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Case Study on Mathematics Pre-service Teachers' Difficulties in Problem Posing

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ABSTRACT
This research is presented in a way that provides useful knowledge for successful problem posing by mathematics pre-service teachers. We present a review of the concept of mathematical creativity (by different authors) and review studies that underline the relevance of problem posing in teaching mathematics, studies that consider problem posing a way to identify students' learning patterns and to test them, and studies that relate mathematical competences to problem posing. Participants in the study were 10 pre-service teachers who were successful in problem solving. Data were gathered through qualitative techniques: classroom observations, sequences of tasks, questionnaires, student focus groups and discussion. The case study illustrated some of pre-service teachers' difficulties in problem posing: creating problems that students recognize as relevant to their everyday lives, problems adapted to the school curriculum at a specific educational level, and problems that can be self-corrected.

Keywords: creativity, pre-service teachers, problem posing, problem solving, competences

INTRODUCTION

This research is presented in a way that we think provides useful knowledge for successful problem posing by mathematics pre-service teachers. It is divided into four parts. In the first part, we present the state of the literature and the research questions. We present the theoretical framework, including a review of the concept of mathematics creativity, and reviews of studies that relate problem posing with problem solving for educational purposes, studies that consider problem posing a way of looking for learning patterns and testing, and studies that relate mathematical competences to problem posing. In the second part, we describe the research design, the objectives, the instruments and the methodological strategies. The aim of this research was to identify aspects that could be improved in problem posing by pre-service teachers who had achieved a suitable academic level in problem solving. In the third part, we present the general results obtained from an analysis of the case study and from classification according to the six research goals. In the fourth part, we present conclusions about academic, social and personal factors that appear to be relevant and need to be improved as they have a negative impact on problem posing, and we present the potential and limitations of this kind of these studies.

THEORETICAL FRAMEWORK

Pásztor, Molnár and Csapó (2015) claimed that creativity plays a crucial role because of its connection with other processes such as problem solving and problem posing. For instance, in a study presented by Bahar and June (2015), creativity and verbal skills were the only variables that contributed to explaining the variation in performance on open-ended problems versus performance on closed problems. Although many authors have tried to define mathematics creativity, there is no single accepted definition of this concept (Mann, 2006). Instead, definitions can be grouped into the following categories: a) those focused on final output; and b) those focused on the process (James, Lederman-Gerard & Vagt-Traore, 2004). Mathematics creativity has been described in several ways in the first category, for example, as the capacity to create an unexpected procedure (Sternberg & Lubart,

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Contribution of this paper to the literature

- According to some authors, problem posing can greatly help teachers to teach their students multiple competences. So, why don't teachers use this strategy more? What difficulties does this strategy involve?
- This research concludes that pre-service teachers find it difficult to create problems that students will recognize as relevant to the reality of their everyday lives, problems that are adapted to the school curriculum at a specific educational level, and problems that can be self-corrected.

2000), exceptional capacity to create useful and original solutions to a problem (Chamberlin, 2005), and in terms of the originality of the output (Sriraman, 2009). In the second category, (Mallart & Deulofeu, 2017) described mathematical creativity as the skill of thinking by drawing on concepts, which involves seven components: originality, flexibility, elaboration, analysis, synthesis, communication and re-definition. Studies on the development of mathematical creativity through a sequence of tasks suggest a positive effect on students' learning (Mann, 2006). Sriraman (2005) claimed that encouraging creativity is not a strategy that teachers commonly use. In light of the research described above, however, we argue that creativity is important in teacher training programmes, and problem posing is a suitable framework to stimulate it.

Many authors have highlighted the importance of using problem posing activities to learn problem solving. In the "understanding a problem" step, the teacher may create another problem that students recognize as being more relevant to the reality of their everyday lives. The "looking back" step (Polya, 1979) also includes re-formulating the original problem. The IDEAL method used by Bransford and Stein (1986) shares the same approach. Murray, Olivier and Human (1998) claimed that students who create their own problems have a greater ability to use appropriate concepts and skills to solve them. Walter and Brown (1977) investigated the connection between problem posing and solving. They found that performance in one activity had an impact on the other, and vice versa. Silver and Cai (1996) also examined the relationship between problem posing and problem solving. They concluded that students' ability to solve problems correlated closely with their ability to pose problems. According to Silver and Cai (1996), problem posing involves formulating new problems and re-formulating given situations. Students can create new problems when they solve a complex problem. They may re-formulate it to reduce the size of the numbers involved, or investigate a specific case of the given situation to better understand the problem (Silver, 1994). This type of activity involving the creation of new problems could be carried out after solving the initial problem. Students could be asked to change the objective, condition or question of the problem, to create new formulations. Leung and Silver (1997) stressed that problem posing helped to improve students' problem solving skills and had a positive impact on solving strategies. English (1997) also addressed problem posing strategies. She claimed that students need knowledge that allows them to deal with the cognitive tasks involved in problem posing. She stated that students must understand what a problem is, recognize its structure, and identify similar structures to be able to create a new problem. She considered that we need to investigate how students understand different types of problem, as well as what type of standard and non-standard strategies they use to solve them. She also indicated that students' ability to understand a mathematical situation from many perspectives may improve their ability to create new problems. Finally, she stated that students' ability to create and solve new problems allows them to discover their own constraints, check their own mistakes, and know their own strategies and difficulties, which will turn them into better problem solvers. Kesan, Kaya and Güvercin (2010) analysed the effects of problem posing within the development of mathematical skills. They concluded that there were significant differences between a group of students who worked on problem posing and another group of students who received regular instruction. In the experimental group, problem posing activities were found to be effective at improving mathematics performance, particularly in terms of solving non-routine problems. Ellerton (1986) showed that problems created by students who had greater mathematical skills required more complex calculations, presented a larger number of operations, involved a more complex number system, and used mathematical language more fluently. Ellerton (1986) also proposed that students with greater mathematical skills created more coherent and consistent formulations. In the same vein, Espinoza (2011) stated that problems posed by gifted students were richer than those created by their peers in a regular classroom in terms of the length of the formulation, the type of question, and the number and quantity of embedded processes and steps required to solve the problem. In addition, Espinoza (2011) found differences in the type of semantic structure and in the quantity of semantic relationships involved in the problem solution. He noted that reading a problem was not enough to immediately identify a procedure to solve it.

Problem posing has also been studied as a way to discover students' abilities and their understanding of the mathematical concepts that they have learnt (Ellerton, 1986; Pelczer & Gamboa, 2008). Problem posing gives students an opportunity to show their knowledge and what they can do with it; but it also allows teachers to observe patterns in their students' learning and to deepen understanding of students' mathematical thinking (Kwek & Lye, 2008). Silver and Cai (2005) considered that if teachers use problem posing as an activity in the process of teaching mathematics, then they could also use it to assess students and check students' understanding or performance. Lin (2004) pointed out that problem posing has an advantage over other types of activity as an instrument to assess students' learning: it is not isolated from the teaching and learning process, but embedded in it. Thus, teachers can use problem posing to grade students' learning of mathematical concepts, once these concepts have been introduced. In the same vein, problem posing could be used as a diagnostic assessment before introducing a new lesson, or as formative assessment for students to become aware of their own learning and focus on it (Espinoza, Lupiáñez & Segovia, 2014).

