Eur. Phys. J. C (2016) 76:228
DOI 10.1140/epjc/s10052-016-4060-6

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Thermodynamical aspects of running vacuum models

J. A. S. Lima[1,a], **Spyros Basilakos**[2,b], **Joan Solà**[3,c]

[1] Departamento de Astronomia, Universidade de São Paulo, Rua do Matão 1226, 05508-900 São Paulo, Brazil
[2] Academy of Athens, Research Center for Astronomy and Applied Mathematics, Soranou Efessiou 4, 11-527 Athens, Greece
[3] High Energy Physics Group, Dept. d'Estructura i Constituents de la Matèria, Institut de Ciències del Cosmos (ICC), Univ. de Barcelona, Av. Diagonal 647, 08028 Barcelona, Catalonia, Spain

Received: 17 February 2016 / Accepted: 1 April 2016 / Published online: 25 April 2016

Abstract The thermal history of a large class of running vacuum models in which the effective cosmological term is described by a truncated power series of the Hubble rate, whose dominant term is $\Lambda(H) \propto H^{n+2}$, is discussed in detail. Specifically, by assuming that the ultrarelativistic particles produced by the vacuum decay emerge into space-time in such a way that its energy density $\rho_r \propto T^4$, the temperature evolution law and the increasing entropy function are analytically calculated. For the whole class of vacuum models explored here we find that the primeval value of the comoving radiation entropy density (associated to effectively massless particles) starts from zero and evolves extremely fast until reaching a maximum near the end of the vacuum decay phase, where it saturates. The late-time conservation of the radiation entropy during the adiabatic FRW phase also guarantees that the whole class of running vacuum models predicts the same correct value of the present day entropy, $S_0 \sim 10^{87}$–10^{88} (in natural units), independently of the initial conditions. In addition, by assuming Gibbons–Hawking temperature as an initial condition, we find that the ratio between the late-time and primordial vacuum energy densities is in agreement with naive estimates from quantum field theory, namely, $\rho_{\Lambda 0}/\rho_{\Lambda I} \sim 10^{-123}$. Such results are independent on the power n and suggests that the observed Universe may evolve smoothly between two extreme, unstable, non-singular de Sitter phases.

1 Introduction

A non-singular early de Sitter phase driven by a decaying vacuum energy density was phenomenologically proposed long ago to solve some problems of the Big-Bang cosmology [1,2]. The basic idea is closely related to early attempts aimed at solving (or at least alleviating) some cosmic mysteries, such as the "graceful exit" problem, which plague many inflationary scenarios (for recent reviews see [3,4]), and also the CCP or cosmological constant problem [5–11], probably, the deepest conundrum of all inflationary theories describing the very early Universe.

Nevertheless, new theoretical developments are suggesting a possible way to circumvent such problems. Results based on the renormalization group (RG) theoretical techniques of quantum field theory (QFT) in curved space-times combined with some phenomenological insights provided a set of dynamical Λ(H)-models (or running vacuum cosmologies) described by an even power series of the Hubble rate [12–18] (cf. [11] for a review). In this line, we have discussed in a series of recent works a unified class of models accounting for a complete cosmological history evolving between two extreme (primeval and late time) de Sitter phases whose space-time dynamics is supported by a dynamical decaying (or running) vacuum energy density [19–25]. In such models, the effective vacuum energy density is a truncated power series of the Hubble rate, whose dominant term is $\Lambda(H) \propto H^{n+2}$, where for the sake of generality the power $n > 0$. Unlike several inflationary variants endowed with a preadiabatic phase, this decaying vacuum is responsible for an increasing entropy evolution since the very early Universe, described by the primeval non-singular de Sitter space-time.

Several theoretical and observational properties of this large class of non-singular running vacuum scenarios have been discussed in the literature. In particular, Mimoso and Pavón shown their thermodynamic consistency based on the generalized second law of thermodynamics (GSLT) by taking into account quantum corrections to the Bekenstein–Hawking entropy [26]. Many details regarding the late-time dynamics can be found in Refs. [27–33], and especially in

[a] e-mail: jas.lima@iag.usp.br
[b] e-mail: svasil@academyofathens.gr
[c] e-mail: sola@ecm.ub.edu

the recent, updated, and very comprehensive works of Refs. [34,35]. More recently, even the solution of the coincidence problem has been discussed in detail in the present framework [36], as well as in generic decaying vacuum cosmologies [37,38].

Finally, let us mention that the running vacuum models under study have recently been tested against the wealth of SNIa + BAO + H(z) + LSS + BBN + CMB data—see [39] for a short review—and they turn out to provide a quality fit that is significantly better than the ΛCDM. This fact has become most evident in the recent works [40,41], where it is shown that the significance of the fit improvement is at $\sim 4\sigma$ c.l. Therefore, there is plenty of motivation for further investigating these running vacuum models from different perspectives, with the hope of finding possible connections with fundamental aspects of the cosmic evolution

In the present work, we focus our attention on the entropy of the cosmic microwave background radiation (CMBR) generated by this large class of non-singular decaying vacuum cosmology. By considering that the decaying vacuum process occurs under adiabatic conditions [in the sense that the specific entropy (per particle) is preserved] this means that the radiation produced satisfies the standard scaling laws, namely, $n_r \propto T^3$ and $\rho_r \propto T^4$ [43]. Under these conditions, the final value of the entropy produced by the decaying vacuum supporting the unstable primeval de Sitter phase is exactly the present radiation entropy existing within the current Hubble radius. Within this framework, the early decaying vacuum process is not plagued with "graceful exit" problem of most inflationary variants and generates the correct number, $S_0 \simeq 10^{88}$ [42], regardless of the power n present in the phenomenological decaying Λ (H)-term. In addition, the ratio between the primeval and the present day vacuum energy densities is $\rho_{\Lambda 0}/\rho_{\Lambda I} \simeq 10^{-123}$, as required by some naive estimates from quantum field theory.

The article is structured in the following manner. In Sect. 2 we justify the phenomenological decaying vacuum law adopted in the paper, whereas in Sect. 3 we set up the basic set of equations and the transition from the early de Sitter to the radiation phase is addressed. How inflation ends and the temperature evolution law are presented in Sect. 4, while in Sect. 5 we discuss the entropy production generated by the decaying vacuum medium. Finally, the main conclusions are summarized in Sect. 6.

