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Constraining the time evolution of dark energy, curvature and neutrino properties with cosmic chronometers

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Abstract. We use the latest compilation of observational Hubble parameter measurements estimated with the differential evolution of *cosmic chronometers*, in the redshift range $0 < z < 2$, to place constraints on cosmological parameters. We used a Markov-Chain Monte-Carlo approach to sample the parameter space for the cosmic chronometers dataset alone and in combination with other state-of-the art cosmological measurements: CMB data from the latest Planck 2015 release, the most recent estimate of the Hubble constant H_0 , a compilation of recent baryon acoustic oscillation data, and the latest type Ia cosmological supernovae sample. From late-Universe probes alone ($z < 2$) we find that $w_0 = -0.9 \pm 0.18$ and $w_a = -0.5 \pm 1.7$, and when combining also Planck 2015 data we obtain $w_0 = -0.98 \pm 0.11$ and $w_a = -0.30 \pm 0.4$. These new constraints imply that nearly all quintessence models are disfavoured by the data; only phantom models or a pure cosmological constant are favoured. This is a remarkable finding as it imposes severe constraints on the nature of dark energy. For the curvature our constraints are $\Omega_k = 0.003 \pm 0.003$, considering also CMB data. We also find that $H(z)$ data from cosmic chronometers are important to constrain parameters that do not affect directly the expansion history, by breaking or reducing degeneracies with other parameters. We find that $N_{\text{eff}} = 3.17 \pm 0.15$, thus excluding the possibility of an extra (sterile) neutrino at more than 5σ , and put competitive limits on the sum of neutrino masses, $\Sigma m_\nu < 0.27 \text{ eV}$ at 95% confidence level. Finally, we constrain the redshift evolution of dark energy by exploring separately the early and late-Universe, and find a dark energy equation of state evolution $w(z)$ consistent with that in the Λ CDM model at the ± 0.4 level over the entire redshift range $0 < z < 2$.

Keywords: dark energy experiments, dark energy theory, neutrino masses from cosmology

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Contents

1	Introduction	1
2	Data	2
2.1	Cosmic chronometers dataset	3
2.2	Additional datasets	3
3	Methodology	4
4	Results	7
4.1	Constraints from the late-Universe probes	7
4.2	Combining early-Universe information	11
5	Conclusions	15
A	Dependence of the results on the assumed evolutionary stellar population synthesis model	17

1 Introduction

Currently, the Λ CDM model represents the simplest framework to describe all available cosmological information. Within this model, the Universe has no spatial curvature, the present day energy is mostly constituted of a dark energy component in the form of a cosmological constant, and there are three massless neutrinos. This model can accurately match at the \sim percent level current observations with the minimal number of parameters.

One approach to make progress in understanding the nature of dark energy is to measure quantities that are independent of the cosmological model. One such technique is to measure directly the expansion history of the Universe: this can be done using massive and passively evolving early-type galaxies as “cosmic chronometers” [1], thus providing *standard(-izable) clocks* in the Universe. The basic idea underlying this approach is based on the measurement of the differential age evolution as a function of redshift of these chronometers, which provides a direct estimate of the Hubble parameter $H(z) = -1/(1+z)dz/dt \simeq -1/(1+z)\Delta z/\Delta t$. The properties of these tracers ensure that they are the best candidates to exploit this technique: they are extremely homogeneous in terms of redshift of formation, assembled their mass at high redshifts ($z \sim 2-3$), are passively evolving since then, hence representing the oldest objects in the Universe at each redshift. An extensive discussion of the method is provided in refs. [2, 3]. There has been significant progress both in the modeling and control of systematics uncertainties [4, 5], and in the improvement of observational data [2, 3, 6]; in particular in these latter works the method was exploited providing new accurate measurements of $H(z)$ in the redshift range $0 < z < 2$, that were used alone [6] and in combination with other probes [5] to constrain cosmological parameters. It is important to underline that the main strength of this approach is that it measures a differential quantity, $\Delta z/\Delta t$, which minimizes many issues and systematic effects (for a detailed discussion, see [2]).

The cosmic chronometer approach relies on the determination of accurate stellar ages for galaxies. These are determined by fitting stellar population models to data, either photometric or spectroscopic. Hence the quality of the derived ages depends on the following

entries and their interplay: the input stellar population model, which in turn depends on the input stellar models; the fitting procedure, e.g. principal component, reconstructed star formation history, parametric star formation history; the wavelength range encompassed by the data. Several studies have recently assessed the impact of various effects on the robustness of ages (e.g. [7, 8]). These works illustrate that spectral fitting is now a robust procedure. Ultimately, the internal clock is dictated by the equation of state assumed in the stellar models and the nuclear reaction rates. For example, ref. [9] finds that using models based on so-called Padova tracks or Frascati tracks lead to a shift of 500 Myr in age for populations about 1 Gyr old, and smaller for older populations.

Because the cosmic chronometer method is independent of the assumed cosmological model, it can provide constraints on the parameters that govern the expansion history of the Universe in a way that can be used to test cosmological models. The most interesting parameters to be constrained are those affecting the background evolution, chiefly the evolution of dark energy and the curvature. However, other parameters which from Cosmic Microwave Background data alone show degeneracies with the background evolution, such as neutrino properties, can also be constrained.

The main aim of this paper is to explore what constraints, on models beyond Λ CDM, can be obtained from the the new dataset of $H(z)$ measurements from ref. [3] both alone and in combination with other data. In particular, we focus on constraints on the time evolution of dark energy, and we demonstrate how early and late-Universe probes give a consistent picture. Further, we demonstrate that the new $H(z)$ data, in combination with the other late-Universe probes, allow for a reconstruction with up to 3 free parameters of the time evolution of dark energy, when dark energy is parameterised via a Chebyshev expansion. We also explore the standard Chevallier-Polarski-Linder (CPL) parameterization, providing accurate constraints on dark energy evolution. The paper is organised as follows. In section 2 we present the data, both the $H(z)$ data and the other state-of-the-art cosmological data and in section 3 we state our underlying assumptions and the methodological approach. In section 4 we present our results, first for the cosmic chronometers data alone, then in comparison with external data sets and finally in combination, discussing our constraints on dark energy time evolution, curvature, and neutrino properties. We conclude in section 5.

2 Data

In this analysis, we compare the constraints on cosmological parameters that can be obtained by probes that map the late-time Universe ($z < 2$) expansion history, as well as the improvement that can be obtained by combining those with early-time Universe probes. The baseline of our analysis is the Hubble parameter measurements obtained with the cosmic chronometers (hereafter CC) technique, but we consider as well more “standard” probes such as Supernovae Type Ia (hereafter SNe), Baryon Acoustic Oscillation distance measurements (hereafter BAO), and local H_0 measurements (hereafter H_0). Early-time Universe probes are the latest Cosmic Microwave Background (CMB) measurements from the Planck mission (hereafter Planck15). We use directly the posterior sampling provided by the Planck collaboration for specific models and for the full combination of temperature and polarisation power spectrum data (in the Planck15 nomenclature TT,TE,EE+lowP).¹ We refer to ref. [10] for more information. In the following we present the other datasets considered.

