Kuhnian Paradigms and the Birth of Quantum Physics: Planck, 1894-1912

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Abstract: A heated controversy about the historical track of quantum discontinuity in Planck's work arose by the end of 1970s. Thomas Kuhn argued in *Black-Body Theory and the Quantum Discontinuity, 1894-1912* (1978) that it did not play a key role in Planck's theory of black-body radiation through 1894-1910 and, indeed, that he did not explicitly, consistently introduced it. Its strong points and overall *internalist* limitations, and a separate historical approach, are discussed.

I. INTRODUCTION

Modern physics is a mere one-century-old field of investigation. Therefore, it is easily understandable the sensitive aspects of different perspectives looking back at its quite young origins, referred to Planck's law of spectral density of electromagnetic radiation in thermal equilibrium and his black-body theory attached to it. A common feature one is expected to find in those perspectives is steering clear of the easily-set idea that all previous theoretical developments had inexorably to converge, and effectively converged, in the current theories and ideas. For it is just far more simple to rebuild previous works from the present vantage point erected upon them than to try to develop the twists and turns that mediate between those past, specific objective and subjective conditions to the contemporary physical models. That is precisely one of the main goals of Thomas Kuhn's *heterodox* view of scientific knowledge's evolution ([1], p.362-4), summed up in *Structure of Scientific Revolutions* [2].

The present paper expects to clarify the up-to-a-part innovative arguments set out by Kuhn in his *Black-Body Theory and the Quantum Discontinuity, 1894-1912* (1978) about the origins of Quantum Physics and Planck's role in them. Some summaries and discussions existing about this issue are also taken into consideration ([3]; [4]; [5]; [6]). Moreover, the paper strives to widen the analysis by focusing in exploring whether Kuhn preserves and pursues or rules out the above remarked linchpin of his *historiographical* approach, and why.

However, owing to reality's complexity, the breach existing between the past physical notions and actual ones cannot be filled out, on the grounds of a seemingly way around, with a resort to individual originalities or brilliant, glimmer-like ideas. Taking into account both methodological cautions, a *historical* perspective ought to deal with different factors beyond internal vicissitudes of theoretical, physical models and scientific community. That is a second important aspect explored in this paper.

To do so, in parallel with the study of Kuhn's approach, it will be outlined the *necessity*, i. e., new pending areas in investigation produced by different factors whose working out goes under duress, and *possibility*, i. e., conditions to accomplish that and for the way in which is carried out, both in fields of scientific theory and society, that lead to the first nascent steps of Quantum Physics.

II. KUHNIAN PARADIGMS

If one had to condense Kuhn's theory of scientific revolutions in just a brief sentence, it could be the emplacement of subjective movement inside scientific activity itself and the holistic environment and nature of that movement. That is, for Kuhn, scientific theories have also a wide variety of conditional relations between them, and their emergence and consolidation cannot be merely explained by their "more accurate" closeness to external reality. Kuhn states that scientific knowledge does not follow a cumulative, linear development process of expansion ([2], p.1-3); this feature is only appropriate for certain periods of scientific activity corresponding to *normal science*. These periodes are set forth by Kuhn as being wrapped by "significant facts", the elucidation of its correspondance with theory and the own "articulation of theory" ([2], p.33). In fact, these general activities require and give birth to a set of commonly shared theoretical principles, experimental framework and methodological norms, conforming a "firm research consensus" in a "strong network of commitments" ([2], p.15, 42). In other words, normal science periods are underpinned by *paradigms*, i. e., "incommensurable ways of seeing the world and of practicing science in it" ([2], p.4).

Then, a *revolution* occurs whenever, for any reason, a paradigm falls into a crisis not possible to overcome by the framework displayed by this prevailing paradigm, and a new one emerges fighting the precedent one. More precisely, a *revolutionary period* in science comes to be a transition up to a new paradigm, characterized by holistic implications on all the previous scientific consensus, special changes in language and alterations in classifying criterions ([7], p.86-9). That kind of periods also arises philosophical, world-view-related questions ([2], p.87-90). To put it another way, Kuhnian theory conceives scientific revolutions as an own attribute of science, an intrinsic necessity of its development linked with the transformation of the existing physical world-view and its practical compromises (paradigm). Science is a zigzagging activity with crucial moments of jumps and rupture.

