Simulating the creation of charmed mesons through electron-positron collisions

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Abstract: The current understanding of particle physics is closely related to our predictions and measurements of particle resonances, even though we are not able to directly access the underlying physics. Acknowledging the relevance of a proper, rigorous visualization of the generation and decay of resonances for scientists working in this area, we simulate the production of charmonium states through electron-positron annihilation, based on the theoretical predictions for their cross sections.

I. INTRODUCTION

The study of electrons-positron collision processes has been vital for understanding particle physics and quantum field theory. Not only was it the basis for the successful Large Electron-Positron Collider [1], but it is regarded as a good approach to higher energies [2] and will be crucial in future projects like the International Linear Collider [3] or the Compact Linear Collider [4]. One of the strengths of this annihilation process is its ability to produce resonances. Understanding this kind of reactions is crucial to make the connection between particle and hadronic physics, and has helped the community develop the current theory of Particle Physics.

As succesful as these experiments have been, however, they remain obscure to seeing everything that happens in the vertices of interactions. It would be of great outreach and instructive interest to depict at our own pace what we currently understand to be going on. By means of a new simulation software, *Phenomena* [5], we visualize these processes and virtual particles to get a physically relevant picture of what happens in an annihilation at a deeper level than the scattering. Although the software is currently used mainly for decays, we have implemented a tool to calculate the outcome of the electron-positron annihilation and its kinematics. In this article, we provide a theoretical background for this phenomenon, explain the nature of the tool, and discuss the simulation. While the main goal of this article is to provide context for one interesting application of this software, we hope to show its great potential for many more interesting features to work on in the future.

II. THEORETICAL BACKGROUND

A. Mesons with Charm

The conservation rules of particle physics are our first tool to understand what we can observe in an e^-e^+ collision. The particle-antiparticle nature of the collision alone fixes most quantum numbers: we can only consider null charged products, with null leptonic and hadronic numbers. The large angular momenta of incoming particles (s=1/2), allows us to access the more massive states with J=1, on which we will focus. The simplest final hadronic states satisfying all of these conditions are of the form $q\bar{q}$.

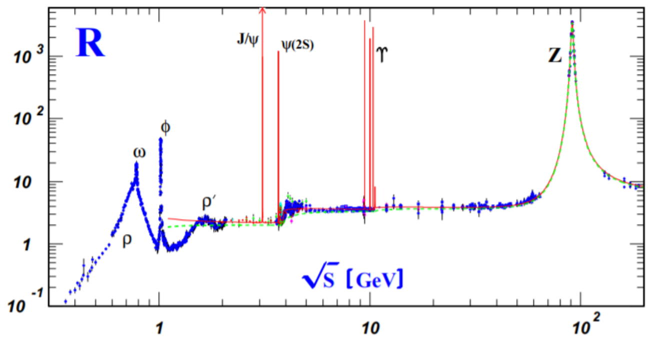

FIG. 1: $R = \sigma_{had.}/\sigma_{\mu^-\mu^+}$ over centre of mass energy $\sqrt{s}$ of the system from [9].

In Fig. 1 we can see the states that we can create in this context starting with a centre of mass energy of 300 MeV/c^2. This plot will be the centre of our discussion, as it allows us to check if the physics used in the simulation is correct. As we discuss later, we focus our visualization on charmonium ($c\bar{c}$) states, J/Ψ and $\Psi(2S)$.

