Theoretical and methodological review of dynamical friction in major mergers

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Abstract: We investigate the process of dynamical friction from the point of view of the dynamics of galaxy mergers. We are particularly interested in providing an accurate formula for the merger time between galactic halos of similar mass, also known as major mergers. We have reviewed the theoretical foundations of this mechanism, as well as used 605 isolated simulations of binary collisions of galaxies to test a new non-separable expression for the merger timescale. Our study reveals some plausible reasons why merger time formulas existing in the literature may not perform successfully for major mergers, related to both the theory and the methodology applied in the analysis of numerical results.

I. INTRODUCTION

In the current Λ cold dark matter (ΛCDM) cosmology, dark matter haloes (DMHs) are the first objects to form. DMHs grow hierarchically, with smaller haloes merging together and giving rise to more massive ones. The baryons inside the parent haloes, which are mostly in gaseous form and well mixed with the DM, become shock heated to the virial temperature of the new halo and reach hydrostatic equilibrium again. In the dense central regions of some DMHs, however, the cooling time of the gas due to radiative processes may become short enough for the gaseous baryons to collapse towards the halo centre, where they give birth to a galaxy. Thus, whenever two or more haloes hosting a galaxy merge into a larger structure, their central galaxies undergo the same process too, resulting often in a remnant with a different morphology than that of its progenitors. Through this phenomenon, galaxies grow and evolve, supermassive black holes coalesce and active galactic nuclei (AGNs) are fed with fresh gas [8].

It is then obvious that galaxy mergers play a key role in the history of the universe and in the genesis and evolution of its structure. In particular, the merger times are of great interest for semi-analytic models of galaxy evolution, since many essential functions such as the luminosity/stellar mass function, the distributions of galaxy sizes, metallicities, colours and morphologies, as well as fundamental quantities such as the amount of gas available to form new stars, the star formation rate, and the abundance of first-ranked galaxies rely on their values. Besides, not all mergers are equally important. The most relevant are those involving pairs of galaxies of similar mass, for they are the ones that produce the most significant changes in the properties of the galaxies and of the stars they host. They are called major mergers and are the ones which shall concern us in this work.

To determine the duration of a DMH merger we ought to understand how self-gravitating extended objects in a bound orbit become progressively close to each other and eventually collide and merge. The process responsible for this happening is known as dynamical friction. Dynamical friction transfers energy and momentum from the relative motion between the interacting galaxies (and their host haloes) to internal degrees of freedom, i.e., converts the energy of the orbital motion into random internal motions of the interacting galaxies. This work focuses on the study of the dynamical friction and the timescales of mergers driven by it, with special attention to all those aspects that can be relevant to major mergers.

II. DYNAMICAL FRICTION: THEORY

Dynamical friction was first studied in 1943 by the Indian astrophysicist Subrahmanyan Chandrasekhar [4]. Chandrasekhar considers an idealised scenario where a massive point-like star is moving through a star field with uniform space density and where the interaction between the massive star and the system is weak enough that the systems response its determined by its properties in the absence of the massive star. The stars in the background are thought to be identical (also point-like) and have peculiar velocities that follow a certain distribution (in [4], Chandrasekhar assumes a Maxwellian probability density function). As the main star moves through the field, its gravity attracts field stars towards its path. A star moving faster than its surrounding stars will leave a wake behind which will tend to slow it down. The decelerating force caused by the wake originated by the passage of the moving star is what is called dynamical friction. We note that Chandrasekhar also uses a two-body approximation to face the problem, i.e., he studies the ideal situation of two-body stellar encounters (where the moving star interacts with only one star at a time of the background stellar system) to estimate the dynamical friction force.

A more recent description of dynamical friction can be found in the book *Galactic Dynamics* [2]. In section 8.1 of this book, Binney and Tremaine consider a test body of mass M (a small galaxy or other stellar system) moving through a star field made by identical stars of mass $m_i << M$. As with Chandrasekhar's study, the subject body is assumed to be point-like and the star field is also considered to constitute an (nearly) infinite and homogeneous system of total mass $\mathcal{M} >> M$. Employing these approximations and assuming the star field obeys an isotropic velocity distribution $f(v_i)$, Binney and Tremaine derive the following expression for the Chandrasekhar's dynamical friction formula

$$\frac{d\mathbf{v}_M}{dt} = -16\pi^2 G^2 M m_i \ln\Lambda \left[\int_0^{v_M} \mathrm{d}v_i v_i^2 f(v_i) \right] \frac{\mathbf{v}_M}{v_M^3}, \quad (1)$$

