Ray tracing for ophthalmic optics. Specific developments for GRIN materials.

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ABSTRACT

In this work we present the developments for the simulation of image formation through an ophthalmic lens. The procedure is, essentially an exact ray tracing taking into account the particular mobility conditions of the exit pupil. This leads to the computation of astigmatism as a function of the field angle considered. In the computer program, the use of a commercial glass with a gradient in the refractive index (GRIN material) is foreseen. This latter feature proves to be useful for two practical objectives: modifying the power and the astigmatism of the lens. To illustrate these points, an example based on a lens made by deforming a plane parallel plate of GRIN material will be presented.

Keywords: Gradient-index lenses, ophthalmic lenses, optical design.

1. Introduction

Graded-Index (GRIN) optics has been widely studied for improving optical properties of many kinds of lenses. The use of GRIN elements in optical design provides powreful additional degrees of freedom for the correction of aberrations [1]. Lenses with GRIN profiles have been adapted to a wide range of applications such as imaging and transforming systems; coupling, connecting and collimating devices in optical communications and so on. Applications of GRIN optical materials include rod lenses, multifocal contact lenses, low chromatic aberration lenses, and high-bandwidth optical fibers.

Most of the applications of GRIN to optical systems deal with the design and manufacture of small lenses and with the correction of aberrations in optical systems consisting of several lenses [2] [3] [4]. There are only a few references where the different aspects related to the design of single (individual) lenses, having higher aperture, have been reported. Several of these applications are intended for ophthatlmic use [5] [6].

The aim of this work is to study the potential use of GRIN materials for the design of ophthalmic lenses. We will show resuls obtained by means of a computer simulation program which implements the image formation through an ophthalmic lens manufactured using a GRIN media. The basis of our program is a ray tracing procedure which tries to reproduce the working conditions of an ophthalmic lens, i.e., the eye (operating under its particular optical requirements) is always placed behind. In fact, the program allows the simulation of lenses either homogeneous or manufactured with different kinds of GRIN conditions. In this case, the program allows the optimization of the lens design in study by means of the modification of the GRIN. This latter feature proves to be useful for two practical objectives: modifying the power and the astigmatism of the lens. To illustrate these points, an example based on a lens made by deforming a plane parallel plate of GRIN material is presented.

2. Image formation

2.1. Working conditions.

The optical properties of an ophthalmic lens are closely related to their conditions of use. Basically, these conditions are defined by the presence of the eye behind the lens and by the allowed relative position between lens and eye. The two main characteristics of the ophthalmic uses of a lens are:

1) The exit pupil of the lens will always be quite small (a few millimeters in diameter), due to the presence of the eye pupil in all practical cases.

2) The position of the exit pupil moves as the eye pupil rotates around the center of the eye, when looking at different points.

The two previous constrains lead to important conclusions:

- An ophthalmic lens must be designed to work with large field angles.
- The most significant aberration in the design of an ophthalmic lens is the astigmatism.

2.2. Determination of the entrance pupil.

Once the optical elements forming the system are geometrically defined, the first step in computations is finding the entrance pupil. This is done by means of an inverse exact ray tracing (i.e., computed from the eye to the object) in the two different cases (on axis and off axis, rotating the eye). This calculation is implemented within the program by means of the following simplified procedure. We take two points in the actual pupil of the eye: one is at the center and the other 1 mm apart. By an exact inverse ray tracing, the two images in the object space are determined. These automatically define the position, orientation and size of the entrance pupil. This procedure is illustrated in Figure 1.

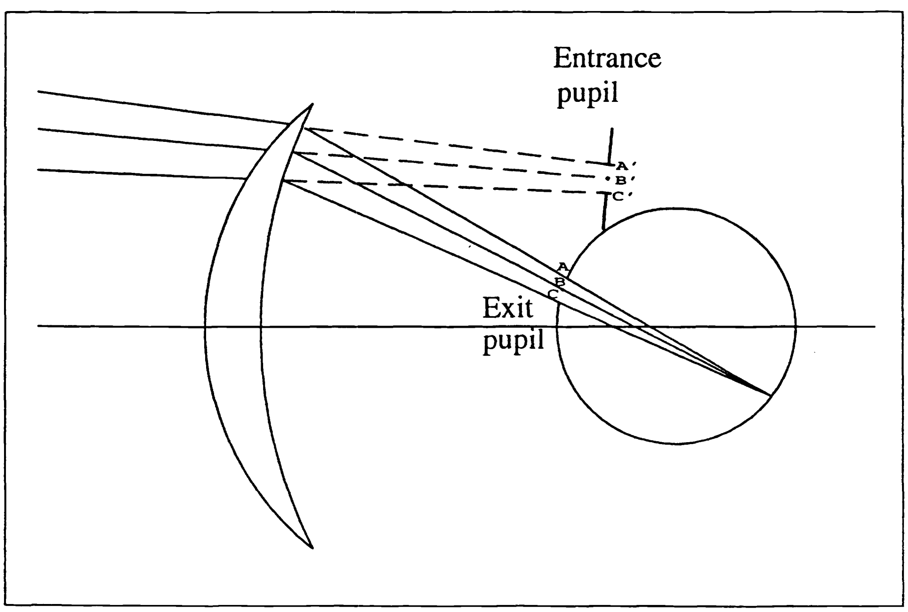

Figure 1: Schematic of ophthamic lens and eye performance.