2 General model for a complete cosmic history

The general $\Lambda(H)$-scenario accounting for a complete chronology of the Universe (from de Sitter to de Sitter) is based on the following expression for the dynamical cosmological term defining the relevant class of running vacuum models under consideration [19–21]:

$$8\pi G\rho_\Lambda \equiv \Lambda(H) = c_0 + 3\nu H^2 + 3\alpha \frac{H^{n+2}}{H_I^n}. \tag{2.1}$$

Here $H \equiv \dot{a}/a$ is the Hubble rate, $a = a(t)$ is the scale factor, and the over-dot denotes the derivative with respect to the cosmic time t. By definition $\rho_\Lambda(H) = \Lambda(H)/8\pi G$ is the corresponding vacuum energy density (G being Newton's constant). The even powers of H (therefore $n = 2, 4, \ldots$) are thought to be of more fundamental origin due the general covariance of the effective action, as required by the QFT treatment in curved space-time [11–18]. In the numerical analysis, however, we will explore also the cases $n = 1, 3$ and 4 for comparison.

The dimensionless free parameters α and ν have distinct status. The first can be absorbed (for each value of n in the arbitrary scale H_I so that it can be fixed to unity without loss of generality (if the scale of inflation is not precisely known) [24,36], whereas ν has been determined from observations based on a joint likelihood analysis involving SNe Ia, baryonic acoustic oscillations (BAO), and cosmic background radiation (CMB) data, with the result $|\nu| \equiv \mathcal{O}(10^{-3})$ [27–35]—see especially the most recent analyses in which the $\nu = 0$ result (associated to the ΛCDM in the post inflationary time) is excluded at $\sim 4\sigma$ c.l. [40,41]. The small value of ν is natural since at late times, the dynamical model of the vacuum energy cannot depart too much from ΛCDM. In this connection, by using a generic grand unified theory (GUT), it has been shown that $|\nu| \sim 10^{-6}$–10^{-3} [17]. Finally, the constant c_0 with the same dimension of Λ yields the dominant term at very low energies, when H approaches the measured value H_0 (from now on the index "0" denotes the present day values of the quantities).

3 Basic equations of the $\Lambda(H)$ model

It is well known that the Einstein field equations (EFE) are valid either for a strictly constant Λ or a dynamical one [19,21]. Therefore, using the vacuum energy density $\rho_\Lambda = \Lambda/(8\pi G)$ and the nominal pressure law $p_\Lambda = -\rho_\Lambda$ one can write the EFE in the framework of a spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) space-time

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \tilde{T}_{\mu\nu} \tag{3.1}$$

where R is the Ricci scalar and $\tilde{T}_{\mu\nu} = (p_m + \rho_m) u_\mu u_\nu - (p_m - \rho_\Lambda) g_{\mu\nu}$ is the total energy-momentum tensor and the index m refers to the dominant fluid component (nonrelativistic matter or radiation). Obviously, in our case, the only difference with respect to the more conventional field equations is the fact that $\Lambda = \Lambda(H)$. In this framework, the local energy-conservation law, $\nabla^\mu \tilde{T}_{\mu\nu} = 0$, which ensures the covariance of the theory, reads

$$\dot{\rho_\Lambda} + \dot{\rho}_m + 3(1+\omega)\rho_m H = 0, \quad (3.2)$$

where we have used $p_m = \omega\rho_m$ for the ordinary cosmic fluid, namely $\omega = 0$ for dust and $\omega = 1/3$ for radiation. In this enlarged framework, the Friedmann equations are given by

$$8\pi G\rho_{\rm T} \equiv 8\pi G\rho_m + \Lambda(H) = 3H^2, \quad (3.3)$$
$$8\pi G p_{\rm T} \equiv 8\pi G P_m - \Lambda(H) = -2\dot{H} - 3H^2. \quad (3.4)$$

By combining the above equations with the class of vacuum models (2.1) one obtains the equation driving the evolution of the Hubble parameter:

$$\dot{H} = -\frac{3}{2}(1+\omega)H^2\left(1 - \nu - \frac{c_0}{3H^2} - \alpha\frac{H^n}{H_I^n}\right). \quad (3.5)$$

A solution of this equation in the high energy regime [where the term $c_0/H^2 \ll 1$ of (3.5) can be neglected] is given by

$$H(a) = \frac{\tilde{H}_I}{[1 + D\,a^{3(1+\omega)\,n\,(1-\nu)/2}]^{1/n}}, \quad (3.6)$$

where $\tilde{H}_I = \left(\frac{1-\nu}{\alpha}\right)^{1/n} H_I$.

The combination of the EFE and the expression for Λ yields

$$\rho_\Lambda(a) = \tilde{\rho}_I\,\frac{1 + \nu\,D\,a^{3n(1+\omega)(1-\nu)/2}}{[1 + D\,a^{3n(1+\omega)(1-\nu)/2}]^{1+2/n}}, \quad (3.7)$$

$$\rho_r(a) = \tilde{\rho}_I\,\frac{(1-\nu)D\,a^{3n(1+\omega)(1-\nu)/2}}{[1 + D\,a^{3n(1+\omega)(1-\nu)/2}]^{1+2/n}}, \quad (3.8)$$

$$\rho_{\rm T}(a) = \tilde{\rho}_I\,\frac{1}{\left[1 + D\,a^{3n(1+\omega)(1-\nu)/2}\right]^{2/n}}. \quad (3.9)$$

The quantity $\tilde{\rho}_I$ in the above equations is the critical energy density defining the primeval de Sitter stage

$$\tilde{\rho}_I \equiv \frac{3\tilde{H}_I^2}{8\pi G}. \quad (3.10)$$

3.1 From initial de Sitter stage to radiation phase

For $\omega = 1/3$ the energy density for the vacuum and radiation read

$$\rho_\Lambda(a) = \tilde{\rho}_I\,\frac{1 + \nu\,D\,a^{2n(1-\nu)}}{[1 + D\,a^{2n(1-\nu)}]^{1+2/n}}, \quad (3.11)$$

$$\rho_r(a) = \tilde{\rho}_I\,\frac{(1-\nu)D\,a^{2n(1-\nu)}}{[1 + D\,a^{2n(1-\nu)}]^{1+2/n}}, \quad (3.12)$$

