t}/(n^m_{j,t})^{\eta^m}}{\hat{\omega}_{e,j,t} + \left(\frac{\theta}{1-\theta}\hat{\omega}_{e,j,t}\right)^{\rho}}, \tag{2.2.5}$$

where $\hat{\omega}_{e,j,t} = (1 - \tau^E_{e,j,t})\hat{w}_{e,j,t}$ is the net (of effective social security tax) wage rate per hour worked for an individual with education e at age j in year t. The term in parenthesis on the right-hand side of Eq. (2.2.5) is the optimal leisure time, while the last term is the optimal time spent on home production. Eq. (2.2.5) indicates that a raise in the effective wage rate $\hat{\omega}_{e,j,t}$ or a fall in the number of equivalent adult consumers of home-production $\eta^h_{j,t}$ decreases the

optimal hours worked in the market. Notice, however, that a raise or a fall over the lifecycle in the number of equivalent adult consumers for market goods does not have an impact on labor, since the right-hand side of Eq. (2.2.4) does not depend on the household size.

At the extensive margin, household heads leave the labor market and specialize in home-production when the marginal rate of substitution between leisure and consumption is greater than the effective wage rate per hour worked

$$\frac{\partial u_e^h(\boldsymbol{c}_{e,j,t})}{\partial z_{e,j,t}} \Big/ \frac{\partial u_e^h(\boldsymbol{c}_{e,j,t})}{\partial c_{e,j,t}^m} > \hat{\omega}_{e,j,t}, \tag{2.2.6}$$

where the left-hand side of Eq. (2.2.6) is equivalent to the marginal utility (measure in monetary terms) of devoting one additional hour in home production; i.e. $p_j^* \frac{\partial f^h}{\partial h_{e,j,t}}$. As a consequence, household heads with lower productivity endowments or higher effective social contribution tax rates have an incentive to leave the labor market. Thus, if productivity endowments are hump-shaped, young and at old household heads are more likely to specialize on home-production.

Similar to Ramey (2009) the ratio of intermediate goods to labor in home production is given by

$$\frac{c_{e,j,t}^x}{h_{e,j,t}} = \left(\frac{\theta}{1-\theta}\hat{\omega}_{e,j,t}\right)^{\rho}, \tag{2.2.7}$$

where the effect of a rise in productivity on the ratio $\frac{c_{e,j,t}^x}{h_{e,j,t}}$ depends on the elasticity of substitution between intermediate goods and labor. Thus, for $\rho = 0$, a raise in productivity does not change the ratio, while the ratio would increase when $\rho > 0$.

2.3 Firms

To have sharp results for the contribution of demography on economic growth, we use a simple production setting. Firms operate in a perfectly competitive environment and produce one homogenous good combining capital and labor, according to a standard Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \tag{2.3.1}$$

where α denotes the share of capital, K_t is the stock of physical capital, A_t is the labor-augmenting technology, and L_t is the stock of human capital. We

exclude land as an input factor in order to guarantee an initial and final steady-state equilibrium when the population grows at a constant rate.[5] Output can be either consumed, used as an intermediary good for home-production, or used as an investment good. Labor-augmenting technology, A_t, is assumed to grow at the exogenous rate of g_t^A, i.e. $A_{t+1} = A_t(1 + g_t^A)$. We assume labor supply of different workers are perfect substitutes. Hence, the stock of human capital is $L_t = \sum_{j=J_0}^{J_R} N_{j,t} \int_{\mathbf{E}} \epsilon_{e,j} \ell_{e,j,t} dE_{t-j}(e)$. The rental price of physical capital (r_t^K) and human capital (r_t^H) equals their marginal products, i.e., $r_t^K = \alpha \frac{Y_t}{K_t} - \delta$ and $r_t^H = (1-\alpha)\frac{Y_t}{H_t}$, respectively, where δ is the depreciation rate of physical capital.

2.4 Government

Our government has two objectives. First, the government runs a simple balanced-budget pay-as-you-go pension system. Thus, in each period the government sets the social tax rate τ_t that workers contribute to guarantee a fraction ψ_{t-j} of the past earnings of pensioners of age j in year t. The budget constraint of the pension system is

$$\tau_t r_t^H L_t = \sum\nolimits_{j>J_R} N_{j,t} \psi_{t-j} \int_{\mathbf{E}} q_{e,j,t} dE_{t-j}(e) \text{ for all } t. \tag{2.4.1}$$

The left-hand side of Eq. (2.4.1) is the total social contributions paid by workers in year t, while the right-hand side of Eq. (2.4.1) is the total pension benefits claimed by pensioners in year t. Second, the government levies 100 percent tax on all accidental bequests and distributes it in a lump-sum fashion among adult individuals –above age 16– who are 28 years younger than the decedents' age. The wealth of those who die before age 44 (=28+16) is assumed to be distributed within the same birth cohort.

The definition of equilibrium is standard and can be found in Appendix B.

3 Data and calibration

3.1 Demographics

The economic model we implement in this paper requires demographic data by single-years of age on death rates, fertility rates, and population size for a time-

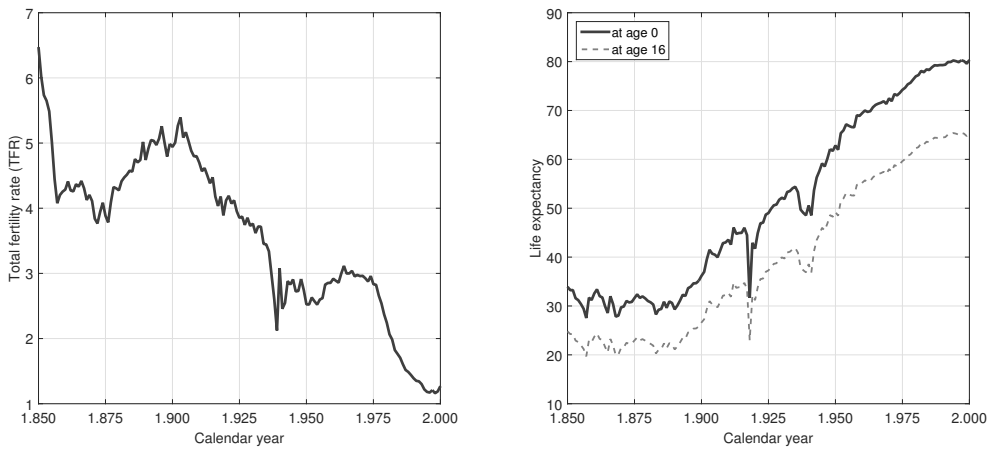
[5] Given a fixed supply of land implies that the price of land varies according to the population density. Recent estimations by Prados de la Escosura and Rosés (2009) suggest that land represented, on average, ten percent of the total output from 1850 until 1950, while it was close to zero by 2000.

span larger than the period analyzed (1850-2000). Historical demographic information from Spain is frequently scatter and many times inconsistent among different sources. Fortunately, important efforts constructing homogeneous demographic time series have already been done by several scholars. For instance, Maluquer de Motes (2008) provides homogeneous total population series from 1850 to 2001, Ramiro Fariñas and Sanz Gimeno (2000) estimates infant and childhood mortality time series from 1790 to 1960, and Reher (1991) calculates for *Castilla La Nueva* vital series of births, marriages, and deaths by using parish registers from 1550 to 1900, among others.

To construct a demographic database consistent with our economic model (i.e., a unisex closed population), we combine two demographic methods widely used in population reconstruction: *Inverse Projection* (IP) and *Generalized Inverse Projection* (GIP) (Lee, 1985; Oeppen, 1993). The IP method is used to calculate net migration rates, while the GIP method is used to reconstruct consistent data on population size by age $N_{j,t}$ and age-specific vital rates; i.e., age-specific fertility rates $f_{j,t}$ and age-specific conditional survival probabilities $\pi_{j,t}$. GIP is a non-linear optimization that produces constrained demographic projections with *a priori* information (Oeppen, 1993). We explain in more detail the GIP method in Appendix C. Both models are implemented using census data for years 1857,1860,1877,1887,1900-1970 from INE (2015b), year 1981 from INE (2015c), and years 1991 and 2001 from INE (2015d). Age-specific fertility rates from 1922 to 2012 are taken from Human Fertility Collection (2015). Total population size from 1787 to 2000 is obtained combining data from INE (2015b), Human Mortality Database (2015), and Maluquer de Motes (2008). Total births and deaths data come from INE (2015b) and from Human Mortality Database (2015).

Figures 1 and 2 show the evolution of three aggregate measures from 1850 to 2000 obtained with our population reconstruction: i) –period– total fertility rate (TFR), ii) –period– life expectancy at birth, and iii) population distributions. Assuming that fertility and mortality patterns in year t prevail throughout life, the TFR is the average number of children that would be born to a surviving woman over her lifetime, while the life expectancy at birth indicates the number of years a newborn infant would live. From Figure 1 we can observe that the levels of fertility and mortality in Spain were similar to those of western Europe before 1800 (Lee, 2003). In this period, TFR ranged between four to five children per women and life expectancy at birth was between 25 to 35 years (Livi-Bacci, 2000). One main consequence of the late onset of the demographic transition in Spain was the slower population growth relative to other western European countries. Closed to the end of the nineteenth century and beginning of the twentieth century, the TFR

started a secular decline from five to values slightly higher than one (Reher, 2011), which was only interrupted during the Franco Regime (1939-1975), and mortality rates decreased –especially in infants and childhood–. The decline in mortality translated into a remarkable increase in life expectancy at birth, which doubled from 40 years up to 80 along the twentieth century. This trend was transitorily stopped because of a few events: the Spanish flu (1918), the Civil War (1936-1939), and its posterior famine. Nowadays, Spain is the country with the second largest overall life expectancy at birth (OECD, 2015b).



(a) TFR

(b) Remaining years lived

Figure 1: Spanish total fertility rate (TFR) and remaining years lived: Period 1850-2000.

Source: Authors' estimates.

The demographic implication of the mortality and fertility processes summarized in Figure 1 can be easily seen in Figure 2. From 1850 to 2000, the Spanish population dramatically changed from a population clearly dominated by children and young adults until 1975 to a population that ages rapidly due to the fertility decline that started in 1975.

