Natural inflation with 2 light quarks

Author: Jon Garí Galíndez
Advisor: Alessio Notari
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Abstract: Natural inflation is one of the most interesting models of inflation, it is generally studied for 1 light quark and is a model that fits with the experimental observations of the CMB within 2 sigma error. We will analyse the behaviour of a model of natural inflation with 2 light quarks and compare with observations. We will find that this model starts to be acceptable within 2 sigma when $m_u/m_d > 4$, otherwise the results are worse than the natural inflation with only 1 light quark.

I. INTRODUCTION

It is known that a primordial inflationary epoch is needed to explain how our universe looks nowadays and to solve the flatness problem, the entropy problem and the horizon problem.

This very fast expansion stretches the perturbations of quantum fields from microscopical to cosmological scales, producing fluctuations in energy density and in the metric. It is believed to be responsible of the galaxy distribution and the anisotropies of the CMB.

This inflationary epoch was due to a slowly rolling scalar field whose energy density dominates until the slow roll conditions are violated and inflation stops [1].

The way we see how realistic is an inflationary model is comparing with the CMB experimental results.

We focus on a generalization of the so-called Natural Inflation (NI) model. NI is a theoretically attractive model, whose potential is a cosine potential, inspired by the QCD axion. We extend this idea by considering a theory with 2 light quarks (similarly to the real QCD axion). We will see if its behaviour is closer to the experimental values, to do it, we will solve the equations with Wolfram Mathematica.

II. INFLATION

The equation of motion of the field is

$$\ddot{\phi}_o + 3H\dot{\phi}_o + V'(\phi_o). \qquad (1)$$

This scalar field is slowly rolling down its potential if $\dot{\phi}_o{}^2 \ll V(\phi_o)$. We define the slow-roll parameters as:

$$\epsilon = \frac{1}{16\pi G_N}\left(\frac{V'}{V}\right)^2 = \frac{M_{Pl}{}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$\eta = \frac{1}{8\pi G_N}\left(\frac{V''}{V}\right) = M_{Pl}{}^2\left(\frac{V'}{V}\right) = \frac{V''}{3H^2}, \qquad (2)$$

where M_{Pl} is the reduced Planck mass.

Inflation only happens if $\epsilon < 1$. When ϵ grows more inflation ends.

Doing some approximations, we can calculate the number of e-foldings:

$$\dot{N} = H \qquad (3)$$

And we know that this inflationary stage had $\Delta N \sim 60$ in order to produce the anisotropies we observed in CMB.

To characterize the properties of the perturbations, we define the power spectrum, on super-Hubble scales of the comoving curvature is

$$\mathrm{P}_R(k) = \frac{1}{2M_{Pl}{}^2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n_R-1}$$

$$A_R{}^2 \equiv \frac{1}{2M_{Pl}{}^2\epsilon}\left(\frac{H}{2\pi}\right)^2 \qquad (4)$$

This is the amplitude of the power spectrum and we can define:

$$A_s \approx \frac{A_R{}^2}{8\pi^2} \qquad (5)$$

By the experimental observations of the CMB, we know that $A_s \approx 2 \cdot 10^{-9}$, 60 e-foldings before inflation ended.

We also have the spectral index:

$$n_s = 1 + 2\eta - 6\epsilon. \qquad (6)$$

And the amplitude of the gravitational waves produced by the inflation is proportional to:

$$r = 16\epsilon. \qquad (7)$$

We will compare our inflationary model using these two last parameters.

III. THE QCD AXION POTENTIAL

To reproduce a natural inflation with 2 light quarks we use the QCD axion potential:

* Electronic address: `jgarigal7@alumnes.ub.edu`

$$V(\phi) = -m_\pi{}^2 f_\pi{}^2 \sqrt{1 - \frac{4m_u m_d}{(m_u + m_d)^2} sin^2\left(\frac{\phi}{f_\phi}\right)} \quad (8)$$

where m_u , m_d are the mass of the 2 light quarks, m_π is the mass of the pion, and f_π , f_ϕ are constants.

This potential is important because of the absence of CP violation in strong interactions [4]. In inflationary scenario we uses the same ideas of QCD, but at much higher energy scales.