where G is the gravitational constant, v_i is the velocity of the field stars and v_M is the net velocity of the moving body relative to the background field. The Coulomb logarithm $\ln\Lambda$ is defined as $\ln\Lambda \equiv \ln(b_{max}/b_{min})$, where b_{min} and b_{max} are, respectively, the minimum and maximum impact parameters of the individual small-angle deflections associated with the passing of the subject body through the star field. This expression is the primary definition of the Coulomb logarithm. As we shall see below, there exist in the literature several alternative expressions for $\ln\Lambda$ that depend on both the system's characteristics and the approach adopted to calculate the two extremal impact parameters.

The analysis of equation (1) shows that only stars moving slower than the subject body of mass M contribute to dynamical friction. Besides, one can observe that the force described by this equation is opposite to the motion of M, as we should expect from the decelerating nature of any ordinary frictional drag. Remember, however, that expression (1) has several internal inconsistencies, which become especially obvious when trying to use it to describe mergers of galaxies. To begin with, it is derived considering that the host system (the star field) is infinite. This means that this equation should work better the greater the difference in mass between the host galaxy (hereafter, the primary halo of mass M_{p}) and the satellite galaxy (hereafter, the secondary halo of mass M_{s}). Moreover, there is the already mentioned problem of the arbitrary definitions that can be adopted for the lower and, especially, the upper cutoffs in the impact parameter b. For example, in [6], b_{max} is set equal to the radius of the primary halo, obtaining the following expression for the Coulomb logarithm (after making other approximations): $\ln\Lambda = \ln(\mathcal{M}/M) \equiv \ln(M_{\mathrm{p}}/M_{\mathrm{s}})$. Nevertheless, in [3] and [8] numerical simulations are used to infer that a better choice for the Coulomb logarithm is $\ln\Lambda = \ln(1 + M_{\mathrm{p}}/M_{\mathrm{s}})$ when dealing with extended bodies. In addition, in the real world interacting galaxies and their haloes constitute far from homogeneous extended stellar fields in which self-gravity cannot be neglected and with velocity distributions $f(v_i)$ that are not necessarily isotropic. These and other drawbacks are accentuated when M_{s} is comparable to M_{p}.

The main conclusion of this section is then that the assumptions made in order to infer equation (1) imply that the resulting expression can offer only an approximation for the true frictional drag mutually exerted between two merging haloes. Furthermore, its accuracy is expected to be positively correlated with the mass ratio $\eta \equiv M_{\mathrm{p}}/M_{\mathrm{s}}$ of the two merging haloes, in the sense that it should perform better the larger the value of η. Thus, it is reasonable to expected that predictions on the merger time based on this formula have only a limited validity for major mergers which, by definition, have values of η close to unity.

III. NUMERICAL SIMULATIONS OF MERGERS

Dynamical friction can also be investigated empirically with the aid of numerical simulations of mergers. Recent works use two different approaches: binary collisions and pure cosmological simulations. The first ones rely on controlled simulations of binary mergers in which pairs of fully isolated bound galaxy haloes are built with pre-set initial conditions. A cosmological simulation recreates a certain large volume of the universe where mergers take place in a self-consistent manner in a continuously evolving environment as structure built up hierarchically.

At the dawn of computer simulations, the study of mergers of galaxies was limited to the modelling of binary collisions, due to the computational limitations of the time. But in more recent times the development of technology and the arrival of more powerful devices has fostered its replacement by cosmological-scale numerical simulations. Yet despite the latter provide a more realistic scenario of merging, the use of pre-prepared simulations to study the role of gravity offers a series of advantages:

- Simulated pairs are at the same dynamical stage, which facilitates the inter-comparison of results.
- In cosmological simulations it is not possible to arbitrarily specify *a priori* the properties of the interacting galaxies.
- The number of groups is not limited by the size of the cosmological box.
- There is no need to rely on a large cosmological-scale run from which galaxy mergers must be first identified and then re-simulated to increase the mass and force resolution.
- It is possible to study galaxy mergers at very high spatial and mass resolutions while keeping the simulations computationally feasible.
- The idealised scenario of two galaxies merging in isolation, although hard to find in the cosmos, is probably much closer to the type of interaction considered by theoretical models.