What is at stake is how Kuhn relates his study of Planck's law and theory to that general, synthesized outlook on scientific revolutions, albeit he openly admits, quite shockingly, that he tries "not to think in these terms when I do history" ([1], p.363); hence a separate wider, historical insight will also be developed.

III. PLANCK, 1894-1901

The modern, quantitative study of heat and radiation in macroscopic physical processes had its starting point when Gustav Kirchoff, in several articles published in 1859-61, after working closely with Robert Bunsen in Heildeberg, generalized that the ratio of energetic emission rate of a body and its absorption coefficient, dimensionless, is an universal function, for all bodies, depending solely on temperature and wavelength ([8], p.27):

$$e_\lambda/a_\lambda = K_\lambda(T) \tag{1}$$

The perfect, ideal body that absorbs all radiation incident upon it, $a_\lambda = 1$, was called by Kirchoff himself a black body (*Körper vollkommen schwarze*). In 1884, Ludwig Boltzmann succeeded at derivating the so-called Stephan-Boltzmann law for the total radiative intensity per unit area, $K = \sigma T^4$, from the general principles of Thermodynamics ([8], p.33). Another German physicist, Wilhelm Wien, obtained in 1893, using that law, the spectral density of electromagnetic energy in a cavity at thermal equilibrium as a function of only one variable, $u_\lambda = \nu^{-3} f(\nu/T)$, which naturally leads to the displacement law, $\lambda_{max} = a/T$ ([8], p.126-7).

This internal development of Thermodynamics, applied to radiation theory, suggested the theoretical necessity for finding the expression of that black-body universal function postulated by Kirchoff. Actually, Planck himself could extract decisive elements from the most advanced scientific theories of that moment: from Thermodynamics, the principle of the increase of entropy and its irreversibility, obtained in his doctoral thesis in 1879; from kinetic theory of gases, a solid conviction that "it cannot be expected to contribute to further progress with this problem" and "will have to be abandoned in favor of the assumption of continuous matter" ([1], p.22-3); and from Electromagnetism, an earnest optimism about dealing with microscopic irreversibility. That is, as Kuhn summs up, when initially facing the black-body problem Planck expected to "retrive irreversibility from continuum mechanics" ([1], p.27). For Planck, the route to thermal equilibrium for an electromagnetic radiation system could be, despite all mechanic simetries, an absolute irreversible process and purely mechanical problem.

These emerging, new fields of theoretical physics, interwined with the mighty Thermodynamics systematically translated into general equations by Rudolf Clausius (*Mechanische Wärmetheorie*, 1864), developed altogether with sweeping changes in economics and politics in Germany and in international order. Some of them are of special interest for our purposes here and are completely left aside by Kuhn's viewpoint. On the one hand, initially stimulated by foreign capital and international markets, German heavy industry of coal and iron and huge plans for railways were boosted since the mid-1830s, becoming rapidly the driving forces of German modernization ([9], p.126). Due to the deeply agrarian economy in Central Europe, that jump forward could only be conducted by a massive and irreversible aperture to technology (for which exists an specific German word, *Herrvortreten der Technik* [10]). For the corresponding know-to-how and other reasons, after 1848 the interest in science among the German intellectuality increased sharply and education was consciously shaped to be up-to-date with engineering necessities and scientific knowledge in general ([9], p.131). The increasingly tight knit between economy, technology and science back then in Germany meant that nearly 75% of state funds for academic research between 1871-1914 went to industrial and military oriented research [11]. Among the main educational institutions of that period stood out the *Realgymnasien* since 1848, for basic education, and the highly professionalized *Technische Hochschulen* (TH) since mid-1880s. In fact, by *Belle Epoque* times German scientific intellectuality had "a range of well-adapted educational sites and research establishments" that resulted in a "cohesive national technical, industrial system"[12].