We will reproduce different cases changing $\frac{m_u}{m_d}$ and we will compare with the case of natural inflation with 1 light quark, whose potential is:

$$V(\phi) = -m_\phi{}^2 f_\phi{}^2 \cos\left(\frac{\phi}{f_\phi}\right),$$

where $m_\phi{}^2 = \frac{m_u m_d}{(m_u+m_d)^2} \frac{m_\pi{}^2 f_\pi{}^2}{f_\phi{}^2}$.

First of all, we compare these 2 potentials varying $\frac{m_u}{m_d}$:

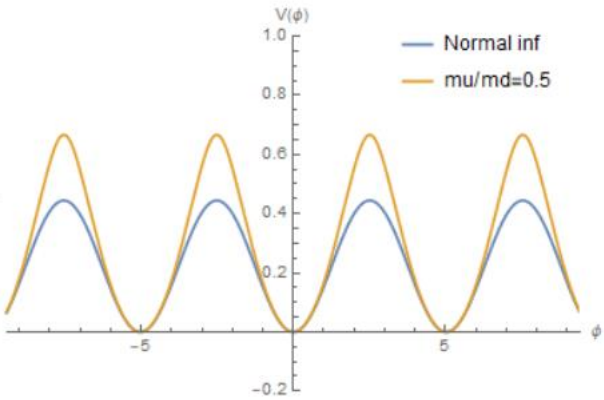

FIG. 1: Representation of the axion potential with $\frac{m_u}{m_d} = 0.5$ and the natural inflation potential.

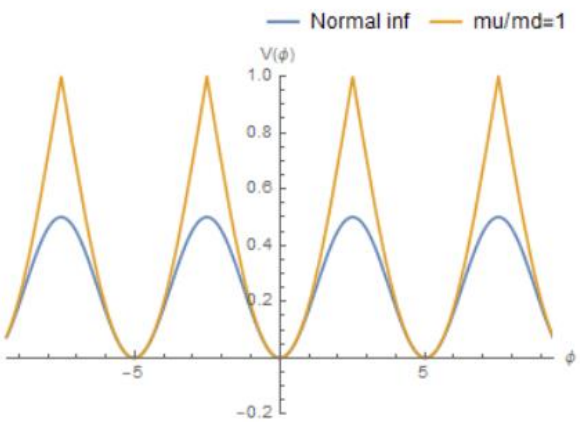

FIG. 2: Representation of the axion potential with $\frac{m_u}{m_d} = 1$ and the natural inflation potential.

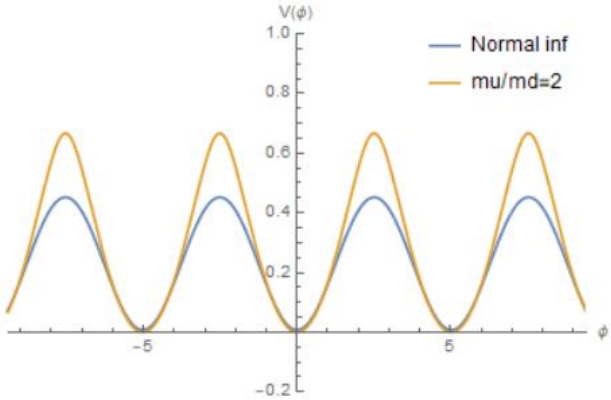

FIG. 3: Representation of the axion potential with $\frac{m_u}{m_d} = 2$ and the natural inflation potential.

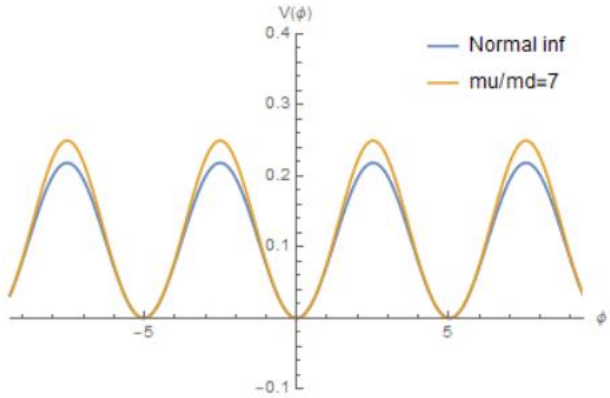

FIG. 4: Representation of the axion potential with $\frac{m_u}{m_d} = 7$ and the natural inflation potential.