¹Downloadable from <http://pla.esac.esa.int/pla/#cosmology> under “Full grid of results”.

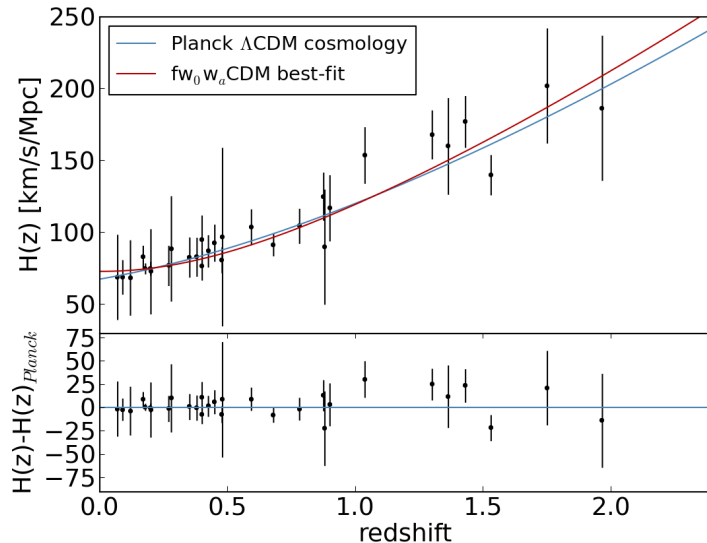


Figure 1. Cosmic chronometers dataset used in this analysis. The red line show the best-fit to the data assuming a fw_0w_a CDM cosmology, while the blue line show the fiducial Planck Λ CDM cosmology ($H_0 = 67.8$ km/s/Mpc, $\Omega_m = 0.308$). Lower panels show the residuals of the data with respect to the fiducial Planck cosmology. CC data have been taken from refs. [2, 3, 6, 11–13].

2.1 Cosmic chronometers dataset

The *cosmic chronometers* approach to measure $H(z)$ was first introduced in ref. [1]; it uses relative ages of the most massive and passively evolving galaxies to measure dz/dt , from which $H(z)$ is inferred. The latest implementation has been explained in detail in ref. [2], where the possible sources of uncertainty and related issues are also discussed; we refer to those references for a comprehensive discussion. We consider the compilation of Hubble parameter measurements provided by [3]. It contains the latest updated list of 30 $H(z)$ measurements [2, 3, 6, 11–13] spanning the redshift range $0 < z < 2$. This sample covers roughly 10 Gyr of cosmic time; the data are presented in figure 1. The CC approach to measure $H(z)$ has the desirable feature of being largely independent on assumptions about the cosmological model (besides isotropy and homogeneity); it does, however, rely on the identification of an optimal tracer of the aging of the Universe with redshift (a cosmic chronometer), and a reliable way to measure the age of a tracer (“dating”). An extended discussion can be found in ref. [3]. Of particular importance is a possible dependence on the adopted evolutionary stellar population synthesis (EPS) model, which is key in determining the age of the chronometer. In this analysis we used the measurements calibrated on [18] (BC03) EPS models, since they provide the largest dataset to date. However, refs. [2, 3, 6] provided measurements also with the newest M11 [19] models for a smaller dataset. We explore the dependence of our results on the adopted EPS model in appendix A.

2.2 Additional datasets

More standard cosmological probes have been also exploited as complementary datasets to this analysis.

Type 1A supernovae (standard candles). We consider the latest “joint light curves” (JLA) sample [14], comprising 740 SNe Ia from the three year Sloan Digital Sky Survey,

Supernova Legacy Survey [20, 21], Hubble Space Telescope [22, 23] and other local experiments (see [24]). Here we use the binned distance modulus provided by ref. [14], with its associated covariance matrix. It is defined as:

$$\mu_b = M + 5 \log_{10} D_L(z), \quad (2.1)$$

where M is a (*nuisance*) normalization parameter and $D_L(z)$ the luminosity distance at redshift z . The luminosity distance at redshift z is related to an integral of the Hubble parameter from redshift 0 to z . As such, it offers sensitivity to the curvature parameter but its integral nature makes it less sensitive than CC to sharp variations in $H(z)$. Also marginalisation over M makes this probe sensitive to the shape of $H(z)$ but not to its overall normalisation (characterised for example by the value at a given redshift like $z = 0$, H_0).

Baryon Acoustic oscillation (standard rulers). Our BAO analysis is based on the (isotropic) acoustic-scale distance ratio $D_V(z)/r_{\text{drag}}$ where r_{drag} is the sound horizon at radiation drag,

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3}, \quad (2.2)$$

and $D_A(z)$ is the angular diameter distance at redshift z .

Our compilation comprises the measurements obtained by 6dFGS [15], SDSS Main Galaxy Sample [16] and BOSS LOWZ and CMASS surveys [17] and is similar to the baseline BAO dataset used in Planck 2015 analysis [10]. D_V is a combination of an integral of $H(z)$ (through D_A) and a direct $H(z)$ measurement,² but the exquisite measurement of r_{drag} provided by CMB data offers a tight constraint on the overall normalisation of the relation (see e.g., discussion in [25, 26]). BAO measurements have been provided also for other surveys (e.g. Wigglez [27]), or for other subsamples (e.g. galaxy clusters, e.g. [28, 29]), but since the covariance between these samples and the dataset used in this analysis has not been estimated, we decided not to use them.

Local H_0 value. Finally in some cases we also include the local distance ladder measurement of the Hubble constant $H_0 = 73.0 \pm 2.4$ km/s/Mpc, as measured by ref. [30] using the recalibration provided by ref. [31].

Since there have been claims of tensions between the local H_0 measurement and the value inferred from early-Universe observations within a Λ CDM Universe (e.g., [10, 32–34] and references therein) when including CMB data we will always report results both with and without the H_0 measurement. In this way the reader can judge whether the inclusion of a possible “discrepant” measurement drives any of the conclusions.

The SNe and BAO data used in this analysis are shown in figure 2.

3 Methodology

Under the assumptions of isotropy and homogeneity, the metric of space-time of the Universe is fully specified by the Friedmann-Lemaître-Robertson-Walker one. The most economic Λ CDM cosmological model can be described by six parameters, but three of them

²Anisotropic BAO measurements can measure D_A and $H(z)$ separately but at present the anisotropic measurements does not yet have significantly more statistical power than the isotropic one [17]. While ref. [10] for CMASS uses the anisotropic measurement from [17], here we use the isotropic one.