2.3. GRIN types.

For GRIN materials three different types have been implemented:

1) Radial variation: the changes in refractive index are of the form $n(r)=n_0+Ar^2+Br^4$, where r is the distance from the optical axis.

2) Axial variation: the refractive index changes as $n(z)=n_0+Az^2+Bz^4$, where z is the position along the optical axis.

3) Depth variation: the index changes in depth, from the surface of the lens, according to $n(d)=n_0+Ad^2+Bd^4$, were d is the distance from the surface. This is the kind of variation one may expect by ion exchange (difussion) of glass within a liquid media. [7].

2.4. Ray tracing and aberrations.

The spot diagrams corresponding to two far point objects (axis and field zone) are computed. This is done by considering an uniform distribution of rays in the entrance pupil and by exact ray tracing through the lens (which may be homogeneous or made with any of the three kind of inhomogeneous media explained).

Exact ray tracing through a GRIN media has been implemented using the Runge-Kutta numerical method, in the form of the algorithm proposed by Sharma et al. [8]. This procedure consists of dividing the ray path into small steps, computing for each one the director cosines and the coordinates of the ray. The problem of computing the intersection of the rays with the surfaces between the different media has been addressed by interpolation, following the idea suggested by Stone et al. [9]. As indicated, our ray tracing procedure always takes into accont the fact that, when looking at a field angle, the eye pupil moves accordingly. Thus, in these conditions the rays will cross the lens obliquely.

The assessment of the quality of the lenses is done by computing the tangential and sagittal foci for a certain field angle (and also the axial focus). These tangential and sagittal focal points are determined as those giving minimum distance between two closely spaced rays, traced in the directions sagittal and tangential, since there will be no actual intersection between two rays traced in a general case. Astigmatism is defined as the distance between the two focal points [10]. Astigmatism and field curvature may be combined to give a figure of merit for the quality of the lens.

2.5. Results from the program.

Here we will present the results corresponding to the working conditions described above, although the same computer program would be useful for other practical applications. For example, designing contact lenses by using GRIN materials would be possible; in this case the merit function to optimize has to be mainly the spherical aberration [6].

Figure 2 a) shows a plot of the meridian rays traced through a lens having a radial variation in refractive index, and the corresponding spot diagrams on axis and field zone. The procedure for finding the entrance pupil is illustrated (image of the eye pupil calculated backwards through the lens). While running, the program allows moving the image plane and the scale for plotting the spot diagrams. Figures 2 b) and c) are spot diagrams on axis and field zone in a plane close to the paraxial focal plane of the lens. The field spot diagram shows the typical shape due to the astigmatism which induces the commented visual conditions (small aperture, large field angle). Figures 2 d) and e) show the spot diagrams in the sagittal plane, whereas f) and g) correspond to the tangential plane. The distance between them is the astigmatism corresponding to the part of the lens where the light rays cross.

