Meson-Baryon molecular states in the (I = 1/2, S = 0, C = 2) sector

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Abstract: In this work we present predictions for meson-baryon molecules in the Ξ_{cc} sector. The states are modelled as either pseudoscalar or vector mesons interacting with ground state baryons in s-wave, through a t-channel vector exchange picture described by effective Lagrangians. Unitarization is implemented using the Bethe-Salpeter equation, from which the molecules emerge as poles of the resulting scattering amplitude. As a result, we get a total of eight states. Four pseudoscalar-baryon states, with \( J^P = 1/2^- \), at 3862, 4135, 4239, and 4240 MeV; and four vector-baryon states, degenerate with \( J^P = 1/2^-, 3/2^- \), with masses of 4380, 4353, 4387 and 4523 MeV.

I. INTRODUCTION

In the past few years there have been observations hard to fit within the usual picture for hadrons, which therefore need a more unorthodox description. This is the case of some baryon resonances, which may have bizarre properties if considered simply as excited three-quark baryons. One of such explanations proposed, and the one considered in this work, is that of quasi-bound states composed of a meson-baryon pair. They belong to a broader family of models called hadronic molecules, (see [1] for a thorough review). A well-known example of this kind of studies is the successful description of the \( \Lambda(1405) \) resonance as a meson-baryon molecule [2].

More recently, following the discovery of five narrow \( \Omega_c^0 \) resonances by the LHCb Collaboration (2017) [3], Monta˜na, Feijoo, and Ramos (2018) [4] showed that two of these states, \( \Omega_{c0}^{(3050)} \) and \( \Omega_{c0}^{(3090)} \), could be described as meson-baryon molecules. Employing a t-channel vector meson exchange model, originally presented in [5], the work successfully reproduced both the mass and the width of the observed resonances. In their model, transition amplitudes were obtained from effective \( SU(4) \) Lagrangians describing the interaction between pseudoscalar mesons and ground-state baryons in the \( C = 1, S = -2 \) and \( I = 0 \) sector. Then the amplitudes were unitarized through the Bethe-Salpeter equation in coupled channels. The sought resonances emerged as poles of the scattering amplitude in the second Riemann sheet of the complex energy plane. Their position is given by \( z_r = M_r + i\Gamma_r/2 \), where \( M_r \) is the mass of the resonance and \( \Gamma_r \) its width. Moreover, their research gave a prediction for the spin-parity of the observed resonances, assigning the values \( J^P = 1/2^- \) to both of them, as the model considered pseudoscalar mesons interacting with baryons in s-wave.

This is specially interesting since quark-based models predict different values for their spin-parity (either \( 3/2^- \) or \( 5/2^- \)). In addition, their work also presented predictions on molecules generated by vector mesons interacting with baryons in the same flavour sector. Similarly, [6] also described the two \( \Omega_{cc} \) resonances mentioned as pseudoscalar-baryon molecules, working with an analogous model which only differed slightly in the Lagrangians employed, as they considered the charm quark as an spectator for the interaction.

The recent discovery of a doubly charmed baryon \( \Xi_{cc}^{++} \) by the LHCb Collaboration [7] (2017), with a mass of 3261 MeV, has also sparked many theoretical studies in an attempt to understand its properties. Since \( \Omega_c \) resonances have been observed, \( \Xi_{cc} \) resonances may exist as well, so there have also been efforts in order to predict possible doubly-charmed molecular states. One of such works is [8], wherein are presented several interesting predictions for quasi-bound states in the \( C = 2, S = 0 \) and \( I = 1/2 \) sector, found following the same approach as in [6]. In the present study we aim to expand the aforementioned method of [4] to this \( \Xi_{cc} \) sector, both for pseudoscalar and vector mesons, considering this new LHCb discovery as the ground state of the \( \Xi_{cc}^{++} \). We will compare the results to those of [8].