Combining the reconstructed fertility and mortality data with NTA estimates, we calculate the number of equivalent adult consumers of market and home-produced goods for a household head at age j in year t using the following formula

$$n^i_{j,t} = 1 + \sum\nolimits_{x=0}^{j} \xi^i_{j-x} \sum\nolimits_{k=0}^{N_C} kP_{x,t}(k), \tag{3.1.1}$$

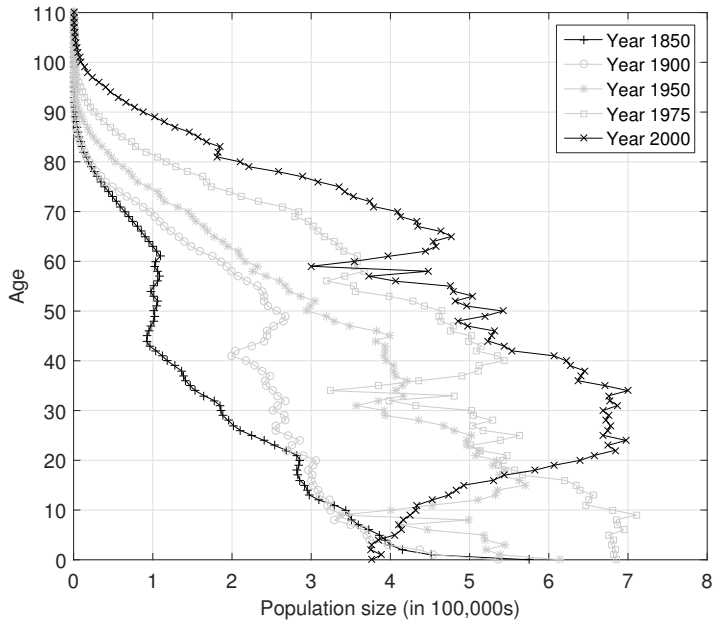
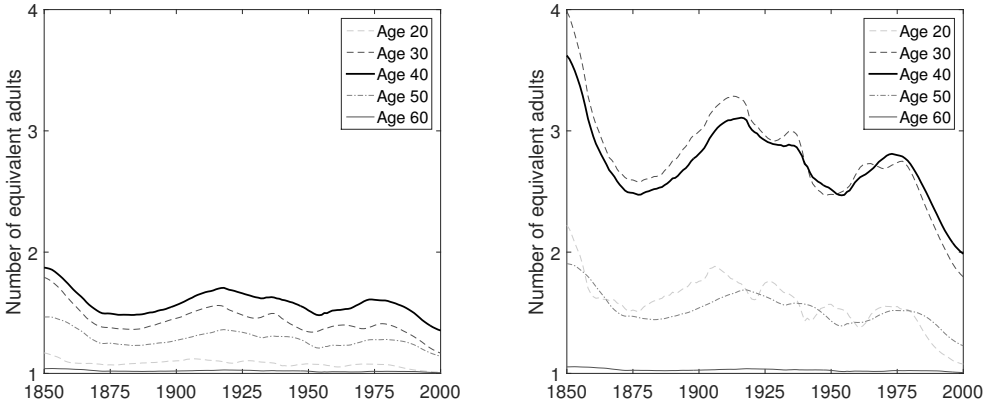


Figure 2: Spanish population distribution (both sexes combined): Selected years 1850, 1900, 1950, 1975, and 2000

Source: Authors' estimates.

where ξ_x^i is the adult-equivalent units of scale of an individual of age x for the good produced in $i \in \{m, h\}$, $P_{x,t}(k)$ is the probability of having k surviving children in year t when the household head was x years old, and N_C is the maximum number of children per year. We use a standard profile for ξ_x^m that equals 0.4 from ages 0 to 4 and rises with age until reaching 1 at the age of 18. For the number of equivalent adult consumer of home-goods ξ_x^h, we pool male and female time-consumption profiles from age 0 to 18 estimated by Renteria, Scandurra, Souto, and Patxot (2015) and rescale it according to the time consumed at home by males and females between ages 30 to 49. Performing this calculation suggests that a recently born baby demands 3.65 times more time than an adult. Afterwards, ξ_x^h exponentially declines to 1 at the age of 16.

Figure 3 shows the evolution from 1850 to 2000 of the number of equivalent adult consumers for market and home-produced goods at six selected ages of the household head (all other ages are omitted for expositional purposes). A value of two in Figure 3 means that a household head must finance the



(a) Market goods, $n^m_{j,t}$ (b) Home goods and services, $n^h_{j,t}$

Figure 3: Number of equivalent adults consumers of market and home goods: Period 1850-2000.

Source: Authors' estimates based on the population reconstruction.

consumption of one additional adult.[6] Thus, over the one-and-a-half centuries adults at the age of 40 were expected to spend on their children less than one time their own consumption on market goods, but more than two times their own consumption on home-produced goods. Recall n^h mainly accounts for the time rearing children, whereas n^m accounts for the goods purchased in the market to keep the caloric demand. Although fertility significantly fall along this period, the number of equivalent consumers did not significantly changed until 1975 because mortality rates were also falling mainly for infants and young ages. Looking across age in the same year, we can observe that the family size increases for both goods until age 40 and declines thereafter. There are, however, some distinctive features between both panels. For instance, the household size for market goods is smaller than that for home-goods. The number of equivalent adult consumers for home-produced goods at age 30 and 40 are very similar. Nevertheless, it is worth noting the increasing gap in n^h between ages 30 and 40 after 1960s due to the fall in fertility and the postponement of child-bearing. The household size shows a clear downward trend but not as step as one might a priori think. This is because not only fertility was high, but also infant mortality.

[6]Since we assume a unisex model, one should multiply by two to approximate the values in Figure 3 to a household run by two adults.

3.2 Stock of physical capital

The stock of physical capital ($\hat{K}_t$) is derived applying the perpetual inventory approach to gross fixed capital formation to the following categories: construction, transport equipment, machinery and equipment, and intangible fixed assets. The information on gross fixed capital formation for each category during the period 1850-2000 is taken from Prados de la Escosura (2003). We apply the standard formula:

$$\hat{K}_t = \sum\nolimits_i K_i(t) \text{ with } K_i(t) = (1-\delta_i)K_i(t) + I_i(t),$$

where $K_i(t)$ is the stock of capital in category i in year t, δ_i is the depreciation rate of category i, and $I_i(t)$ is the gross capital formation in category i in year t. The depreciation rates applied to each category are 2.1, 18.2, 13.8, and 15%, respectively (Hulten and Wykoff, 1981).[7] The capital stock at the beginning of the period, $K_i(1850)$, is computed so as to minimize the variance of the corresponding time series along the first fifty years.

3.3 Stock of human capital

We reconstruct the stock of human capital $\hat{L}_t$ taking into consideration the following components:

$$\hat{L}_t = \mathrm{HW}_t \sum\nolimits_{j=16}^{65} \mathrm{N}_{j,t} \int_{\mathbf{E}} \mathrm{WP}_{e,j}\mathrm{LF}_{e,j} dE_{t-j}(e), \tag{3.3.1}$$

where HW_t = average annual hours worked per worker in year t, $\mathrm{N}_{j,t}$ = population size at age j in year t, $\mathrm{WP}_{e,j}$ = productivity per hour worked at age j with education e –or efficient labor units–, $\mathrm{LF}_{e,j}$ = labor participation rate at age j for individuals with education e, $\mathrm{E}_{t-j}(e)$ = educational distribution for the cohort born in year $t-j$.

Time series on annual hours worked per worker are taken from Prados de la Escosura and Rosés (2009) and adjusted from 1977 to 2000 with OECD (2015) data on hours work. The productivity per hour worked by educational attainment is calculated deterministically by fitting a quadratic function to MTAS (2010) data on the average wage rate per hour worked across age by educational attainment (see Appendix **??** for further information). We represent these functions in Figure 4(b). Using data from INE (2015), we calculate LF

[7] A more sophisticated calculation of the stock of productive capital, which distinguishes between capital input and capital quality along the period 1850-2000, can be seen in Prados de la Escosura and Rosés (2009).

profiles by educational attainment as the average labor force participation rate between 1987 to 2013 for each educational group. The LF profile by educational attainment is depicted in Figure 4(c). We choose this period because it includes two periods in which the unemployment rate declined (1987-1991 and 1995-2007) and another two periods in which the unemployment rate increased (1991-1994 and 2008-2013). The educational distribution by birth cohort E_c in Spain is taken from Wittgenstein Centre for Demography and Global Human Capital (2015) for two reasons. First, this database offers information on historical reconstruction of educational attainment for the 20th century and second, it provides harmonized projections until 2100 of the population by age and educational attainment (Lutz, Butz, and Samir, 2014). We extract information for Spain on the shares of population by levels of education for the period 1970 to 2100, which allow us to calculate the educational distribution for each birth cohort born after year 1868.[8] Since the available data is grouped by five-years age groups, we apply lineal splines to interpolate the educational distribution for intermediate cohorts. Last, but not least, in oder to guarantee an initial steady-state, the educational attainment for cohorts born before 1868 are assumed to coincide to that of the cohort born in year 1868.

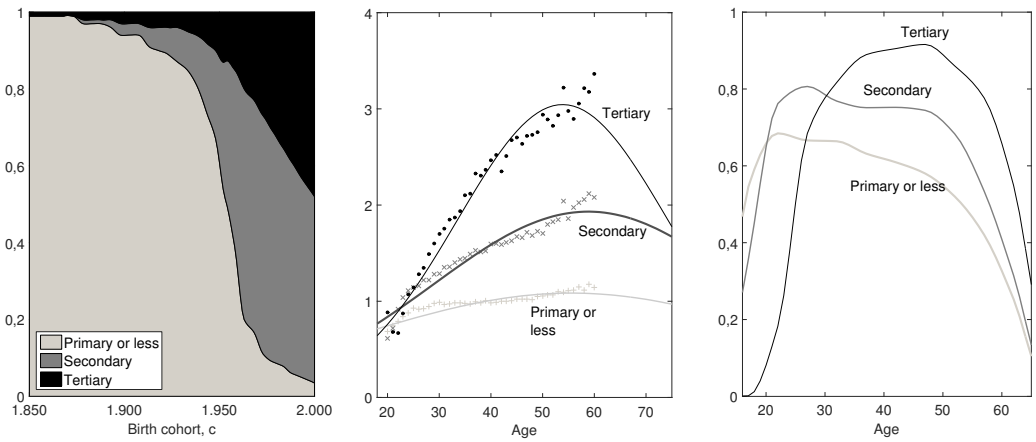
To check the validity of our reconstruction of the stock of human capital $\hat{L}_t$, we compare the total number of workers, that results from applying the formula $\sum_{j=16}^{65} \mathrm{N}_{j,t} \int_{\mathbf{E}} \mathrm{LF}_{e,j} dE_{t-j}(e)$, to existing estimates of historical workers from 1850 to 2000. Figure 5 shows that our total employment estimates before 1975 are very similar to those reported by Nicolau (2005), which validates our strategy. The lack of fitness to the actual data for the last 25 years is due to the high unemployment rate that has not been modeled for simplicity reasons.

3.4 Labor-augmenting technology

The labor-augmenting technology is calculated using reconstructed input factors in sections 3.2 and 3.3. To account for the rise in unemployment, the total number of workers from 1956 is based on OECD (2015c). Our estimation of the labor-augmenting technological progress is calculated applying the formula

$$\Delta \ln A_t = \Delta \ln(\hat{Y}_t/N_t) - \Delta \ln(\hat{L}_t/N_t) - \frac{\alpha}{1-\alpha} \Delta \ln(\hat{K}_t/\hat{Y}_t), \tag{3.4.1}$$

[8] Given the positive correlation between educational attainment and life expectancy, individuals born close to year 1868 that reached tertiary education are likely to be overrepresented. Nevertheless, due to the late onset of the educational expansion in Spain, our main results are not affected.



(a) The educational distribution, $E_c(e)$

(b) The endowment of efficient labor units, WP

(c) Labor force participation rates, LR

Figure 4: Decomposition of the stock of human capital by educational attainment (both sexes combined)

Sources: Educational distribution data is taken from Wittgenstein Centre for Demography and Global Human Capital (2015), the endowment of efficient labor units comes from MTAS (2010), and the average labor force participation rate between 1987 and 2013 is calculated using data from INE (2015).

where $\hat{Y}_t$ is the value added, $\hat{L}_t$ is the stock of human capital, and $\hat{K}_t$ is the stock of physical capital. Figure 6 depicts the percentage change in the estimated labor-augmenting technological progress A_t. Before 1850 and after 2012 we assume no productivity growth.