From the perspective of mathematics competences, posing mathematical problems for educational purposes is a great challenge to teaching competences (Tichá & Hošpesová, 2013; Milinkovié, 2015). Teachers may systematically introduce problem posing tasks to achieve different goals. Analysing students' understanding of the mathematical concepts embedded in the problems that are posed may be a way for teachers to determine students' understanding of concepts, their mathematical skills, creativity, in-depth understanding of concepts and their relations, patterns, and use of numbers and quantities. In fact, problem posing could be used to improve students' problem solving competences. Problem posing may be complementary to problem solving, because it adds greater mathematical representation to it. In addition, teachers may ensure that the mathematical contents are focused on situations that are familiar to students. However, problem posing activities are not common practice in mathematics classrooms. Instead, problem solving activities prevail. This may be because teachers lack confidence in using problem posing activities (Leung & Silver, 1997). We consider that a teacher must not only be able to solve mathematical problems, but also choose, modify and pose problems for educational purposes (Tichá & Hošpesová, 2013). Teachers must develop analysis and intervention skills to evaluate the problems they use (Breda, Pino-Fan & Font, 2017). These skills involve reflecting on the mathematical practice of problem solving and problem posing, and assessing problems based on certain educational criteria. In teacher training, this competence can be developed with tasks that involve managing the educational analysis. One of these tasks consists of posing problems and reflecting on them from a teaching perspective. We assume that problem posing is a process through which a new mathematical problem is obtained by varying a given problem or creating a new problem. This process is undertaken in response to a certain situation or a specific request of a mathematical or educational nature (Malaspina, Mallart & Font, 2015).

The research presented here is part of a group of studies about pre-service mathematics teachers. We were interested in future teachers who had a certain level of success in problem solving, selected after passing a course of mathematical reasoning. Our intention was to detect weak aspects in problem posing, which could then be improved through an intervention carried out at the university. This perspective was very interesting because it offered useful tools for educational interventions as preventive measures.

OBJECTIVES

The goal of this research was to identify which difficulties associated with the academic background of mathematics teachers could cause them to avoid problem posing. Participants were pre-service teachers who were successful at problem solving. The aim was to determine which aspects needed to be stressed to encourage teachers to use problem solving in class. The identification of difficulties would allow us to establish guidelines for action in university courses, which could then be used to enhance the education of pre-service mathematics teachers and mathematics teachers' refresher courses.

Considering these questions, the research considered had the following specific objectives:

a) To discover pre-service teachers' ideas about the meaning of posing a problem before they create their own problems.
b) To analyse what type of problems pre-service teachers pose when they are given certain guidelines, and to assess the extent to which they follow the guidelines.
c) To find out pre-service teachers' ideas about the meaning of posing a problem after they have created two problems of their own. This objective draws on objective (a) above.
d) To identify how to improve a problem, starting from a set of problems that have already been created, and to choose one that is considered original.
e) To identify pre-service teachers' difficulties in creating new problems (working with the problem selected in objective (d) above, in order to address the negative aspects that have already been identified, and enhance the processes.
f) To analyse in detail the case of the pre-service teacher who poses the most creative problem, according to the variables introduced by Author (2017).

CASE SELECTION

Considering that mathematics teachers are not self-confident enough to use problem posing in the classroom (Leung & Silver, 1997), in our study we selected future mathematics teachers before they had started working so that they were not influenced by their experience of a job (i.e. of a specific school and its class groups and rules). The inclusion criteria were pre-service teachers who were finishing their academic education as teachers of mathematics, who knew the curriculum, and who had passed a course on problem solving and mathematics reasoning. The selected students were from the final stretch of their university degree (they were researchers' students) and had not failed any mathematics subjects. Although we are aware that passing mathematics subjects does not imply a high level of mathematics competence in problem posing, for our research it could be used as a selection criterion. Moreover, considering that our goal was to identify relevant difficulties in problem posing in class, we had to choose an indicator linked to the success of problem solving, despite its limitations. Problem solving and problem posing are linked, as stated in the theoretical framework.

We focused on students of the Faculty of Education, University of Barcelona (Spain) in 2017. We selected students who had a certain level of academic success, determined by various indicators:

1. They had passed a course on problem solving and mathematical reasoning.
2. They had passed all the mathematics subjects before the last one.
3. They were enrolled on a Geometry course.
4. They knew the curriculum.

On the basis of these indicators, we obtained an initial list of 58 students. After an interview with each of them, the researchers (who were the teachers at the same time) selected the 10 most successful, representative, interesting students for the research. The selection was done by different researchers, so the perspectives were different (this speaks in favour of validity).

METHODOLOGY

The design used to carry out this research follows the same rules as a case study. In fact, as pre-service mathematics teachers play an active role in their learning, and as they are considered mediators between their own learning and their environment (Wang, Peverly & Catalano, 1987), we decided to draw up a case study. We collected qualitative data to analyse the subjects' perceptions about their learning environment and their own learning process. In case studies, a phenomenon is researched in a daily life context using multiple sources of evidence (Yin, 1994).

We were interested in finding out pre-service teachers' ideas about problem posing before they created any problems of their own. To achieve this first objective, we decided that the best instruments for gathering data were two open questionnaires (Tasks 1 and 2) as a starting point (see Task 1 in Appendix A, and Task 2 in Appendix B). Rather than use an objective measure of the variables, we wanted to determine the participants' feelings about problem posing. Our aim was to obtain relevant data about the meaning of problem posing for the participants. With these fourteen open questions, our interpretation of their thought was very close to their opinion (reliability in the context of qualitative research).

The second objective was to analyse what types of problem pre-service teachers posed when they were given certain guidelines, and the extent to which they followed the guidelines. We thought that the best instrument for gathering data for the second objective was an open questionnaire in which students could freely express their ideas about the proposed task, regardless of the set of general guidelines that were established. Thus, we introduced a third task (Task 3) for pre-service teachers, which involved creating and solving two geometry problems according to specific requirements:

> *"Pose and solve two problems in geometry that take into account the following: 1) 2D shapes, 2) measuring surfaces, 3) graphic representation and 4) everyday life. The problems should be aimed at upper primary school students."*

The third objective was to discover pre-service teachers' ideas about the meaning of posing a problem after they had created two problems of their own, according to specific guidelines (Task 3). We found that the best instrument for gathering data for the third objective was an open questionnaire (See Task 4 in Appendix C). This questionnaire highlighted aspects of the difficulties pre-service teachers had in posing problems, so that we could compare their ideas at this point with their initial thoughts. With these seven open questions, our interpretation of their thought was very close to their opinion (reliability in the context of qualitative research).

Tasks 3 and 4 were introduced after half a semester working on problem solving. Drawing on Abrantes' (1996) concept of a participatory classroom, we introduced several slightly altered problems, to spark discussion in the classroom among the pre-service teachers.

The fourth objective was to identify how pre-service teachers improve a problem. For this task, we chose an original problem out of the set of problems that had already been created. We considered that the most original problem was the one that used a shape (a hexagon) that did not appear in any other problem. The other shapes (triangles, quadrilaterals and circles) were included in more than one problem. Next, we presented the selected problem:

> *"In a school playground, we found a honeycomb. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape and calculate the area, if one side measures 4 cm."*

Before we gave this problem to the pre-service teachers, they read an article about the characteristics of a suitable problem, in educational terms (Mallart, Font & Malaspina, 2016). We discussed this article with the pre-service teachers, using their own problems (Task 3) as examples. The discussion was Task 5 (focus group).