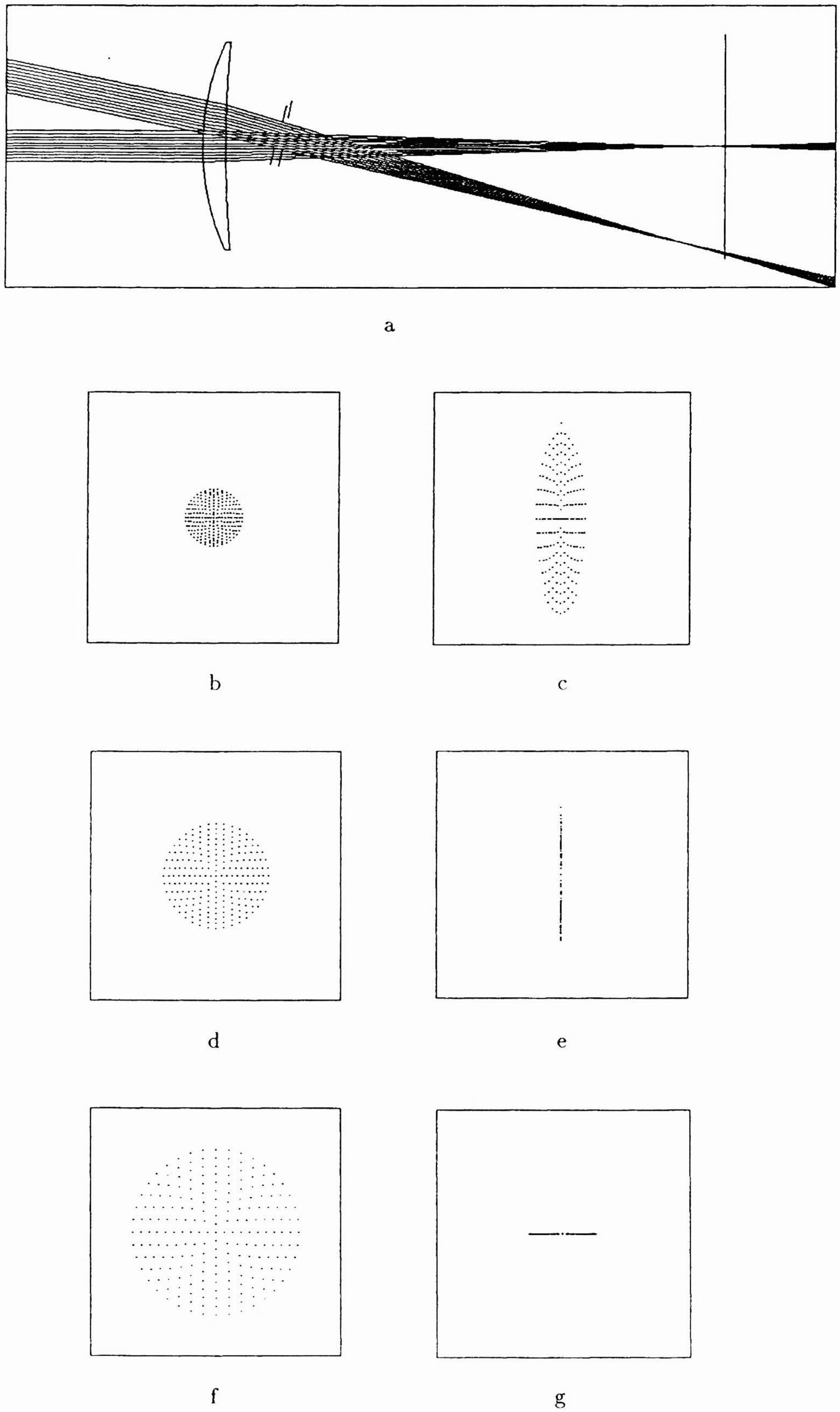

Figure 2.- Ray tracing through a GRIN lens.

3. Designing a GRIN ophthalmic lens

To show the potential use of GRIN materials for manufacturing ophthalmic lenses, an example case has been developed. A plane-parallel slab of material of 2mm thick (say plastic or glass) is curved to give two 5.25 dioptre surfaces and 60 mm diameter. We will work with a field angle of 25 deg. We will assume a radial variation in the refractive index given by $n(r)=1.5+Ar^2+Br^4$. Here, the second order term introduces a variation in optical path length which is equivalent to a spherical surface, thus inducing a variation in optical power similar to a common change of surface curvature for homogeneous materials. The fourth order term is equivalent to the substitution of a spherical surface by an aspherical one, thus modifying the resulting aberrations (mainly astigmatism).

The design procedure has been oriented to find the two parameters A and B which define the GRIN. The results obtained show that A is mainly related to the final power of the lens, whereas B controls the astigmatism. For example, A=-0.001 gives a lens of about 4 dioptres; if B=0, the astigmatism is -0.007 dioptres, wile making B=8E-8 the astigmatism reverses sign (0.007 dioptres), indicating that B may virtually eliminate it with proper adjustment.