We can see from (3.11) that the value (3.10) just provides the vacuum energy density for $a \to 0$, namely $\rho_\Lambda(0) = \tilde{\rho}_I$. As $|\nu| \ll 1$ we also see that $\tilde{\rho}_I/\rho_I \sim \alpha^{-2/n}$, thereby effectively showing that the constant α can be absorbed in the scale H_I, as remarked before. Let us also emphasize from the previous formula that for $a \to 0$ we have $\rho_r/\rho_\Lambda \propto a^{2n(1-\nu)} \to 0$, i.e. the very early Universe is indeed vacuum dominated with a negligible amount of radiation. In the rest of the paper, we neglect the effects proportional to ν (which, since it is the coefficient of H^2, is not essential for the study of the early Universe, the epoch where the H^4-term is fully dominant). Thus, without loss of generality, H_I will be hereafter rescaled so that $\alpha = 1$ and we set $\nu = 0$ in all the formulas. Within this framework, we have $\tilde{H}_I = H_I$ and Eq. (3.10) becomes

$$\tilde{\rho}_I \equiv \rho_I = \frac{3H_I^2}{8\pi G}. \quad (3.13)$$

In addition, from Eq. (3.6) it follows that the scale factor of the Universe takes on an exponential form $a(t) \sim a_i e^{H_I t}$ as long as the condition $Da^{3(1+\omega)} \ll 1$ is fulfilled. Obviously, this means that the Universe is initially driven by a pure non-singular de Sitter vacuum state, and therefore is inflating. However, the above mentioned de Sitter inflationary phase is only ephemeral. Indeed it is easy to check that in the post-inflationary regime, i.e. for $a \gg a_i$ (with $Da^{3n(1+\omega)} \gg 1$), we are led to $H \propto a^{-2}$ (or $a \propto t^{1/2}$) for $\omega = 1/3$ [see Eq. (3.6)]. Therefore, the present model evolves smoothly from inflation toward the conventional radiation stage, thereby insuring that the initial very large amount of vacuum energy density does not preclude the standard picture of the primordial Big-Bang nucleossynthesis.

On the other hand, using Eq. (3.5) one may check that the decelerating parameter, $q = -\ddot{a}/aH^2 = -1 - \dot{H}/H^2$, for arbitrary values of the power index n, reads

$$q(H) = 2\left[1 - \left(\frac{H}{H_I}\right)^n\right] - 1, \quad (3.14)$$

Naturally, the existence of the radiation stage is not enough to identify precisely the end of inflation since the deceleration parameter varied from $q = -1$ (de Sitter) to $q = 1$ (radiation) and the inflationary period finished when $q = 0$, or equivalently, $\ddot{a} = 0$. We will discuss this point in the next section.

3.2 When exactly inflation ends?

In order to answer this question we first combine Eqs. (3.11) and (3.12) so as to obtain the ratio of the radiation energy density ($\omega = 1/3$) to the vacuum energy density:

$$\frac{\rho_r(a)}{\rho_\Lambda(a)} = Da^{2n}. \quad (3.15)$$

In principle, inflation must end when both components—the decreasing vacuum energy density and the created radiation energy density—contribute alike. Assuming that the scale factor at the point of vacuum–radiation "equality" is $a = a_{\rm eq}$ the above expression implies that $D\, a_{\rm eq}^{2n} = 1$. This relation enables us to rewrite the Hubble parameter (3.6) in the following way:

$$H(a,n) = \frac{H_I}{\left[1 + \left(a/a_{\rm eq}\right)^{2n}\right]^{1/n}}. \tag{3.16}$$

It follows that the value of the Hubble function at the vacuum–radiation equality depends on the value of the parameter n and the initial scale H_I:

$$H_{\rm eq} \equiv H(a_{\rm eq}) = \frac{H_I}{2^{1/n}}. \tag{3.17}$$

Now, by inserting this value of $H_{\rm eq}$ into Eq. (3.14) we obtain effectively that $q = 0$ as should be expected. Hence, once the arbitrary scale H_I is fixed, the energy scale or the moment for which the inflation ends is readily defined.

As remarked before, the start of the radiation phase in this model is not characterized by the canonical radiation value $q = 1$, as one might have naively expected. Due to the continuous energy exchange between vacuum and radiation there is a short period of time in which q goes from $q = 0$ to the standard result $q = 1$. In the begin of the standard adiabatic radiation regime the deceleration parameter is written as

$$q(H_{\rm rad}) = 2\left[1 - \left(\frac{H_{\rm rad}}{H_I}\right)^n\right] - 1. \tag{3.18}$$

In the approach to the radiation phase one may safely assume that it started when $q(H_{\rm rad}) \sim 0.9999$ with the Hubble parameter given by $H_{\rm rad} = H_I/(2\times 10^4)^{1/n}$ [see Eq. (3.18)], and from Eq. (3.16) we obtain $a_{\rm rad}/a_{\rm eq} \simeq (2\times 10^4)^{1/2n}$.

At this point it is appropriate to make the following comments concerning the cosmic history. First, in the full radiation era the value of the scale factor of the Universe $a_{\rm rad}$ becomes at least one order of magnitude larger than the corresponding value at the vacuum–radiation equality. Actually, the total entropy generated by the vacuum decaying process does not depend on the exact value of the ratio $a_{\rm rad}/a_{\rm eq}$. Second, when the radiation epoch is well left behind, the Universe goes into the cold dark matter dominated era (Einstein–de Sitter, $a(t) \propto t^{2/3}$); and, after some billion years (∼7 Gyrs) it enters the present vacuum dominated phase, in which $\Lambda \simeq \tilde{\Lambda}$ =const—confer [29,34,40,41].

In the upper panel of Fig. 1 we provide the evolution of the normalized Hubble parameter $E(a,n) = H(a,n)/H_I$ for the following vacuum models, namely $n = 1$ (dashed line), $n = 2$ (solid line), $n = 3$ (long dashed line) and $n = 4$ (dotted line) [see Eq. (2.1)]. We observe that for $a \ll a_{\rm eq}$ the cosmic evolution begins from an unstable inflationary phase [early de Sitter era, $H \simeq H_I$] powered by the huge value H_I presumably connected to the scale of a Grand Unified Theory (GUT). Obviously, when the primeval inflationary era is left behind, particularly for $a \gg a_{\rm eq}$, the cosmic evolution enters smoothly in the standard radiation period $H \propto a^{-2}$. Overall, we would like to emphasize that the above natural mechanism for graceful exit is universal for the whole class of vacuum models which obey the restriction $n > 0$. Subsequently when the c_0/H^2 quantity in Eq. (3.5) starts to dominate over H^n/H_I^n (where $H \ll H_I$) the radiation component becomes sub-dominant and the matter dominated era appears. This implies that Eq. (2.1) reduces to $\Lambda(H) = \tilde{\Lambda} + 3\nu(H^2 - H_0^2)$, which generalizes the traditional ΛCDM model. In this case the vacuum at the present time (cosmological constant) becomes $\tilde{\Lambda} = 3c_0 + 3\nu H_0^2$, where H_0 is the Hubble constant. More details regarding the late-time dynamics can be amply found in Refs. [27–29,32,33], and especially in the recent, updated, and very comprehensive works of Refs. [34,35].