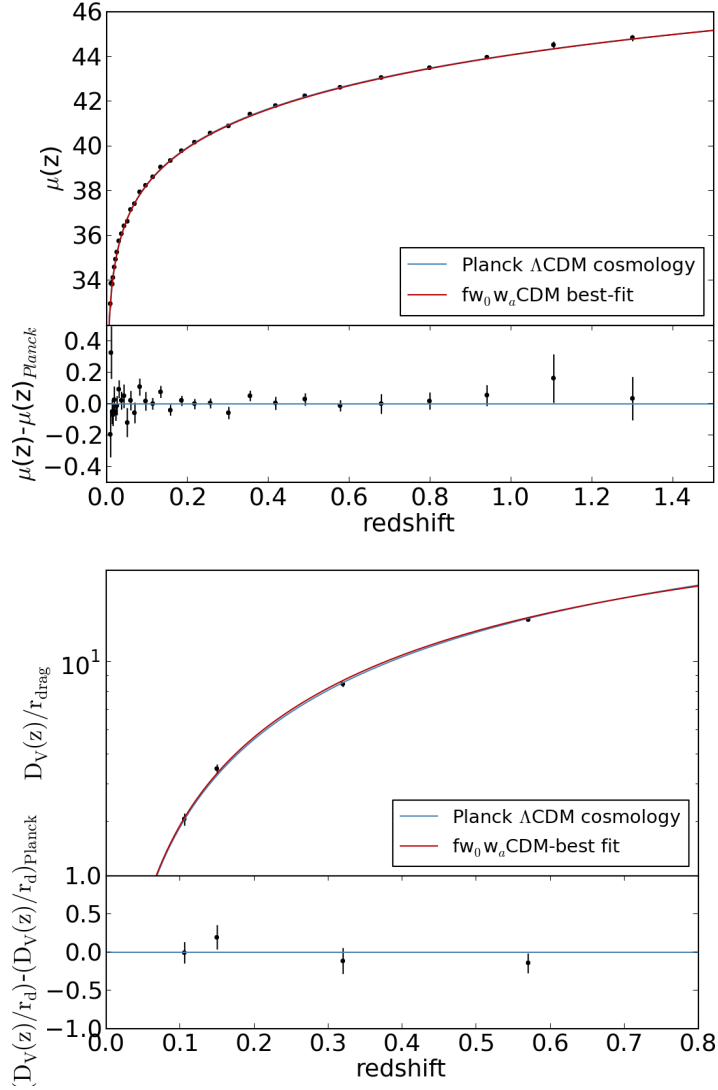


Figure 2. Same as figure 1, but for BAO and SNe datasets; SNe data have been taken from ref. [14], and BAO data from [15–17].

univocally define the background evolution described by the Hubble parameter $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_{\text{DE}}}$. Extensions to this model have been proposed by relaxing one or more of its assumptions, such as flatness, dark energy Equation-of-State (EoS) parameter evolution, number of relativistic species in the Universe, total sum of neutrino masses; in this context, a generic model for the expansion rate of the Universe adopts a generic form for the equation of state parameter of dark energy $w(z)$, and is:

$$H(z) = H_0 \left\{ \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{\text{DE}}(1+z)^3 \left(\int_0^z \frac{w(z')}{(1+z')} dz' \right) \right\}^{1/2}, \quad (3.1)$$

where Ω_i denote the energy density parameter for the various species in the Universe (matter, curvature, dark energy and radiation) at $z = 0$. Given that the contribution to the total energy due to radiation is not significant at late time where Ω_{DE} is important, we can safely neglect it; the relation between the energy density parameters is $1 = \Omega_m + \Omega_{\text{DE}} + \Omega_k$.

A popular model for the expansion rate of the Universe is given by the CPL [35, 36] parameterisation of the equation of state for dark energy, $w(z) = w_0 + w_a(z/(1+z))$, yielding for the expansion rate:

$$H(z) = H_0 \left\{ \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{\text{DE}}(1+z)^{3(1+w_0+w_a)} e^{-3w_a \frac{z}{1+z}} \right\}^{1/2}. \quad (3.2)$$

The stronger dependence of $H(z)$ is on the Hubble constant, matter density and curvature parameter, while the dependence on dark energy EoS parameters is less significant, particularly for w_a which is the most difficult parameter to constrain.

We follow two approaches. First, we analyze the constraints that can be put on cosmological parameters, and in particular on dark energy time evolution, with late-Universe probes (i.e. at $z < 2$, not considering CMB data), such as CC, SNe and BAO (with and without H_0), first separately then combined; then we explore how the constraints can be narrowed down by adding early-Universe information from CMB observations. Therefore, we will be able to test how the late and early Universe agrees with each other, being this fully determined within a given cosmological model, like Λ CDM.

We analyze the goodness of fit of CC, SNe and BAO with a standard χ^2 approach; this is possible as the reported errors on the data are Gaussianly distributed. For SNe, we use the full covariance matrix, while for the other probes, measurements are uncorrelated and the covariance is diagonal; the χ^2 have been estimated accordingly.

For CC we have that:

$$\chi^2 = \sum \frac{(H_{\text{th}}(z) - H_{\text{obs}}(z))^2}{\sigma_{H_{\text{obs}}(z)}^2}, \quad (3.3)$$

where $H_{\text{th}}(z)$ is taken from eq. (3.2) and H_{obs} and $\sigma_{H_{\text{obs}}}$ from ref. [3].

For SNe we have:

$$\chi^2 = (\mu_b - M - 5 \log_{10}(D_L(z)))^t C^{-1} (\mu_b - M - 5 \log_{10}(D_L(z))), \quad (3.4)$$

where D_L is the luminosity distance and C the covariance matrix associated with distance modulus measurements μ_b .³

For BAO we have:

$$\chi^2 = \sum \frac{(D_{V,\text{th}}(z)/r_{\text{drag}} - D_{r,\text{obs}}(z))^2}{\sigma_{D_{r,\text{obs}}(z)}^2}, \quad (3.5)$$

where $D_r = D_V/r_{\text{drag}}$, and $D_{V,\text{th}}$ is given by eq. (2.2). When CMB information is not included, r_{drag} is treated as a nuisance parameter i.e., marginalised over.

We sample the distribution of parameters with the public python package `emcee` [37], which is an implementation of the affine-invariant ensemble sampler for Markov chain Monte Carlo (MCMC) proposed by ref. [38]. Most uses of MCMC in the astrophysics literature are based on slight modifications to the Metropolis-Hastings method. The performance of this sampler is very sensitive to the choice of the proposal distribution and in particular its covariance matrix and there is no easy and straightforward method to optimize this. However, the affine-invariant ensemble sampling algorithm is much more efficient and does not depend on the aspect ratio in highly anisotropic distributions. The method evolves simultaneously an

³http://supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html.

ensemble of many ($K \gg 10$) walkers where the proposal distribution for one walker k is based on the current positions of the $K - 1$ walkers. To update the position of a walker at position X_k , a walker X_j is drawn randomly from the remaining walkers and its new position will be a step away from the position of walker X_j with a specific proposal distribution with one tunable parameter (“stretch move”). This approach can be further tuned to be parallelized.

We considered uniform priors on the following variables: $H_0 = [50, 100]$ km/s/Mpc, $\Omega_m = [0.01, 0.99]$, $w_0 = [-3, 0]$, $w_a = [-5, 5]$, $r_{\text{drag}} = [100 - 200]$.

