

# On-axis joint transform correlation based on the interferometric acquisition of the output plane

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## ABSTRACT

In this work we propose a method to obtain single centered correlations with an optical setup based on a joint transform correlator. This approach is a modification of a previous procedure that required displaying devices with a full  $2\pi$  phase modulation. The displaying requirements are less restrictive than before, allowing the use of many modulators and configurations. This new method is based on a binary power spectrum and it needs an interferometric process to obtain a single detection peak. To validate our new procedure, we propose an optical setup and we present the experimental results achieved.

**Keywords:** pattern recognition; optical correlation; joint transform correlator; liquid crystal displays.

## 1. INTRODUCTION

Joint transform correlator (JTC)<sup>1,2</sup> has been widely used as an architecture to implement correlation in optical pattern recognition field. Many improvements have been proposed to obtain real-time detection and to increase the discrimination capability,<sup>3,4</sup> but there is still a remaining issue in JTCs: to obtain an isolate output correlation, due to the zero-order term and the two symmetrically non centered correlation terms that appear at the output plane. Different approaches have been proposed to deal with this problem: to focus each diffracted term on a different plane,<sup>5</sup> to use phase-shifting methods to get a non-zero-order spectrum<sup>6,7</sup> or to remove the zero-order diffraction term.<sup>8</sup>

Recently we proposed an approach not to have only a non-zero-order joint transform correlation, but also obtain a single centered detection term at the output plane using a JTC.<sup>9</sup> The method was based on a four-level power spectrum requiring a phase-only configuration to be displayed on a modulator. In the present work, we introduce a modification of our previous approach using now a binary power spectrum. The requirements to display this binary distribution are less restrictive than the needed for the four-level, allowing the use of different configurations. This method can be useful for liquid crystal displays (LCD) that can not reach a  $2\pi$  phase modulation, as many of the ones removed from commercial videoprojectors, widely used in optical setups. Moreover, we propose a suitable setup based on an interferometric architecture to implement our method and we show some experimental results we have obtained.

## 2. JOINT TRANSFORM CORRELATION REVIEW

In a JTC, scene and reference are jointly displayed on the input plane and, as a consequence, multiple diffracted terms appear at the output plane. A simple scheme of the JTC architecture is shown in Figure 1.

Analyzing mathematically all the correlation process in a JTC, first scene and reference are displayed at the input plane with a separation between them:

$$f_R(x + x_0/2, y + y_0/2) + f_S(x - x_0/2, y - y_0/2) \quad (1)$$

assuming that the scene is located at  $(x_0/2, y_0/2)$  and the reference is at  $(-x_0/2, -y_0/2)$ . The intensity of the optical Fourier transform performed by a lens system is registered by a CCD camera. This distribution is called the joint power spectrum (JPS) and it is described by:

$$I(u, v) = |F_R(u, v)|^2 + |F_S(u, v)|^2 + 2|F_R(u, v)||F_S(u, v)| \cos\{2\pi(x_0u + y_0v) + \phi_R(u, v) - \phi_S(u, v)\} \quad (2)$$

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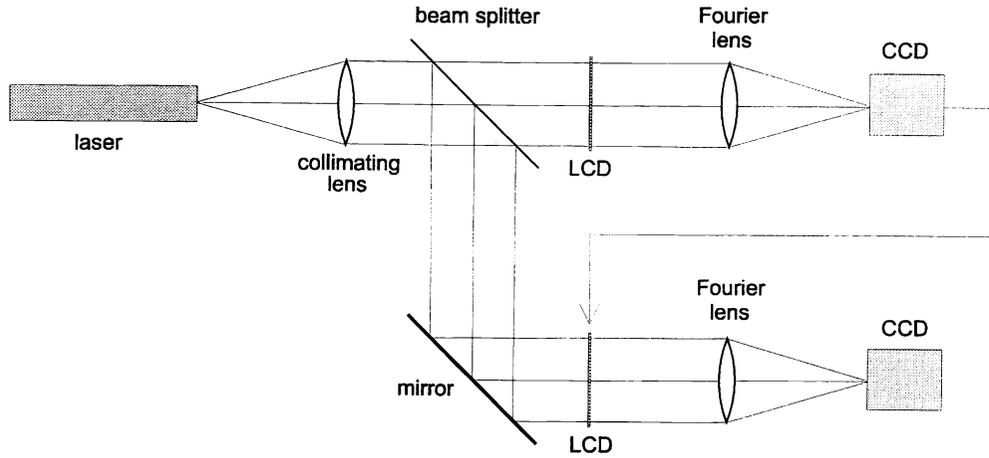