Of course, there is still the important caveat of the *ad hoc* initial conditions systematically adopted in traditional controlled simulations, such as strongly radial orbits and small pericentric distances, with the obvious intention of leading to fast mergers that save CPU time. The assignment of initial conditions is a problem that has been solved guided more by common wisdom than by specific evidence. Nevertheless, this does not have to be the case. If the results of binary mergers served at the time as a guide for the first fully cosmological experiments, it is now possible to use the latter as feedback to establish realistic initial conditions for the former and thus to compare the results of both types of simulations on a roughly equal footing. This is exactly the strategy adopted in [9]. These authors built a suite of 605 high-resolution N-body simulations of isolated major mergers, whose set-up relied entirely on initial conditions inferred from large-scale-structure simulations carried out in the frame-work of the standard concordant ΛCDM model, including those that incorporate baryon physics. The experiments were used to investigate the dependence of the dynamical friction timescale for merging, $\tau_{\rm mer}$, on a range of orbital parameters, mass-ratios, spins, and morphologies representative of such systems. In the next section, I will use the outcomes of these runs to expand on some of the aspects that were mentioned but not addressed in that study.

IV. DYNAMICAL FRICTION: EXPERIMENTS

The most popular fitting formulas for the merger timescale adopted in simulations are adaptations of the analytic expression proposed by Lacey & Cole [7]. This formula in turn is inspired by Chandrasekhar's idealised treatment of dynamical friction described in section II. In fact, Lacey & Cole begin their deduction of the merger time using the expression for the dynamical friction force included in the first edition of [2].

Both Lacey & Coles formula and its subsequent developments in [3], [6], [8], [5] and [9], calculate the merger time from three magnitudes which define the main merger characteristics: the initial mass ratio of progenitors, orbital energy and orbital circularity. We have already defined the mass ratio η in section II. The orbital energy is given by the radius of a circular orbit with same orbital energy of the merger, $r_c = r_c(E)$, where E would be the initial energy of the merger orbit. On the other hand, the orbital circularity is measured by the angular momentum of the orbit relative to that for a circular orbit with the same initial energy E, $\epsilon \equiv J/J_c(E)$

As stressed in [9] when the expressions tuned from simulations are applied to estimate $\tau_{\rm mer}$ for major mergers they led to considerable discrepancies both among them and with the experimental values. One could imagine that this happens because of the inherent approximations included in Chandrasekhar's description of dynamical friction that we have highlighted in section II. Nevertheless, the discrepancies are of such magnitude (between 30% and 40% in most cases) that they lead us to believe that it may not be the only cause. For instance, the most common forms of the fitting formulas, i.e., those following the Lacey & Coles prescription, are a product of three different functions, one for each of the defining magnitudes of the merger (mass ratio η, orbital energy r_c and orbital circularity ϵ). This factorisation is adopted just as a convenient approach for the fitting (it allows one to fit each parameter independently), but it could not be the case. For this reason, and since according to [9] the dependence on mass ratio appears to be the most robust one, in this work we have decided to investigate the adequacy of merger timescale formulas which are a non-separable form of ϵ and r_c. Specifically we propose:

$$\frac{\tau_{mer}}{\tau_{dyn}} = \mathbf{A}\frac{\eta^{\mathbf{B}}}{\ln(1+\eta)} f\left[\mathbf{C}\left(\frac{r_c}{R_p}\right) + \mathbf{D}\epsilon\right], \qquad (2)$$

where $R_{\rm p}$ is the virial radius of the primary galactic halo, $\tau_{\rm mer}$ is the merger time and $\tau_{{\rm d}yn} \propto R_{\rm h}^{3/2}/M_{\rm h}^{1/2}$ is the halo-independent dynamical timescale [9]. For the function f, we shall use the hyperbolic trigonometric sinh and cosh functions for their resemblance with the exponential function used in [3], [8] and [9] for fitting the dependence on the orbital circularity ϵ and the inseparable nature of their argument.

Aside from the functional dependence adopted to define the merger timescale, something as seemingly simple as the very definition of the merger time is not without controversy. Take the end of a merger. It should mark the final coalescence of the two galactic haloes. In most simulations this moment is usually determined, as we see in [8], when a specific quantity, namely the specific orbital angular momentum or its number of bound particles, falls below an arbitrary threshold considered to be sufficiently small. However, there is no guarantee that a threshold condition, no matter how small, can be violated momentarily during the approach of both halos and before coalescence is truly reached, which would invariable result in an underestimate of the true value of the merger time. This is why in [9] they choose to follow the temporary evolution of the behaviour of a function that measures the relative separation of the galaxies in phase space—instead of the evolution of its value—to determine the end of a merger.