We obtained that the annual average labor-augmenting technological progress from 1850 to 2000 is 1.43%, which is equivalent to a total factor productivity (TFP) of 0.95% (=0.66× 1.43%). Our TFP growth is lower than the 1.1% estimated by Prados de la Escosura and Rosés (2009). The difference is to a large extent due to our lower TFP growth after the Spanish's admission into the European Union.[9] This is explained by several reasons: the progressive reliance of the Spanish economy on low productivity sectors with an increasing

[9]According to our assumptions, the rise in the average output per hour worked after the Spanish's admission in the European Union is due to the increasing proportion of workers with tertiary education, whose productivities grow faster than the other educational groups. Note, however, in Figure 4(b) that the endowments are likely affected by seniority. Hence, although our main results won't change, our TFP value should be taken with caution.

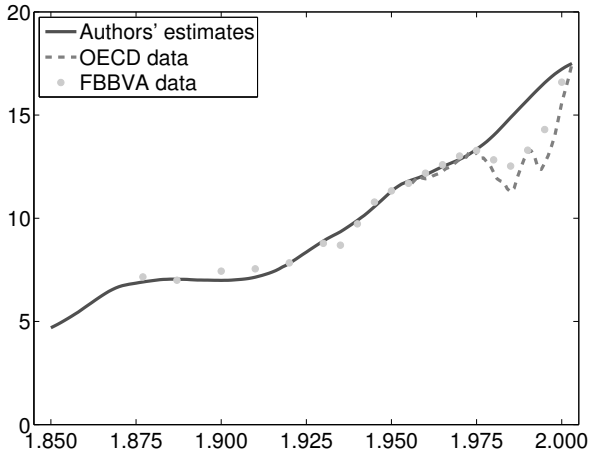


Figure 5: Total employment (both sexes combined, in millions)

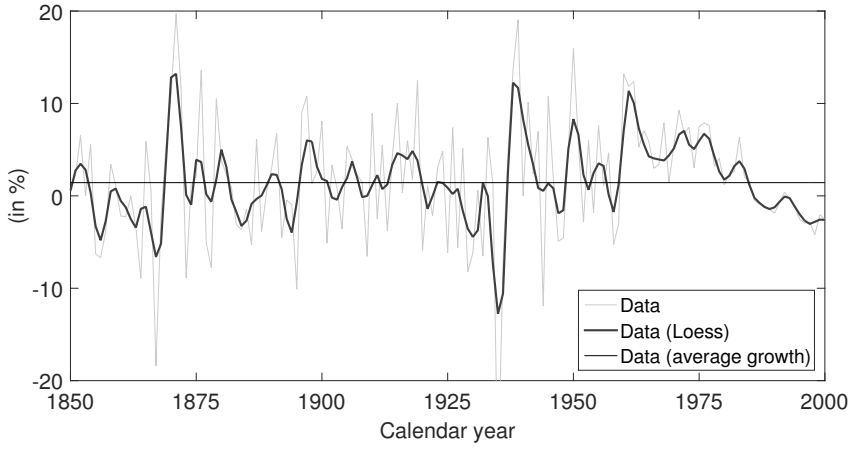


Figure 6: Labor-augmenting technological progress, Spain 1850-2000

Source: Authors' calculations. *Notes*: The 'Loess' smoothing profile was computed using a bandwidth of 0.05 and rescaled to give an annual average growth rate of 1.43%.

number of temporary workers, the lack of investment in R&D, and the lack of competition, among others.

3.5 Calibration

To perform our quantitative experiment we need to find the parameters such that the model is capable of reproducing some key historical facts of the Spanish economy. We proceed as follows.

In our baseline, we set the annual physical capital depreciation rate δ and the labor share $1-\alpha$ to the standard values 0.05 and 0.66. Our modeled depreciation rate is close to 0.057, which is the average depreciation rate from 1850 to 2000 according to our stock of physical capital reconstruction. According to Prados de la Escosura and Rosés (2009) the average labor share for the period 1850-2000 is 0.68, which is also close to our value 0.66. However, since we are aware that the Spanish labor share was not stable along the period 1850-2000 (Prados de la Escosura and Rosés, 2009), we run two alternative simulations with changing labor shares in order to gauge the quantitative importance of this assumption in the context of our model.

Since there is no information on the average pension replacement rate across cohorts, we indirectly calculate ψ_t as follows. First, we calculate the average age of pensioners (A_b) assuming all people older than 65 are pensioners. We obtain A_b is close to 72 years in 1850 and increases to 76 years by 2000. Thus, we consider the average pension benefit at time t is the average pension benefit received by the cohort born in year $t-72$. We keep this number fixed at 72 because the mean age of the representative pension earner would be younger than A_b when workers' productivity increase and pension benefits are only adjusted by consumer prices. Second, we calculate the average public pension benefit per retiree (B_t) and the average labor income per individual between ages 16 to 64 (YL_t) from 1850 to 2000 –assuming that workers receive 2/3 of the value added–. Third, we apply the formula

$$\psi_t = B_{t+72} \Big/ \sum\nolimits_{s=t+N_b-1}^{t+64} \frac{\mathrm{YL}_s}{N_b} \,,$$

where N_b is the number of years entering the computation of the benefit base. Following Boldrin, Jimenez-Martin, and Peracchi (1999), we assume $N_b = 2$ for cohorts retiring before year 1985, $N_b = 8$ for cohorts retiring between 1985 and 2001, and N_b equals 15 when retirement occurs after 2002. Figure 7 shows our estimation of the average replacement rate for birth cohorts born between 1835 and 1940. It can be observed a gradual increase in the replacement rate across cohorts with a hump for those cohorts born between 1918 and 1927. This hump is due to the introduction in 1990 of noncontributory pensions for elderly people aged sixty-five and over and the pension law introduced in 1997 that made the replacement rate less generous. To complete the model, we set

the replacement rate at nine percent for those cohort born before 1835 and at sixty-two percent for those born after 1940.

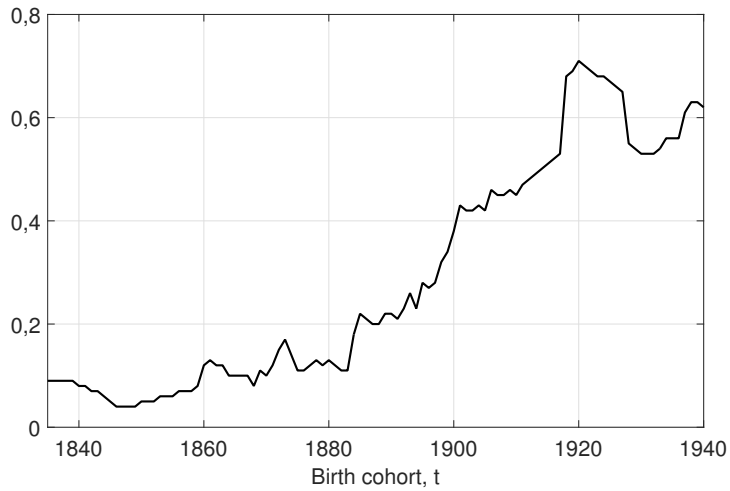


Figure 7: Average replacement rate (ψ_t), Spain: Birth cohorts 1835-1940

All remaining behavioral parameters are structurally estimated using the model. Let us denote by $\boldsymbol{\lambda}$ the 11×1 vector of parameters left to be determined:

$$\boldsymbol{\lambda} = [\lambda_e, \nu, \epsilon, \bar{c}, \theta, \rho]' \text{ with } \lambda_e = \{\sigma_e, \beta_e\} \text{ for } e \in \mathbf{E}.$$

We impose the following set of restrictions to these parameters, $\boldsymbol{\lambda} \in \Lambda$. First, the intertemporal elasticities of substitution σ_e must lay between 0 and 1. Second, the weight on utility of household produced goods β_e must be non-negative. Third, we let the degree of economies of scale in market purchased goods ν and home produced goods ϵ to range between 0 and 1. A value of one would imply that an expected increase in the household size would have no impact on the average consumption of household members. While a value of zero implies that all members would reduce their consumption when a new member enters in the household. Fourth, the minimum consumption level of market goods cannot be negative. Fifth, the shares of intermediate goods and hours worked at home θ must lay between zero and one, and the elasticity between intermediate goods and labor in home production ρ is restricted to values equal or lower than one.

For a given $\boldsymbol{\lambda}$, we solve the model in order to obtain the optimal labor supply of individuals over their lifecycle and the human capital stock. Then,

we compute function $F(\boldsymbol{\lambda})$ defined by

$$F(\boldsymbol{\lambda}) = \sum_{j=16}^{65} \sum_{e \in \mathbf{E}} \Phi^1_{e,j}(\boldsymbol{X};\boldsymbol{\lambda})^2 + \sum_{t=1851}^{2000} \sum_{i=\{l,y,c\}} \Phi^i_t(\boldsymbol{X};\boldsymbol{\lambda})^2, \quad (3.5.1)$$

where $\boldsymbol{X}$ denotes the exogenous information set of the economic model. The first term corresponds to the difference between the labor supply by level of education shown in Figure 4(c) and the individual labor supply by level of education obtained from the model, i.e. $\Phi^1_{e,j}(\boldsymbol{X};\boldsymbol{\lambda}) = \gamma \mathrm{LF}_{e,j} - \frac{1}{27}\sum_{t=1987}^{2013} \ell_{e,j,t}$. To transform the participation rate to actual hours worked, we set γ to 0.32, which is equivalent to working 36 hours per week out of a total of 112 hours per week. The second term captures the difference between the growth from 1850 to 2000 of the observed stock of human capital, income per capita, and consumption per capita and those obtained with the model. Thus, our target for $\boldsymbol{\lambda}$ is to minimize the function $F(\boldsymbol{\lambda})$.

Table 1 reports the parameter values assumed as well as those structurally estimated with the model. From Table 1 we can emphasize two key parameters: the elasticity of substitution on labor in home production (ρ) and the subsistence level ($\bar{c}$). The model yields a ρ value of 0.60, which is line with Ramey (2009) that shows a lower ρ than a Cobb-Douglas home production. As a consequence, a rise in the effective wage rate $\hat{\omega}$ leads to a fall in the market hours worked relative to the time spent on home-production. The subsistence level $\bar{c}$ we get is 0.16. If we multiply our $\bar{c}$ by the household size n^m we get values close to 0.30 in 1850, which is similar to that obtained by Restuccia and Vandenbroucke (2013) for the US in 1870. The increase in the technological progress and the decline in the household size will reduce the relative importance of $\bar{c}$ in the household consumption, and consequently a fall in the market labor supply at the micro level.

Figure 8 shows the in-sample performance of the baseline model with respect to the targeted time series. In Figure 8(a), we can see how well the model replicates the per capita hours worked by level of education between 1987 and 2013. Figure 8(b) compares the stock of human capital obtained in the baseline to the observed stock of human capital. The discrepancy is explained by the fact that we do not consider the risk of unemployment in the model, whereas the unemployment rate rose to values over twenty percent from 1976 to 2000.[10] Thus, the calibration procedure compensates the large unemployment during the last 25 years of our analysis by reducing the increase in the stock of human capital from 1850 to 1975.[11] In Figures 8(c) and 8(d)

[10] Mainly due to the lack of reliable data for the entire period.