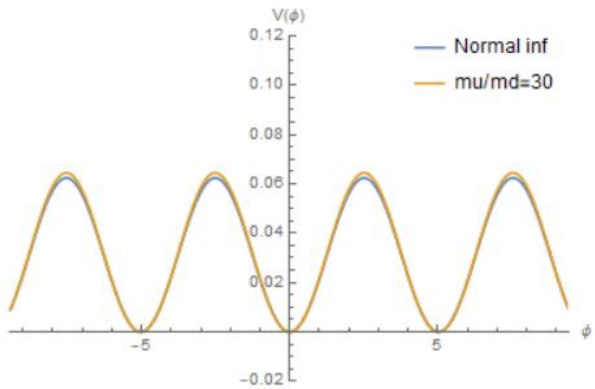

FIG. 5: Representation of the axion potential with $\frac{m_u}{m_d} = 30$ and the natural inflation potential.

We can see how for the axion potential, the bigger $\frac{m_u}{m_d}$ the closer to the natural inflation potential.

Now, we are going to solve the Eq. (1), we use the program Wolfram Mathematica to calculate the solution. We also solve the Eq. (2), (3) to calculate ϵ, η and N.

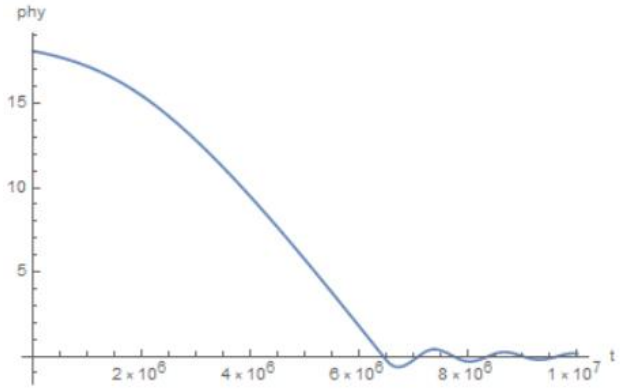

FIG. 6: Representation of the scalar field ϕ in function of time in conditions of inflation.

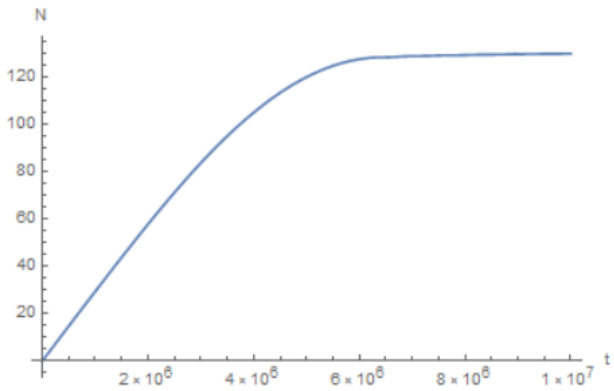

FIG. 7: Representation of the number of e-foldings in function of time in conditions of inflation.

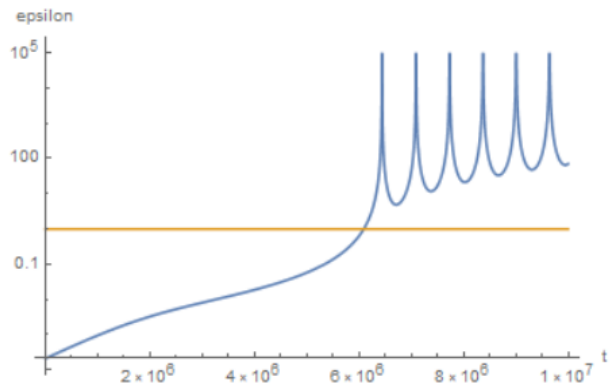

FIG. 8: Representation of epsilon in function of time in conditions of inflation and a line in $\epsilon = 1$, the condition of the end of inflation.

To do these plots, we use: $M_{Pl} = 1$, $f_\pi = 1$, $\frac{m_u}{m_d} = 2$, $f_a = 6$, $m_\pi = 6.3 \cdot 10^{-5}$.