Defining the beginning of mergers is not free of difficulties either. In [3], [6] and [8] it is stated that the merger begins when the secondary halo enters the virial radius of the primary galactic halo, without specifying any particular point. This is a vague definition since haloes are extended objects. In [9], following [1], they clearly specify that the start of a merger is the instant when the centre of mass of the secondary halo first crosses the virial radius of the primary galactic halo. However, it is not evident that the definition of [1] is necessarily the most adequate for correctly calculating the merger time, especially for a major merger. The difference between considering the start of a merger the instant when the centre of mass of the secondary halo first crosses the virial radius of the primary one and considering it the instant when both virial radii first cross is negligibly small for a minor merger, but it becomes significant as the mass, and consequently, the size of the secondary progenitor grows, i.e., as we approach the major merger condition.

In this work, we will consider both possibilities when fitting equation (2) to the numerical data. To do this, we have modified the original Fortran90 code the authors used in [9] so that it gives to us both simulated merger times: the one estimated using the definition in [1] and the one calculated from the first crossing of the virial radii. The program also gives the defining magnitudes of the merger η, r_c and ϵ at both points. The $t = 0$ point of the simulations, which coincides with the first crossing of the virial radii, provides directly the initial values, while what we hereafter call the virial values of these magnitudes are calculated numerically at the first crossing of the centre of mass of the secondary halo with the virial radius of the primary halo. There is one exception, the halo mass ratio, which is always taken to be the ratio of virial mass of the progenitor haloes before they start merging.

V. RESULTS AND DISCUSSION

For every one of the 605 pairs of haloes, we have calculated the merger time using equation (2) in four different configurations, which correspond to the possible combinations of f being equal to sinh or cosh and the merger defining magnitudes being the initial values or the virial values, as illustrated in Figure 1. The values of the free parameters **A, B, C** and **D** of equation (2) have been fit applying a sequential procedure that seeks to minimise the sum of squared residuals Q = $\frac{1}{N}\sum_{i=1}^{N}(\tau_{mer}/\tau_{dyn} - \tau_{sim}/\tau_{dyn})^2$, where N is the number

sinh	A	B	C	D	**Q**
initial	1.13	0.80	0.30	1.60	0.851612
virial	0.12	1.03	0.62	3.59	1.501504
cosh	A	B	C	D	**Q**
initial	0.91	0.79	0.27	1.87	0.361081
virial	0.12	1.03	0.62	3.58	1.391230

TABLE I: Best-fit parameter values and least squares value **Q** from fits of equation (2) to the entire set of our major merger simulations.

of simulated pairs of haloes and τ_{sim} is the merger time computed by our code from the simulations. The values of this figure of merit obtained from the entire set of 605 pairs are listed in Table I, while Table II reports the results inferred when the merger times of individual simulations of pairs that share the same initial defining magnitudes (but differ for instance in the haloes spin) are averaged. In this last case, we obtain 24 data points, which emerge from all possible combinations of the initial values of the defining magnitudes of the merger: $\eta = 1, 2, 3$, $r_c = 1.3, 2.0, 2.7$ and $\epsilon = 0.20, 0.45, 0.70$, with the exception there are no simulations with $r_c = 2.7$ and $\epsilon = 0.70$ (see Figure 2). We also note that no all averages contain the same number of simulations, which may introduce a

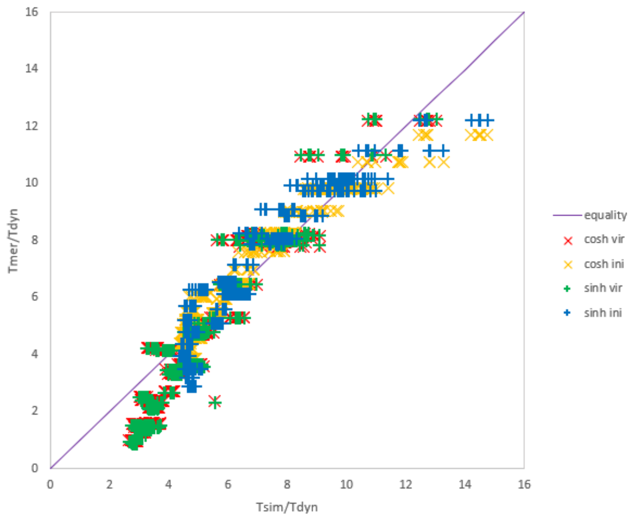

FIG. 1: Calculated merger times of the entire set of our major merger simulations using formula (2) compared against the respective merger time computed by our code

small bias in the calculations. As in the case of individual runs, the optimisation of the coefficients of equation (2) that deals with merger averages is done sequentially.