B. $e^-e^+ \to \mu^-\mu^+$ scattering

Although we want to calculate the cross section for the annihilation into hadrons, we will present the result corresponding to the leptonic $e^+e^- \to \mu^+\mu^-$ process first. It will be seen later on that, in the energy regime we are interested in, the hadronic cross section can be expressed in terms of the asymptotic value for the leptonic one, recovering the ratio R presented in Fig. 1. Therefore, the rules of quantum electrodynamics (QED) suffice to begin: we can draw the diagram for this interaction (see Fig. 2) which, by using the Feynman rules [6], we can translate into the amplitude of the process in terms of the

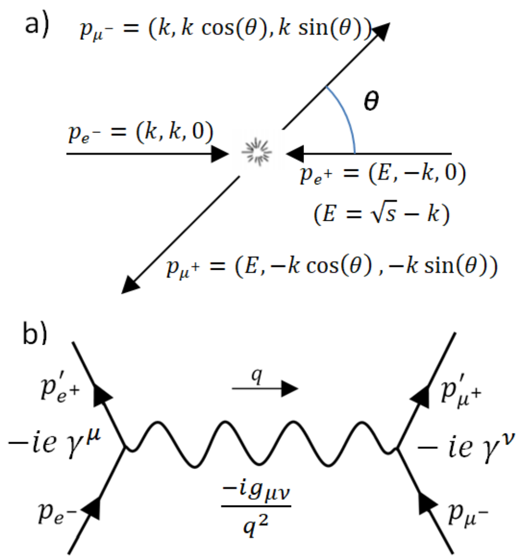

FIG. 2: Feynman (a) and kinematic (b) diagrams corresponding to the annihilation into a muon antimuon pair. The flow of time is shown with the total momentum $q = p_{e^+} - p_{e^-}$.

particle spinors, the vertices and the energy propagator:

$$i\mathcal{M} = \bar{v}^{s_{e^+}}(p_{e^+})(-ie\gamma^\mu)u^{s_{e^-}}(p_{e^-})\left(\frac{-ig_{\mu\nu}}{(p_{e^+}-p_{e^-})^2}\right) \quad (1)$$
$$\bar{u}^{r_{\mu^-}}(p_{\mu^-})(-ie\gamma^\nu)v^{r_{\mu^+}}(p_{\mu^+}).$$

The treatment of γ elements follows the rules of the Clifford Algebra $\mathcal{C}l_{1,3}(\mathbb{R})$ [6, 7]. Considering also the completeness relations, in the limit $m_e \to 0$ (which is justified by the range of E_{cm} we consider) we get

$$\frac{1}{4}\sum_{spins}|\mathcal{M}|^2 = \frac{8e^4}{q^2}\Big[(p_{e^-}p_{\mu^-})(p_{e^+}p_{\mu^+}) + (p_{e^-}p_{\mu^+})(p_{e^+}p_{\mu^-})$$
$$+ m_\mu^2(p_{e^-}p_{e^+})\Big] = e^4\Big[(1+\frac{m_\mu^2}{E^2}) + (1-\frac{m_\mu^2}{E^2})\cos^2\theta\Big], \quad (2)$$

where we sum over outgoing spins and average over incoming. Also, in the second equality we already used kinematic variables from Fig. 2a and momentum conservation. It can be seen [4] that the cross section and the scattering amplitude relate as

$$\frac{d\sigma_{\mu^-\mu^+}}{d\Omega} = \frac{1}{2E^2}\frac{|p_{\mu^-}|}{16\pi^2 E}\Big(\overline{\sum_{spins}}|\mathcal{M}|^2\Big), \quad (3)$$

which, after performing the angular integration becomes:

$$\sigma_{\mu^-\mu^+} = \frac{4\pi\alpha^2}{3E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\Big(1+\frac{1}{2}\frac{m_\mu^2}{E^2}\Big). \quad (4)$$

Note that, given the energy scale set by the first charmonium resonances ($E/m_\mu > 30$), on expanding Eq. (4) in the limit $E \to \infty$, keeping only the leading term (order 0) $\frac{4\pi\alpha^2}{3E^2}$ will be a good approximation. We will call this term $\sigma^0_{\mu^-\mu^+}$.

C. Cross section of $e^-e^+ \to hadrons$

We are now in a position to spot the effects of quantum chromodynamics (QCD) in Fig. 1. As seen more rigorously in Ref. [6], QCD being an SU(3) non-Abelian gauge theory implies that the coupling constant of its lagrangian interaction term is suppressed at high energies. This feature of the theory, called asymptotic freedom, allows us to ignore quark interaction effects for most of the energy range in Fig. 1. Our hadronic cross section can therefore be obtained by slightly modifying our previous expression for the leptonic scattering as done below.