II. THEORETICAL FRAMEWORK

Interacting particles forming a quasi-bound state cannot be studied using perturbation theory, since from the point of view of quantum field theory the particles interact infinitely many times so all diagrams must be considered. The Bethe-Salpeter equation implements a summation of loop diagrams to infinite order (see Chapter 6 in [9] for a detailed presentation of this equation). In order to simplify the calculations, usually only the most relevant diagrams are taken into account.

A. Interaction model

The interaction between a meson and a baryon has three different tree-level diagrams possible, but we shall only consider a t-channel interaction taking place due to the exchange of a vector meson. The other two diagrams, the u-channel and the s-channel, are not taken into account, since their contribution to the interaction is much smaller that of the t-channel, as seen in [2] and [4].

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Moreover, as we want to study the possibility of bound states and the mass of the exchanged vector meson is relatively large, we will take the limit at low $t$, eventually reducing the $t$-channel diagram to a contact term.

The fields of the sector we are interested in are those of pseudoscalar mesons, vector mesons, and baryons built up from $u$, $d$, $s$, and $c$ quarks, and their respective antiquarks in case of the mesons. We will use the identification $(1, 2, 3, 4) = (u, d, s, c)$. We will denote by $\phi$ the 16-plet matrix of the $\pi$, by $\rho_{i}$ the 16-plet matrix of the $\rho$, and by $B$ the tensor of baryons of the 20-plet of the proton. For their explicit expressions see [4].

Let us first describe the case of a pseudoscalar meson interacting with a baryon, which we will call $PB$ scattering. The Lagrangians related to each of the two vertices were first presented in [5] and found using the hidden gauge formalism assuming $SU(4)$ symmetry. The $VPP$ vertex coupling the exchange vector meson to the in and out pseudoscalar mesons is described by

$$\mathcal{L}_{VPP} = ig \text{tr}[(\partial_\mu \phi, \phi) V^\mu],$$

where $\text{tr}$ is taken over the $SU(4)$ matrices in flavour space; while the $VBB$ vertex coupling the vector meson to the baryons is

$$\mathcal{L}_{VBB} = \frac{g}{2} \sum_{i,j,k,l=1}^{4} \bar{B}_{ij} \gamma^\mu (V_{\mu i}^k B^{jkl} + 2V_{\mu i}^j B^{ikl}).$$

The factor $g$ is the coupling constant, which we take $g = \frac{m_V}{2\pi}$, where $m_V$ is a representative mass of the exchanged meson (usually taken around the $\rho$-meson mass), and $f = 93$ MeV is the pion decay constant.

Now, using the given effective Lagrangians to calculate $-i\mathcal{L}_{\text{vertex}} = i\mathcal{L}_{\text{vertex}}$ and working out the diagrams with Feynman rules, one obtains an identical transition amplitude for all possible channels $i$ coupling to channels $j$ through a vector meson exchange $v$, except for a coefficient $C_{ij}$. We denote by $m_v$ the mass of the exchanged vector meson, and by $p_i, p_j$ ($k_i, k_j$) the four-momenta of the baryons (mesons) of the $i,j$ channels. The resulting transition amplitudes, which we shall call potentials, are

$$V_{ij} = -\frac{1}{4f^2} \sum_{v} C_{ij}^v \bar{u}_{p_i} \gamma^\mu u_p \frac{m_V^2}{m_v^2} (k_i + k_j)_\mu,$$

where we have already taken the limit $t = (p_i - p_j)^2 = (k_i - k_j)^2 \ll m_v^2$ in order to reduce the diagram to a contact term. The $C_{ij}^v$ coefficients for all sectors involving $u, d, s$, and $c$ quarks have already been calculated and presented in [5]. Now, we can define new coefficients by adding up all $C_{ij}^v$ for all possible exchanged $v$, each weighted according to its mass:

$$C_{ij} := \sum_{v} C_{ij}^v \left(\frac{m_V}{m_v}\right)^2.$$  

We break $SU(4)$ symmetry by considering $(m_V/m_v) \approx 1$ if $v$ is a light (uncharmed) vector meson, $(m_V/m_v) = \kappa_c \approx 1/4$ if $v$ is a singly charmed meson ($D^\ast$ family), and finally $(m_V/m_v) \approx 1/16$ for a doubly charmed vector meson ($J/\psi$). Finally, expanding equation (3) up to order $O(p^2/M^2)$, we obtain

$$V_{ij}(\sqrt{s}) = -C_{ij} \left[2\sqrt{s} - M_i - M_j \right] \frac{m_V}{8f^2} \sqrt{(E_i + M_i)(E_j + M_j)} M_i M_j,$$

where $s = (p_i + k_i)^2 = (p_j + k_j)^2$ represents the square of the total energy in the center of mass frame, $M_i, M_j$ represent the masses of the baryons of the $i,j$ channels, and $E_i = p_i^0, E_j = p_j^0$.

In our case, which is that of the $C = 2, S = 0$ and $I = 1/2$ sector, the possible channels are those in Table I. The resulting $C_{ij}$ coefficients are given in Table II.

### TABLE I: Available pseudoscalar-baryon channels ($J^P = 1/2^-$) and their threshold mass, in MeV.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\pi^{\pm}$, $D_A$, $\eta^{\pm}$, $K\Omega$, $D\Sigma\Xi$, $D\Xi^\ast\eta$, $\eta'\Xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{\pm}$</td>
<td>2, $\sqrt{2}c^\ast,$ 0</td>
</tr>
<tr>
<td>$D_A$</td>
<td>$1 - \xi_c$</td>
</tr>
<tr>
<td>$K\Omega$</td>
<td>1, $\sqrt{2}c^\ast,$ $\sqrt{2}c^\ast$</td>
</tr>
<tr>
<td>$D\Sigma\Xi$</td>
<td>3, $\xi_c$</td>
</tr>
<tr>
<td>$D\Xi^\ast\eta$</td>
<td>$1 - \xi_c$</td>
</tr>
<tr>
<td>$\eta'\Xi$</td>
<td>0</td>
</tr>
</tbody>
</table>

### TABLE II: $C_{ij}$ coefficients for the $PB$ scattering.

The interaction of vector mesons with baryons, which we will call $VB$ scattering, is calculated similarly. One need only substitute the $VPP$ vertex for a $VVV$ vertex, which is described by the effective Lagrangian

$$\mathcal{L}_{VVV} = ig \text{tr}[(V^\mu, \partial_\nu V) V^\nu].$$

The resulting $VB$ interaction is identical to the $PB$ one (eq. 5) multiplied by the factor $\bar{c}_i \cdot \bar{c}_j$ of polarizations, except for some $C_{ij}$ coefficients. The available channels are shown in Table III. As done in [4], following [10], the new $C_{ij}$ coefficients can easily be obtained from those in Table II by identifying $\pi \rightarrow \rho$, $K \rightarrow K^\ast$, $D \rightarrow D^\ast$, $D_s \rightarrow D_s^\ast$, $\sqrt{2}\eta' \rightarrow \omega$, and $-\sqrt{2}\eta' + \sqrt{2}\eta' \rightarrow \phi$.

In comparison, [8] presents an almost identical model. Their $C_{ij}$ coefficients mostly coincide to ours if $\kappa_c = \xi_c = 0$, as they treat the charm quark as spectator. A few non-diagonal terms are mismatched due to different $SU(4)$ symmetry breaking. The only other difference is that the potential is simplified to order $O(p/M)$ in [8].
to a contact term. The resulting subtraction constants are given in Tables IV, V. We have also studied the case with \( \Lambda = 650 \text{ MeV} \), as it is the cut-off considered in [8].

\[
\begin{array}{llllllll}
\text{Channel} & \pi \Xi_{cc} & DA_{cc} & \eta \Xi_{cc} & K \Omega_{cc} & D_{\Sigma} \Xi_{cc} & D_{\Xi} \Xi_{cc} & \eta \Xi_{cc} & D_{\Xi} \Xi_{cc} \\
\hline
a_1^{\Lambda=800} & -2.92 & -2.21 & -2.75 & -2.81 & -2.27 & -2.31 & -2.34 & -2.67
\end{array}
\]