We first explore a flat model ($\Omega_k = 0$) with the CPL parameterisation for $w(z)$ (fw_0w_a CDM hereafter) to compare the results with the ones that can be obtained from CMB [10].

Subsequently, to include early time (CMB) constraints we have used the posterior samples provided by the Planck 2015 data release, and importance sampled them with CC measurements, comparing the results with the ones obtained from the combination of Planck15+BAO and Planck15+SNe provided by the Planck team [10]. The joint posterior for two data sets (D_1 and D_2) can be estimated easily if the posterior for e.g. D_1 has already been simulated with an MCMC. The weight of each point in the chain is proportional to the posterior for D_1 which is the product of the likelihood for D_1 and the prior. The joint posterior for D_1 and D_2 is proportional to the product of the likelihood for D_1 , the likelihood for D_2 and the prior, which can be obtained by reweighting the weight of the chain by the likelihood of D_2 . This procedure is called importance sampling. The simulated posterior so obtained is a good approximation if data set D_2 is consistent with D_1 .

4 Results

We start by discussing the results obtained from late-Universe probes: cosmic chronometer analysis alone, and then in combination with other probes (BAO, SNe). We then combine late-Universe probes with early-Universe observations (CMB) to improve constraints also on parameters that do not have a direct effect on $H(z)$, such as the number of the relativistic species in the Universe N_{eff} and the sum of the neutrino masses Σm_ν , by breaking parameter degeneracies.

4.1 Constraints from the late-Universe probes

Figure 3 shows constraints on a flat w_0w_a CDM cosmology using CC, SNe, BAO individually and their combination. Marginalised constraints for each parameter are reported in table 1. From this analysis, it is evident that CC and BAO have a similar constraining power in the w_0-w_a plane, with CC providing better constraints on H_0 ($H_0 = 73.2^{+10}_{-8.1}$) and slightly better constraints on Ω_m ; SNe, while insensitive to H_0 , provide stringent constraints on the dark energy EoS evolution. The poor constraining power on H_0 of the BAO analysis results from the fact that r_{drag} is treated as a nuisance parameter (i.e. is left as a free parameter, without any CMB prior, and is marginalised over). Most interestingly, the directions of the degeneracy between parameters in the various probes are different, and hence their combination can provide more stringent constraints, as shown by figure 3 bottom right panel. More quantitatively, the constraints obtained with CC are $\sim 70\%$ and $\sim 50\%$ better than the ones from BAO respectively for H_0 and Ω_m , while just $\sim 8\%$ for the dark energy EoS parameters. The constraints from SNe are better than the ones for CC of a factor ~ 3.5 for w_0 , and of $\sim 60\%$ for w_a , while being worst by $\sim 50\%$ on Ω_m . The combination of all late-Universe probes yields to a significant improvement of $\sim 40\%$ on Ω_m with respect to SNe alone, and helps

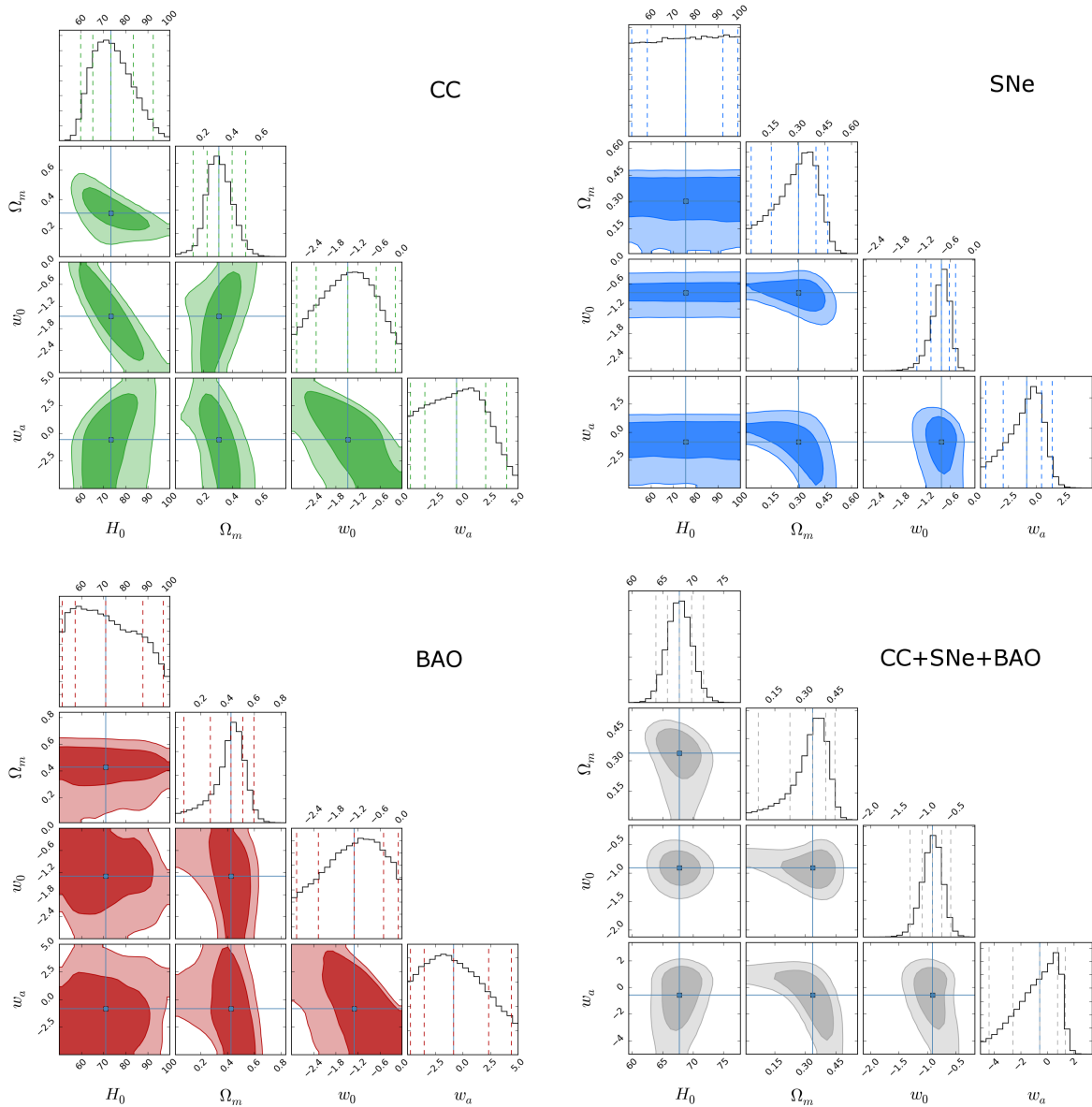


Figure 3. Constraints on a flat w_0w_a CDM cosmology from cosmic chronometers (in green, upper-left panel), SNe (in blue, upper-right panel), BAO data (in red, lower-left panel), and from the combination of the various probes (in grey, lower-right panel) obtained with a MCMC approach. In each panel the contour plots are shown at 68% and 95% confidence level, and the posterior distribution of H_0 , Ω_m , w_0 and w_a , with the 68% and 95% confidence level limits.

to partially remove the degeneracy in the w_0 - w_a plane, improving by $\sim 25\%$ the measurements of w_0 . In particular, it is interesting to note the orthogonality between the constraints provided by CC and SNe. We actually verified that most of the constraining power of the combination CC+SNe+BAO comes from CC+SNe, as they provide constraints for most parameters almost as stringent as when BAO data is added. For w_a , which is the more difficult parameter to be constrained, the addition of BAO proves to be helpful to reduce errorbars.