Figure 1. Classic JTC.

where  $|F_R(u, v)| e^{i\phi_R(u, v)}$  and  $|F_S(u, v)| e^{i\phi_S(u, v)}$  are the Fourier transforms of the reference  $f_R(x, y)$  and the scene  $f_S(x, y)$  respectively. In turn, the JPS is subsequently displayed on the LCD so a second Fourier transformation ( $\mathcal{F}$ ) gives:

$$c(x, y) = \mathcal{F}\{I(u, v)\} = f_R(x, y) \otimes f_R(x, y) + f_S(x, y) \otimes f_S(x, y) + f_S(x, y) \otimes f_R(x, y) * \delta(x - x_0, y - y_0) + f_R(x, y) \otimes f_S(x, y) * \delta(x + x_0, y + y_0) \quad (3)$$

where the symbols  $\otimes$  and  $*$  stand for the correlation and the convolution product, respectively. Therefore, at the output plane there are two cross-correlation terms centered at points  $(x_0, y_0)$  and  $(-x_0, -y_0)$ , and two other terms that give non-useful information as they are self-correlation products.

### 3. ON-AXIS JOINT TRANSFORM CORRELATION

Our previous work was based on the use of a four-level power spectrum and the correlation results emulated the performance of a VanderLugt correlator.<sup>9</sup> That procedure required an LCD with a full  $2\pi$  phase-only modulation.

Now we propose to obtain a single centered correlation with an LCD that can only achieve a  $\pi$  phase-only modulation. The procedure is similar to the binarization of the joint power spectrum (JPS),<sup>10</sup> but in our case the final result is just a single centered correlation term.

To obtain this single detection peak we display scene and reference superimposed on the input plane with a phase shift of  $\pi/2$  between them,

$$f_R(x, y) + f_S(x, y)e^{i\pi/2} \quad (4)$$

It is possible to achieve the required phase shift between these two images using the modulation properties of the LCDs, which are described by its operating curves, as we will further explain in Section 4.

When scene and reference are introduced at the input plane of a JTC, following Relation 4, the intensity registered by a CCD camera placed at the Fourier plane is described by:

$$I_S(u, v) = |F_R(u, v)|^2 + |F_S(u, v)|^2 + 2|F_R(u, v)||F_S(u, v)| \sin(\phi_R(u, v) - \phi_S(u, v)) \quad (5)$$

Once the intensity distribution  $I_S$  is captured, we can define the binary function  $I_{BS}$ :

$$I_{BS}(u, v) = \begin{cases} i & \text{if } I_S(u, v) \geq I_T(u, v) \\ -i & \text{if } I_S(u, v) < I_T(u, v) \end{cases} \quad (6)$$

and using the following threshold function:

$$I_T(u, v) = |F_R(u, v)|^2 + |F_S(u, v)|^2 \quad (7)$$

the distribution  $I_{BS}(u, v)$  can be reduced to<sup>11</sup>:

$$I_{BS}(u, v) \approx i \sin(\phi_R(u, v) - \phi_S(u, v)) \quad (8)$$

The aforementioned threshold function (Equation 7) can be obtained by evaluation of the intensity of the Fourier transform of the scene and the reference separately.

Once this binary distribution is computed it can be displayed on the LCD and, performing a second Fourier transformation, we get at the correlation plane the following result:

$$c(x, y) \propto i(\delta(x - x_r, y - y_r) - \delta(x + x_r, y + y_r)) \quad (9)$$

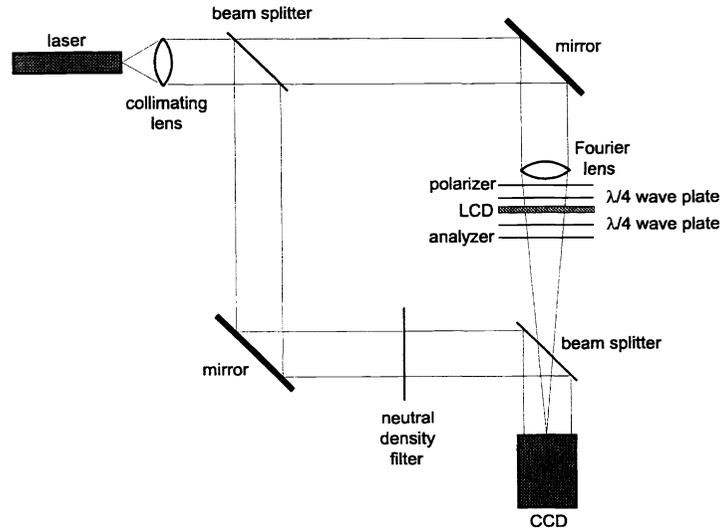
when the reference is included into the scene at the position  $(x_r, y_r)$ .