Before that, it is relevant to point out that energy regions where the resonances appear are precisely those where the strong interaction cannot be neglected. When we reach the energy value for the appearance of a new quark flavour, at first the $q\bar{q}$ pair is created with such low relative energy that they can "see" each other, interacting and forming the bound resonant state. Only when the pair is created with larger energies does asymptotic freedom play a relevant role.

For the new expression we must take into account the charge of quarks, $Q|e|$ (as opposed to e for muons), and the 3 quark color possibilities which represent the symmetry of the theory. We recall that we had to sum over all possible outcomes, and so at high energies

$$\sigma_{had.} = \sum_{color}\sum_{q_{charge}}(Q_q|e|)^2\frac{4\pi\alpha^2}{|e|^2 3E^2} = 3\sigma^0_{\mu^-\mu^+}\sum_q Q_q^2. \quad (5)$$

This expression divided by $\sigma^0_{\mu^-\mu^+}$ is precisely the outline of R in Fig. 1 without the peaks. As we should expect, it fails for lower energies and, most importantly, it shows very clearly how we 'unlock' quarks levels: once we have enough energy to produce a more massive quark, σ rises by $3Q_q^2\sigma^0_{\mu^-\mu^+}$ from the previous level. The initial level in Fig. 1, before the charmonium peaks, is up to the strange quark: $3\big[(\frac{-1}{3})^2_d + (\frac{2}{3})^2_u + (\frac{-1}{3})^2_s\big]\sigma^0_{\mu^-\mu^+} = 2\sigma^0_{\mu^-\mu^+}$.

D. Virtual particles and peaks

The feature of Fig. 1 we are most interested in are the peaks. When $\sqrt{s}$ reaches certain values, the probability to produce particles is increased by orders of magnitude; this is what we refer to as resonances. These resonant cross sections can be related to the creation of particles that do not correspond to asymptotic states, but to what we call virtual particles. As very short lived particles, ranging from 10^{-20}s for some charmonium states to 10^{-25}s for the Z boson, the uncertainty in their masses is high enough that we have to consider them off-shell, that is, not always obeying the relationship $\|p\|^2 = m^2c^4$; they can be understood to be the propagators seen in the diagram of Fig. 2b. To see these virtual particles in our simulation, we have to treat the cross section in a different way such that it reproduces the observed peaks.

A consequence of the unitarity of S matrices is the optical theorem, which states that the cross section σ and the imaginary part of the amplitude $\mathcal{M}$ in a forward scattering are related as follows:

$$\mathrm{Im}\mathcal{M}(p_1p_2, p_1p_2) = 2E_{cm}p_{cm}\sigma(p_1p_2 \to anything), \quad (6)$$

and it holds for any $\mathcal{M}$ defined from Feynman diagrams. We know that the propagator function for a vector particle is proportional to $i/(p^2 - m^2 - M^2(p^2))$, where $-iM^2(p^2)$ is the self-energy (and can be calculated as the sum of all 1 particle irreducible insertions, in diagramatic terms [6]). When there is an open decay channel into virtual particles, $\mathcal{M}$ has an imaginary part. Considering that an imaginary $M^2(p^2)$ will have a pole associated, the LSZ [8] reduction formula for S-matrices (see also chapter 7 of Ref. [6]) relates our amplitude $\mathcal{M}$ to the self-energy with the strength Z of the pole as $\mathcal{M}(p \to p) = -ZM^2(p^2)$. In this situation, the propagator is displaced from the real axis proportional to

$$\frac{iZ}{p^2 - m^2 - iZ\,\mathrm{Im}M^2(p^2)}. \quad (7)$$

This will be the case precisely when considering our annihilation creating a single hadron, which means our cross section will be

$$\sigma \propto \left|\frac{1}{E^2 - m^2 - iZ\,\mathrm{Im}\,M^2}\right|^2. \quad (8)$$