In order to gain further insights into the time evolution of dark energy, it is useful to explore beyond the two-parameter $w_0 - w_a$ fit. Ideally, one would like to reconstruct $w(z)$ in

MARGINALIZED 1D CONSTRAINTS				
$f w_0 w_a$ CDM model				
	H_0	Ω_m	w_0	w_a
CC	$73.2^{+10}_{-8.1}$	$0.31^{+0.09}_{-0.08}$	$-1.46^{+0.77}_{-0.87}$	$-0.5^{+2.6}_{-2.9}$
SNe	—	$0.31^{+0.1}_{-0.15}$	$-0.82^{+0.19}_{-0.26}$	$-0.9^{+1.3}_{-2.1}$
BAO	70^{+17}_{-14}	$0.42^{+0.09}_{-0.16}$	$-1.3^{+0.81}_{-0.97}$	$-0.8^{+3.2}_{-2.7}$
CC+SNe	$68.2^{+2.1}_{-2}$	$0.34^{+0.06}_{-0.1}$	$-0.9^{+0.17}_{-0.19}$	$-1.1^{+1.7}_{-2.1}$
CC+SNe+BAO	67.8 ± 2	$0.33^{+0.06}_{-0.12}$	$-0.9^{+0.17}_{-0.19}$	$-0.5^{+1.3}_{-2}$

Table 1. Constraints on H_0 , Ω_m , w_0 and w_a (68% confidence limits) obtained for a flat Λ CDM cosmology with equation-of-state parameter for dark energy parameterised as $w(z)=w_0+w_a(z/(1+z))$.

a non-parametric way; unfortunately, this is not possible (e.g. see ref. [11]). So, alternatively, one needs to use a parameterisation that is the least possible model dependent. Ref. [11] proposed an expansion with Chebyshev polynomials, T_i , without any further assumptions on the shape of the dark energy EoS evolution:

$$w(z) = \sum_{i=0}^N \omega_i T_i(x(z)), \quad (4.1)$$

where $T_i(x)$ are the Chebyshev polynomials: up to the second order $T_0 = 1$, $T_1 = x$, $T_2 = 2x^2 - 1$, $x = (2z/z_{\max}) - 1$ and ω_i are the parameters to be constrained by the data.

In this framework, it is possible to write the expansion history (for a flat Universe) as:

$$H(z) = H_0(1+z)^{3/2} \sqrt{\Omega_m + (1 - \Omega_m) \exp\left(\frac{3}{2} z_{\max} \sum_{i=0}^N \omega_i G_i(z)\right)}, \quad (4.2)$$

where the functions G_i can be obtained iteratively from the equation $J_i = 1/nb[(2z/z_{\max} - 1)^i - (-1)^i] - (a/b)J_{i-1}$, $J_0 = 1/b \log(1+z)$, $a = 1 + (z_{\max}/2)$ and $b = z_{\max}/2$. Up to the first two orders, it can be obtained that $G_0 = J_0$, $G_1 = J_1$ and $G_2 = 2J_2 - J_0$ [11].

In table 2 we report the results on the constraints of ω_i up to both first and second order obtained by fitting jointly CC, SNe and BAO data to eq. (4.2), and in figure 4 we show the contours for the fit up to the second order. Because of the increased freedom in the EoS parameterisation, in this analysis we always include the H_0 measurement.

The present-day value of w in this expansion can be estimated as:

$$w_{z=0} = \sum_{i=0}^N (-1)^i \omega_i, \quad (4.3)$$

from which we find from the combined constraints $w_{z=0} = -1.21^{+0.69}_{-0.81}$ with the first order, and $w_{z=0} = -1.5^{+1.4}_{-1.56}$ with the second order expansion.

In this analysis by combining SNe and BAO data with the latest CC measurements (and the H_0 prior), we are able to close for the first time the 2σ contours on the ω_i parameters up

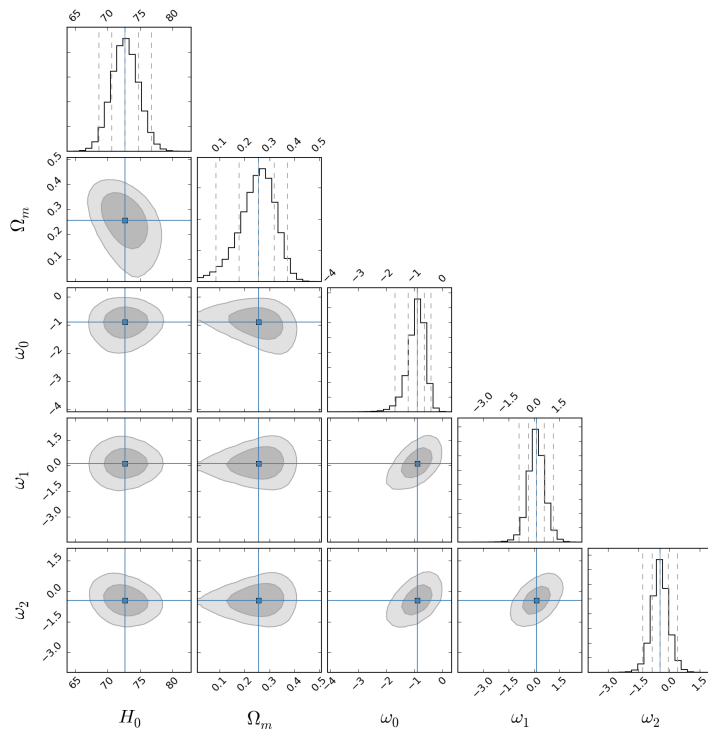


Figure 4. Constraints for the Chebyshev expansion fit obtained from the combination of cosmic chronometer, SNe and BAO data. In each panel the contour plots are shown at 68% and 95% confidence level, and the posterior distribution of H_0 , Ω_m , ω_0 , ω_1 and ω_2 , with the 68% and 95% confidence level limits. For these constraints we assumed a Gaussian prior on $H_0 = 73 \pm 2.4$ [30, 31, 39].