Note that this amplitude distribution will lead us to capture two indistinguishable intensity detection peaks when we register it by means of a CCD camera. The differences between this result and the one obtained in a conventional JTC are that the two cross-correlation terms are centered on-axis and there is no zero-order term. However, our goal is to achieve an output plane with a single centered detection peak, therefore, we have to remove one of these two peaks to get a single term.

Before the correlation plane is captured by a CCD camera we can add a plane wave that will allow, in the ideal case, the elimination of one of the terms of Equation 9 due to negative interference. When the amplitude of this plane wave is not exactly the same as the one of the terms, the duplicated correlation peak is just reduced. By binarizing the final output plane using a high enough threshold, the resulting background and the possible remaining useless peak can be removed.

#### 4. OPTICAL SETUP

To perform the whole process optically, we propose the architecture sketched in Figure 2, that is based on a Mach-Zehnder interferometer. To obtain the required power spectra we only use one of the arms of this setup, while both arms are used to achieve the final correlation.



**Figure 2.** Optical setup. Polarizers and wave plates are needed to obtain the desired LCD configurations

Images are displayed on an LCD that modulates both the phase and amplitude of the transmitted light, depending on the applied voltage and the polarization of light. This is controlled by the output signal of a VGA video card, that transforms different grey levels of images on a computer in electrical signals. It is then necessary to obtain the operating curves of the panel, which give the relationship between the grey level of the input image and the amplitude and phase modulation. Depending on the orientation of the polarizer and analyzer placed before and after the modulator and on the values of some potentiometer controls available on the videoprojector (brightness, contrast and color), different kinds of response are obtained.

First, we have to display the reference and the scene images at the input plane (Relation 4). For that purpose, we could require two LCDs. Both of them should only modulate the amplitude but their operating curves should give a phase shift of  $\pi/2$  between them. In this way, after a Fourier transform, we would obtain  $I_S$  (Equation 5) by means of the CCD camera, so we could compute  $I_{BS}$  (Equation 6). As we pointed out in previous works, we can use a single panel with a high-contrast (HC) configuration and binary input images.<sup>7,9</sup>

To perform the second Fourier transform and get the desired centered correlation plane, we need a single modulator with a configuration with the required two values of  $I_{BS}$ . This can be easily achieved with a phase-only response, which has a constant transmittance and a wide range of phase variation.

## 5. EXPERIMENTAL PROCEDURE AND RESULTS

The scene chosen for the experimental procedure is shown in Figure 3, and the verification of the method consists in the detection of each satellite. In the optical setup, we have used a single LCD removed from an Epson EMP-3000 videoprojector. This panel has a VGA resolution (640x480 pixels) with a pixel pitch of 42x42  $\mu\text{m}$ . The whole process has been carried out by use of different configurations of a single modulator. These panels cannot reach a full  $2\pi$  modulation due to its low birefringence.

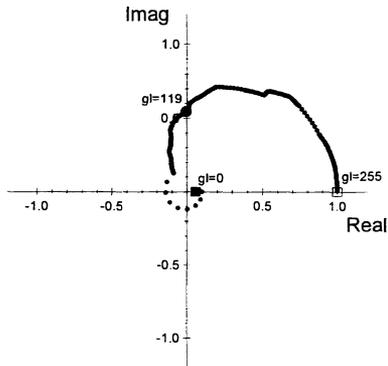


**Figure 3.** Image used as scene.

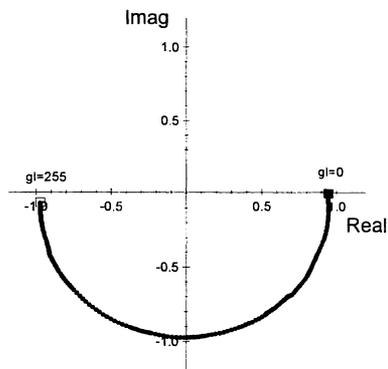
To display the required distribution of Equation 4, we have chosen a HC configuration (Figure 4) that gives a high contrast between the grey level  $gl=255$  and  $gl=0$ , and a phase shift between  $gl=255$  and  $gl=119$  of  $\pi/2$ , as needed.