We can observe that when inflation ends, the scalar field starts to oscillate, and N stops growing too. The oscillations of epsilon after inflation doesn't matter because this parameter only has importance during inflation.

According to the experimental observations of the CMB [2], the amplitude: $A_s = 2 \cdot 10^{-9}$, 60 e-foldings before the end of inflation. With Eq. (5) we calculate A_s, and we change the parameters m_π and f_ϕ until our A_s is equal to the experimental value. When we obtain the same value, we calculate r and n_s with Eq. (6), (7).

We do that for different values of m_π and f_ϕ to get a line, and doing this also for different $\frac{m_u}{m_d}$ we obtain the next plot:

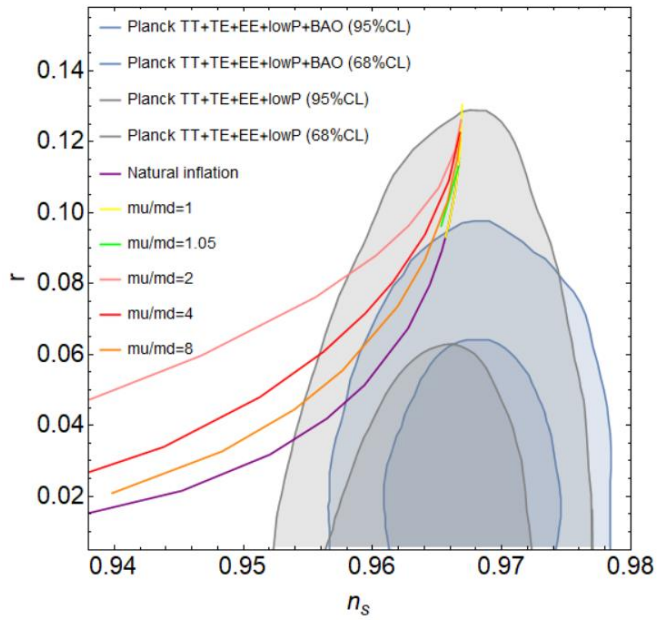

FIG. 9: Representation of the results for different potentials and the experimental Plank results.

We can see how the natural inflation is the model that fits the best the CMB observations.

To analyse the axion potentials results, we divide these potentials in two groups: the first when $\frac{m_u}{m_d} \geq 2$, where we can see how the results approach natural inflation when $\frac{m_u}{m_d}$ grows, but are not as good as the normal inflation. The other group is when $1 \leq \frac{m_u}{m_d} < 2$, where we find that the closer $\frac{m_u}{m_d}$ is to 1, the more similar to the natural potential, and in the case of $\frac{m_u}{m_d} = 1$, the axion potential and the NI have the same result. But there is a problem with this second group, we can't find lower values of ns and r due to the fact that we can't reach the lower limit of $N \geq 60$. Then unless it can be as good result as normal inflation for big values, it isn't a useful model.

Finally, we find that the axion potential fits observations within 2 sigma when $\frac{m_u}{m_d} < 4$, because some part of the line is inside the region of the experimental observations.

IV. CONCLUSIONS

- We have built a general model of inflation for 2 light quarks, analyse how its behaviour is for different $\frac{m_u}{m_d}$ and compare with the experimental observations of the CMB.
- We see that $\frac{m_u}{m_d} > 4$ is needed to have a result that fits the experimental observations range.
- The presence of a second light quark does not help in improving the fit to experimental observations.

Acknowledgments

I would like to thank to my advisor for his guidance and all the help he gave me answering all my questions and solving my problems. I would also like to thank to my family and friends for all the support they gave me.

[1] A. Riotto, "Inflation and the Theory of Cosmological Perturbations", Sezione di Padova, 2002.

[2] Plank Collaboration "Plank 2018 results. X. Constraints on inflation", 2018

[3] F. C. Adams, J. Richard, K. Freese, J. A. Frieman and A. V. Olinto, "Natural inflation: Partil

[4] G. Grilli, E. Hardy, J. Pardo and G. Villadoro, "The QCD axion, precisely", Trieste, 2016.

[5] R. Z. Ferreira, A. Notari, G. Simeon, "Natural Inflation with a periodic non-minimal coupling", Universitat de Barcelona, 2018