MARGINALIZED 1D CONSTRAINTS					
Chebyshev expansion					
	H_0	Ω_m	ω_0	ω_1	ω_2
CC+ H_0	—	0.36 ± 0.04	$-7.6^{+6.2}_{-6.5}$	$-6.1^{+8.7}_{-9.3}$	$-0.2^{+6.3}_{-3.5}$
CC+SNe+BAO+ H_0 - 1 st order	—	$0.27^{+0.05}_{-0.06}$	$-0.68^{+0.16}_{-0.21}$	$0.53^{+0.30}_{-0.21}$	—
CC+SNe+BAO - 2 nd order	$71.9^{+4.2}_{-3.9}$	$0.27^{+0.08}_{-0.1}$	$-0.91^{+0.27}_{-0.36}$	$0.12^{+0.49}_{-0.49}$	$-0.43^{+0.51}_{-0.43}$
CC+SNe+BAO+ H_0 - 2 nd order	—	$0.26^{+0.06}_{-0.08}$	$-0.91^{+0.26}_{-0.33}$	$0.13^{+0.47}_{-0.45}$	$-0.46^{+0.4}_{-0.39}$

Table 2. Constraints on H_0 , Ω_m , and ω_0 , ω_1 and ω_2 parameters (from eq. (4.1)) at 68% confidence level obtained from the fit to the Chebyshev expansion of $w(z)$ up to the second order as modeled in eq. (4.2). When a Gaussian prior on H_0 is assumed ($H_0 = 73 \pm 2.4$ [30, 31, 39]), no constraints are provided for H_0 , because we will be simply recovering the prior.

to the second order, significantly improving on previous measurements [11] where unbounded constraints were obtained, and only up to the first order. This is an important result, since it provides a measurement of the evolution of dark energy EoS with a completely independent parameterisation. To check whether the results are being affected by the inclusion of the prior on H_0 , we performed the analysis also without adding it, finding no significant differ-

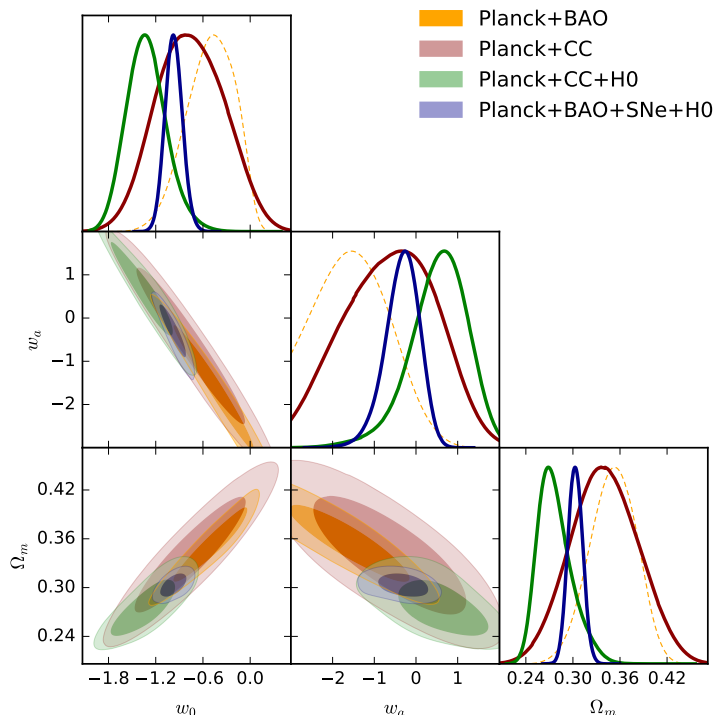


Figure 5. Constraints on Ω_m , w_0 and w_a for a fw_0w_a CDM model obtained with different combinations of data sets. Cosmic chronometers and BAO have a similar constraining power in the w_0 – w_a plane, as shown in figure 3, and the combination of CMB+BAO and CMB+CC provides comparable results. We used $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

ences, but, as expected, larger errorbars; these results are reported in table 2. In figure 6 we compare the estimated evolution of $w(z)$ with different parameterisations and different datasets combinations, finding, remarkably, results in very good agreement, and compatible with a cosmological constant. We note that $w(z)$ in the Chebyshev expansion appears to slightly deviate from a cosmological constant at the highest redshifts ($z \sim 2$); this deviation, however, is not significant, and well below two sigma.

4.2 Combining early-Universe information

CC measurements are extremely useful in breaking CMB parameters degeneracies for models beyond the “minimal” Λ CDM model. In particular we will show their fundamental importance in models that for allow a time-variability of the dark energy equation of state and for models where CMB constraints show parameter degeneracies with the expansion history.

We start extending the “minimal” Λ CDM where the additional parameter affects directly the late-time expansion history. We first explore a flat w_0w_a CDM model which constraints are presented in figure 5 and table 3. For the fw_0w_a CDM model, the base analysis provided by the Planck collaboration include Planck15 and BAO data; as demonstrated in section 4.1, CC and BAO have a similar constraining power on the w_0 – w_a plane, hence when combining Planck15+BAO+CC we obtain a little gain in the constraints, still however being able to close the 95% confidence level contours for w_a . When including also H_0 , the errorbars shrink considerably, and we obtain $w_0 = -1.09^{+0.19}_{-0.21}$ and $w_a = -0.02^{+0.61}_{-0.49}$ (at 68% confidence level); in comparison, the combination Planck15+BAO+CC+SNe+ H_0 obtains $w_0 = -0.98 \pm 0.11$ and $w_a = -0.30^{+0.42}_{-0.34}$ (at 68% confidence level).

MARGINALIZED 1D CONSTRAINTS			
$f w_0 w_a$ CDM model			
	Ω_m	w_0	w_a
Planck15+BAO	$0.351^{+0.032}_{-0.028}$	$-0.51^{+0.36}_{-0.26}$	$-1.47^{+0.78}_{-1.0}$
Planck15+CC	$0.314^{+0.039}_{-0.052}$	-0.81 ± 0.40	$-0.8^{+1.2}_{-1.0}$
Planck15+CC+ H_0	$0.273^{+0.017}_{-0.020}$	$-1.18^{+0.20}_{-0.25}$	$0.01^{+0.63}_{-0.44}$
Planck15+BAO+CC+ H_0	$0.294^{+0.016}_{-0.018}$	$-1.09^{+0.19}_{-0.21}$	$-0.02^{+0.61}_{-0.49}$
Planck15+BAO+SNe+ H_0	0.3031 ± 0.0093	-0.97 ± 0.11	$-0.34^{+0.45}_{-0.35}$
Planck15+BAO+CC+SNe+ H_0	0.3032 ± 0.0092	-0.98 ± 0.11	$-0.30^{+0.42}_{-0.34}$

Table 3. Marginalised constraints on Ω_m , Ω_{DE} , Ω_k , w_0 and w_a at 68% confidence level obtained for a flat Λ CDM cosmology with equation-of-state parameter for dark energy parametrized as $w(z) = w_0 + w_a(z/(1+z))$ ($f w_0 w_a$ CDM). We adopted $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