[11] Nonetheless, an alternative strategy matching the stock of human capital from 1850 to 1975 gives a similar contribution of demography to per capita income growth.

Table 1: Model parameters

Parameter	Symbol	Level of education		
Firms technology				
Capital share	α		0.33	
Capital depreciation rate	δ		0.05	
Home production				
Factor share	θ		0.67	
Elasticity of substitution on labor	ρ		0.60	
Household preferences				
Age at parental leave	J_0		16.0	
Retirement age	J_R		65.0	
Weight of current labor income on pensions	ϱ		0.02	
Subsistence level	$\bar{c}$		0.16	
EOS on market goods	ν		0.49	
EOS on home goods	ϵ		0.13	
		Primary or less	Secondary	Tertiary
IES on leisure	σ_i	0.52	0.63	0.35
Weight of home goods	β_i	3.96	2.85	1.72
Weight of leisure	ζ_i	9.10	7.23	3.00

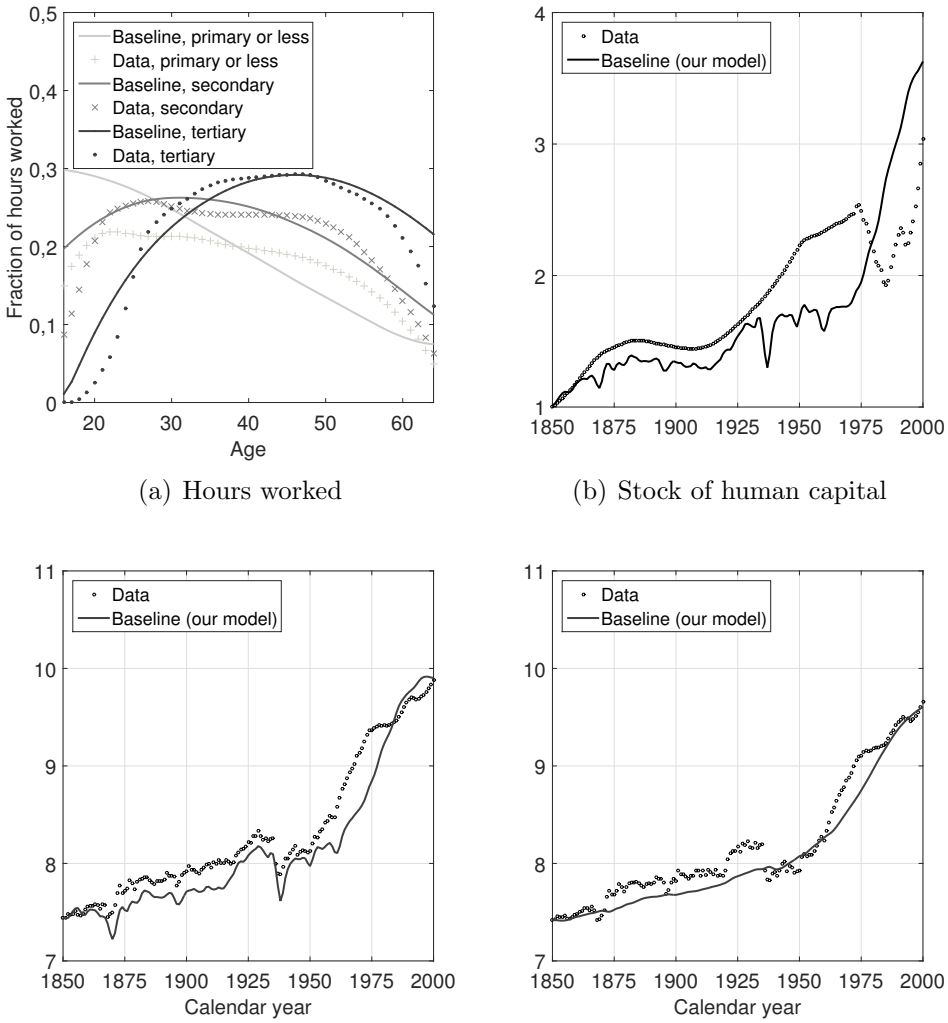
we show how the model replicates well the evolution of the income per capita (or economic growth) and the consumption per capita from 1850 to 2000.

The next section used the data and provides the main results obtained from the OLG model.

4 Results

In this section we quantify the *Spanish demographic dividend* or, equivalently, the contribution of the Spanish demography to its economic growth from 1850 to 2000. In so doing, we first need to realize that assessing the demographic dividend over a long period of time by using the naïve demographic model

$$(Y/N)_{\text{gr}} = (Y/W)_{\text{gr}} + (W)_{\text{gr}} - (N)_{\text{gr}} \,,$$



(a) Hours worked

(b) Stock of human capital

(c) Income per capita (in logs)

(d) Consumption per capita (in logs)

Figure 8: In-sample performance of the model, Spain 1850-2000

Source: See text on Figure 4 for panels 8(a) and 8(b). National accounts data are taken from OECD (2015c) and Prados de la Escosura and Rosés (2009).

where W stands for workers and 'gr' denotes the average growth rate, gives an incorrect measure, since $(W)_{\text{gr}}$ may tend to be equal to $(N)_{\text{gr}}$ in the long run. Table 2 shows the decomposition of the growth rate of per capita output in Spain from 1850 to 2000. A naïve calculation suggests that only 5% (i.e. =(.80-.72)/1.63) of the Spanish economic growth from 1850 to 2000 is explained by

demographic changes. However, long-run demographic changes are translated into economic growth through productivity effects –known as the *productivity component*– (Kelley and Schmidt, 2005). For instance, some possible channels for demography to impact on productivity are: scale economies, population density, life-cycle savings, and human capital accumulation, among others. Unfortunately, in order to disentangle these effects using convergence models one must face the possible endogeneity problems in the factor accumulation variables (Feyrer, 2007). Indeed, this problem is even more severe with longer time spans. To avoid this problem, Sánchez-Romero (2013) shows, under the correct experiments, that OLG models give similar demographic dividends than convergence models. For this reason, given the frequent lack of historical data to apply in convergence models, it seems reasonable to use OLG models.

Table 2: Per capita output growth in Spain: 1850-2000 (annual average logarithmic rates in percent)

Period	Output per capita Y/N	Output per worker Y/W	Workers W	Population N
1850-2000	1.63	1.54	0.80	0.72
1850-1950	0.68	0.48	0.88	0.68
1951-1974	5.05	5.48	0.69	1.13
1975-2000	2.06	1.88	0.69	0.50

Source: Authors' estimations and Prados de la Escosura and Rosés (2009).

Here, we follow the same strategy as in Sánchez-Romero (2013) to assess the Spanish demographic dividend. First, we show that our model is capable of reproducing the evolution of per capita income along the period 1850-2000. In this regard, Figure 8(c) shows that our model mimics well the small growth of per capita income for the period 1850-1925, the period of stagnation from 1925-1950, the *Golden Age* of rapid economic growth from 1950-1975, and the growth from 1975 to 2000. Second, based on the vital rates obtained for year 1800, we built a set of different demographic scenarios (from now on experiments) to disentangle the effect of demography for the period 1850 to 2000. In Experiment 1 we cancel the effect of fertility and mortality. Life expectancy at birth is fixed at 31.5 years and the TFR is fixed at a value slightly about 5, which implies that the annual population growth rate is 0.5 percent (see the dotted line in Figure 9). By comparing the economic outcomes of the baseline simulation to those of Experiment 1 give us the contribution of demography

to economic growth. In Experiment 2 we cancel the effect of the increase in longevity. As a consequence, the population would have increased until 1920 and it would have declined afterwards at an accelerated rate (see the triangle line in Figure 9). In Experiment 3 we shut down the effect of the decline in fertility. Thus, the population growth rate would have continuously raised over the twentieth century until reaching a rate of 3 percent (see circled line in Figure 9). Setting Experiments 2 and 3 allow us to separate, as completely independent factors, the effect of fertility and mortality on economic growth.

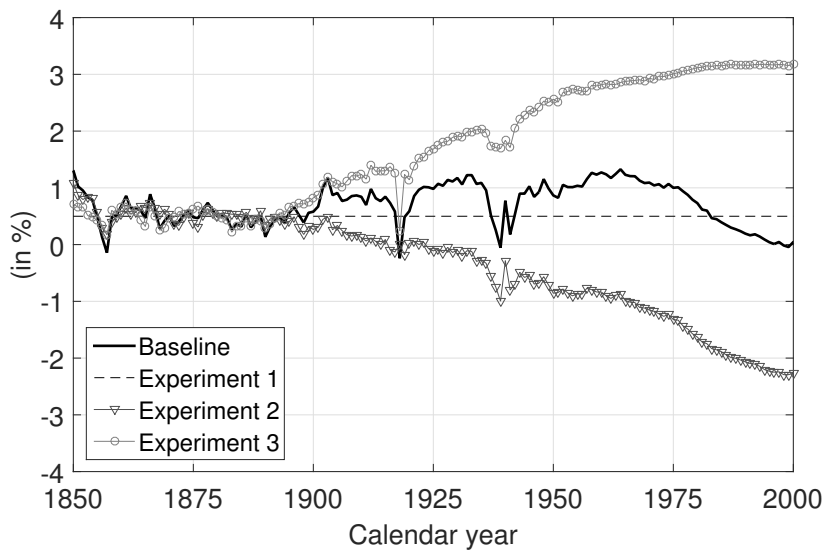


Figure 9: Annual population growth rates under different experiments, Spain 1850-2000

Source: Authors' calculation. *Notes*: Experiment 1 assumes a fixed TFR around 5 and a life expectancy at birth of 31.5 years. Experiment 2 assumes a fixed life expectancy at birth of 31.5 years and a TFR as in the Baseline. Experiment 3 assumes TFR is fixed around 5 and life expectancy evolves as in the Baseline.

4.1 Mortality and fertility effects

Our theoretical model implies that demography affects per capita income through three main channels: i) the difference between the growth rate of workers and the growth rate of the population –or *translation component*–, ii) changes in the market labor supply caused by the rise in life expectancy and

the fall in the number of equivalent adult consumers –for market goods and for home-produced goods–, and iii) changes in household savings. Realize that by adding ii) and iii) gives the *productivity component*– (Kelley and Schmidt, 2005).

Given our economic setup, these three components are well-captured by the following decomposition of per capita income growth

$$(Y/N)_{\text{gr}} = \underbrace{\frac{\alpha}{1-\alpha}(K/Y)_{\text{gr}} + (L/W)_{\text{gr}}}_{\textit{Productivity component}} + \underbrace{(W/N)_{\text{gr}}}_{\textit{Translation component}} + A_{\text{gr}}. \tag{4.1.1}$$

By comparing per capita income growth in the baseline to that in the experiments, the first term on the right-hand side of Eq. (4.1.1) captures the change in household savings, the second term L/W reflects the change in the average hours worked per worker, since we are controlling for the educational transition, and the third term W/N reflects the change in the employment rate. This is so because our household heads optimally choose when to leave the labor market and specialize in home-production. In addition, since the productivity growth is exogenously given, the last term in Eq. (4.1.1) cancels out by comparing the baseline simulation to the experiments.