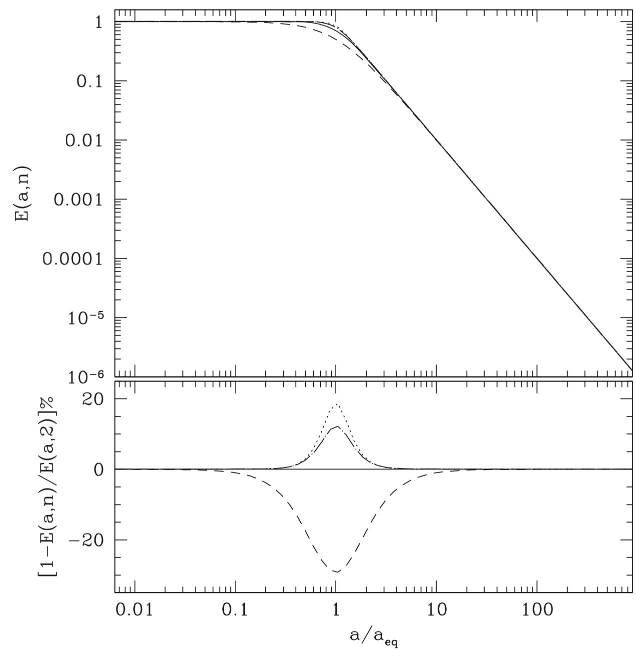

Fig. 1 *Upper panel* Evolution of the normalized Hubble parameter $E(a,n) = H(a,n)/H_I$ during the inflationary epoch and its transition into the FLRW radiation era. The Hubble parameter is normalized with respect to H_I, and the scale factor with respect to $a_{\rm eq}$, the value for which $\rho_\Lambda = \rho_r$ (see the text). The *lines* correspond to the following $\Lambda(H) \propto H^{n+2}/H_I^n$ scenarios [see Eq. (2.1)], namely $n = 1$ (*dashed*), $n = 2$ (*solid*), $n = 3$ (*dot-dashed*) and $n = 4$ (*dotted*). *Lower panel* We provide the relative deviation $[1 - E(a,n)/E(a,2)]$ % of the normalized Hubble parameter for the $n = 1, 3, 4$ vacuum models with respect to $n = 2$

Finally, by using $n = 2$ (corresponding to $\Lambda(H) \propto H^4/H_I^2$ in the early Universe) as a fiducial model in the large class (2.1), we can appreciate in the bottom panel of Fig. 1 the relative deviation of the normalized Hubble parameters

$E(a, n)$ with respect to the $n = 2$ solution $E(a, 2)$. Obviously, when we are far away from the epoch of the vacuum–radiation equality the deviations from the fiducial $E(a, 2)$ case are extremely small. On the other hand there is a visible deviation from the latter around the transition region $a \to a_{\rm eq}$. This deviation becomes at the level of $\sim -30\,\%$ and $\sim +20\,\%$ for $n = 1$ and $n > 2$ respectively.

4 Radiation and its temperature evolution law

Assuming an "adiabatic" decaying vacuum it has been found that the specific entropy of the produced massless particles remains constant, despite the fact that the total entropy can be increasing.[1] This implies that the energy and the number density as a function of the temperature are given by the standard expressions, namely: $\rho_r \propto T_r^4$ and $n_r \propto T_r^3$, but the temperature does not obey the scaling relation $T_r(t) \sim a(t)^{-1}$. Such results were first derived based on a covariant nonequilibrium thermodynamic description [43,44]), and, more recently, using a kinetic theoretic approach [45,46]. In the following, we discuss the temperature evolution law in the present framework along these lines.

Let us start with Eq. (3.12), which is rewritten in terms of $a_{\rm eq}$ as follows:

$$\rho_r = \rho_I \frac{(a/a_{\rm eq})^{2n}}{\left[1 + (a/a_{\rm eq})^{2n}\right]^{1+2/n}} = \frac{\pi^2}{30} g_* T_r^4, \qquad (4.1)$$

where in the last equality we included the degrees of freedom (d.o.f.) of the created massless modes through the g_*-factor (see [42]). Now, by solving for the temperature we find

$$T_r = 2^{\frac{n+2}{4n}}\, T_{\rm eq} \frac{(a/a_{\rm eq})^{n/2}}{\left[1 + (a/a_{\rm eq})^{2n}\right]^{\frac{n+2}{4n}}}, \qquad (4.2)$$

where $T_{\rm eq} = T_r(a_{\rm eq})$. Obviously, $T_{\rm eq}$ is the maximum value of the radiation temperature (4.2) which is defined by

$$T_{\rm eq} = \left(\frac{15\,\rho_I}{2\pi^2 g_*}\right)^{1/4} = \left(\frac{45\,H_I^2}{16\pi^3 G g_*}\right)^{1/4}. \qquad (4.3)$$

As expected, $T_{\rm eq}$ is given in terms of the arbitrary initial scale of the primeval de Sitter phase (ρ_I, or equivalently H_I). Unlike the value of $H_{\rm eq}$ (see Eq. (3.17)), this value of $T_{\rm eq}$ is valid for the whole class of models since is does not depend on the power n. In particular, this unique maximum temperature suggests that the total entropy generated within the horizon and presently observed is basically the same for all models (see next section).