In Task 6, we asked the pre-service teachers to point out three aspects of the original problem that could be improved, according to their own criteria. The best data collection instrument for the fourth objective was an open questionnaire that let participants freely give their suggestions for improvement.

The fifth objective was to identify pre-service teachers' difficulties in creating new problems (working with the selected problem in the fourth objective), so that we could then address these negative aspects, and enhance the richness of the processes. The best data collection instrument for the fifth objective was an open questionnaire. The pre-service teachers received the following instruction: *Change the problem to make its solution more complex, by involving more processes and concepts. Solve the variations of the problem as well* (Task 7).

The sixth objective was to analyse in detail the case of the pre-service teacher who posed the most creative problem, according to the variables introduced by (Mallart & Deulofeu, 2017). This objective was reached by analysing all the previous open questionnaires.

The four questionnaires (used for Tasks 1–4), complemented with classroom notes, some of them written during discussion of the article (Task 5), and combined with monitoring of the suggested problems presented as improvements of previous problems (Task 7) by the researchers (ourselves), allowed us to triangulate the information. As a result, we could build an overall, comprehensive image of each of the ten cases in relation to the aspects that we were studying (the detection of difficulties in problem posing).

The data analysis was carried out using content analysis techniques (Bardin, 1986). First, a report was prepared about each of the cases. Then, the results were assessed with respect to the objectives: what thoughts pre-service teachers had about problem posing before they were asked to create their own problems; what kind of problems students posed when they were given guidelines and the characteristics of these problems; what thoughts students had about problem posing after being asked to create two problems; what aspects they thought could be improved in a problem, and how they implemented changes.

Based on this assessment, we obtained evidence of relevant variables that could be improved in the use of problem posing for academic and educational interventions. Because of space constraints, all possible implications of the twenty-one open questions have not been studied.

PRESENTATION AND DISCUSSION OF THE RESULTS

A summary of the outcomes of the cases is given below. Due to their variety and extent, we have organized the answers depending on the objectives of the analysis, following the ideas of cross-case analysis. Unfortunately, we cannot include all the results obtained for each case, due to space limitations.

The Meaning of Posing Problems before Students Create their Own Problems

The analysis of Tasks 1 and 2 led us to conclude that pre-service teachers' initial ideas about the meaning and usefulness of problem posing, before they were asked to create any problems, could be grouped into two main categories. The first refers to assessment: pre-service teachers considered that posing problems would allow them to check whether their future students had acquired the required knowledge and knew how to use it. The second category relates to their future students' creativity. Below is an example that includes both categories:

> *Pre-service teacher 1: [Is it necessary to teach how to pose problems?] "Yes, because students can check whether they have learnt the lesson, they can understand the usefulness of the theory, and improve their creativity."*

Pre-service teachers mentioned "creativity" many times, more than other types of variable such as "motivational strategy". Given that references to creativity are not generally common among pre-service teachers, a reasonable explanation is that the questions led them to use this term.

Type of Problems Posed when Certain Guidelines are Given

In Task 3, the first notable result is that none of the pre-service teachers followed all the instructions for both problems (created by them) at the same time. For instance, in the example below, the pre-service teacher did not follow the instruction to "include more than one 2D shape":

> *Pre-service teacher 5: "Some kids were playing basketball. When they threw the ball, it got stuck in the hoop. So, the children will have to buy another smaller ball. However, first, they must figure out the area, if the hoop has a diameter of 50 cm. Find out the area of the hoop and then represent the equivalent shape".*

The most difficult instruction to follow was to pose a problem for upper primary education level. In fact, only one pre-service teacher achieved this for both problems. Out of a total of 20 problems, only three were suitable for upper primary education level, as requested in the task. Fifteen problems were much too easy (lower primary education level), and two were more suitable for secondary school. Below is an example of a problem corresponding to lower primary education level, and another aimed at a higher educational cycle.

> *Pre-service teacher 6: "Look! Imagine that you have moved from your house, so now you have another room. You want to buy a carpet that fully covers your new room. How much carpet do you need to buy?"*

> *Pre-service teacher 4: "We want to organize a 100-metre race in our neighbourhood and the only street long enough is the boulevard. We know that diagonal to diagonal measures 102 metres and the street's width is 9 metres. How long is the street?"*

When we say that a problem is suitable for a specific educational level, what we mean is that the normal solution involves the use of content from this level. However, we can also use problems that are not suitable for primary school *a priori*, but aim to stimulate students' creativity. In other words, one way to encourage children's creativity is to ask them to solve problems that are usually aimed at higher levels of education, because they may be able to solve them if they are creative enough. However, this is not an appropriate way to encourage creativity. Creativity can be encouraged by drawing on the right curricular level for each child or, more generally, through the instructions provided within the task.

The Meaning of Posing Problems after Students Created Two Problems

The third objective focused on understanding pre-service teachers' ideas about the meaning of problem posing after they had performed this action in Task 3. These ideas were then compared with pre-service teachers' initial feelings on the subject (Objective 1). We also aimed to assess whether the subjects were sufficiently aware of their own degree of understanding to create problems of educational interest, and aware of the advantages of posing problems for teaching. To examine these aspects, we analysed the answers to Task 4.

Pre-service teachers found it hard to adapt the difficulty of a problem to their future students (most pre-service teachers mentioned this point). In addition, it was difficult for them to link the formulation with an everyday context, to create a non-ambiguous formulation with non-redundant information or no missing information, to connect data and to motivate the students.

In general terms, the subjects claimed that they were not ready to create new problems (they considered that they had insufficient mathematical knowledge and they did not feel motivated to create new problems). They stated that they had been taught to solve problems, but not to pose them. They also said that they did not have the educational tools required for the task.

Factors that they considered crucial to posing problems included "being creative" and "being motivated". They claimed that their main weakness was a lack of mathematical training. They also highlighted a lack of teaching knowledge, a shortage of strategies to teach how to create new problems, and inability to see mathematics in everyday situations.

Some of the pre-service teachers believed they could learn how to create new problems in mathematics through courses on teaching resources. They considered that mathematical content should be provided and pre-service teachers supported, to help them to observe their environment in mathematical terms. Some also believed that starting from problem solving (or re-formulating existing problems), they could be taught how to create problems.

The pre-service teachers also thought that knowing how to create new problems could be a teaching strategy. They considered that this tool could increase their understanding, based on the needs of students. They also thought that knowing how to create new problems increased their knowledge of mathematics and was good for practice.

They claimed that working on closed problems and not analysing problems from different perspectives prevented them from creating new problems. Likewise, if they did not have a lecturer listening to them and supporting them, it was hard for them to solve problems using different approaches. They wanted to study geometry in greater depth, and also focus more on studying the use of different methods and strategies to solve problems. They appreciated the opportunity to connect everyday life situations to mathematics. They also claimed that they had never created new problems before; they were used to solving problems, and therefore did not have skills to create new ones.

According to the pre-service teachers involved in this study, a mathematical problem is interesting (in educational terms) when its difficulty matches the student's ability. The problem must be recognized by the students as being relevant to the reality of their everyday lives, with some kind of applicability. It must have a coherent formulation and there must be various ways in which it can be solved.

The analysis of data on Objective 3 concludes with a comparison of these results with those obtained in Objective 1. Before they had created new problems, participants thought that posing problems was a tool to test future students' knowledge of mathematics. After they had created their own problems, they argued that problem posing was a tool to introduce good problems to future students, thus helping students to understand a problem. The pre-service teachers also thought that creating new problems helped to increase their mathematical knowledge as well as its use.