As we mentioned before, we focus only on charm quarkonia, since, while they span a wide range of masses and are thus interesting enough, most of their peaks are narrow enough that we can approximate $M^2(p^2)$ with the constant $M^2(m^2)$. By using the identification

$$\Gamma = -\frac{Z}{m}\mathrm{Im}\,M^2(m^2), \quad (9)$$

the exact dependence is known [9] to be:

$$\sigma = \frac{12\pi\Gamma_{ee}\Gamma E^2/M^2}{(E^2 - m^2)^2 - E^2\Gamma^2}, \quad (10)$$

where Γ is the total width of the hadron and Γ_{ee} is the electronic width. The shape of the peaks can be described by a Breit-Wigner (BW) distribution, which we can reproduce separately for every possible outcome of the annihilation. When focusing on charmonium states, we are effectively avoiding particles with wider peaks, for which the dependence of M on s makes the BW fit with experimental data worse. One should note that [10] for Z or ω bosons, the parameters of the distribution Γ and M are not related to the width and the mass of the particle in the same trivial way. This gives further motivation to our choice of energy scale.

III. THE PHENOMENA SOFTWARE

Phenomena [5] as a software has two parts. One handles the graphical representation and draws the visualization, and is written in Java. The other, written in Python, performs all the physically relevant calculations, and is currently structured mainly around decaying particles. Our work focused on this part, with the goal of adding new features such as the charmonium production. Using data from the Particle Data Group (PDG) [11], it has access to all known particles and any of their measured properties. These are used to perform weighted selections on decays or particle productions, with the purpose of simulating real statistics and watching the probabilistic nature of particle physics. Simulations can be seen in a particle like environment, as in Fig. 3, which we used to represent our virtual particles, or in a bubblechamber like representation, as in Fig. 4, which shows the usual trace representation.

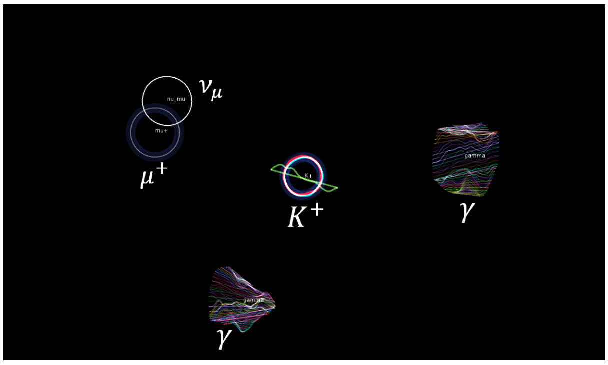

FIG. 3: *Phenomena* particle visualization of a K^+ about to decay, and a ρ^+ that already decayed into a μ^+, a ν_μ and two photons γ.

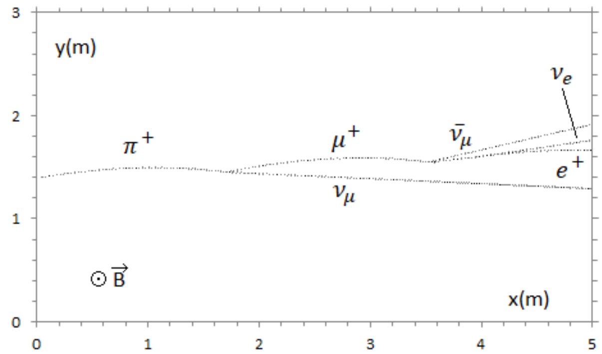

FIG. 4: *Phenomena* bubblechamber visualization of a π^+ decay into ν_μ and μ^+, which then decays into e^+, ν_e and $\bar{\nu}_\mu$. Magnetic field points outwards from the paper. Neutral charged particles are also drawn.