In the top panel of Fig. 2 we present the temperature evolution for several values of n. Note that in the very beginning of the evolution (when $a \mapsto 0$), the temperature of the created photons is also zero in accordance with the fact that $\rho_r = 0$ (see Eq. (4.1)). However, for finite values of $a \neq 0$, we observe the existence of two regimes. In the first, $a \ll a_{\rm eq}$, the radiation temperature (4.2) increases as $T_r \propto a^{n/2}$ (it is linear for $n = 2$ [23]), reaching of course its maximum value at $a = a_{\rm eq}$. In this non-adiabatic regime, the vacuum instability guides constantly the model to the standard radiation epoch from a process that started in the non-singular de Sitter phase. Note that the evolution is different from inflationary models where a highly non-adiabatic "reheating" process happens immediately after the adiabatic evolution of the inflaton field (see [42] and references therein). In the opposite regime, $a \gg a_{\rm eq}$, we are well within the radiation epoch when all running vacuum models decrease in the classical way, that is, following an adiabatic scaling law, $T_r \propto a^{-1}$. As we shall discuss below, the power-law increasing of the temperature up to the vacuum–radiation equality, beyond which the Uni-

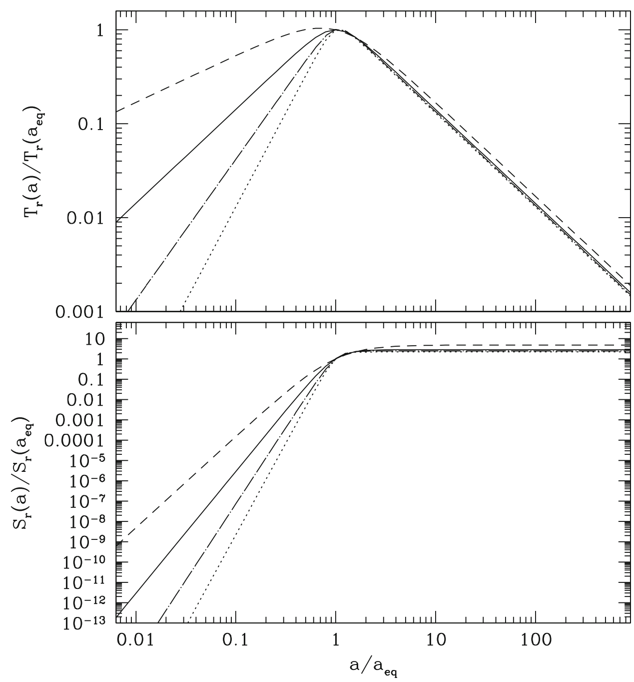

Fig. 2 *Top panel* The evolution of the radiation temperature (normalized with respect to its maximum value) during the inflationary period for several values of the free parameter n (see caption of Fig. 1 for definitions) and its transition into the FLRW radiation era. As in Fig. 1, the *lines* correspond to the following scenarios, namely $n = 1$ (*dashed*), $n = 2$ (*solid*), $n = 3$ (*dot-dashed*) and $n = 4$ (*dotted*). *Bottom panel* The evolution of the normalized comoving entropy from the inflationary period (where it increases) until reaching the saturation plateau for $a/a_{\rm rad} \geq 1$. The asymptotic value corresponds the total entropy at present

[1] More precisely, the constancy of the specific entropy (per particle) of the produced particles defines the "adiabatic" vacuum decaying process (see Refs. [43,44]).

verse enters the standard temperature regime, is the reason of the large radiation entropy observed in the present epoch.

For the purpose of the present study it is also important to find a way to calculate the primeval parameters (ρ_I, H_I) from first principles and indeed one may use the following approaches (from now on we consider natural units, $\hbar = k_B = c = 1$). A first possibility is to consider that the initial de Sitter energy density ρ_I is related with the Planck scale M_P through $\rho_I = M_P^4$. An alternative approach is to connect ρ_I to the GUT energy scale $M_X \sim 10^{16}$ GeV and thus $\rho_I = M_X^4$ (see Refs. [24,25]). A third possibility is to use the event horizon (EH) of the de Sitter space-time. From the analysis of Gibbons and Hawking [47] we know that the temperature of the de Sitter EH in natural units is $T_{\rm GH} = H_I/2\pi$, where H_I is the (constant) Hubble parameter at the de Sitter epoch. In our case, following the third approach and combining the last equality of Eqs. (4.1) and (3.13) we obtain

$$T_I = \frac{H_I}{2\pi} = \left(\frac{45}{\pi g_*}\right)^{1/2} M_P, \quad \rho_I = \frac{135}{2g*} M_P^4. \tag{4.4}$$

Then inserting the above expressions into Eq. (4.3) it is easy to show that the characteristic radiation temperature $T_{\rm eq}$ becomes

$$T_{\rm eq} = \left(\frac{45}{2\pi\; g_*}\right)^{1/2} M_P. \tag{4.5}$$

It should be stressed that all the above characteristic scales are slightly below the corresponding Planck scale, which means that we are inside in the semi-classical QFT regime. For instance, by taking $g_* = 106.75$, which corresponds to the particle content of the standard model of particle physics we find from Eq. (4.5) that $T_{\rm eq} \simeq 0.26\, M_P$ which as expected does not depend on the power n. In this respect if we take into account the number of light d.o.f in the GUT then we find that the characteristic temperature is still smaller (nearly 10% of Planck mass). At this point it should be stressed that the primordial Gibbons–Hawking thermal bath was used only as a peculiar initial condition to fix the arbitrary scale H_I. As we shall see in the next section, the ratio between the very early- and late-time vacuum energy densities depends only on the pair (H_I, H_F) characterizing the extreme de Sitter phases. It has the expected magnitude thereby also contributing to alleviate the so-called cosmological constant problem in the context of the such models (see Eq. (5.7)).

5 The radiation entropy

The total entropy of the radiation included in the present Hubble radius ($d_H \simeq H_0^{-1}$) reads

$$S_0 = \frac{2\pi^2}{45}\, g_{s,0}\, T_{r0}^3\, H_0^{-3} \simeq 2.3h^{-3} 10^{87} \sim 10^{87} - 10^{88}, \tag{5.1}$$

where $T_{r0} \simeq 2.725\,^\circ$K $\simeq 2.35 \times 10^{-13}$ GeV is the CMB temperature at the present time and $H_0 = 2.133\, h \times 10^{-42}$ GeV (with $h \simeq 0.67$) is the present day Hubble parameter.[2]

In order to check the viability of our model we need to take into account that the total entropy should be measured from the initial entropy generated by the decaying vacuum. Since in our vacuum model the BBN proceeds fully standard, the equilibrium entropy formula remains valid because only the temperature law is modified [43–46]. Therefore, the radiation entropy per comoving volume is given by the well-known expressions

$$S_r \equiv \left(\frac{\rho_r + p_r}{T_r}\right) a^3 \equiv \frac{4}{3}\frac{\rho_r}{T_r} a^3 = \frac{2\pi^2}{45} g_s T_r^3 a^3, \tag{5.2}$$

where g_s is the entropy factor at temperature T_r (at very high temperature g_s is essentially equal to the effective number of massless species, g_*. However, for lower values there is a correction related to the freeze out of neutrinos and electron–positron annihilation).