Ways of Improving a Problem to Take the Maximum Advantage of it Educationally

The results of Task 6 focus on the main aspects that could be improved in the most original problem. The selected problem was: "*In a school playground, we found a honeycomb. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape and calculate the area, when one side measures 4 cm.*" Pre-service teachers considered that the problem would be improved (from a teaching perspective) if its solution involved more processes and a wider range of answers, and if it could not be solved using a single formula. This conviction is illustrated by the quote below:

> *Pre-service teacher 2: "It does not allow other answers, because we need to use a specific formula to calculate the area".*

They also believed that it would be a better problem if it prompted students to ask good questions or if it helped in the creation of a new problem, as the quote below shows:

> *Pre-service teacher 1: "It* [the problem] *does not prompt you to ask easy questions to reach the solution; you just need to use the formula."*

Pre-service teachers thought that the problem was unfamiliar to students (students would not be able to measure a real honeycomb) and that there was a lack of resources to work with it (or the resources would be very dangerous if they were available). For instance:

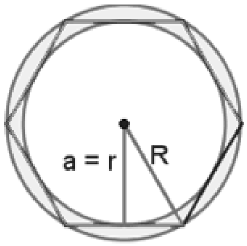
> *Pre-service teacher 4: "I think that in order to solve this problem we would need resources that are not available or would be very dangerous to work with."*

> *Pre-service teacher 2: "It does not reflect a real situation, because not everyone would have a real opportunity to measure a honeycomb."*

Difficulties in Creating Problems after Correcting the Aspects the Respondents had Chosen as Being Possible to Improve

We analysed Task 7 to attain the fifth objective and to find out the difficulties pre-service teachers experienced in creating new problems. The pre-service teachers detected negative aspects of the problem defined as the "most original". We then analysed their ability to modify this problem to address all the negative aspects that had been mentioned. We grouped the difficulties into four blocks.

The honeycomb in the playground is the shape of a regular hexagon whose sides measure 4 cm each. We draw a circle within this hexagon, and then another one outside it. What is the area of the circular ring that we have created?
To understand the problem: a) the honeycomb is a regular hexagon whose sides measure 4 cm each; b) there is a circle drawn within the hexagon; c) there is another circle drawn outside the hexagon; d) calculate the area of the circular ring created.
Strategies to solve the problem (devise a plan): a) graphic representation; b) search for a formula.
Implementation of the strategy (carry out the plan): R = side = 4 cm



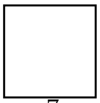
r^2 =apothem^2 =4^2-2^2 = 12;Area=PI· (4^2-12) = 4· PI=12.567 cm

[mistake: 12.567 cm^2]

R: radius of the outer circle
r: radius of the inner circle
a: apothem
Solution: the area of the circular ring created is 12.567 cm (Pre-service teacher 7)

Figure 1. Example of a version of the problem that has been changed to involve more processes, but with a solution that is too difficult for the educational level

If we have a square garden that measures 7 m per side, what is its area? If we have 3 roses in each squared metre, how many roses do we have in total?



area = side x side = 7 · 7 = 49 m^2
3 roses each m^2, 49· 3 = 147 roses (Pre-service teacher 3)

Figure 2. Example of a version of the problem that has been changed to involve more processes, but with a solution that is too easy for the educational level

The first difficulty was how to make the problem more relevant to the reality of the students' everyday lives. Pre-service teachers did not know how to create problems that were associated with students' everyday lives. The quote below illustrates this:

> *Pre-service teacher 1: "In the playground, I found a honeycomb. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape in your notebook and calculate the area, considering that one side measures 4 cm. What would be the area if each side measured 5 cm? Calculate the difference in both cases, in terms of the area".*

A second group of difficulties centred on how to enrich the problem by including more processes and concepts in its solution, according to educational level. Pre-service teachers tried to achieve this goal by adding more questions to the formulation of the problem. Of course, solving a problem does not just involve a straight formula. However, the difficulty of the problem was not properly gauged: in some cases it was too difficult (**Figure 1**), in others too easy (**Figure 2**), and in others the participants did not know how to solve it, and made mistakes (**Figure 3**).

We include an example of the first situation: a problem that is more difficult than the level it was designed for (upper primary school cycle). The solution involved the use of Pythagoras' theorem to find the radius of the inscribed circumference. We noted a mistake in the measurement of the area, because it was not expressed in powers of two (see **Figure 1**).

Figure 2 shows an example of the second situation: the solution involves more processes, but is not suitable because it is much too easy for the students. The hexagon has been changed to a square, which makes the polygon area too easy to calculate. The area has been linked to the number of roses that fit within 1 m², which increases the number of processes embedded in the problem, as requested.

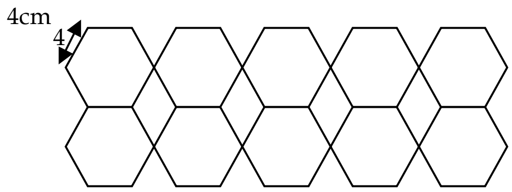
Next, we include an example that is too difficult; even the pre-service teacher who created it was not able to solve it. His most serious mistake was not considering the space between hexagons to calculate the total area of the honeycomb. There is also a mistake in the measurements of the area, because they are not expressed in powers of two (see **Figure 3**).

We have found a honeycomb in the playground. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape and calculate the area, considering that each side measures 4 cm. What is the total area of the honeycomb considering that it has 10 hexagons?

Understanding the problem: a) each side of the hexagon measures 4 cm; b) what is its area? c) the honeycomb is made up of 10 identical hexagons; d) what is the total area for the 10 hexagons?

Strategies to solve the problem (devising a plan): a) draw the hexagon; b) use the formula for the area; c) to figure out the total area for the 10 hexagons, I will draw them and multiply the area of one by 10.

Implementation of the strategy (carrying out the plan):



Area of the hexagon: (P·ap)/2=(24·3.2)/2=38.4 cm

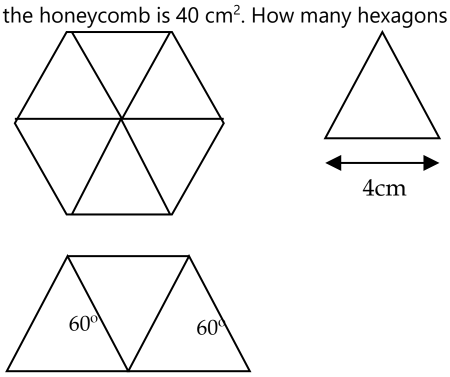
Perimeter: 4·6=24 cm

Total area of the hexagon: 38.4·10=384 cm

Looking back: In order to find out the area of the hexagon, we need to know the total perimeter. When we know the perimeter (24 cm), then we can figure out the area (38.4 cm). Finally, because the honeycomb is formed of 10 hexagons, we just need to multiply the area of one hexagon by 10, which is the total number, thus we will obtain the total area for the honeycomb (384 cm). (Pre-service teacher 2)

Figure 3. Example of a problem that has been changed to involve more processes, but has a more difficult solution that even the author cannot obtain

We have found a honeycomb in the playground. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape and calculate the area, considering that each side measures 4 cm. In addition, we know that the total area of the honeycomb is 40 cm^2. How many hexagons will fit in?