TABLE IV: Subtraction constants for the PB channels.

\[
\begin{array}{llllllll}
\text{Channel} & D^* \Lambda_c & \rho \Xi_{cc} & D^* \Sigma_c & D^* \Xi_c & K^* \Omega_{cc} & \phi \Xi_{cc} & D^* \Xi_c & D^* \Xi_c \\
\hline
a_1^{\Lambda=800} & -2.25 & -2.70 & -2.70 & -2.31 & -2.34 & -2.72 & -2.67 & -2.39
\end{array}
\]

TABLE V: Subtraction constants for the VB channels.

Solving for \( T \), we find

\[ T = (1 - VG)^{-1}V. \]

The resonances will appear as poles of the equation above, that is, when \( G = 1/V \). However, they will do so in the second Riemann sheet of the complex plane [11], so we need to rotate the on-shell momentum of the loop meson from \( q_l \) to \(-q_l\) whenever the real part of the complex energy \( \sqrt{s} \) is larger than the channel threshold. Equivalently, [12] showed that in that case \( G_{II}^I \) can also be calculated as

\[ G_{II}^I(s) = G_{I}^I(s) + i2M_{l}q_{l}/4\pi\sqrt{s}. \]
the dominant term of the Laurent series about them,

\[ T_{ij}(s) \approx \frac{g_{ij}}{\sqrt{s} - z_r}. \]  

(11)

Therefore we can calculate the couplings for the diagonal terms through Cauchy’s residue theorem

\[ g_i^2 = \frac{1}{2\pi} \int_{0}^{2\pi} T_{ii}(z_r) e^{ig} d\theta, \]

(12)

where \( R \) is the chosen radius of integration.

Fig. 2 shows an example of a \( \sum_j |T_{ij}| \) graph for \( i = \pi\Xi_{cc}, D\Sigma_{cc}, D_0\Xi_{cc}. \) The two narrow peaks correspond to two different bound states, respectively coupled to \( D_0\Xi_{cc} \) and \( D_0\Xi_{cc}. \) The smooth local maximum of the \( \pi\Xi_{cc} \) channel on the left indicates the presence of a pole in the second Riemann sheet of the complex plane with \( \text{Re}(\sqrt{s}) \approx 3850 \) MeV. The same behaviour can be observed in \( D_0\Xi_{cc} \) around 4250 MeV, indicating the presence of a fourth pole, also in the complex plane.

![Graph showing coupling strengths](image)

FIG. 2: The solid lines represent the sum over all \( j \) channels of the module of the PB scattering amplitude \( T_{ij} \), for \( \Lambda = 800 \) MeV and \( i = \pi\Xi_{cc}, D\Sigma_{cc}, D_0\Xi_{cc}. \) The dashed vertical lines correspond to the threshold masses of the respective channels.

We first describe the results for the pseudoscalar-baryon interaction. Table VI shows the PB poles found for \( \Lambda = 800 \) MeV. Since they consist of a pseudoscalar meson and a ground state baryon interacting in s-wave, they have \( J^P = 1/2^+ \). Following the local \( \pi\Xi_{cc} \) maximum in Fig. 2, we first find a wide resonance at 3862 MeV, with a width of 158 MeV, coupled mainly to the \( \pi\Xi_{cc} \) channel but also to \( D\Lambda_{c} \) and \( K\Omega_{cc}. \) The large width is due to its mass being well over the \( \pi\Xi_{cc} \) channel threshold, to which it couples most. The second pole found is a bound state at 4353 MeV, corresponding to the \( D_0\Xi_{cc} \) peak in Fig. 2, and it couples mainly to \( D\Lambda_{c} \), also to \( K\Omega_{cc}, D_0\Xi_{cc}, \) and \( \eta\Xi_{cc}. \) As for the \( D\Lambda_{c} \) peak we find a narrow resonance coupled exclusively to \( D\Lambda_{c} \) and \( D_0\Xi_{cc} \) with mass and width equal to 4239 MeV and 0.8 MeV respectively. These two states are bound states with negligible width due to their mass being lower than all thresholds of the channels to which they couple most.