With the help of Eq. (4.2) the comoving entropy (5.2) can be expressed as a function of the scale factor as follows:

$$S_r = \frac{2^{(7n+6)/4n} \pi^2 g_s}{45}\, T_{\rm eq}^3 a_{\rm eq}^3 f_n(r), \tag{5.3}$$

where $r \equiv a/a_{\rm eq}$ and the function $f_n(r)$ depends on the parameter n as given below:

$$f_n(r) = \frac{r^{\frac{3(n+2)}{2}}}{(1 + r^{2n})^{\frac{3(n+2)}{4n}}}. \tag{5.4}$$

The obtained result for the comoving entropy boils down to the one derived in Ref. [23] for $n = 2$, as it should. Note also that $\lim_{a\to 0} S_r = 0$, as it ought to be expected from the fact that the initial de Sitter state is supported by a pure vacuum (no radiation fluid). We also see that during the inflationary phase ($a \ll a_{\rm eq}$), that is, at the early stages of the evolution, the total comoving radiation entropy of the Universe increases very fast; in fact, proportional to $a^{3(1+n/2)}$. For instance, for $n = 2$ (corresponding to H^4-driven inflation) the initial entropy raises as $\sim a^6$. Note also that for $a = a_{\rm eq}$, $f_n(r = 1) = 1/8^{(n+2)/4n}$ and the associated value $S(a_{\rm eq})$ is still not the total comoving entropy that the decaying vacuum is able to generate (see discussion

[2] Following standard lines, we have assumed the coefficient $g_{s,0} = 2 + 6 \times (7/8)\left(T_{\nu,0}/T_{r0}\right)^3 \simeq 3.91$ is the entropy factor for the light d.o.f. at the present epoch, in which we have used $T_{\nu,0}/T_{r0} = (4/11)^{1/3}$ [42].

below Eq. (3.14)). This occurs only when $a = a_{\rm rad}$ so that $r^{2n} = (a_{\rm rad}/a_{\rm eq})^{2n} \gg 1$ and $f_n(r) \simeq 1$ for all practical purposes. At this point the generated comoving entropy S_r reaches the final value, $S_{\rm rad}^f = S_r(a_{\rm rad})$, when the standard radiation phase begins.

It thus follows that the asymptotic (*adiabatic*) value of (5.3), defined by $f(r) \simeq 1$ for $r \gg 1$, is given by:

$$S_r \to S_{\rm rad}^f = \left(\frac{2^{(7n+6)/4n}\pi^2 g_s}{45}\right) T_{\rm eq}^3 a_{\rm eq}^3, \tag{5.5}$$

a saturated value that must be compared with the present day entropy since the subsequent evolution of the model is isentropic.

In the bottom panel of Fig. 2 we show the entropy as a function of the ratio $r = a/a_{\rm eq}$ for several values of n. Notice that the entropy is scaled to its value at the vacuum–radiation equality. Initially, for $a \ll a_{\rm eq}$ the amount of entropy differs among the vacuum models but when $a \to a_{\rm eq}$ the corresponding entropies start to converge and subsequently they reach a plateau, namely $S_r(a)/S_r(a_{\rm eq}) \to 8^{(n+2)/4n}$ for $a \gg a_{\rm eq}$, which characterizes the standard adiabatic phase, which is sustained until the present days because the bulk of the vacuum energy $\Lambda(H) \propto H^{n+2}/H_I^n$ already decayed.

Armed with the above expressions we now compute the prediction of the total entropy inside the current horizon volume $\sim H_0^{-3}$. Using the temperature evolution law [see Eq. (4.2)] one may see that the expression $T_{\rm eq}^3 a_{\rm eq}^3$, which appears in the final entropy as given by (5.5), is equal to $2^{-3(n+2)/4n} T_{\rm rad}^3 a_{\rm rad}^3$, where $T_{\rm rad} = T_r(a_{\rm rad})$ and $a_{\rm rad}$ was defined in Sect. 3.1 [see discussion below Eq. (3.18)]. Note that the n-dependence cancels out and we arrive at the same final result (the power n is important only in the inflationary phase since it determines the time scale in which the radiation equilibrium phase is attained):

$$S_{\rm rad}^f = \left(\frac{2\pi^2 g_s}{45}\right) T_{\rm rad}^3\, a_{\rm rad}^3 = S_0, \tag{5.6}$$

where S_0 is given by (5.1). In the last step we used the entropy conservation law of the standard adiabatic radiation phase, which implies that $g_s\, T_{\rm rad}^3\, a_{\rm rad}^3 = g_{s,0}\, T_{r0}^3\, a_0^3$.

It should be stressed that in the very early de Sitter era, the radiation entropy is zero. However, it increases steadily as $S_r \sim a^{3(1+n/2)}$ and, finally, as shown in the bottom panel Fig. 2, deep inside the radiation stage becomes constant and approaching its asymptotic present day observed value, S_0.

In other words the running vacuum model provides an overall past evolution to the present ΛCDM cosmology by connecting smoothly between two extreme cosmic eras (early inflation and dark energy) driven by the vacuum medium [see Eq. (3.5)]. Specifically, using Eq. (4.4) we find that the ratio between the associated vacuum energy densities becomes

$$\frac{\rho_{\Lambda 0}}{\rho_{\Lambda I}} \equiv \frac{\rho_{\Lambda F}}{\rho_{\Lambda I}} = \frac{H_F^2}{H_I^2} = \frac{H_0^2\Omega_{\Lambda_0}}{H_I^2}$$
$$= \frac{g_*}{180\pi}\,\frac{H_0^2\Omega_{\Lambda_0}}{M_P^2} \mapsto \rho_{\Lambda 0} \simeq 10^{-123}\rho_{\Lambda I}\,, \tag{5.7}$$

in agreement with traditional estimates based on quantum field theory (see Refs. [5,7]). Note that the late time de Sitter scale, $H_F = H_0^2\Omega_{\Lambda_0}$, was used in the above expression [36]. To conclude, the decaying vacuum model explains the amount of the radiation entropy and simultaneously it also alleviates the so-called cosmological constant problem.

Needless to say, for a final resolution of the problem one needs to understand the ultimate origin of the current value of the cosmological constant. Remarkably, the obtained description of the cosmic history is based on a unified dynamical $\Lambda(H)$-term accounting for both the vacuum energy of the early and of the current Universe. An alternative approach to such description, based on the Grand Unified Theory framework, can be shown to provide similar results; see [24,25]. Somehow this shows that the obtained results are truly robust and independent of the initial conditions reigning in the primeval Universe.