$A = b \cdot h/2 = 4 \cdot h/2$

$A = \text{Perimeter} \cdot \text{apothem} / 2 = (4 \cdot 6) \cdot 4/2 = 24 \cdot 4/2 = 48\ cm^2$

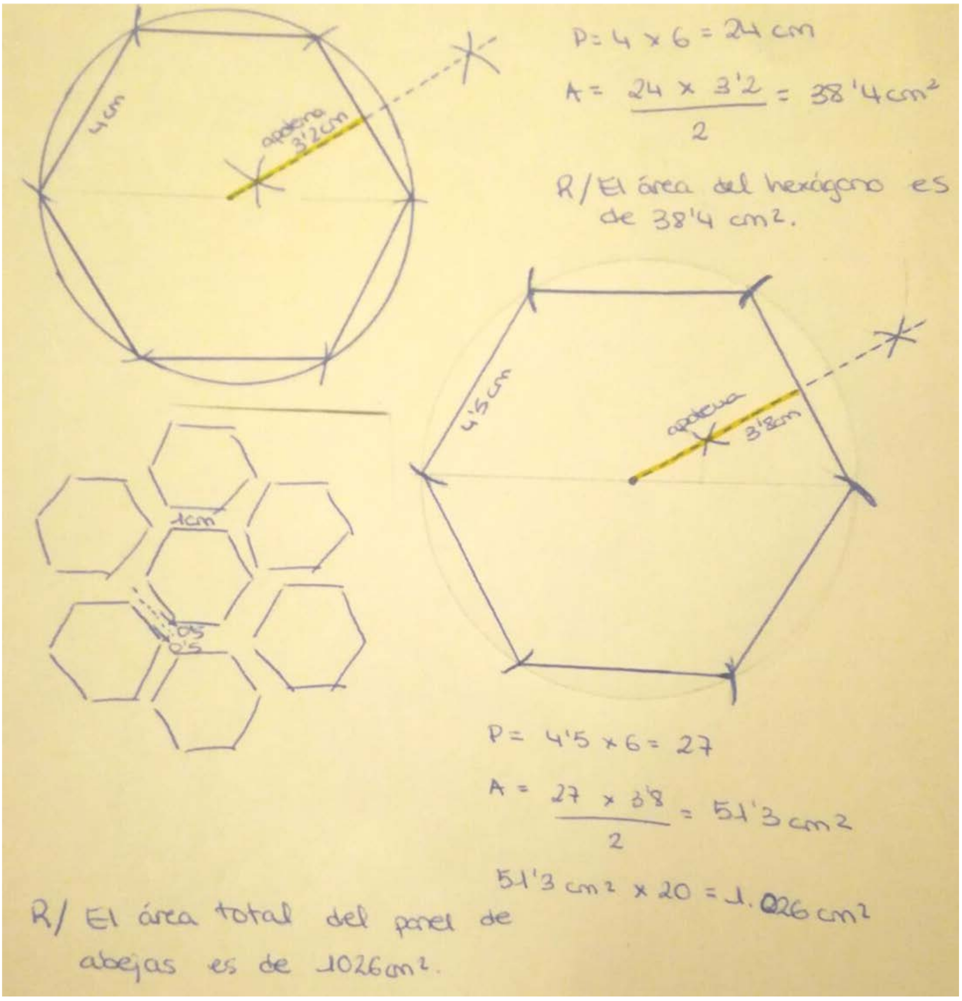
(If the students can draw it correctly, they can also figure out the solution, because all angles measure 60°)

No hexagons will fit in the honeycomb, because the area of one hexagon would be bigger than the area of the entire honeycomb. The problem is not formulated correctly (Pre-service teacher 4).

Figure 4. Example of a problem that has been changed to try to prompt the student to formulate good questions, but unsuccessfully

The third group of difficulties is related to producing a statement that prompts students to formulate good questions to solve the problem. Although the pre-service teachers tried to follow this instruction by adding more questions to the initial statement, as in the example (**Figure 4**), they did not meet the goal. We can even appreciate self-criticism in the example below, in which a pre-service teacher realized that his problem was not properly formulated. In addition, there was a mistake in the reasoning, because the apothem did not measure 4 cm.

The fourth group of difficulties are related to self-correction of the problem. Pre-service teachers tried to change the original problem so that it could be self-corrected, by adding comments and new questions. However, they were unsuccessful. The quote below illustrates the case of Pre-service teacher 10. This teacher added two more conditions to the original problem (the existence of 20 hexagons and a 1-cm distance between them) and one more question (total area), but he failed to transform the problem into a self-corrected one. In addition, the solving process is quite difficult, because students must understand that considering 1 cm between each hexagon involves an increase of 0.5 cm per side in each hexagon.



Translation
Answer:
The area of the hexagon is 38.4 cm^2

Answer:
The total area of the bees nest is 1026 cm^2

Figure 5. Example of a problem that has been changed so that it can be self-corrected, but unsuccessfully

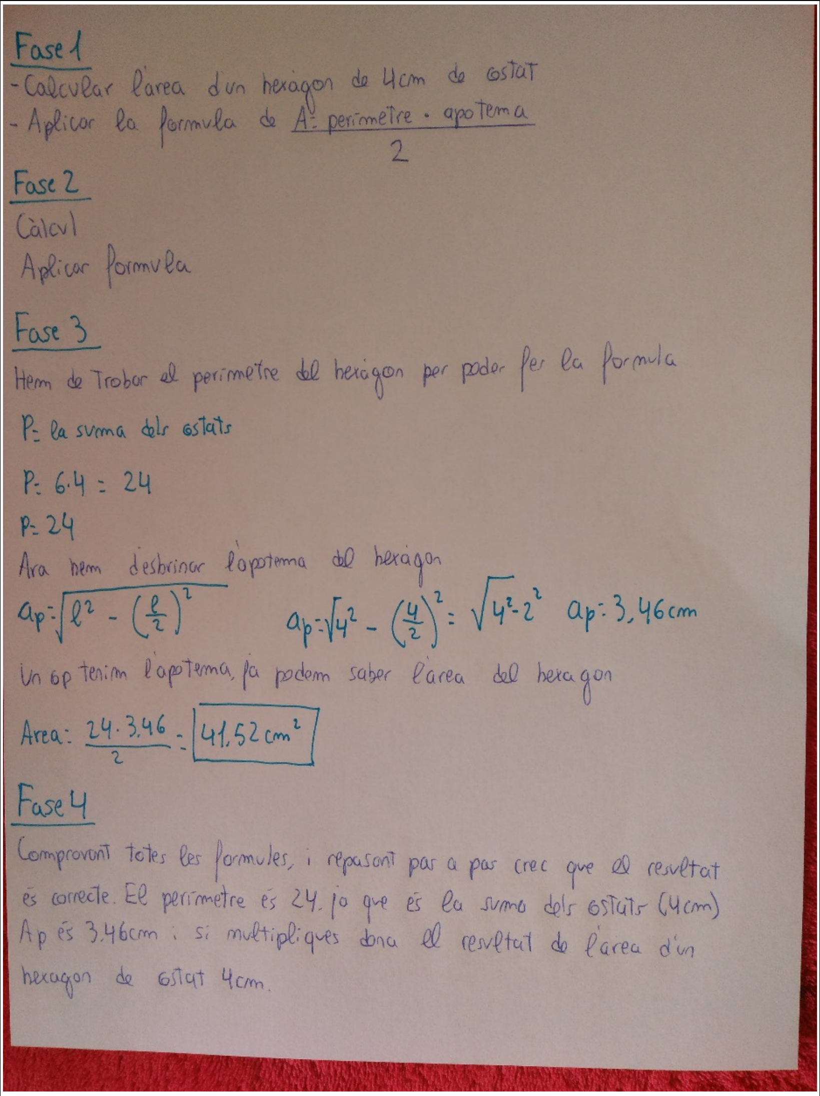
> *Pre-service teacher 10: "We have found a honeycomb in the playground. We noticed that the geometric shapes within the honeycomb were hexagons. Draw this shape and calculate the area, considering that one side measures 4 cm. If there are 20 hexagons within the honeycomb and the distance between them is 1 cm, calculate the total area of the honeycomb".*

The Case of the Pre-service Teacher Who Posed the Most Creative Problem (According to the Variables Introduced by Mallart & Deulofeu (2017))

The last objective discussed in this article is the analysis of the learning trajectory of a pre-service teacher (Pre-service teacher 9). To discuss her trajectory, we analysed all her answers to the proposed tasks. We selected this pre-service teacher because her answers were the most original.