Finally, we find another wide resonance at 4240 MeV with a width of 166 MeV. It is greatly coupled to \( D_0\Xi_{cc} \), as the local maximum on the right-hand side of Fig. 2 suggests. \( D_0\Xi_{cc} \) has a threshold mass greater than that of the resonance, however, the state presents a large width since it is also weakly coupled to channels \( \pi\Xi_{cc}, D\Lambda_{c}, \eta\Xi_{cc}, K\Omega_{cc}, \) all of which are open for decay.

![Graph comparing resonances](image)

TABLE VI: PB results \( (J^P = 1/2^-) \) for \( \Lambda = 800 \) MeV.

As for the results concerning the \( VB \) scattering, associated to degenerate \( J^P = 1/2^-, 3/2^- \), we also find 4 poles. The first pole found is a resonance at \( M_r = 4280 \) MeV with \( \Gamma_r = 6.4 \) MeV, coupled to the \( D^*\Lambda_{c}, \eta\Xi_{cc}, \) and \( D_0\Xi_{cc} \) channels. Even though its mass is lower than any threshold mass, it shows some width due to that of the \( \rho \). We then find another resonance related to similar channels, but at a mass of 4353 MeV and a greater width of 22 MeV, since it is already over the \( D^*\Lambda_{c} \) threshold. It is also predominantly coupled to \( \rho\Xi_{cc} \) and \( D_0\Xi_{cc} \) as the previous pole, but less coupled to the \( D^*\Lambda_{c} \) channel and notably coupled to \( K\Omega_{cc} \) instead. The third pole is a narrow resonance strongly coupled to the \( D^*\Sigma_{cc} \) channel, and also mildly to \( D_0\Xi_{cc}. \) Both thresholds are higher than the pole mass, which is approximately 4387 MeV. Therefore its width is only 1.3 MeV. Comparing to the \( D\Sigma_{cc} \) \( PB \) resonance, it seems that they could be spin partners of the same molecule. Finally, one last wide resonance with \( M_r = 4523 \) MeV and \( \Gamma_r = 54 \) MeV, coupled to \( D^*\Sigma_{cc}, K\Omega_{cc}, \phi\Xi_{cc}, \) and slightly to \( \omega\Xi_{cc}, \) which is open for decay.

![Graph comparing resonances](image)

TABLE VII: VB results \( (J^P = 1/2^-, 3/2^-) \) for \( \Lambda = 800 \) MeV.

\[ \text{Re}(z) = M_r \quad \begin{array}{c|cc|cc} \hline \text{MeV} & \text{4270.8} & \text{4352.6} & \text{4386.8} & \text{4522.6} \\ \hline \text{Im}(z) = \Gamma_r/2 \quad \begin{array}{c} \text{3.2} \quad 11 \quad 0.65 \quad 27 \end{array} \\ \end{array} \]

| \( g_{\pi\Xi_{cc}} \) | \( 4293 \) & 1.67 & 0.86 & 0.08 & 0.21 \\ | \( g_{D\Lambda_{c}} \) | \( 4393 \) & 1.57 & 1.91 & 0.21 & 0.36 \\ | \( g_{\rho\Xi_{cc}} \) | \( 4404 \) & 0.30 & 0.15 & 0.19 & 0.92 \\ | \( g_{D_0\Xi_{cc}} \) | \( 4460 \) & 0.15 & 0.62 & \text{3.12} & 0.02 \\ | \( g_{K\Omega_{cc}} \) | \( 4581 \) & 1.45 & 1.76 & 0.23 & 1.64 \\ | \( g_{\eta\Xi_{cc}} \) | \( 4606 \) & 0.37 & 1.54 & 0.34 & 2.02 \\ | \( g_{\rho\Sigma_{cc}} \) | \( 4641 \) & 0.43 & 0.21 & 0.27 & 1.30 \\ | \( g_{D_0\Xi_{cc}} \) | \( 4689 \) & 0.04 & 0.16 & \text{1.89} & 0.20 \\ \hline \end{array} \]
The results obtained using $\Lambda = 650$ MeV, both for the $PB$ and $VB$ interaction, are qualitatively the same as the ones described. A lower cut-off limits the intermediate states, which translates into less binding interaction. Therefore, the poles with $\Lambda = 650$ MeV show larger masses, approximately between 15 and 40 MeV heavier. As a result, they also present larger widths.