If this brief panoramic view serves as a background for the clamoroulsy German-based advances and theories in Thermodynamics, first, and Quantum Physics, later, the internal difficulties of the steadfast ascent of German industrial class can shed light on why radiation problems became a fulcrum for Quantum Theory. So for the heavy industry what really plays a crucial role are the big-scale processes, both for extracting materials from mines and assembling huge numbers of workers in cooperative production. Obviously it was prone to fuel those general, macroscopic abstractions underlying Thermodynamics, first; after and secondly, to draw German scientific intellectuals' attention to fill out preciseness-related, microscopic problems appeared as the industry diversified, due to the "decline of precision engineering" in the first decades of German industrialization [11]. If to this seemingly *national urgence* one considers the *boom* of electricity at the end of 19th century, the apparently pure-intellectual, personal obstinate interest of Planck in black-body radiation theory by mid-1890s makes quite more sense. Here, then, we can roughly locate the social, historical necessity for being solved that problem.

It is also easier to understand his approach to this question if one takes into consideration another derivative of that epoch. All these lines of accelerated economic modernization were welded into archaic political structures, based on the Prussian Hohenzollern monarchy and the eastern, traditional landlords' (*Junkers*) agrarian nucleus of power ([9], p.128). Instead of being a brake on the industrialization, those political relations adapted to and boosted them, by promoting the Prussian customs union (1818), the German customs union known as *Zollverein* (1834), the German national political union (1866), and its constitution as an Empire (*Reich*) in 1871 and as an international power, rubbing shoulders with and confronting UK, Italy and France, then on-the-wane old powers. Finally, one cannot shun the vigorous German intellectual tradition condensed in the classical German philosophy (Kant, Fichte, Schelling, Hegel, etc.) and Romantic literature (Schiller, Goethe, Schlegel, Hölderlin, etc.), both usually seen as the culmination of Western universalist intellectualism. Then, that unstoppable looking-forward economic process, easily overcoming apparently contradictory old forms of political power, and this universalist, rich philosophical inheritance and theoretical tradition, can help to explain the usually unexplained Planck's adherence to, what himself called, the search for "the absolute, the universally valid, the invariant" ([13], p.47). Besides, Planck's gathering of pivotal elements from classical theories for a radical new attempt seems to be entirely connected with that environment of upturned-to-science, industrial-entrenched German intellectuality with roots in *universalist* thought amid a thorougly *contradictory* modern development. These different factors can help to explain how Planck undertook the black-body related, unresolved questions at the end of 19th century, i. e., the socio-historical possibility to do so, as long as Kuhn does not tackle it.

More specifically, Planck's initial model assumed an empty cavity with perfect reflective walls, full with an arbitrary all-spectrum electromagnetic field (*primary field*) that would interact with one-dimensional linear, harmonic electric resonators, taken as field variables able to re-emit certain radiation as *secondary field*. That interaction, Planck thought, would lead irreversibly to a situation of equilibrium with a certain redistribution of radiative energy among different wavelengths. In his first two papers dealing with it in 1896-97, Planck wrote the equations governing a conservative damped resonator:

$$kf + \frac{2k}{3c^3L}\dot{f} + L\ddot{f} = E \tag{2}$$

$$U_o = \frac{k}{2}f^2 + \frac{L}{2}\dot{f}^2 \tag{3}$$

where f is its dipolar momentum, U_o its energy, E is the total neat field it receives, and k, L are constants. In a five-installments serie of articles published between 1897-99, later compiled as *Über irreversible Strahlungsvorgänge*, Planck stepped forward to consider equilibrium conditions. However, he faced soon a key setback: resonators' thermalizing effect was not compatible with all posible initial conditions. The sinusoidal amplitude coefficients of their energy had to be reduced to "slowly varying average values" in order to allow to compute the time average for eq. 2 ([1], p.82). It forced Planck to restrict resonators' interaction with primary field to those frequencies equal to theirs, condition known as *natural radiation*. It abstracly reminds of Boltzmann's *molecular disorder*, described in 1895 as a specific condition upon the average free path of molecules essential when obtaining the collision rate for a gas as the grounding for the famous H-theorem (1872).