Regarding Task 1, the data suggest that Pre-service teacher 9 expected to receive from the Geometry Education course resources and tools to teach mathematics in different ways. According to her, knowledge of mathematics is "*what everyone knows, but not everyone knows how to use it. It is similar to drawing. Everyone knows how to draw, but in different ways. With mathematics it is the same*". For Pre-service teacher 9, learning mathematics is "*opening your mind and not understanding mathematics as a mechanical process*".

Regarding Task 2, Pre-service teacher 9 could distinguish the level of reasoning involved in solving either an exercise or a problem, although she considered that solving a problem did not involve any formulae: "*The exercise is mechanical and the problem is not. You need formulae for the first one, not for the second one.*" According to her, exercises were for practising, whereas problems were "*to think, to reason, to justify and make mistakes trying to answer a proposed question*". She considered that a problem was an excellent option from the teachers' point of view when "*the formulation is understandable, and all students have the resources to solve it*". Pre-service teacher 9 thought that problem


Fase 1
-Calcular l'area d'un hexagon de 4cm de costat
-Aplicar la formula de A: $\frac{\text{perimetre} \cdot \text{apotema}}{2}$
Fase 2
Càlcul
Aplicar formula
Fase 3
Hem de Trobar el perimetre del hexagon per poder fer la formula
P: la suma dels costats
P: 6·4 = 24
P: 24
Ara hem desbrinar l'apotema del hexagon
$ap: \sqrt{l^2 - \left(\frac{l}{2}\right)^2}$ $ap: \sqrt{4^2 - \left(\frac{4}{2}\right)^2} = \sqrt{4^2 - 2^2}$ $ap: 3{,}46\,cm$
Un cop tenim l'apotema, ja podem saber l'area del hexagon
Area: $\frac{24 \cdot 3.46}{2} = 41{,}52\,cm^2$
Fase 4
Comprovant totes les formules, i repasant pas a pas crec que el resultat és correcte. El perimetre és 24, ja que és la suma dels costats (4cm) Ap és 3.46cm i si multipliques dona el resultat de l'area d'un hexagon de costat 4cm.

Translation
Phase 1:
(a) Calculate the area of a hexagon with sides of 4 cm.
(b) Apply the formula: Perimeter · apothem/2
Phase 2:
Calculate and apply the formula.
Phase 3:
Apply Pythagoras' theorem to calculate the apothem!
Phase 4:
Checking the formulae and reviewing the calculations, I think that the result is correct. The perimeter is 24, because it is the sum of the sides. The apothem is 3.64 cm, and if we multiply it we get the area of a hexagon measuring 4 cm in each side.

Figure 6. Solution of Problem 1 by Pre-service teacher 9

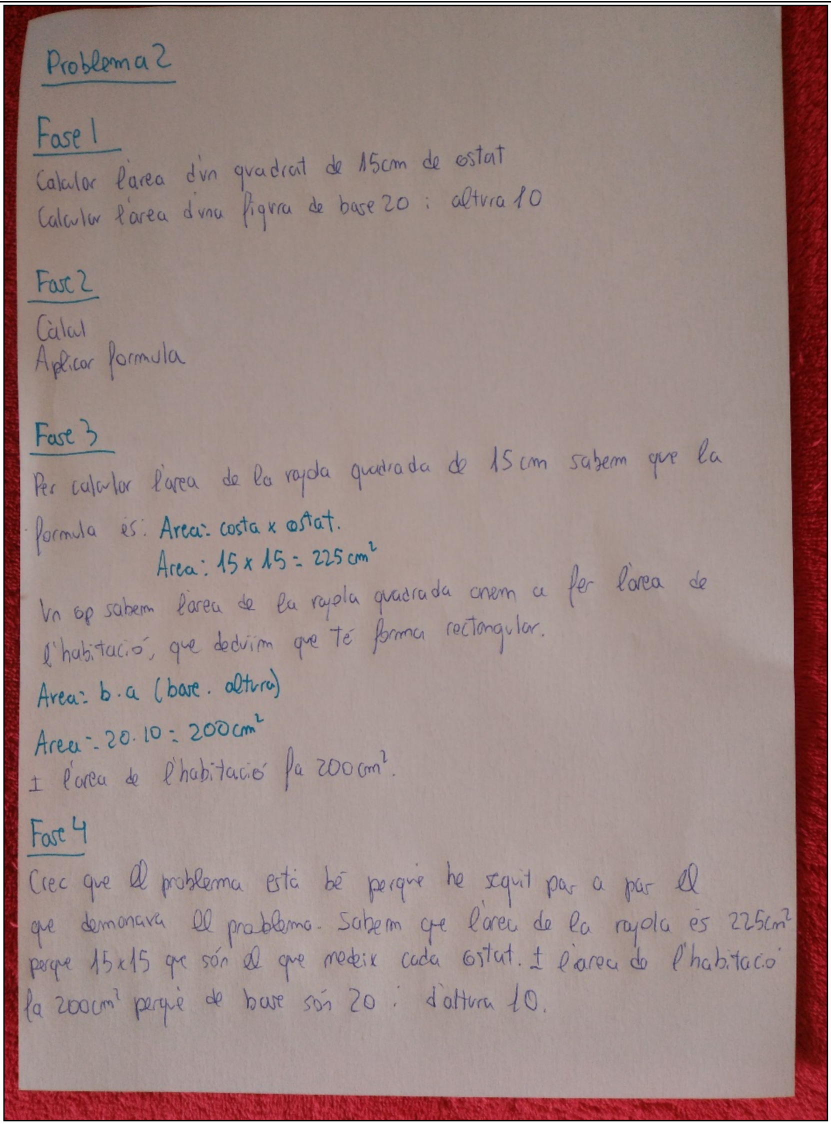
posing was interesting: "*because it is a way to approach mathematics, making sense of it, that is, you do not get a question ready for you. In addition, you learn in a fun and different way.*" She thought that mathematics pre-service teachers should learn how to create new problems because "*you have to follow some rules in order to make (the problem) understandable so it can be solved*".

In Task 3, we asked her to create and solve two problems aimed at students in upper primary level. The problems had to be relevant to everyday life and focused on geometry, connecting 2D polygons, calculating areas and representing results graphically. Her first problem was selected in the final questionnaire as the most original of all those created by pre-service teachers. It was considered the most original problem because its statement described a 2D polygon (hexagon) that did not appear in the other questions. Notably, in the formulation: (a) there are no other 2D polygons (only hexagons); (b) the link with everyday life is not evident (because some students may not be familiar with honeycombs); (c) the solution proposed by Pre-service teacher 9 involves using Pythagoras' theorem, which is not part of primary school curriculum content (**Figure 6**).