With respect to the results of [8], there is qualitative agreement on the nature and couplings between the states found, although not quantitative. They only presented three states for each $I=1/2$, $S = 0$, and $C = 2$ sector. Regarding the $PB$ scattering, our $\pi\Xi_{cc}$ pole appears at $3861.9 + i79$, whereas it is found at $3837.3 + i100$ in [8]. Even a greater discrepancy is encountered for the $D\Sigma_c$ bound state, as it appears 150 MeV lower than in our results, with a mass of 4092 MeV. Regarding the states coupled to $\Lambda\Delta_c$ and $D_s\Xi_c$, only one state is found in [8], at 4083 MeV, as opposed to the two states we present. A similar comparison holds for the $VB$ molecules: general qualitative agreement, but quantitative disagreement on the values of the masses and widths. Again, only three $VB$ states were found in [8], with comparable couplings to those of our states. Further research is needed to better understand these discrepancies, but as of now, we believe that the quasi-bound states found following our approach qualify better as molecular states, since they have higher energies and are closer to thresholds.

IV. CONCLUSIONS

We have studied the possibility of dynamically generated hadronic molecules in the $\Xi_{cc}$ sector, i.e. $(I, S, C) = (1/2, 0, 2)$, for ground state baryons interacting in s-wave with both pseudoscalar and vector mesons. The interaction is modelled through effective Lagrangians describing a t-channel vector exchange picture. Unitarization is implemented via the Bethe-Salpeter equation, from which the possible molecules emerge as poles of the resulting scattering amplitude. The loops have been calculated through dimensional regularization choosing suitable subtraction constants.

We find four poles for $PB$ scattering, with $J^P = 1/2^-$. A wide resonance at 3862 MeV, mainly coupled to $\pi\Xi_{cc}$; a $\Lambda\Delta_c$ bound state at 4135 MeV, slightly below its threshold mass; a narrow resonance at 4239 MeV mainly coupled to $D\Sigma_c$ but also $D_s\Xi_c$; and finally a wide $D_s\Xi_c$ resonance with a mass of 4240 MeV.

Regarding the $VB$ scattering, which produces molecules degenerate with $J^P = 1/2^-, 3/2^-$, we also find four possible states. A narrow resonance at 4280 MeV, slightly below the threshold of channel $D^*\Lambda_c$, to which it couples the most; a wider resonance coupled to $\rho\Xi_{cc}$ at 4353 MeV; a narrow state strongly coupled to $D^*\Sigma_c$ with a mass of 4387 MeV; and finally a wide resonance at 4523 MeV mainly coupled to $K^*\Omega_{cc}$.

The observation of resonances close to these eight molecules predicted would give support to the composite nature of these states, and give deeper understanding on the structure of heavy hadrons and their spin-parity. In order to provide further information to guide experimental searches, we shall deepen this study on the near future. Moreover, we plan to extend it to equivalent sectors substituting the $c$ quark for the heavier $b$ quark.

Acknowledgments

I would like to express my deep gratitude to Prof. Angels Ramos for her useful advise and guidance, and most of all her dedication and time given. Also, I wish to thank Irina, my parents, my sister, and my family and friends in general for all the support and patience. This work is partly supported by the Spanish Ministerio de Educación through a department collaboration grant.