6 Discussion and conclusions

In this paper we have addressed a fundamental issue concerning the thermodynamics of the early Universe. It is well known that within the context of the concordance cosmological model, or ΛCDM model, the thermal history is incomplete and it leads to inconsistencies with the present observations. One of the main problems is the well-known horizon problem, which can be rephrased thermodynamically as the entropy problem. The concordance model, in fact, cannot offer an explanation for the large entropy enclosed in our Hubble sphere, which is tantamount to say that it cannot explain the large amount of causally disconnected regions contained in it.

The well-known solution to these problems is inflation, a patch that has to be added to the incomplete ΛCDM description. While in the more traditional approach inflation is accomplished by postulating the existence of a new fundamental scalar field, called the inflaton, in the present work we have proposed an entirely different (but no less efficient) framework. It is based on the properties of a large class of non-singular decaying vacuum models whose structure is that of a truncated power series of the Hubble rate, $\Lambda(H)$. The involved powers to describe inflation must be higher than H^2 since the latter can be relevant only for the current Universe.

In this work we have explored an inflationary dynamics where the decaying vacuum is triggered by an arbitrary higher power of the Hubble rate, H^{n+2} $(n > 0)$ recently proposed [19–21].

In such a unified model of the vacuum energy, the effective cosmological term is a dynamical quantity, which evolves extremely fast in the early Universe and goes through an approximate de Sitter phase in our time—the dark energy epoch. In principle, the late time $\Lambda(H)$-Universe as described by Eq. (2.1) remains very close to the concordance model, but it stills carries a mild vacuum evolution (hence a mildly evolving cosmological term) compatible with observations. In principle, such a term may act as a smoking gun of its lengthy and energetic history; indeed, a much richer history than that associated to the idle Λ-term inherent to the ΛCDM model. However, since the late-time entropy production is very small, for the sake of simplicity we have taken the ν parameter equal to zero. Therefore, the model discussed here can be seen as a primeval non-singular phase of the standard ΛCDM model. Once we leave early times, the ν-parameter recovers an important role which, in point of fact, makes the running vacuum models not only compatible with the current cosmological data but fully competitive with the ΛCDM description [40,41].

We have studied in detail some important thermodynamical aspects of that class of dynamical vacuum models. Most noticeably we have focused on the issue of the radiation entropy, its origin, and generation in the first stages of the primeval Universe, and then its final impact on the current epoch. Our calculations were based on the assumption that the produced radiation from vacuum decay satisfy the standard relations, $n_r \propto T^3$ and $\rho_r \propto T^4$ [43], a hypothesis related with the idea of an "adiabatic" decaying vacuum and the fact that the specific entropy is preserved during the process [43]. The basic result is that at early times the temperature of the radiation increases ($T_r \propto a^{n/2}$) until its maximum value, determined by the equilibrium temperature $T_{\rm eq}$ of the vacuum–radiation transition (see Fig. 2) and the same happens with radiation energy density. As a consequence, the entropy also increases at very early stages ($S_r \propto a^{3(n+2)/2}$) being later on conserved during the radiation epoch (neglecting the photon entropy produced in the electron–positron annihilation). In this context we have found that the large amount of radiation entropy now ($S_0 \sim 10^{87}$–10^{88} in natural units) can be fully accountable in our dynamical vacuum context. We have first shown that the entropy was produced during the inflationary process itself at the expense of the continuous vacuum decay. Subsequently, its production stagnated and this occurred shortly after the vacuum had lost its energetic power and the Universe entered the standard radiation phase. From this point onwards the adiabatic evolution of the cosmos carried the comoving entropy unscathed until our days. Overall, the wide class of running $\Lambda(H)$-vacuum models provides not only an alternative scenario for inflation (beyond the traditional inflationary scalar field models, some of them in serious trouble after the analysis of *Planck* 2015 data [48]), but also a new clue for graceful exit, which is indeed fully guaranteed within the $\Lambda(H)$-cosmology context and does not depend on the power n of the Hubble rate. The remarkable independence of both the graceful exit and the entropy prediction from the power n singles out such class of dynamical vacuum models from the rest.

Finally, as originally pointed out, the thermodynamical solution ensures that no horizon problem exists because all points of the current Hubble sphere remained causally connected as of the early times when the huge entropy was generated by the decaying dynamics of the primeval vacuum. Interestingly enough, the above features are not only universal for the entire class of $\Lambda(H)$-models but also independent of the initial conditions of the early Universe. They provide a rather robust basis for the dynamical $\Lambda(H)$-cosmology, which is currently being tested and will continue being tested against the increasingly accurate observations.

Acknowledgments JASL is partially supported by the Brazilian Research Agencies, CAPES, CNPq, and FAPESP (PROCAD2013, INCT-A, and LLAMA projects). JS has been supported in part by FPA2013-46570 (MICINN), CSD2007-00042 (CPAN), 2014-SGR-104 (Generalitat de Catalunya) and MDM-2014-0369 (ICCUB). SB also acknowledges support by the Research Center for Astronomy of the Academy of Athens in the context of the program "*Tracing the Cosmic Acceleration*". JASL and SB are also grateful to the Department ECM (Universitat de Barcelona) for the hospitality and support when this work was being finished.

Funded by SCOAP^3.

A FRW radiation phase: Wien's law and the relation $T_{\rm rad}/T_I$

In this appendix a simple argument based on the equilibrium Wien's law is adopted to show that the decaying vacuum drives the model progressively to the radiation phase without reheating period (no exit problem).

To begin with we recall that the initial conditions in our picture are $T_r = 0$ and $T_{\rm GH} = T_I = H_I/2\pi$. However, due to the evolution of the Universe, the Hubble parameter and the horizon temperature diminish while the temperature of the created radiation increases due to the continuous vacuum decay.

The model evolves out of equilibrium because the entropy generation is concomitant with the inflationary process. In principle, the standard radiation FRW phase is reached when the Wien law becomes strictly valid. In the following we show that it provides a useful constraint on the value of the temperature in the begin of the FRW phase thereby suggesting the physical consistency of the model.