Figure 10 shows the evolution of the capital-to-output ratio (left panels), the average hours work (middle panels), and the employment rate (right panels) both in the baseline and in the three experiments. The capital-to-output ratio follows two consecutive U-shapes in the periods 1850-1939 and 1939-2000. However, the rise in the capital-to-output ratio between 1925 and 1950 is due to the economic stagnation, rather than to an increase in the stock of physical capital. Besides, our simulation unrealistically amplifies the increase in the capital-to-output ratio because we assume no additional capital destruction during the Civil War (1936-1939).[12] The gap between the solid line and the dashed line in Figure 10(a) suggests a positive effect of the demographic transition on household savings. If we compare Figures 10(d) and 10(g), we can see that the positive effect of demography on capital-to-output is almost exclusively explained by the rise in longevity. Therefore, the rise in the stock of physical capital is due to the increase in the average age of workers and the higher incentive to save for retirement motive.

From 1850 to 2000 the baseline simulation gives an average decline of forty percent in hours work per worker, which is close to thirty six percent estimated by Prados de la Escosura and Rosés (2009). Comparing the solid line to the

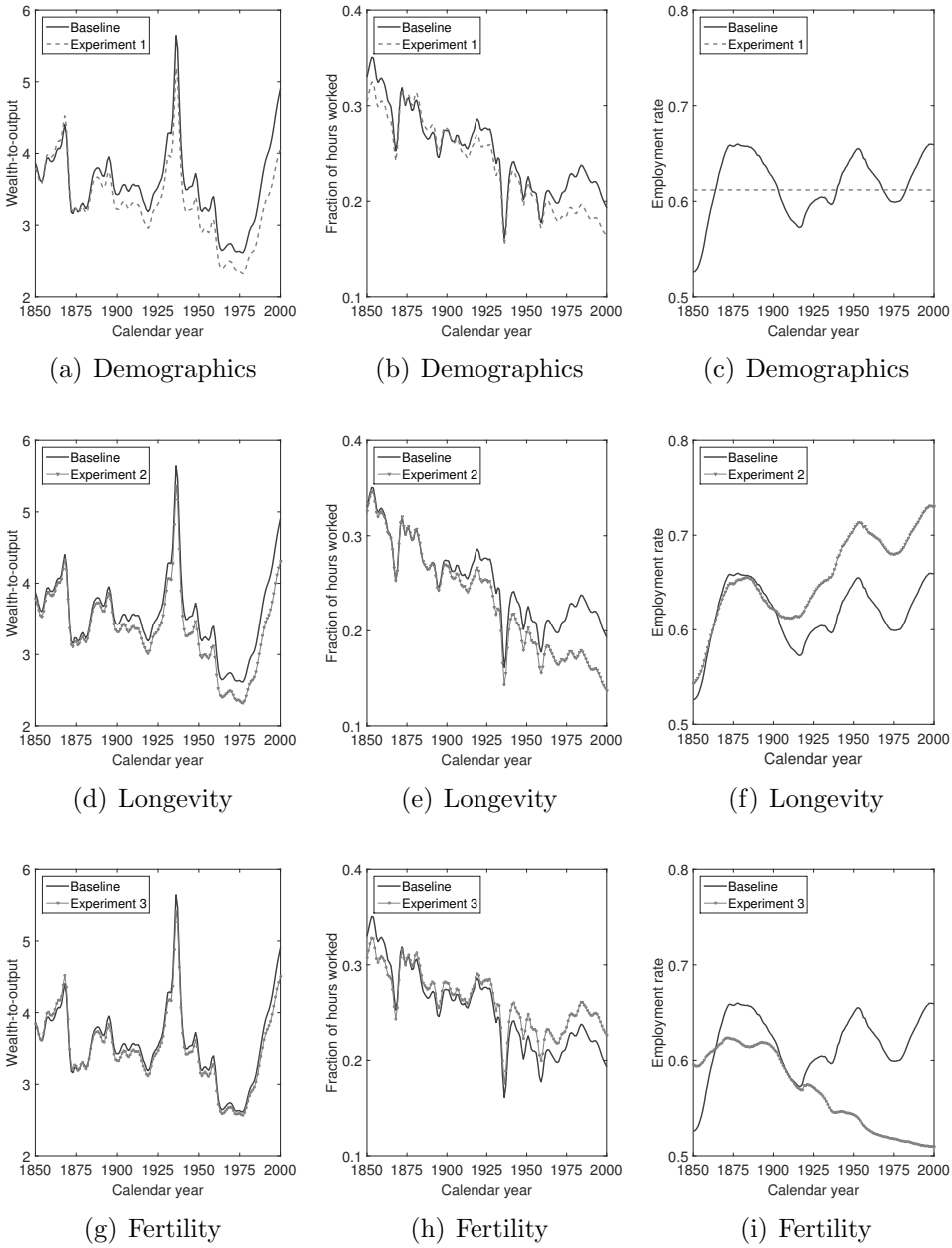
[12]We opted for a constant capital depreciation rate and labor share in order to have sharper results with respect to the effect of demography on economic growth. Moreover, we have not found good estimates about how much physical capital was destroyed.

dashed line in Figure 10(b) suggests that the demographic transition yields an increase in the average hours worked. Hence, the downward trend in hours worked from 1850 to 2000 is explained by the increase in workers' productivity, rather than by the fall in the family size. Indeed, according to our simulations the demographic transition had almost no impact on the total hours spent on home production from 1850 to 2000. This result is consistent with Ramey (2009). We can see looking at Figures 10(e) and 10(h) that the rise in longevity has led to an increase in the average hours worked, while the fall in fertility has caused a decline in hours worked (both sexes combined). The intuition is as follows. A fall in fertility raises consumption because there are more resources per household member, while the rise in longevity causes a decline in consumption due to a longer retirement span. Given that consumption and leisure are both normal goods, individuals react to the fall in fertility by decreasing their labor supply, while the rise in longevity leads individuals to work more hours.

According to our baseline shown in the left panels in Figure 10, the employment rate increased from fifty three percent in 1850 to sixty six percent in year 2000. There are two important remarks to make about this measure. First, our model does not include market frictions that give rise to unemployment. As Figure 5 shows, this assumption, although convenient, is unrealistic during the period 1975-2000, in which unemployment rate overpassed twenty percent. Nonetheless, since our calculation of the demographic dividend is based on the number of workers in 2000, our error is not significant. Second, our ratio does not take into consideration the number of hours worked by each worker. This has already been captured by the average number of hours worked. As a consequence, our measure of employment rate is closer to the ratio working age population to total population. In Spain, labor market statistics from the OECD (2015) reports a working age population relative to the total population value slightly below sixty nine in year 2000, which is close to our sixty six percent. Our rate is smaller than that of the OECD because we take into consideration that individuals with higher educational attainment postpone their entrance to the labor market.

The overall impact of demography on employment rate has been positive (c.f. dashed and solid lines in Figure 10(c)). The effect is given by the difference between the average age of the population (A_p) and the average age of workers (A_y).[13] This is a well-known property shown in mathematical demography (Coale, 1972; Keyfitz, 1977; Lee, 1980). Assuming workers are in

[13] Under a stable population, the growth rate of the working age population to the total population ratio is given by $\frac{\int_{J_0}^{J_R} e^{-nx} l(x)dx}{\int e^{-nx} l(x)dx}(A_p - A_y)$ where $A_p = \frac{\int_0^{\omega} x e^{-nx} l(x)dx}{\int_0^{\omega} e^{-nx} l(x)dx}$ and $A_y =$



(a) Demographics (b) Demographics (c) Demographics

(d) Longevity (e) Longevity (f) Longevity

(g) Fertility (h) Fertility (i) Fertility

Figure 10: Decomposition of per capita output by demographic factor, Spain 1850-2000.

Source: Authors' calculations.

$\frac{\int_{J_0}^{J_R} xe^{-nx}l(x)dx}{\int_{J_0}^{J_R} e^{-nx}l(x)dx}$.

the age range 17-65, the difference between A_p and A_y was close to -9 years in 1850 (i.e., $A_y > A_p$), while the difference between A_p and A_y was 3 years in 2000 (with $A_p = 42$ and $A_y = 39$). The rise in $A_p - A_y$ is equivalent to say that the proportion of dependents in the population fell. If we analyze the effect of longevity and fertility independently we observe two opposite effects. In particular, the rise in longevity had a negative effect on employment rate (c.f. the triangle line and the solid line in Figure 10(f)). This is because the decline in mortality initially induces the population to grow faster, and thus the number of dependents increase (i.e. more surviving children). In contrast, the fall in fertility on employment rate had a positive effect (c.f. the circled line and the solid line in Figure 10(i)), since the number of dependents decrease due to the slower population growth.

Table 3: Demographic contribution to the growth rate of per capita output $(Y/N)_{gr}$ from 1850 to 2000 by demographic component (in percent)

	Contribution	Relative contribution		
	$(Y/N)_{\text{gr}}$	$\frac{\alpha}{1-\alpha}(K/Y)_{gr}$	$(L/W)_{gr}$	$(W/N)_{gr}$
Experiment 1 (*Demography*)	**23.1**	*15*	*45*	*40*
Experiment 2 (*Longevity*)	**13.1**	*17*	*106*	*-22*
Experiment 3 (*Fertility*)	**15.3**	*11*	*-12*	*102*

Source: Authors' estimates. Notes: The contribution rate is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Baseline}}$. The relative contribution is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Contribution}}$.

Combining all the effects shown in Figure 10 gives the total contribution of the demographic transition on income growth. As already said, this is equivalent to studying the difference of Eq. (4.1.1) between the baseline and the experiments gives the contribution of demographics to per capita output growth. Table 3 shows that the Spanish demography contributed 23.1 percentage points to the Spanish per capita output growth from 1850 to 2000 (see the first column of Table 3 under Experiment 1), instead of 5 percent estimated using the naïve demographic model. The rise in longevity and the fall in fertility had a positive and significant effect on per capita income growth (see the last two rows in the first column of Table 3). As far we know, this is the first time the decline in mortality clearly shows a significant effect on economic growth.[14] In particular, the rise in longevity and the fall in fertility explain 13.1 and 15.3 percentage points, respectively, of the observed income growth

[14]This result should however be expected since the fall in fertility and mortality have a

from 1850 to 2000. Notice that since each demographic factor has a non lineal effect on the economic outcomes, the sum of both demographic components is not equal to the total effect. Moreover, the relative contribution of the translation component and the productivity component is reported in the last three columns of Table 3. The first row shows that the translation component accounts for 40 percent of the demographic dividend along the period 1850-2000, while the productivity component explains 60 percent. If we study the relative contribution by demographic component, the second and third rows show that the rise in longevity mainly affected the productivity component, whereas the fall in fertility mainly affected the translation component.

In the next section we investigate how our model specifications may have affected our calculation of the impact of demography on per capita income growth.