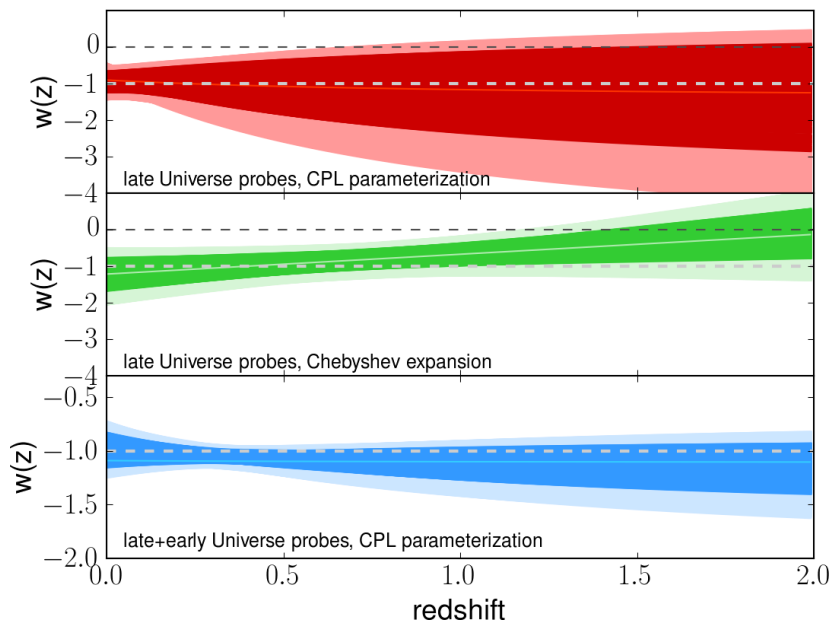


Figure 6. Reconstruction of the time evolution of dark energy EoS obtained from different probes. Red contours the constraints obtained from late-Universe probes with a CPL parameterisation (see eq. (3.2) and table 1), green contours show the constraints obtained from late-Universe probes with Chebyshev decomposition up to the first order (see eq. (4.1) and table 2), and blue contours the constraints obtained adding also CMB information with the CPL parameterisation (see table 3). The darker regions show in each case the 68% confidence level contours, while the light ones the 95% confidence level contours. These constraints assume $H_0 = 73 \pm 2.4$ km/s/Mpc, taken from refs. [30, 31, 39].

In figure 6 we show the evolution of $w(z)$ up to $z = 2$ (which is the maximum redshift covered by our datasets) for different parameterisations. We compare the constraints obtained from Chebyshev reconstruction and from a CPL parameterisation from late-Universe

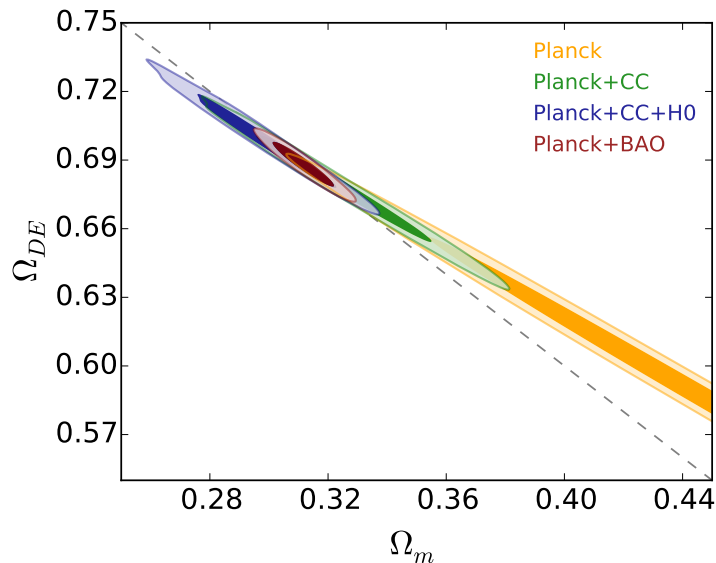


Figure 7. Constraints in the Ω_{DE} - Ω_m plane for different data-set combinations. Note that combining cosmic chronometer data-set with CMB data helps in constraining the contours close to the $\Omega_k = 0$ region, indicated with the dashed line. Similar results are obtained when combining the BAO with the CMB. We used $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

probes alone, and from a CPL parameterisation combining also Planck15 measurements. For homogeneity with CPL, we show here the results from Chebyshev reconstruction up to the first order, to compare results with similar degrees of freedom. We find that all the best-fit constraints show deviations from Λ CDM only up to $\sim 25\%$ up to $z = 2$ for the CPL parameterization, and up to $\sim 60\%$ up to $z = 1.5$ for the Chebyshev reconstruction. Quite remarkably, all measurements are compatible at 2σ with a constant dark energy EoS $w = -1$ in the entire redshift range. The most accurate measurement is obtained when combining both late- and early-Universe probes together, providing a constraint on $w(z)$ which is consistent with the Λ CDM model at the 40% level over the entire redshift range $0 < z < 2$ (at 68% confidence level).

We then consider an open Λ CDM model, to explore the constraints that can be obtained on curvature from the different probes. The results are shown in figure 7 and table 4. We find that cosmic chronometer are fundamental to break the Ω_m - Ω_{DE} degeneracy present in CMB only data, constraining $\Omega_k = -0.0030^{+0.0055}_{-0.0042}$. The error bars can be reduced when also the H_0 data is included, obtaining $\Omega_k = 0.0028^{+0.0035}_{-0.0031}$. For comparison, BAO data provide a similar result, $\Omega_k = 0.0002 \pm 0.0021$.

It is possible to constrain neutrino properties from the combination of CMB data with late-Universe measurement of the expansion rate (e.g., see refs. [30, 40, 41]). In this work, we consider a simple flat Λ CDM Universe where the effective number of relativistic species N_{eff} is let free, and not fixed to the standard value of $N_{\text{eff}} = 3.04$; we analyze this model studying the combined dataset of Planck15 and CC and H_0 .

Results are shown in figure 8 and table 5. Planck15+CC+ H_0 have the same constraining power as Planck15+BAO. They restrict $N_{\text{eff}} = 3.17^{+0.29}_{-0.30}$. Clearly, CC are not able to measure N_{eff} directly, but provide constraints by breaking the degeneracy between the number of relativistic species and the parameters that fix the matter-radiation equality and the expansion history (e.g., see ref. [41]). We then fit a flat Λ CDM Universe where the total neu-

MARGINALIZED 1D CONSTRAINTS			
$\omega\Lambda$ CDM model			
	Ω_m	Ω_{DE}	Ω_k
Planck15	$0.47^{+0.063}_{-0.086}$	$0.57^{+0.064}_{-0.047}$	$-0.040^{+0.023}_{-0.016}$
Planck15+CC	$0.325^{+0.018}_{-0.023}$	$0.678^{+0.018}_{-0.015}$	$-0.0030^{+0.0055}_{-0.0042}$
Planck15+CC+ H_0	0.298 ± 0.015	0.699 ± 0.013	$0.0028^{+0.0035}_{-0.0031}$
Planck15+BAO	0.3117 ± 0.0070	0.6881 ± 0.0065	0.0002 ± 0.0021

Table 4. Constraints on Ω_m , Ω_{DE} and Ω_k at 68% confidence level obtained for an open Λ CDM cosmology. For H_0 , we considered the value 73 ± 2.4 km/s/Mpc from refs. [30, 31, 39].