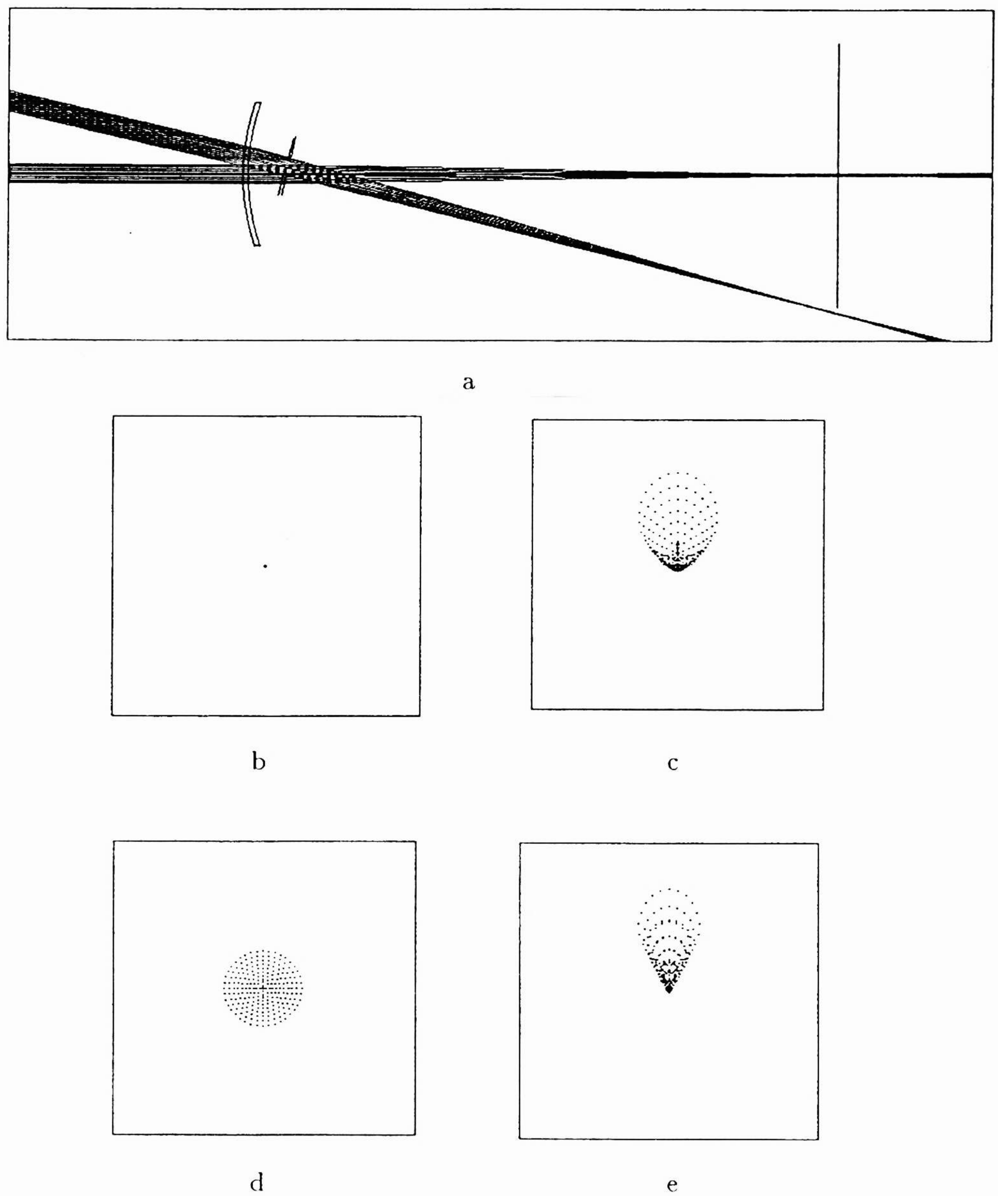

Figure 3.- Ray tracing through a curved slab.

Figure 3 illustrates the behaviour of the lens in the initial case (A=-0.001 y B=0). For the other case (A=-0.001 y B=8E-8) the plots are very similar. Figure 3a corresponds to the ray tracing through the lens (in fact this is only a curved slab with a radial variation in the refractive index). Figures 3b and 3c are the spot diagrams (on axis and in field zone, respectively) in the focal plane of the lens. Finally, Figure 3d and 3e are the spot diagrams in an intermediate plane between the sagittal and tangential foci. One may note, in the focal plane, that the axial image is virtually a point while in field zone shows the typical comma shape. The same kind of phenomenon, even more noticeable, is visible in 3 c. Both images evidence the absence of astigmatism, which implicitly leads to a small comma, practically a third order one.

4. Conclusions

In this work we have addressed several aspects related to the potential use of GRIN materials for the manufacture of ophthalmic optics. First, we have analyzed the working conditions of ophthalmic lenses, according to the eye characteristics, and the relevance of the different aberrations. Consequently, a computer program was developed to implement the above ideas: taking into account the role of the eye in the process and evaluating the astigmatism as the more important merit function.

The program begins by finding the entrance pupil, from the actual position of the eye pupil, performing an exact ray tracing for lenses which may be manufactured with several different kinds of variations in the refractive index (either radial, axial or in depth). Then, the program computes the aberrations (particularly astigmatism) and allows the user to take this factor into account in the design of the opthalmic lens.

As an example, we present the results obtained with a lens formed by a curved slab (i.e., initially a plane parallel piece) of material having a radial variation in the refractive index. We show that this kind of GRIN allows a significant increase in optical power (by variation of the first coefficient) while the second coefficient allows a good control of the astigmatism (it may be virtually zero).

5. Acknowledgements

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