5 Discussion of the results

Besides demography, in our model we have assumed three other exogenous sources of variation: the labor-augmenting technological progress, the increase in the number of years of education –or educational expansion—, and the pension system. From 1850 to 2000, one of the engines of growth was the progressive conversion of Spain from a rural economy to an economy in which the industry and the tertiary sector play the most important role. Although most of the consequences of the industrialization and tertiarization of the economy has already been captured by the labor-augmenting technological progress, this process is also partly reflected by the change in the labor share. To capture the effect of a constant capital share on our results, we re-run our baseline and Experiment 1 using a changing labor share in Eq. (2.3.1). The new labor share is based on Prados de la Escosura and Rosés (2010b). Although there is no significant difference, the technological progress, A_t, is also re-calculated by applying a changing capital share in Eq. (3.4.1). Table 4 reports the contribution of demography to per capita income growth using a fixed capital share of 1/3 and a changing capital share. Table 4 illustrates that our assumed capital share slightly underestimates the contribution of demography to per capita income growth, although the difference is small enough to not significantly change our results.

To assess the impact of the educational expansion on our results, we have

delayed positive impact on economic growth (Bloom and Williamson, 1998, p. 420). Hence, the longer the time horizon, the greater the positive impact of these two demographic processes would be.

Table 4: Contribution of demography to per capita income growth $(Y/N)_{gr}$ from 1850 to 2000 (in percent)

	Contribution	Relative contribution		
	$(Y/N)_{\text{gr}}$	$\frac{\alpha}{1-\alpha}(K/Y)_{gr}$	$(L/W)_{gr}$	$(W/N)_{gr}$
Fixed capital share	**23.1**	*15*	*45*	*40*
Changing capital share	**23.6**	*16*	*45*	*39*

Source: Authors' estimates. Notes: The contribution rate is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Baseline}}$. The relative contribution is calculated as $100 \times \frac{\text{Baseline}-\text{Experiment}}{\text{Contribution}}$.

compared our baseline simulation to a counterfactual experiment in which we fix the educational levels of all cohorts to the educational distribution of those born in 1850. The result suggests that in our model only 9.3% of the total increase in per capita income growth from 1850 to 2000 is due to the educational reform. This result does not imply that the contribution of education is generally small, but that the expansion of education in Spain was quite late relative to other European countries. Thus, if we extend the time horizon beyond 2000, we can obtain that the contribution of the educational expansion on economic growth has the same impact as the demographic transition. Moreover, this value is, of course, on the lower bound since we have assumed that education has no impact on the technological progress.

The main driver for the accumulation of capital is due to the lack of income to finance the consumption during retirement, i.e. *savings for retirement motive*. To avoid an unrealistic savings profile, we have introduced the historical evolution of the social security system through our indirect estimation of the replacement rate. Thereby, by setting the replacement rate to zero, we can have a rough assessment of the importance of the pension system on our results. Comparing the baseline and Experiment 1 without a pension system, we get that demography accounts for 23.8% of the total per capita income growth. The positive difference between the demographic contribution in the baseline and that without a pension system gives support to the second demographic dividend hypothesis (Mason and Lee, 2006).

It is also interesting to study what would be the impact of demography when the translation component $(W/N)_{\text{gr}}$ is close to zero. Recall the translation component in the steady-state equilibrium is, by definition, zero, because the growth rate of workers is equal to the growth rate of the population. Hence, if we increase the time span of our analysis from 1850-2000 to 1850-2400, we

obtain that the contribution of demography to per capita income will slightly decrease from 23.1% to 22.8%. Performing the same exercise without a pension system would give a total contribution of 28.6%.[15] If we analyze how the productivity component is distributed between physical and human capital, we obtain that the demographic transition raises the capital-output ratio by 25-30% and the average human capital stock per worker by 70-75%.

Finally, despite the fact that our result is robust to changes in many of the assumptions and consistent with the literature, our estimated contribution of demography to the Spanish economic growth from 1850 to 2000 should not, however, be taken as a fixed number. We believe the value of 23.1 percentage points is likely to be the minimum contribution of demography to per capita income growth. Indeed, there is a growing evidence showing that changes in longevity may explain to a large extent the educational transition and the change in labor supply (Cervellati and Sunde, 2013; Restuccia and Vandenbroucke, 2013; Sánchez-Romero, d'Albis, and Prskawetz, 2015). Thus, if we assume that eighty percent of the increase in years of education is explained by the rise in longevity, then demography may account up to thirty one percent of per capita income growth (=23.1+.8×9.3). Furthermore, this figure could be even higher if we consider that population density may boost technological progress (Lee, 1988; Kremer, 1993; Galor and Weil, 2000; Jones, 2001; de la Croix, Lindh, and Malmberg, 2008).

6 Conclusion

To study the contribution of changes in the demographic structure on per capita income growth, we considered an OLG model populated by heterogenous household heads by level of education, which optimally choose the consumption of market and home-produced goods, and the time spent working in the market and in home-production. We find that the rise in longevity and the fall in fertility account for 23.1% of the increase in per capita income growth from 1850 to 2000. An analysis by demographic component gives that the fall in fertility explains 15.3% of the economic growth, while the rise in longevity explains 13.1%. Moreover, we have studied the contribution of the demographic transition in terms of the *translation component* and the *productivity component*. Our results for Spain suggest that the translation component

[15] The rising difference in the contribution of demography to per capital income growth between a model with and without a pension system suggests that the second demographic dividend will be temporary in the case of Spain. A similar result has been obtained by Sánchez-Romero, Patxot, Renteria, and Souto (2013).

accounts for 40% of the total income growth from 1850 to 2000, and hence the productivity component accounts for 60%.

We also show that the model is capable of replicating the observed increase in income and consumption per capita, and the pronounced decline in hours of work, even when the rise in technological progress positively affects home-production, and the labor supply across age at different levels of education. Therefore, the model is suitable to analyze the impact of future changes in demography on labor supply.

We are aware of the limitations of the model to tackle some important aspects of demography. For instance, we have abstracted from the introduction of migration, modeling human capital investment and the retirement decision endogenously, the introduction of unemployment risk, the impact of housing on savings and the evolution of asset prices, or the impact of population density on technological progress, among others possibilities. We plan to incorporate some of these important features in our future research.

References

Attanasio, O, P., Weber, G., 2010. Consumption and saving: Models of intertemporal allocation and their implications for public policy. *Journal of Economic Literature*, 48 (3): 693–751.

Auerbach, A. J., Kotlikoff, L. J., 1987. *Dynamic Fiscal Policy*. Cambridge University Press, Cambridge.

Barro, R., 1991. Economic growth in a cross section of countries. *Quarterly Journal of Economics* 106 (2): 407–43.

Barro, R., 1997. *Determinants of Economic Growth: A Cross-country Empirical Study*. Cambridge, Mass.: MIT Press.

Barro, R. J., Lee, J. W., 2013. A new data set of educational attainment in the World, 1950-2010. *Journal of Development Economics*, 104: 184–198.

Bloom, D., Williamson, J., 1997. Demographic transitions, human resource development, and economic miracles in emerging Asia. In *Emerging Asia: Change and Challenges*. Manila: Asian Development Bank.

Bloom, D. E., Williamson, J. G., 1998. Demographic transitions and economic miracles in emerging Asia. *The World Bank Economic Review*, 12 (3), 419–455.

Boldrin, M., Jimenez-Martin, S., Peracchi, F., 1999. "Social Security and retirement in Spain". In *Social Security and retirement around the world*, edited by J. Gruber and D. A. Wise, 305–353. University of Chicago Press.

Braun, A. R., Ikeda, D., Joines D. H., 2009. The saving rate in Japan: Why it has fallen and why it will remain low. *International Economic Review*, 50 (1): 291–321.

Browning, M., Ejrnæs, M., 2009. Consumption and children. *Review of Economics and Statistics*, 91 (1), 93–111.

Browning, M., Lusardi, A., 1996. Household saving: Micro theories and micro facts. *Journal of Economic Literature*, 34 (4): 1797–1855.

Cervellati, M., Sunde, U., 2013. Life expectancy, schooling, and lifetime labor supply: Theory and evidence revisited. *Econometrica*, 81 (5), 2055–2086.

Coale, A., 1972. *The growth and structure of human populations: A mathematical investigation*. Princeton, Princeton University Press.

Crespo Cuaresma, J., Lutz, W., Sanderson, W., 2014. Is the demographic dividend an education dividend. *Demography*, 51: 299–315.

Curtis, C. C., Lugauer, S., Mark, N. C., 2015. Demographic patterns and household saving in China. *American Economic Journal: Macroeconomics*, 7 (2): 58–94.

de la Croix, D., Lindh, T., Malmberg, B., 2008. Swedish economic growth and education since 1800. *Canadian Journal of Economics/Revue canadienne d'conomique*, 41 (1), 166–185.

Feyrer, J., 2007. Demographics and productivity. *Review of Economics and Statistics*, 89 (1): 1000–1009.

Galor, O., Weil, D., 2000. Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond. *American Economic Review* 90, 806–828.

Greenwood, J., Seshadri, A., and Yorukoglu, M., 2005. Engines of liberation. *Review of Economic Studies*, 72 (1), 109–133.

Human Mortality Database, 2015. University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany). Available at *www.mortality.org* or *www.humanmortality.de* (data downloaded on June, 2015).

Human Fertility Collection, 2015. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at *www.fertilitydata.org* (data downloaded on June, 2015).

Hulten, C. R., Wykoff, F. C., 1981. "The measurement of economic depreciation". In: Hulten C. R. (ed) *Depreciation, inflation, and the taxation of income from capital*. The Urban University Press, Washington DC, 81–125.

Instituto Nacional de Estadística, 2015a. *Encuesta de Población Activa*. Madrid: INE, 1987-2013.

Instituto Nacional de Estadística, 2015b. *Censos de Población del periodo 1857-1970*. Madrid, INE.

Instituto Nacional de Estadística, 2015c. *Censos de Población de 1981*. Madrid, INE.

Instituto Nacional de Estadística, 2015d. *Censos de Población y Viviendas 1991, 2001*. Madrid, INE.

Jones, C. I., 2001. Was an Industrial Revolution inevitable? Economic growth over the very long run. *Advances in Macroeconomics*, 1 (2), 1–45.

Kelley, A., 1988. Economic Consequences of Population Change in the Third World. *Journal of Economic Literature*, 27: 1685–728.

Kelley, A., Schmidt, R., 1995. Aggregate population and economic growth correlations: The role of the components of demographic change. *Demography*, 32 (4): 543–55.

Kelley, A., Schmidt, R., 2005. Evolution of recent economicdemographic modeling: A synthesis. *Journal of Population Economics*, 18 (2): 275–300.

Keyfitz, N., 1977. *Applied Mathematical Demography*. New York, John Wiley and Sons.

Kremer, M., 1993. Population growth and technological change: One million B.C. to 1990. *Quarterly Journal of Economics*, 108 (4), 681–716.

Lee, R. D., 1980. Age structure intergenerational transfers and economic growth: An overview. *Revue économique*, 31 (6): 1129–1156.

Lee, R. D., 1985. Inverse projection and back projection: a critical appraisal, and comparative results for England, 1539 to 1871. *Population Studies*, 39 (2),233–248.

Lee, R. D., 1988. Induced population growth and induced technological progress: Their interaction in the accelerating stage. *Mathematical Population Studies*, 1 (3), 265–288.

Lee, R. D., 2003. The demographic transition: Three centuries of fundamental change. *Journal of Economic Perspectives*, 17 (4), 167–190.