Armed with that statistical condition, Planck could come to an end for his 1894-99 program by, first, obtaining a quantitative relation between spectral density of radiative energy in the cavity, u_ν, and the average energy of a resonator, U_ν; secondly, by maximizing up to equilibrium an entropic function depending on U_ν picked up by him, so $\partial S/\partial U = 1/T$ drove directly to the distribution for U_ν, with c being the speed of light:

$$u_\nu = \frac{8\pi\nu^2}{c^3}U_\nu \tag{4}$$

$$S = -\frac{U_\nu}{a\nu}log\left(\frac{U_\nu}{eb\nu}\right) \quad \Rightarrow \quad U_\nu = b\nu e^{-a\nu/T} \tag{5}$$

where a and b are universal constants. Planck's result in 1899, eq. 5, was Wien's distribution law precariously formulated in 1896 but proved correct in infrarred measurements (0.7-6 μm) carried out until then at the *Physikalisch-Technische Reichanstalt* (PTR, Imperial Institute) in Berlin, by Otto Lummer and Alfred Pringsheim ([14], p.149). If TH were the "linchpin of German educational service to industry"[12], PTR was its crowning, national-imperial project, being defended since 1872 by Hermann Helmholtz and Werner Siemens, among others, and finally set up in 1887 [11]. Photometry, arouse by government-endorsed requests by industrial big firms ([8], p.125), played an effective part in the development of modern radiation theory with inmediate checkings of the different distribution laws suggested in 1895-1900. It can be identified as another derivative of social-economic, technical possibility for those theoretical developments.

In march 1900 Planck polished his selection of the entropic function, eq. 5, but already in november 1899 a "systematic pattern of deviations" was obtained in PTR when measuring spectral intensities up to 8.4 μm ([1], p.94). Planck rapidly modified, somehow *ad hoc*, his criterion for picking up the entropic function and presented before the German Physical Society in 19 October 1900 a distribution law now known as Planck's law, whith h and k as universal constants related to a and b in eq. 5:

$$U_\nu = \frac{h\nu}{e^{h\nu/kT} - 1} \tag{6}$$

A new, thorough derivation was presented in 14 December 1900, and a drastically changed one with combinatorial arguments was published in 7 January 1901, in his currently most known paper.

IV. PLANCK, 1901-1912

Though being extremely summarized, this was Planck's route to his formula for black-body's energy spectral distribution, quickly checked by Heinrich Rubens at PTR for long wavelengths where Wien's one failed ([14], p.154). But here we must halt in order to appreciate several important aspects of his route. Not only the formal similarity between Boltzmannian *molecular disorder* and Planckian *natural radiation* suggests certain parallelism. Also Planck's purpose until the end of 1900 of explaining equilibrium from strictly mechanical principles, despite not entirely accomplished, and his proposal to maximize a self-defined logarithmical entropic function conveys an analogy with Boltzmann's work. In fact, Planck's 1895-1900 articles seemed to seek an H-electromagnetic theorem, as Kuhn points out in chapter III, as long as Boltzmann himself had originally pursued a mechanical-deterministic approach for his kinetic theory [15]. As chief editor of Kirchoff's posthumous *Lectures* in 1894, Planck had had an open discussion with Boltzmann concerning the Kirchoffian rudimentary statistical foundations of his own kinetic theory ([1], p.61-5).

On these grounds, Kuhn asserts that Planck, after 1896, had a "detailed acquaintance with Boltzmann's statistical theories" and pushes it further to state that "Planck was following Boltzmann extremely closely" ([1], p.106, 270). Planck himself, in his *Autobiography*, maintains that in 1900 he *explicitly* sought "to pursue the line of thought inaugurated by Boltzmann" ([13], p.41). It is reasonable to understand that, somehow, but effectively, Boltzmann's H-based model was the fruitfullest theoretical possibility Planck found in contemporary science, interlaced with the essential principles of Thermodynamics and Electromagnetism. Therefore, Kuhn argues that, like in the case of Boltzmannian molecules with continuous energy albeit its heuristic restriction to elements ϵ, "the concept of restricted resonator energy played no role in [Planck's] thought" in 1894-1901: it is his central thesis, described as an "historiographic heresy" ([1], p.126).