Her second problem had the following formulation:

"*We are doing some building work at home that involves tiling the floor with square tiles, measuring 15 cm per side. Calculate the area of the tile and then calculate the area of the whole room if there are 20 tiles (length) and 10 tiles (width)*".

In this problem, the pre-service teacher refers to two 2D figures (the square tile and the rectangular floor). The problem involves reasoning to calculate the areas, and students can recognize it as relevant to the reality of their everyday lives. However, there is no explicit reference to the graphic representation; and the problem is not appropriate to the educational level either (it is too easy because it is just about calculating the area of a square and a rectangle). As **Figure 7** shows:


Problema 2

Fase 1
Calcular l'area d'un quadrat de 15cm de costat
Calcular l'area d'una figura de base 20 i altura 10

Fase 2
Càlcul
Aplicar formula

Fase 3
Per calcular l'area de la rajola quadrada de 15 cm sabem que la formula és: Area: costat x costat.
Area: 15 x 15 = 225 cm²
Un cop sabem l'area de la rajola quadrada anem a fer l'area de l'habitació, que deduïm que té forma rectangular.
Area: b · a (base · altura)
Area: 20 · 10 = 200cm²
I l'area de l'habitació fa 200cm².

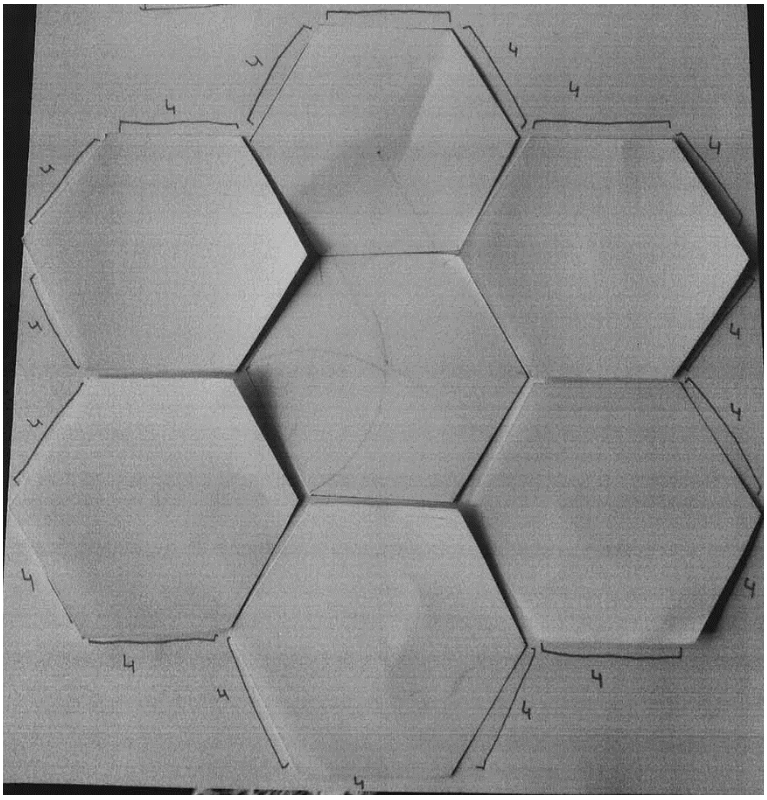
Fase 4
Crec que el problema està bé perquè he seguit pas a pas el que demanava el problema. Sabem que l'area de la rajola és 225cm² perquè 15x15 que són el que medeix cada costat. I l'area de l'habitació fa 200cm² perquè de base són 20 i d'altura 10.

Translation
Phase 1:
(a) Calculate the area of a square of 15 cm per side.
(b) Calculate the area of a polygon with base 20 and width 10.
Phase 2:
Calculate and apply the formula.
Phase 3:
The student calculates the area of the square tile (225 cm^2). Then, she calculates the area of the room, which is 200 cm^2!
Phase 4:
She checks the result and states (without noticing the mistake): room, 200 cm^2; tile, 225 cm^2.

Figure 7. Solution of Problem 2 by Pre-service teacher 9

Regarding Task 4, Pre-service teacher 9 considered that there are certain difficulties when we create a new geometry problem, i.e. understanding that someone does not have the strategies or the prior knowledge; formulating the problem correctly; realizing that in some cases the person cannot solve the problem. She claimed that she was not competent enough to pose new problems, mainly for three reasons: "*a) I have no training to know how to formulate a question that can be understood by the student; b) I don't think I have the geometry knowledge to pose or solve a problem; c) geometry is not attractive to me, so I'm less receptive to this topic.*" Pre-service teacher 9 claimed that one of her strengths was her creativity to come up with new problems; however, her weakness was her lack of mathematics knowledge. She stated that a teacher could teach how to create new problems by offering strategies and tools. She also thought that pre-service teachers benefit from creating new problems in three ways: "*a) being able to create your own problems; b) being able to teach your students how to create new problems as well; c) when teaching mathematics, you may feel more confident and probably you will be less dependent on the textbook, for instance.*" Pre-service teacher 9 affirmed that she had not worked on geometry enough during her undergraduate degree. She felt that she was not able either to create new problems for every educational level or to teach students how to solve these problems in different ways. She felt that all these elements were crucial to be able to create new problems. She also claimed that these problems might be interesting for the students when: "*the problems are familiar to them; they are challenging for the students; there are many ways to solve them.*"

After providing Pre-service teacher 9 with a research article about all the characteristics of a good problem in educational terms (Mallart, Font & Malaspina, 2016), and discussing it with the group (Task 5), Task 6 involved asking this participant how her problem, which had been selected as the most original, could be improved. She



"We know that each side of the hexagon is 4 cm. We want to know the perimeter of the flower. We just need to count the sides of the flower, and then multiply them by 4.
P = 18 · 4 = 72 cm".

Figure 8. Solving the new formulation by Pre-service teacher 9

stated: "*a) The level of the problem is not appropriate to the students for whom it was designed; b) it is not about a familiar situation for the student; c) it is not possible to create new problems starting from it.*"

Regarding Task 7, we observed that Pre-service teacher 9 changed the original problem into another that was more attractive to students, as it involved them actively:

"We have found a honeycomb in the playground. We noticed that the geometric shapes within the honeycomb were hexagons. Using a compass, draw 7 hexagons, and cut them out. Then, try to create the shape of a flower and calculate the total perimeter for the new shape, if each side of the hexagon is 4 cm".

This problem can be self-corrected, and new problems can be created from it, as illustrated in **Figure 8**. It also meets the requirement of involving a larger number of processes. However, the new formulation does not ask for calculation of the area.

CONCLUSIONS

This research focuses on analysing pre-service mathematics teachers' difficulties in problem posing at the end of their university studies. The research approach is interesting because it provides meaningful knowledge to design preventive strategies and thus increase teachers' confidence in using problem posing in mathematics classes. Brown and Walter (1993) showed that if students are given the opportunity to pose and solve their own problems, they will become more active in their learning process. There is also evidence that this approach increases students' engagement in constructing their knowledge (Cunningham, 2004). Akay and Boz (2010) looked at the effect of problem posing on 82 pre-service teachers' attitudes towards mathematics. They concluded that this type of activity may improve students' attitudes towards mathematics, decrease their anxiety and encourage them, even when the students are pre-service teachers with little background in the subject.