Leslie, P. H., 1945. On the use of matrices in certain population dynamics. *Biometrika*, 33: 183–212.

Livi-Bacci, M., 2000. *The population of Europe*. Oxford, Blackwell.

Livi-Bacci, M., Reher, D. S., 1991. Otras vías hacia el pasado: De series vitales a dinámicas demográficas en poblaciones históricas. *Boletín de la Asociación de Demografía Histórica*, 9 (3), 87–108.

Lleras-Muney, A., 2005. The relationship between education and adult mortality in the United States. *Review of Economic Studies*, 72 (1), 189–221.

Lutz, W., Butz, W. P., Samir, K. C. (Eds.), 2014. *World population and human capital in the twenty-first century*. OUP, Oxford.

Maluquer de Motes, J., 2008. El crecimiento moderno de la población de España de 1850 a 2001: Una serie homogénea anual. *Investigaciones de Historia Económica*, 10, 129–162.

Maluquer de Motes, J., Llonch, M., 2005. "Trabajo y relaciones laborales". In *Estadísticas Históricas de España, siglos XIX y XX,* edited by A. Carreas and X. Tafunell, 1157–1245. Bilbao: Fundación BBVA, 2005.

Mason, A., Lee, R. D., 2006. Reform and support systems for the elderly in developing countries: Capturing the second demographic dividend. GENUS LXII (2): 11–35.

MTAS, 2010. *La muestra continua de vidas laborales*. Coleccin de Informes y estudios de la Seguridad Social, N24, Madrid: Ministerio de Trabajo y Asuntos Sociales.

Nicolau, R., 2005. "Poblacin, salud y actividad". In *Estadísticas Históricas de España, siglos XIX y XX,* edited by A. Carreas and X. Tafunell, 1157–1245. Bilbao: Fundación BBVA, 2005.

Nuñez, C. E., 2005. "Educación". In *Estadísticas Históricas de España, siglos XIX y XX,* edited by A. Carreas and X. Tafunell, 155–244. Bilbao: Fundación BBVA, 2005.

OECD, 2015. *Hours worked (indicator).* doi: 10.1787/47be1c78-en (Accessed on 19 November 2015).

OECD, 2015b. *Health at a glance 2015: OECD indicators.* OECD Publishing, Paris. DOI: `http://dx.doi.org/10.1787/health_glance-2015-en`

OECD, 2015c. *OECD.Stat (database).* (Accessed on 19 November 2015).

Oeppen, J., 1993. Back projection and inverse projection: members of a wider class of constrained projection models. *Population Studies*, 47 (2), 245–267.

Prados de la Escosura, L., 2003. El progreso económico de España, 1850-2000. Bilbao: Fundación BBVA.

Prados de la Escosura, L., Rosés, J. R., 2009. The sources of long-run growth in Spain, 1850-2000. *Journal of Economic History* 69 (4), 1063–1091.

Prados de la Escosura, L., Rosés, J. R., 2010. Capital accumulation in the long run: The case of Spain, 1850-2000. In Alexander J. Field (ed.) *Research in Economic History, Volume 27.* Emerald Group Publishing Limited, 141–200.

Prados de la Escosura, L., Rosés, J. R., 2010. Human capital and economic growth in Spain, 1850-2000. *Explorations in Economic History*, 47 (4), 520–532.

Preston, S. H., Heuveline, P., Guillot, M., 2002. *Demography: measuring and modeling population processes.* Blackwell, Oxford

Ramey, V. A., 2009. Time spent in home production in the twentieth-century United States: New estimates from old-data. *Journal of Economic History* 69 (1), 1–47.

Ramiro Fariñas, D., Sanz Gimeno, A., 2000. Childhood mortality in Central Spain, 1790–1960: Changes in the course of demographic modernization. *Continuity and Change*, 15 (2), 235–267.

Reher, D. S., 1991. "Dinámicas demográficas en Castilla La Nueva, 1550-1900: Un ensayo de reconstrucción". In *La Evolución Demográfica Bajo los Austrias*, edited by J. Nadal Oller, 17–75. Alicante: Instituto de Cultura Juan Gil-Albert.

Reher, D. S., 2011. Economic and social implications of the demographic transition. *Population and Development Review*, 37, 11-33.

Renteria, E., Scandurra, R., Souto, G., Patxot, C., 2015. Household bargaining along the lifecycle in Spain: Money and time transfers by gender. AGENTA working paper 8/2015.

Restuccia, D., Vandenbroucke, G., 2013. A century of human capital and hours. *Economic Inquiry*, 51 (3), 1849–1866.

Sánchez-Romero, M., 2013. The role of demography on per capita output growth and saving rates. *Journal of Population Economics*, 26 (4): 1347–1377.

Sánchez-Romero, M., Patxot, C., Renteria, E., Souto, G., 2013. On the effects of public and private transfers on capital accumulation: Some lessons from the NTA aggregates. *Journal of Population Economics*, 26 (4): 1409–1430.

Sanchez-Romero, M., d'Albis, H., Prskawetz, A., 2015. Education, lifetime labor supply, and longevity improvements. Economic Development - Theory and Policy 112937, Verein für Socialpolitik.

Vargha, L., Gál, R. I., Crosby-Nagy, M. O., 2015. Household production and consumption over the lifecycle: the National Time Transfer Accounts in 14 European countries. AGENTA working paper 4/2015.

Williamson, J. G., 2013. Demographic dividends revisited. *Asian Development Review*, 30 (2): 1–25.

Wittgenstein Centre for Demography and Global Human Capital, 2015. Wittgenstein Centre Data Explorer Version 1.2. Available at: `http://www.wittgensteincentre.org/dataexplorer`.

Appendices

A Household problem

For notational simplicity and without lost of generality we get rid of subscripts that denote the level of education e and time t. The optimal allocation of time and consumption of a household, whose head is of age j, is obtained by

maximizing Eq. (2.2.1) with respect to c_j^m, c_j^h, c_j^x, h_j, and z_j, subject to Eqs. (2.2.2)-(2.2.3), and the boundary condition.

The first-order conditions (FOC) are:

$$c_j^m: \quad \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^m} = \pi_{j+1} \frac{\partial V_{j+1}(\boldsymbol{x}_{j+1})}{\partial a_{j+1}}, \tag{A.1}$$

$$c_j^h: \quad \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^h} = \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^m} p_j^*, \tag{A.2}$$

where p_j^* is the shadow price of home-produced goods and services in a household whose head is age j.

$$c_j^x: \quad \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^h} \frac{\partial f^h}{\partial c_j^x} = \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^m}, \tag{A.3}$$

$$h_j, z_j: \quad \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^h} \frac{\partial f^h}{\partial h_j} = \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial z_j} \text{ for all } j. \tag{A.4}$$

If household heads supply their labor in the market, the time spent on home production satisfies

$$h_j: \quad \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^h} \frac{\partial f^h}{\partial h_j} = \frac{\partial u_e^h(\boldsymbol{c}_j)}{\partial c_j^m} \left(1 - \tau_j^E\right) \hat{w}_j, \tag{A.5}$$

where $1 - \tau_j^E = 1 - \tau + \Psi_{j+1}\varrho$ is the effective labor income tax at age j and $\Psi_j = \frac{\partial V_j(\boldsymbol{x}_j)}{\partial q_j} \Big/ \frac{\partial V_j(\boldsymbol{x}_j)}{\partial a_j}$ is the marginal rate of substitution between the social security wealth and assets at age j.

The envelope conditions (EC) are

$$a_j: \quad \frac{\partial V_j(\boldsymbol{x}_j)}{\partial a_j} = \pi_{j+1} \frac{\partial V_{j+1}(\boldsymbol{x}_{j+1})}{\partial a_{j+1}} R_j, \tag{A.6}$$

$$q_j: \quad \frac{\partial V_j(\boldsymbol{x}_j)}{\partial q_j} = \pi_{j+1} \begin{cases} \frac{\partial V_{j+1}(\boldsymbol{x}_{j+1})}{\partial q_{j+1}} & \text{for } J_0 \leq j \leq J_R, \\ \frac{\partial V_{j+1}(\boldsymbol{x}_{j+1})}{\partial a_{j+1}} \psi + \frac{\partial V_{j+1}(\boldsymbol{x}_{j+1})}{\partial q_{j+1}} & \text{for } j > J_R. \end{cases} \tag{A.7}$$

B Clearing conditions

Let $j \in \mathbf{J} = \{0, \ldots, 100\}$, $t \in \mathbf{T} = \{1500, \ldots, 2500\}$, and $e \in \mathbf{E}$. Given initial values $\{\eta^m, \eta^h, \bar{c}, \beta_e, \zeta_e, \sigma_e, \theta, \rho, \alpha, \delta, g_t^A, \varrho, \psi_t, J_0, J_R\}_{e\in\mathbf{E}, j\in\mathbf{J}, t\in\mathbf{T}}$, demographics $\{N_{j,t}, n_{j,t}^m, n_{j,t}^h, \pi_{j,t}\}_{j\in\mathbf{J}, t\in\mathbf{T}}$, the educational distribution $E_t(e)$ for cohorts born at time $t \in \mathbf{T}$, and the age-specific productivity endowment by educational attainment $\{\epsilon_{j,e}\}_{e\in\mathbf{E}, t\in\mathbf{T}}$, a recursive competitive equilibrium is a sequence of a set of household policy functions $\boldsymbol{c}_{e,j,t} \in \mathbf{C}$, government policy functions $\{tr_{j,t}, \tau_t\}_{j\in\mathbf{J}, t\in\mathbf{T}}$, and factor prices $\{r_t^H, r_t^K\}_{t\in\mathbf{T}}$ such that

1. Factor prices equal their marginal productivities.

2. The government's budget constraints (2.4.1) is satisfied and all accidental bequests equal all transfers given

$$\sum\nolimits_{j=17}^{100} N_{j,t}(1-\pi_{j,t}) \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e) = \sum\nolimits_{j=17}^{100-28} N_{j,t}\pi_{j,t} tr_{j,t}$$

   where

$$tr_{j,t} = \begin{cases} \frac{N_{j+28,t}(1-\pi_{j+28,t})}{N_{j,t}\pi_{j,t}} \int_{\mathbf{E}} a_{e,j+28,t} dE_{t-j-28}(e) + \frac{1-\pi_{j,t}}{\pi_{j,t}} \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e) & \text{if } 17 \leq j < 44, \\ \frac{N_{j+28,t}(1-\pi_{j+28,t})}{N_{j,t}\pi_{j,t}} \int_{\mathbf{E}} a_{e,j+28,t} dE_{t-j-28}(e) & \text{if } j > 44, \end{cases}$$

3. Given the factor prices and government policy functions, household policy functions satisfy Eqs. (2.2.4)-(2.1.2), and the commodity of home production clears:

$$\sum\nolimits_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} f^h(c_{e,j,t}^x, h_{e,j,t}) dE_{t-j}(e) = \sum\nolimits_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} c_{e,j,t}^h dE_{t-j}(e)$$

4. The stock of physical capital and the labor input are given by:

$$K_t = \sum\nolimits_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} a_{e,j,t} dE_{t-j}(e)$$

$$L_t = \sum\nolimits_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} \epsilon_{e,j} \ell_{e,j,t} dE_{t-j}(e)$$

5. The commodity market clears:

$$Y_t = C_t + S_t$$

   where the total consumption of market goods $C_t = \sum_{j=17}^{100} N_{j,t} \int_{\mathbf{E}} c_{e,j,t}^m + c_{e,j,t}^x dE_{t-j}(e)$ and S_t is gross savings in year t.