If we draw our attention to Planck's derivations in 1899-1901, we find that in *all* of them, except the last one in 1901, his methodology bore on maximizing a function of total resonators' entropy at different frequencies. In this case, the restriction $\epsilon = h\nu$ came to be an *a priori* hypothesis used to get the expression $S(\nu)$ and Wien's distribution law. On the contrary, in 1901, Planck supposed thermal equilibrium as given to select one frequency ν resonators, and imposed on the resultant distribution function U_ν Wien's displacement law. Here, restriction $\epsilon = h\nu$ appeared as a result. With this unsteady place of discontinuity in Planck's theory, secondary to the self-sufficient criterion of entropic irreversibility, which was the true physical meaning given by him to $h\nu$?

In his famous 1901 article, that apparent energetic discreteness was not in Planck's crosshairs, as he focused on the close relation between h and the ideal gases constant appearing in spectral density law. Furthermore, in his 1906 canonical *Lectures*, Planck explicitly located resonators uniformly distributed in elliptical regions of a phase space for each frequency, with constant areas of value equal to h delimited by rings of energy equal to $nh\nu$. Then, resonators were in regions of continuous energy limited by definite values and only their *average* energy in nth-region is quantized ([16], p.153):

$$\int\int dq dp = h \quad ; \quad U_n \propto \left(n - \frac{1}{2}\right)h\nu \tag{7}$$

For Kuhn this is an "essential clarification" ([1], p.128). Only after 1909-10 Planck would explicitly admit a clear quantization in his so-called *second theory* (*ca.* 1911-13), emplacing it in the emitting process for resonators but not in absorption. Even then he was extremely reluctant to it, acknowledging in 1910 that "I have located the discontinuity at the place where it can do the least harm" and in 1915 that "I hate discontinuity of energy even more than discontinuity of emission" ([1], p.236-53).

Consequently, Kuhn's main arguments for his daring thesis -no discontinuity existing in Planck's theory between 1894-1909- are [5]: a) Planck adopted Boltzmann's mechanical-statistical fundamental ideas and methods, extrapolating them into his radiation theory; b) then, in his combinatorial proof of 1901, he did not confer physical reality on his formal result of energy elements $\epsilon = h\nu$, as had not done so Boltzmann; c) during 1901-09, Planck retained the continuum-oriented conception of his theory, so that his *Lectures* (1906) ensured and systematized it. His *second theory* was actually a "radical step" forward for him, contrary to the extended idea of a conservative "retreat" from his *advanced* positions in 1901 ([1], p.244).

On the opposite historiographical pole, Klein puts a stress on dangerous Kuhn's obstinacy in establishing "the internal consistency of Planck's position"[3]. Klein argues that Planck was not really moving with ease across Boltzmann's deeply complex theoretical principles and could not rigurously follow him as Kuhn deffended, so that Planck *really* introduced energetic discontinuity for resonators' energy in 1901, though "he was not aware of it at that time"[3]. While Kuhn passes physical quantum discontinuity introduction to Paul Ehrenfest and Albert Einstein in 1906, for they found the missing link in Planck's criterion for entropic function (Ehrenfest) or elaborated a general model where discontinuity was forcefully necessary (Einstein), Klein states that they and others got already inspired by Planck's theory of 1901.