In this study, pre-service teachers considered that problem posing was an instrument for testing mathematical knowledge, and for assessing creativity. In relation to the use of problems for assessment, pre-service teachers were asked to pose problems according to some fixed guidelines that in the future could reflect the intentions of an exam. However, the problems they posed did not consider all the guidelines. A first finding of the study is that pre-service teachers considered it most difficult to meet the requirement of matching the level of difficulty to the educational level asked. Although they proposed that one way to enrich a problem is by adding more processes and concepts in its solution, they did not know how to calibrate difficulty. They added more questions to the statement of the problem, but they made the problem into another that was easier or more difficult.

The kinds of problems posed by pre-service teachers, following the instruction of keeping the statement in the problem relevant to the students' daily lives, revealed a second conclusion of this research: participants had serious difficulties in finding a familiar context. In other words, pre-service teachers had difficulties in: connecting data, expressing a well-formulated and intelligible statement of a problem with no redundant or missing information, engaging students in the solution, allowing several solutions, and offering the possibility of solving the problem by manipulative resources.

This research reveals that there is a third conclusion to be drawn which relates to the singularity of the characteristics of the problems posed: it was difficult to create problems that could be solved in different ways or, at least, self-corrected.

A further finding obtained in this research is that the pre-service teachers did not feel prepared to pose mathematics problems, because they had only been trained to solve them. They confessed that they did not have suitable resources or tools. This idea was also described in studies by Sriraman (2005) and Breda et al. (2017), who stated that pre-service teachers must develop the competence of assessing the problems they could use, and that this competence must be developed in teacher education. A way of teaching this competence is with tasks that involve managing the educational analysis, for example, posing problems by varying a given problem or by creating a new problem, whether a certain situation is faced or due to a specific request (Malaspina, Mallart & Font, 2015).

Finally, we present a case study of a pre-service teacher who had a high level of creativity according to the seven indicators (originality, flexibility, elaboration, analysis, synthesis, communication and re-definition) developed by (Mallart & Deulofeu, 2017). After analysing the pre-service teacher's answers in the case study, we can conclude that her final problem is original (it is novel and non-standard, because manipulative work must be used to solve it); flexible (it reframes the original problem to create a flower with the hexagons); elaborate (it adds elements such as measuring the perimeter of the flower, drawing on the hexagons of the honeycomb); expresses a high analytical ability (because it considers breaking down the figure, and analysing the sides that form part of the perimeter); uses a high level of synthesis to compare the different parts in which the problem can be split, in order to solve it (construction of hexagons with a compass, identification of the external sides of the figure); involves a high degree of communication (she communicates the solution clearly and convincingly); and, lastly, satisfies the indicator of redefinition (starting from information that is already known, which is the hexagons within the honeycomb, which she transforms into flower petals).

This case is interesting to determine the pre-service teachers' thoughts about problem posing and to identify difficulties that problem posing represents to a future creative teacher. This pre-service teacher does not understand mathematics as a group of mechanical processes. In her opinion, mathematics problems are useful for thinking, reasoning, justifying and making mistakes. Consequently, she considers that problem posing is interesting because it represents an enjoyable and motivating way of bringing mathematics to students. She considers that all pre-service teachers should know how to create problems, because some rules need to be followed to obtain intelligible and solvable problems. She cannot follow all the fixed guidelines simultaneously to pose two problems. In fact, she shows difficulty obtaining two solvable problems using suitable methods according to a fixed curriculum (she suggests solutions by using methods that are more complicated or easier). She points out some difficulties in problem posing: lack of previous mathematical knowledge, lack of problem solving strategies, and lack of knowledge about creating appropriate problems. She recognizes that she does not know how to create problems with educational intentions, because she is out of practice. She would like to know how to solve more problems, and she explains that mathematics is not interesting enough for her. She thinks she is very creative, but she needs to know more mathematics.

She considers that pre-service mathematics teachers have a very useful tool in problem posing, not only for testing students, but also for teaching them. She thinks that if teachers know how to pose problems, they will be able to meet the needs of their students at any time and independently of textbooks. She believes that problem posing helps her to learn more. She thinks that a problem will be interesting for students if it is more closely related to their reality, if its solution represents a challenge, and if it can be resolved in different ways. When she reviewed her own problems, she detected that the educational level did not correspond with the level requested; the context was not relevant enough to the daily life of the students, and her problems did not encourage self-correction.

However, when she rectified her problems: a) she actively involved the solver, to make the solution more interesting; b) she provided problems that could be self-corrected by manipulative resources; c) she presented problems that could lead to the generation of new ones; d) she involved more processes in the solutions. But she was not able to follow all the fixed guidelines: she did not ask for any areas to be calculated.

To sum up, this research focuses on determining pre-service mathematics teachers' characteristics and their academic background, and on identifying strategies and education interventions that encourage the use of problem posing. Therefore, our study is a first step to finding out what aspects should be enhanced through actions in the context of training future teachers to overcome possible difficulties. We must be open-minded to detect and correct difficulties (our study variables) during the preventive education intervention. Consequently, the research could help us to verify whether actions to remedy pre-service teachers' difficulties reduce the risk of avoiding problem posing. We must consider the limitations of our study and the instruments we used, to find signs of difficulties when pre-service teachers use problem posing.

We believe that this study contributes to a better understanding of problem posing in mathematics' classrooms. The findings are similar to the conclusions of previous national and international studies: problem solving is very much a part of mathematics teaching classes, but problem posing is not addressed to the same extent. We recognize that there are still more aspects to be studied, and consequently we must be extremely careful with our conclusions and with any generalizations. For these reasons, we consider that it is important to continue to work in this research area, to identify the difficulties teachers have in using problem posing in mathematics classes, and to determine how to prevent the emergence of these difficulties by including certain suitable modifications in the method of training pre-service teachers.

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APPENDIX A

Task 1

Task 1: Questionnaire 1

1) What do you expect from the course "Mathematics Education"?
2) Try defining "Mathematics Education".
3) What do you understand by "knowing mathematics"?
4) What do you understand by "learning mathematics"?
5) What do you understand by "teaching mathematics"?

APPENDIX B

Task 2

Task 2: Questionnaire 2

1) What is an exercise?
2) What is a problem?
3) Could you explain three differences between exercises and problems?
4) What are exercises for?
5) What are problems for?
6) Could you describe three characteristics of an excellent plan from the teacher's point of view?
7) What is the "process of solving a problem"?
8) Do you find it interesting to create new problems?
9) Do you think that it is necessary to teach problem posing to pre-service teachers? If so, could you explain why, using three different reasons?

APPENDIX C

Task 4

Task 4: Questionnaire

1) Discuss at least three difficulties that may appear when you create a new problem in mathematics.
2) Do you think that you have enough background knowledge to create new problems in mathematics? Justify your answer with at least three reasons.
3) What are your strengths in problem posing? And your weaknesses?
4) How can a teacher teach problem posing?
5) Explain at least three reasons why it is positive for pre-service teachers to know how to create new problems in mathematics.
6) Explain at least three aspects that you never studied in your academic life before coming to university, but you think are crucial for problem solving in mathematics.
7) Describe at least three characteristics of an educationally interesting mathematics problem, for the mainstream.