sinh	A	B	C	D	**Q**
initial	0.14	0.95	0.72	3.39	3.795005
virial	0.11	1.04	0.63	3.64	1.281085
cosh	A	B	C	D	**Q**
initial	0.14	0.95	0.72	3.39	3.619303
virial	0.11	1.05	0.63	3.63	1.195825

TABLE II: Best-fit parameter values and least squares value **Q** from fits of equation (2) to the representative average samples of our major merger simulations.

A first result from Tables I and II is that f=cosh always performs better than f=sinh, though the differences in the values of **Q** are small for the averaged experiments. It can also be observed that the best overall fits are obtained when using the complete set of 605 individual simulations, though virial magnitudes lead to slightly better fits with the averaged runs. However, the most striking result is that the values of the coefficients **{A, B, C, D}** corresponding to the two best fits (Table I) are quite different from the rest, which also turn out to be quite similar to each other. Although the minor discrepancies in the fitted coefficients obtained when using virial and initial values suggest that the differences in the instant adopted as the start of the merger are, after all, not very important, the last discrepancy seems to have an origin more mathematical than physical. The explanation we find the most plausible is that equation (2) has more than one minimum, something which is not unlikely given that we are working in a four-dimensional parameter space. If

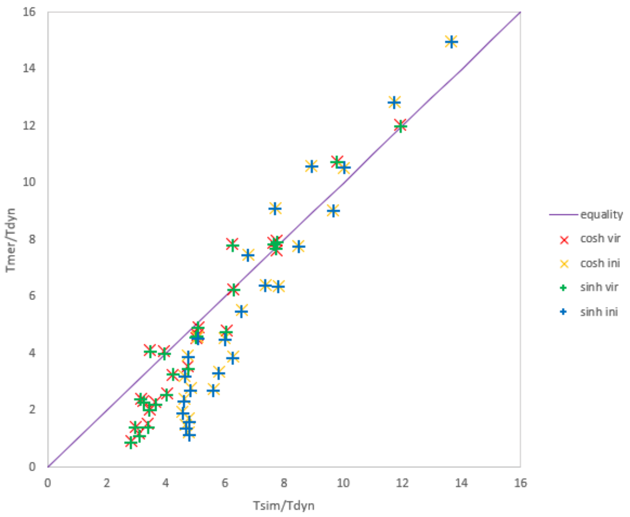

FIG. 2: Calculated merger times of the 24 representative major merger simulations using formula (2) compared against the respective merger time computed by our code

this is the case, it could well be that any of our sets of fitted parameters corresponds to the absolute minimum, since we started the sequential adjustment from an arbitrary point. It is, therefore, advisable in this type of multiparametric minimisations to scan the entire parameter space in search of the values that determine the optimal fit.

Finally, we want to stress that even though the fits we have obtained are not better than the ones in [9] with the same numerical data, the values of **Q** are sufficiently low to not discard the proposed merger time formula (2). In fact, all except the two highest values of **Q** are in the range of those corresponding to the fits shown in [9].

VI. CONCLUSIONS

In this work we have reviewed the foundations of dynamical friction, the loss of momentum and kinetic energy of moving bodies through gravitational interactions with surrounding matter in space. This gravitational drag governs many astrophysical process including the dynamics of galaxy mergers. We are interested in providing an accurate formula for the timescale of major mergers which involve galactic halos of comparable mass. With this aim, we have gone through the main hypotheses involved in the theoretical treatment of dynamical friction, identifying those that may not be fulfilled by major galaxy mergers. We have also adopted an experimental approach to this question by using 605 isolated binary mergers to test a new, non-separable merger time formula and evaluating two different definitions of the moment that defines the beginning of a merger.

Our study has highlighted some of the weaknesses of present calculations of the merger timescale reported in the literature, including some usually overlooked as those related to the mathematical treatment of the problem that involve the fitting method or the very definition of the duration of merger. We have also revealed some theoretical issues, including the limitations of the hypotheses traditionally adopted in the description of dynamical friction that become critical for major mergers, a matter which in order to be solved requires a profound and solid comprehension of galactic dynamics.

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