The sine distribution  $I_S$  (Equation 5) has been obtained by displaying the binarized reference image with values  $gl=255$  and  $gl=0$ , and the binarized scene with  $gl=119$  and  $gl=0$ . To obtain the threshold function ( $I_T$ ) we have displayed separately both images with the respective aforementioned grey levels.

After computing  $I_{BS}$  (Equation 6), we display it on the LCD with a configuration obtained with elliptically polarized light that gives  $0-\pi$  phase-only modulation (Figure 5). The two grey levels chosen to display  $I_{BS}$  are  $gl=0$  and  $gl=255$  because they have approximately the same transmittance and a relative phase shift of  $\pi$  radians between them.



**Figure 4.** High-contrast configuration.



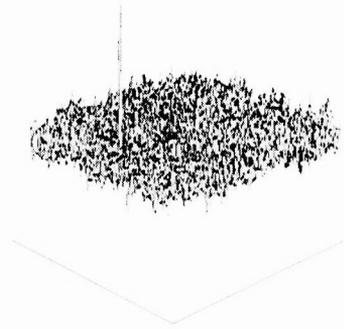
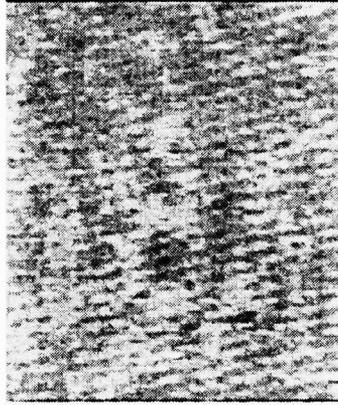
**Figure 5.**  $0-\pi$  phase-only configuration.

Displaying  $I_{BS}$  on the LCD and performing a second Fourier transformation, we have achieved the desired final correlation by adding the suitable plane wave at the output plane before it has been captured by the CCD camera. Figures 6, 7 and 8 show the correlation planes and their respective 3D plots when each satellite is detected. It can be seen that the detection peaks are higher than the background whose mean value is approximately half the value of the detection peak. Removing this background the detection peaks can be seen clearer than before, as shown in Figures 9, 10 and 11.

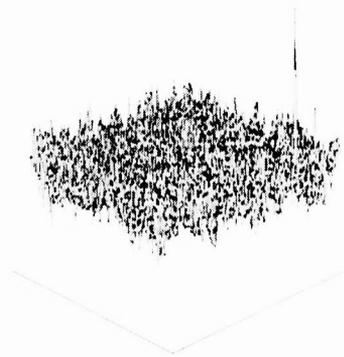
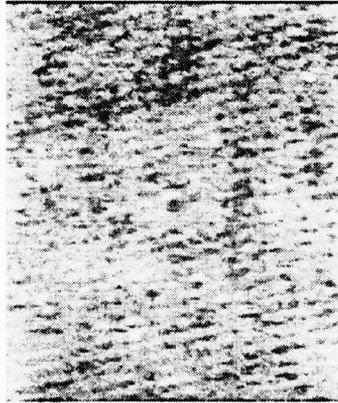
If we compare these present results with the ones achieved with the four-level spectrum we see that they are similar but now we have increased the resolution of the panels, we have a pixel by pixel addressing of the LCDs, the needed operating curves are less restrictive than before, and we have reduced the required power spectra to compute the final intensity distribution from five to three.

## 6. CONCLUSIONS

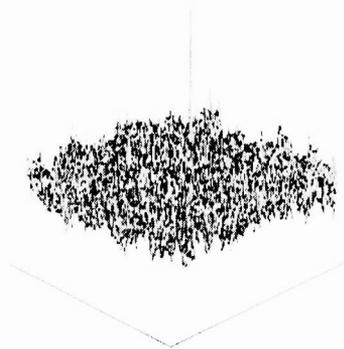
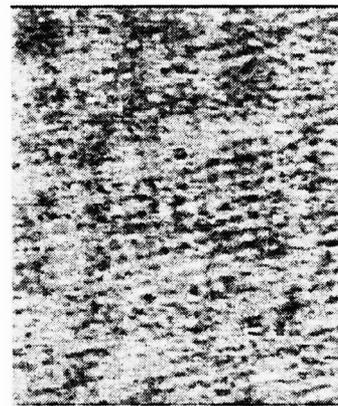
We have presented a modification of a previous method to obtain a single centered correlation term. In this new approach, the requirements in the configurations of the LCD are more flexible than before because only two grey levels are needed. Therefore, now we can use modulators with less than  $2\pi$  phase-only modulation. However, the modified method requires an optical setup based on an interferometric architecture, involving an accurate experimental adjustment. Comparing this method with the previous one, we have reduced the number of power spectra required to compute the final distribution from five to three, and as a consequence of using VGA panels, we have increased the devices resolution and we have a real pixel by pixel addressing of the modulators. Finally, we have carried out an experimental verification of this new approach achieving good detection results with the proposed interferometric setup.