V. CONCLUSION

- On the one hand, differing from Klein and Shimony [3], we do think there is a common background for *Blackbody radiation* and Kuhn's theory of paradigms and scientific revolutions. The whole questions falls on which sense or mode does it exist. For instance, Kuhn prolifically explains (chapters VIII-IX) how a holistic change occured in physics since 1906-07, concerning different fields as distant as chemical physics. Moreover, it was precisely from this area that a vigorous impulse to Quantum Physics was given by Einstein and Walther Nernst, when studying low-temperature behaviour of specific heats. Noticeable changes in language also occured, for which Planck exchanged "resonators" for "oscilators" and "energy element" for "energy quanta" since 1910 ([1], p.201). Furthermore, original Planck's continuum-based theory was suppressed of public scientific knowledge once new Quantum Mechanics flared and a new *paradigm* had definetely to be established and reinforced.
- On the other hand, Kuhn offers a convincing body of arguments for Planck's originary thought not managing discontinuity of energy. Nevertheless, History is not tricky and Planck's 1894-1901 work cannot be isolated from scientific outbreak of interest in radiation theory and quantum models in 20th century dawn. But, concerning the other end of the historiographical spectrum, it does not mean that quantum principles were neat but hidden, born but unknown in Planck's law in 1901. In fact, continuum Mechanics and Quantum were not on a quandary by then nor their controversy was an open issue to be resoundingly solved [4]. Both Kuhn and Klein reduce the whole complex historical matter of Quantum' birth to a mere chronological, theoretical question.
- Besides, the extent up to which Planck *really* followed Boltzmann is relatively less important than at first glance: the historical crucial fact is that Boltzmann's de-

terminist background could be overcome only because Planck explored and exhausted all possibilities in it referred to radiation theory. But Kuhn, Klein and others tend to evaluate Boltzmann's influence on Planck as an expensive price he finally had to pay for, whereas historically it mediated in pursuing new physical principles.

- Finally, contrary to Kuhn's obstinacy in determining Planck's thought's total coherence or Klein's twisted remark about *unconscious but real* quantum formulation by Planck, we think contradictions also exist in, and actually govern, scientific world-view and theories, specially in revolutionary periods, like the decades 1900-1910. Reluctance to assimilate contradiction as an objective, leading factor -historically determined, socially conditioned and reflexively examined- that fuels subjective movement is latent in Kuhn's view. Theoretical justification for his aversion towards contradiction can be easily found through his work: Kuhn understands "scientific revolution" as "a revolution of *ideas*", circumscribed to "episodes in which a scientific community abandons one time-honored way of regarding the world" ([17], p.41-226), i. e., presupposing the social soil of a scientific community where *revolutions of ideas* can occur *preserving* the community itself. Then, instead of attaining its relations with society, Kuhn must arbitrarily direct to its internal, psychological nuances, easily overlooking or, at most, attenuating its contradictions. His adscription to neo-Kantian philosophy is eloquent enough ([1], p.361).

Therefore, while we do agree with Pinch's adjectivization of Kuhn as "entrenched firmly within the internalist camp"[3], we cannot agree with his description of *Black-Body Theory* as "the final stage of a process of retraction" by Kuhn. Because this book concretely reproduces and openly exposes the abstract limitations existing across his *Structure*, briefly mentioned above. Pitfalls encountered in Planck's scientific thought have been confronted with an exhausting exegetic procedure by Kuhn. On the contrary, the contradictons through which navigated Planck and the ambiguities with which he rowed can be naturalized when connected to the social-economic storm Germany was passing then; besides, we think those were solved by the conjunction of those conditions that matched prevailing necessities, both socio-economic and theoretical. In fact, one can think of those Planck's contradictions about *irreversibility for classical mechanics* as a particular expression, subjectivized, of those material contradictions underlying German modernization -combining imposing new industrial forces with old, surviving political institutions. From this perspective, one final aspect of Quantum emergence seems evident. As Newtonian mechanics in 17th century shaken Britain (which began to stand out as *the workshop of the world* for its colonial power and manufacturing capacity [18]), and French Revolution's impulse to mathematization of up-to-then experimental fields ([17], p.61), Quantum Physics arose in the country firmly directing outstanding social-economic transformations in late 19th century, Germany. In other words, a general pattern Quantum Physics' birth seems to prove is the tight dependence of science on production and political context, and vice versa; even a more specific dependence, on the deciding episodes in seizing power and ascent development by the industrial, bourgeois class, can be appreciated.

However, Kuhn dismisses an approach of that kind by saying that its "main drawbacks have always been that it attempts to explain too much" ([17], p.59). In that case, it is maybe this *historian of ideas*' own paradigm what must also be overthrown and revolutionarized.

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