This program is a very powerful tool for outreach, but also for graduate students who want to digest and work with interactions that they already know, but have trouble picturing. Watching decays, annihilations, the appearance of virtual particles or a bubble chamber trace picture in a slowed down timeframe, where we show aspects of the theory that cannot be seen in real experiments, is a practical way to raise interesting questions and push the bounds of oneself's understanding of the theory. Having this in mind, the graphical part is meant to reflect the real properties of particles with as much accuracy as possible. For example, mass is reflected by the intensity of the lines and the shape of the particle deforms more or less over time to represent its coupling to the mass field; hadrons show their quark components changing color as they interact through gluons, which are drawn as colored lines around them; fermions are represented by particle-like shapes while bosons are oscillation-like. Some of these can be seen in Fig. 3, although attributes like shape deformation are difficult to read from a static picture.

IV. SIMULATION AND RESULTS

A. Crafting the visualization

To produce the visualization, we first programmed a Breit-Wigner distribution to calculate the cross section as in Eq. (10) for a given $\sqrt{s}$. For each possible outcome, as discussed in the first section, we used the available data from PDG. To focus on the charmonium states the simulation was performed at the range of 3 to 11 GeV. However, the process has been checked to be able to produce Z bosons, ω mesons and other particles when working at the energies of other peaks in the *Phenomena* framework, regardless of the accuracy of their cross sections. In fact, it is capable of processing any energy input, although different effects have to be taken into account then. Firstly, we can obtain particles away from their resonance peaks due to the BW tails, even though it happens with very small probabilities (a biased selector has been implemented if this feature wants to be accentuated). Most importantly, however, it brings forward that we cannot always explain the process through an intermediate virtual state. In between resonances, most of the contribution to the cross section will not have an adequate representation in our visualization, because it focuses on particles by definition.

The individual cross sections are used to make a weighted choice to decide the particle we produce, with the higher values being more probable than the smaller ones. The difference in magnitude between the leading cross sections make each particle appear almost exclusively under their resonance energy range. Then, to decide if a hadronic particle is produced at all, we use the total sum of cross sections and the luminosity L of the beam to compute how likely it is statistically. Initially, we aimed at following the parameters of real colliders (L $\approx 10^{34}$ cm^{-2}s^{-1}). However, the values of the cross sections at the peaks with this luminosity would then reach the order of 10^5 particles produced, far exceeding the graphical capacities of the software. While the low probabilities of production make it unsuitable to approximate the statistics through the central limit theorem the lower the amount of interactions is, we chose to keep this approximation for a smaller luminosity (L$\approx 10^{29}$cm^{-2}s^{-1}) that would produce a visually understandable ($\sim 10^0$) amount. Also, the current speed of the program restricts the number of collisions we can simulate to a few per second. Still, the value of $\sqrt{s}$ changes the outcome by a factor of 10^3, and so the luminosity is kept as an input parameter together with the energy.

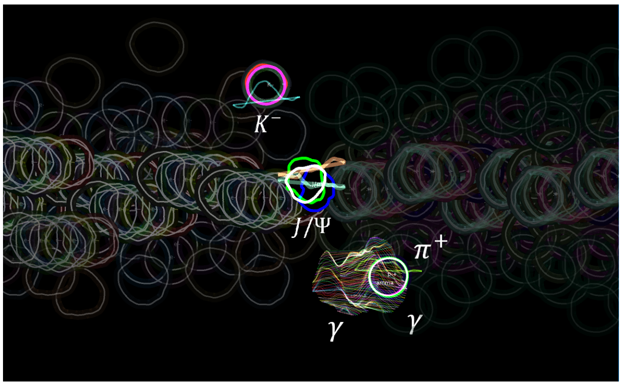

FIG. 5: *Phenomena* visualization of the production of a J/Ψ through our hadron production simulation at $\sqrt{s} = 3.096$ GeV. We can see a J/Ψ, also K^-, π^+ and 2 γ as the final products of the decay of a previous J/Ψ production.