In natural units, the standard Wien's law ($\lambda_m T = 0.290$ cm K) reads:

$$\lambda_m T = 1.27. \tag{A.1}$$

In order to equalize the horizon temperature, the wavelength at the maximum black body intensity (in the begin of the FRW phase, $H = H_{\rm rad}$, $T = T_{\rm rad}$) is expected to be $\lambda_m < H_{\rm rad}^{-1}$. Hence, the Wien law takes the form

$$1.27 = \lambda_m T < T_{\rm rad}/H_{\rm rad}. \tag{A.2}$$

Now, in our model we know that $H_{\rm rad} \sim H_I/(2.10^4)^{1/n}$ (see Sect. 3.2) and $H_I = 2\pi T_I$. Thus, it follows that

$$1.27 < T_{\rm rad}/H_{\rm rad} < \frac{(2.10^4)^{1/n} T_{\rm rad}}{2\pi T_I}, \tag{A.3}$$

which can be rewritten as

$$\frac{T_{\rm rad}}{T_I} > \frac{7.98}{(2.10^4)^{1/n}} \tag{A.4}$$

The above relation shows two interesting aspects of the model: (i) $T_{\rm rad}$ is much smaller than $T_{\rm eq} = T_I/2^{1/n}$, but it can assume values not dramatically too low in comparison with the both characteristics scales (T_I and $T_{\rm eq}$); a result in agreement with the demonstrated progressive approach to the FRW phase (see Eq. (4.2) and the associated comments), and (ii) it also suggests that there is no exit problem in our model (the reheating process is not needed for this class of decaying vacuum models). For instance, by taking n= 1, 2 we see that $T_{\rm rad} > 4.10^{-4} T_I$, and $T_{\rm rad} > 0.056 T_I$, respectively. Note also that the inequality is still more safely satisfied for values of $n \leq 2$. This is comprehensible because inflation ends faster for higher values of n.

References

1. J.A.S. Lima, J.M.F. Maia, Phys. Rev. D **49**, 5597 (1994)
2. J.A.S. Lima, M. Trodden, Phys. Rev. D **53**, 4280 (1996)
3. A.H. Guth, D.I. Kaiser, Y. Nomura, Phys. Lett. B **733**, 112 (2014)
4. A. Linde, Lectures at the Les Houches School "Post-Planck Cosmology 2013". arXiv:1402.0526
5. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
6. V. Sahni, A.A. Starobinsky, Int. J. Mod. Phys **A9**, 373 (2000)
7. T. Padmanabhan, Phys. Rept. **380**, 235 (2003)
8. P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003)
9. J.A.S. Lima, Braz. J. Phys. **34**, 194 (2004). arXiv:astro-ph/0402109
10. E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)
11. J. Solà, J. Phys. Conf. Ser. **453**, 012015 (2013). arXiv:1306.1527
12. I.L. Shapiro, J. Solà, JHEP **0202**, 006 (2002)
13. I.L. Shapiro, J. Solà, Phys. Lett. B **475**, 236 (2000)
14. I.L. Shapiro, J. Solà, Phys. Lett. B **530**, 10 (2002)
15. I.L. Shapiro, J. Solà, Nucl. Phys. Proc. Suppl. **127**, 71 (2004)
16. I.L. Shapiro, J. Solà, PoS AHEP **2003**, 013 (2003). arXiv:astro-ph/0401015
17. J. Solà, J. of Phys. A. **41**, 164066 (2008)
18. I.L. Shapiro, J. Solà, Phys. Lett. B **682**, 105 (2009). arXiv:0808.0315
19. J.A.S. Lima, S. Basilakos, J. Solà, Mon. Not. R. Astron. Soc. **431**, 923 (2013)
20. S. Basilakos, J.A.S. Lima, J. Solà, Int. J. Mod. Phys. D **22**, 1342008 (2013)
21. E.L.D. Perico, J.A.S. Lima, S. Basilakos, J. Solà, Phys. Rev. D. **88**, 063531 (2013)
22. S. Basilakos, S., J. A. S. Lima and J. Solà. Int. J. Mod. Phys. D **23**, 1442011 (2014)
23. J.A.S. Lima, S. Basilakos, J. Solà, Gen. Rel. Grav. **47**, 15 (2015)
24. J. Solà, A. Gómez-Valent, Int. J. Mod. Phys. D **24**, 1541003 (2015)
25. J. Solà, Int. J. Mod. Phys. D **24**, 1544027 (2015). arXiv:1505.05863
26. J.P. Mimoso, D. Pavón, Phys. Rev. D **87**, 047302 (2013)
27. C. España-Bonet et al., Phys. Lett. B **574**, 149 (2003). JCAP **0402** 006 (2004)
28. C. España-Bonet et al., JCAP **0402**, 006 (2004)
29. S. Basilakos, M. Plionis, J. Solà, Phys. Rev. D. **80**, 3511 (2009)
30. S. Basilakos., Mon. Not. R. Astron. Soc. **395**, 234 (2009)
31. S. Basilakos., Astron. Astrophys. **508**, 575 (2009)
32. J. Grande, J. Solà, S. Basilakos, M. Plonis, JCAP **08**, 007 (2011)
33. S. Basilakos, D. Polarski, J. Solà, Phys. Rev. D. **80**, 3511 (2012)
34. A. Gómez-Valent, J. Solà, S. Basilakos, JCAP **01**, 004 (2015)
35. A. Gómez-Valent, J. Solà, Mon. Not. R. Astron. Soc. **448**, 2810 (2015)
36. G. Zilioti, R.C. Santos, J.A.S. Lima, Solving the Coincidence Problem in a Large Class of Running Vacuum Cosmologies (2015). arXiv:1508.06344
37. J. Grande, J. Solà, H. Stefancić, JCAP **08**, 011 (2006)
38. J. Grande, A. Pelinson, J. Solà, Phys. Rev. D **79**, 043006 (2009)
39. J. Solà, Running vacuum in the universe: current phenomenological status. In: Proc. of the 14th Marcel Grossman Meeting, Rome (2015) **(to appear)**. arXiv:1601.01668
40. J. Solà, A. Gómez-Valent, J. de Cruz Pérez, First evidence of running cosmic vacuum: challenging the concordance model. arXiv:1602.02103
41. J. Solà, A. Gómez-Valent, J. de Cruz Pérez, Astrophys. J. **811**, L14 (2015)
42. E.W. Kolb, M.S. Turner, The Early Universe. Addison-Wesley Publishing Company, Boston (1990)
43. J.A.S. Lima, Phys. Rev. D **54**, 2571 (1996)
44. J.A.S. Lima, Gen. Rel. Grav. **29**, 805 (1997)
45. J.A.S. Lima, I. Baranov, Phys. Rev. D **90**, 043515 (2014)
46. I. Baranov, J.A.S. Lima, Phys. Lett. B **751**, 338 (2015)
47. G.W. Gibbons, S.W. Hawking, Phys. Rev. D **15**, 2738 (1977)
48. P.A.R. Ade et al. [Planck Collaboration] (2015). arXiv:1502.0211