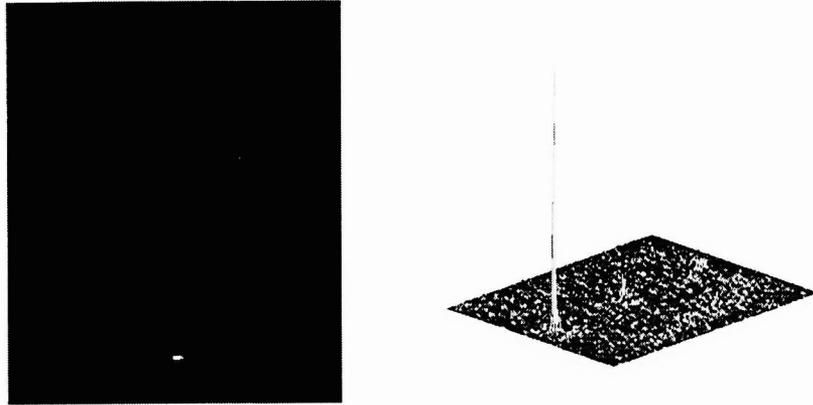
**Figure 6.** Total correlation plane. Detection of the bottom satellite.



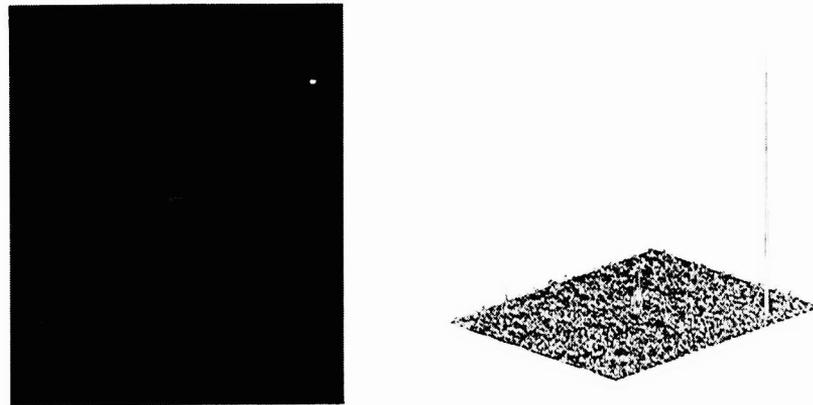
**Figure 7.** Total correlation plane. Detection of the upper right satellite.



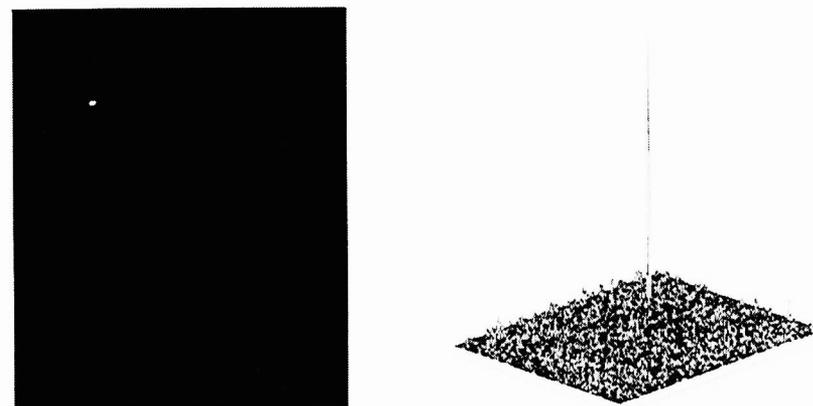
**Figure 8.** Total correlation plane. Detection of the upper left satellite.



**Figure 9.** Total correlation plane with background removed. Detection of the bottom satellite.



**Figure 10.** Total correlation plane with background removed. Detection of the upper right satellite.



**Figure 11.** Total correlation plane with background removed. Detection of the upper left satellite.

## ACKNOWLEDGMENTS

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