To maintain physical rigorousness in the simulation, we must add the kinematic calculations of a collision. In regard to an upcoming more general implementation of collisions, we implemented code to calculate the momenta of outgoing particles from different sets of incoming data (incoming particle momenta, incoming particle energies, centre of mass system energy...), although for graphical limitations we decided on always representing the centre of mass frame of reference, so the created particle apears at rest. We also drew a static depiction of incoming particle beams, with a representation of the density of particles across its width; this way, we can represent how in actual colliders there is a constant stream of particles but only some interact. The final result can be seen in Fig. 5.

In order to make our visualization clearly discernible, other concessions were made. *Phenomena* uses every particle's real lifetime value to decide how long it takes to decay, with an exponential refactoring shortening the order of magnitude span of lifetimes from $(\sim 10^{-13}, 10^{14})$ to $(\sim 10^{-1}, 10)$, so unstable particles are visible for a timeframe of 10^{-1}s at least. The refactoring was insufficient for the very short lives of charmonium, and their time on screen had to be artifically extended by $\sim 10^{10}$ to bring them closer to the shorter ones without cluttering every other particle into an even smaller range of magnitudes.

B. Accuracy of cross sections

The BW approximation has been used for many years; we can see in [12], for example, that they are indeed precise for the charmonium resonances. Taking the rest of the data points of 1 from [11], we can compare the cross sections in *Phenomena* to the measurements:

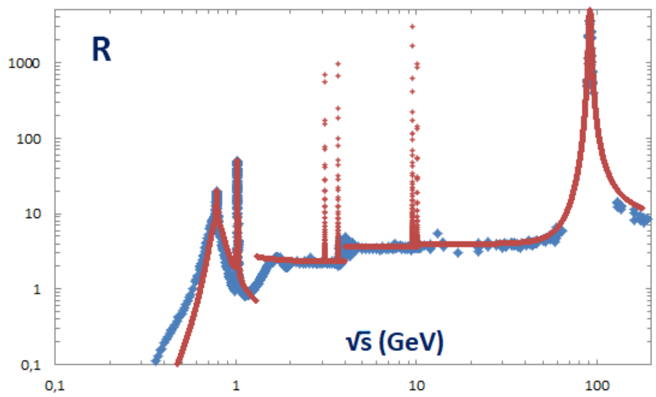

FIG. 6: $R = \sigma_{had.}/\sigma_{\mu^-\mu^+}$ over centre of mass energy $\sqrt{s}$ obtained with our simulations in red, compared to measurements from PDG data [11] that were used for Fig. 1.

One of the main shortcomings in recreating the real cross section data is that we are restrained to counting only those particles we know well enough through PDG data. This discrepancy does not have any impact in the peaks we are studying, but would show up everywhere else. However, in Fig. 6 it is masked by the addition of the QCD baseline in the computation of the cross section.

V. CONCLUSIONS AND FUTURE WORK

- We were able to simulate the production of charmonium particles in an electron-positron annihilation for different energies. Focusing on the resonances for which the behaviour could be modeled more accurately, we retrieved the expected outputs according to PDG data [11], although certain aspects such as the rate of produced particles and their lifetimes had to be adapted to the limitations of the graphical side of *Phenomena*.
- This work is a starting point, inside the *Phenomena* project, for the implementation of collisions in a universal way. It has led to contributions in the development of new kinematic calculations and the visualization of virtual particles too, projects that we will continue to develop. The theoretical discussions and decisions on the approach that sparked during the development of this work show that simulating particle physics in a way that is graphically interesting and full of information cannot be done without having a strong hold on the physics that explain these phenomena, both mathematically and conceptually.
- *Phenomena* can be potentially used to study any kind of collisions between particles. While most of the work was checked through the behaviour of the particle-like type of drawing, we will be moving our efforts into the bubblechamber appearance as the picture that most scientists are familiar with. With that in mind, we are adding features like energy loss, dynamics under magnetic fields or interactions with the medium, so that we are ultimately able to perform a vertex reconstruction using only the information on the representation.

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