C Population reconstruction

Time is discrete. Individuals are assumed to live for a maximum of 100 years. Let the survival probability to age j in year t be

$$S_{j,t} = \prod_{x=0}^{j-1} \pi_{x,t-x} \text{ with } S_{0,\cdot} = 1,\ S_{100,\cdot} = 0, \tag{C.1}$$

where $\pi_{j,t}$ is the conditional probability (of being alive at age j in year t) of surviving to age $j+1$ (with $\pi_{j,\cdot} = 0$, for all $j \geq 100$). Let $N_{j,t}$ be the size of the population at age j in year t. We assume a closed population. Thus, the population at time $t+1$ is given by the population in year t plus the total number of births in year t, denoted B_t, less the total number of deaths during the year D_t. The dynamics of the population can be written in matrix notation using a Leslie matrix (Leslie, 1945; Preston, Heuveline and Guillot, 2002)

$$\mathbf{N}(t+1) = \mathbf{\Gamma}(t)\mathbf{N}(t), \tag{C.2}$$

with

$$\begin{aligned} \Gamma_{j,1}(t) &= \frac{L_{0,t}}{2S_{0,t}} \left(f_{j,t} + f_{j+1,t} \frac{L_{j+1,t}}{L_{j,t}} \right) f_{fab}, \\ \Gamma_{j+1,j}(t) &= \frac{L_{j+1,t}}{L_{j,t}}, \text{ for } j \in \{1, \ldots, 99\} \text{ at time } t, \end{aligned} \tag{C.3}$$

where $L_{j,t} = \frac{S_{j,t}+S_{j+1,t}}{2}$ is the person years lived by the cohort between ages j and $j+1$ in period t, $f_{j,t}$ is the age-specific fertility rate at age j in year t, f_{fab} is the fraction of females at birth (I assume $f_{fab} = 0.4886$, which is the standard value in the demographic literature).

To reconstruct the population in Eq. (C.2) and (C.3), we use a simplified version of a GIP model that matches the specific characteristics of our economic model: one gender without distinction between parity and region of birth, among others. The objective function used to solve the problem is:

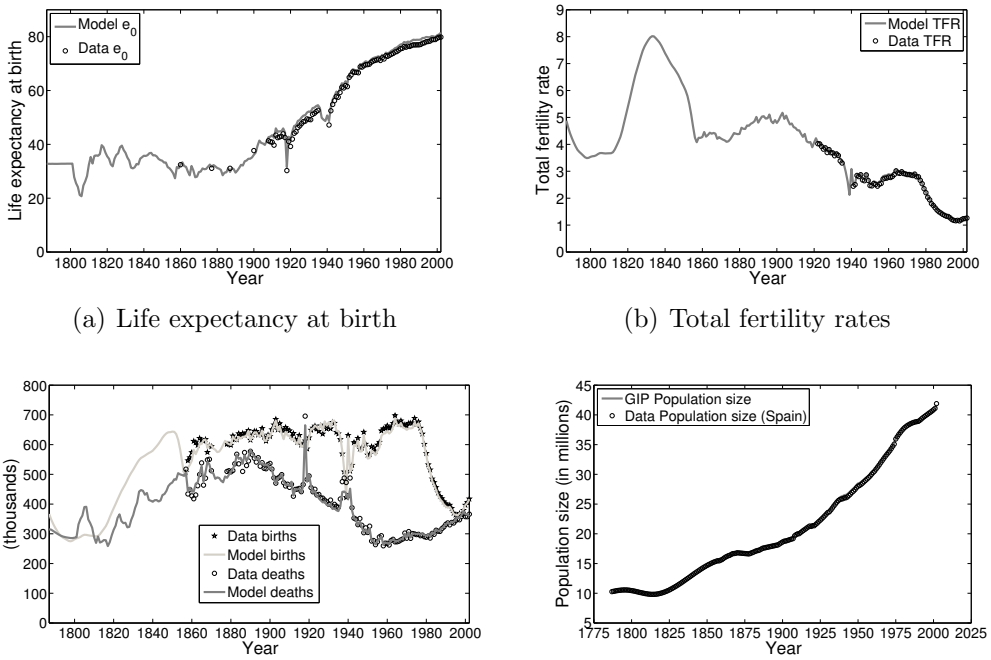
$$\begin{aligned} \min_{\{\alpha_t^1, \alpha_t^2, \alpha_t^3, \mu_t, \beta_t\}} & \sum_{t \in \mathbb{D}} \left(1 - \hat{\mathrm{D}}_t/\mathrm{D}_t\right)^2 + \sum_{t \in \mathbb{B}} \left(1 - \hat{\mathrm{B}}_t/\mathrm{B}_t\right)^2 + \sum_{t \in \mathbb{N}} \left(1 - \hat{\mathrm{N}}_t/N_t\right)^2 \\ & + \sum_{t \in \mathbb{E}} \left(1 - \hat{\mathrm{e}}_{0,t}/\mathrm{e}_{0,t}\right)^2 + \sum_{t \in \mathbb{T}} \left(1 - \mathrm{T}\hat{\mathrm{F}}\mathrm{R}_t/\mathrm{TFR}_t\right)^2 + \sum_{t \in \mathbb{C}} \sum_{a=0}^{\Omega-1} \left((\mathrm{N}_{a,t} - \hat{\mathrm{N}}_{a,t})/\mathrm{N}_t\right)^2 \\ & + \sum_{t=t_0}^{T} \sum_{i=0}^{2} \left(\alpha_{t+1}^i - \alpha_t^i\right)^2 + \sum_{t=t_0}^{T} \left(\mu_{t+1} - \mu_t\right)^2 + \sum_{t=t_0}^{T} \left(\beta_{t+1} - \beta_t\right)^2, \end{aligned} \tag{C.4}$$

subject to equations (C.2)-(C.3) and to

$$\sum\nolimits_{k=1}^{3} \alpha_t^k f_j^{(k)} = f_{j,t}, \qquad \prod_{x=0}^{j-1} \pi_{x,t} = \frac{e^{2(\mu_t+\beta_t Y^1(x)+(1-\beta_t)Y^2(x))}}{1+e^{2(\mu_t+\beta_t Y^1(x)+(1-\beta_t)Y^2(x))}} \tag{C.5}$$

$$\sum\nolimits_{k=1}^{3} \alpha_t^k = 1, \qquad \beta_t \in [0,1], \tag{C.6}$$

where $\{\alpha_t^1, \alpha_t^2, \alpha_t^3, \mu_t, \beta_t\}$ are the corresponding parameters for fertility and mortality, respectively; $f_x^{(i)}$ and $\{Y^1(\cdot), Y^2(\cdot)\}$ are actual age-specific fertility rates and two Brass logit model standards –where $Y^1(\cdot), Y^2(\cdot)$ are associated to high mortality and low mortality rates, respectively–; and $\mathbb{I} \equiv \{\mathbb{D}, \mathbb{B}, \mathbb{N}, \mathbb{E}, \mathbb{T}, \mathbb{C}\}$ are the sets of deaths, births, total population, life expectancy, total fertility rates, and censuses used in the calculation. Crude migration rates are obtained using *inverse population projection* and are exogenous to the GIP model. Since GIP suffers from weak ergodicity, we use an initial population growth rate consistent with historical data prior to 1800 based on Livi-Bacci and Reher (1991) and Reher (1991).



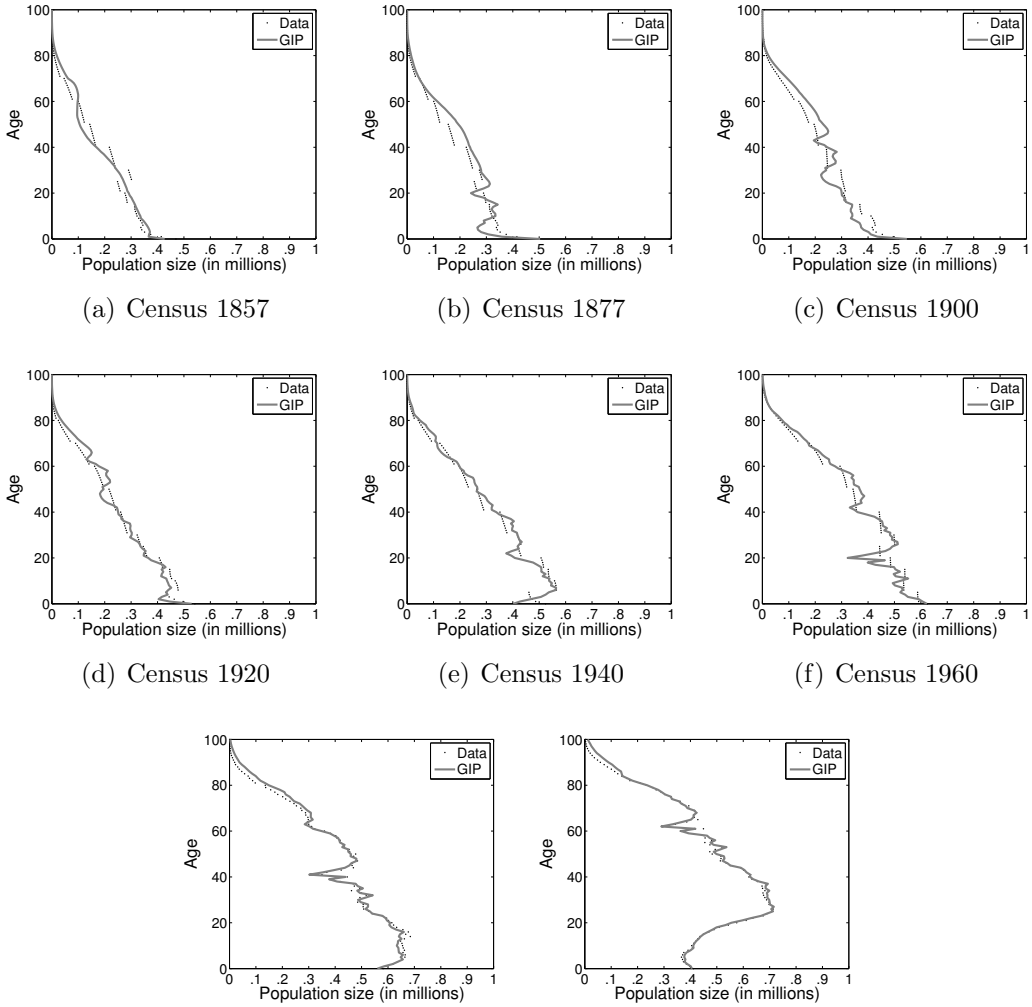
(a) Life expectancy at birth

(b) Total fertility rates

(c) Total births and deaths

(d) Total population

Figure 11: In-sample performance of the GIP model to existing demographic data. Spain: Selected year between 1787 and 2000



(a) Census 1857 (b) Census 1877 (c) Census 1900

(d) Census 1920 (e) Census 1940 (f) Census 1960

(g) Census 1980 (h) Census 2002

Figure 12: In-sample performance of the GIP model to existing census data. Spain: Selected year between 1857 and 2002