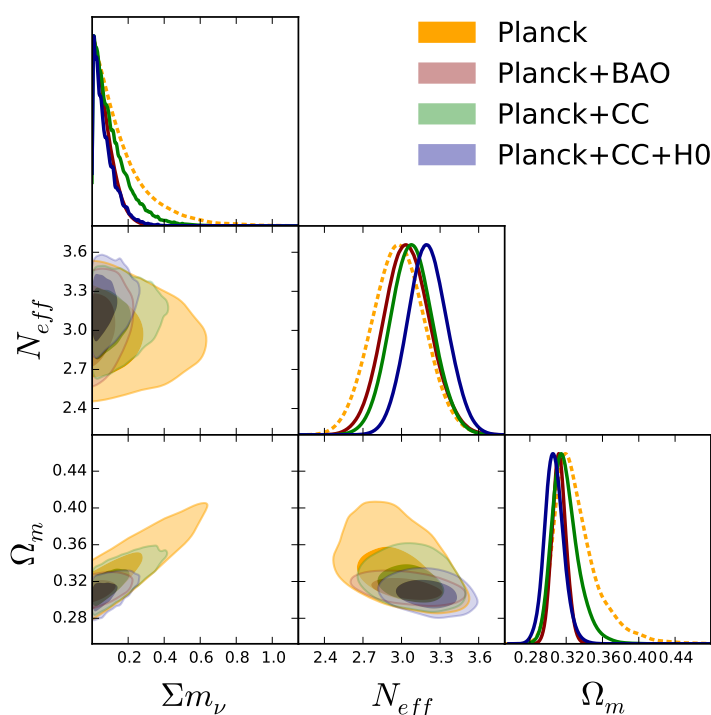


Figure 8. Constraints on number of effective relativistic species N_{eff} and their total mass Σm_ν obtained with different combination of probes. Note that cosmic chronometers data-set in combination with the CMB provide comparable constraints to those of CMB+BAO. We adopted $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

trino mass is left as a free parameter, and looked at the constraints on the sum of neutrino masses.⁴ We obtain a constraint of $M_\nu < 0.27$ at 95% confidence. We note how the addition of CC data helps to significantly reduce the errorbars both on neutrino mass and N_{eff} with respect to Planck data alone.

Finally, to exploit the constraining power of our new results in the w_0 - w_a plane, in figure 9 we compare our measurements with some theoretical models taken from refs. [36, 42].

⁴We assume three degenerate neutrino species; current data have no sensitivity on the hierarchy, so this is a very good approximation.

MARGINALIZED (95%) 1D CONSTRAINTS		
Λ CDM+ Σm_ν model		
	Σm_ν	N_{eff}
Planck15	< 0.577	$2.93^{+0.39}_{-0.38}$
Planck15+CC	< 0.423	$3.05^{+0.33}_{-0.32}$
Planck15+CC+ H_0	< 0.269	$3.17^{+0.29}_{-0.30}$
Planck15+BAO	< 0.222	$2.99^{+0.37}_{-0.34}$

Table 5. Constraints on Σm_ν (in units of eV) and N_{eff} at 95% confidence level obtained for a flat Λ CDM cosmology. For the Planck15+CC+ H_0 combination, we assumed a Gaussian prior 73 ± 2.4 km/s/Mpc from refs. [30, 31, 39].

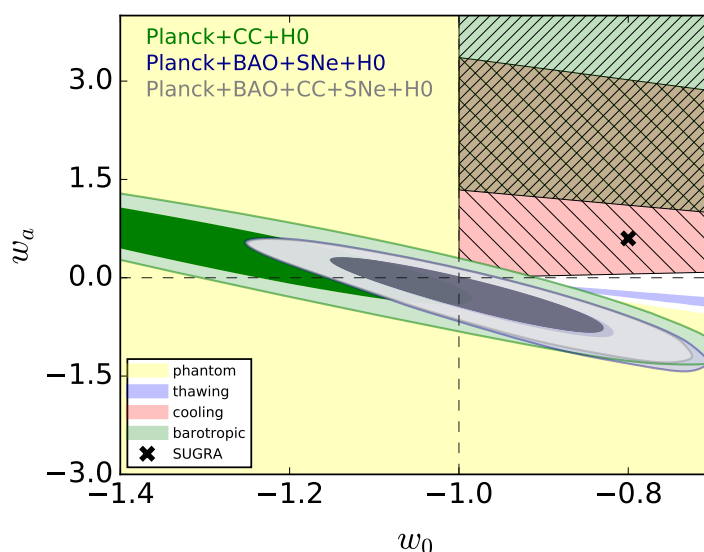


Figure 9. Constraints on the time evolution of dark energy equation of state compared to several theoretical models (the brown region is the overlap between barotropic and cooling quintessence models). Models have been taken from refs. [36, 42]. Dashed lines correspond to the position of the cosmological constant, $(w_0, w_a) = (-1, 0)$. The Hubble constant considered for these constraints is $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

They comprise a large variety of dark energy models, including barotropic models, phantom models, two different quintessence models and a SUGRA model. We refer to refs. [42, 43] for a comprehensive description of the models. Here we note that thank to the newly analysed data, the 95% allowed region in the w_0 - w_a plane is significantly reduced. The new constraints allow not only to reject at 95% confidence level the barotropic models, but also to exclude almost all quintessence models and the SUGRA model.

5 Conclusions

Measurements of the expansion history are the only direct indication that the Universe is undergoing an accelerated expansion, therefore suggesting that it should be dominated by

a dark energy component [3]. Given the magnitude of the implications of the accelerated expansion, it is important to have different, independent and complementary observations of the expansion history.

In this work we have used the latest cosmic chronometers measurements to set constraints on the time evolution of dark energy. We have shown that cosmic chronometers alone are able to set constraints on cosmological parameters comparable to other state-of-the-art late-Universe probes, namely BAO and SNe. However, unlike BAO and SNe, the cosmic chronometer data provide a direct measurement of the expansion rate $H(z)$, without having to assume a cosmological model, and thus can be used to test it. In particular, the constraints on cosmological parameters from cosmic chronometers and SNe demonstrated to be almost orthogonal one to another. Furthermore, their joint constraints prove to be as stringent as those obtained adding BAO data.

Remarkably, we find that constraints on dark energy are fully consistent with a cosmological constant up to $z \sim 2$, and when combining late- and early-Universe probes we find deviations from the Λ CDM model smaller than 40% over the entire redshift range $0 < z < 2$ (at 68% confidence level). We tested these results considering two independent parameterizations, the standard CPL parameterization and an expansion of the dark energy EoS based on Chebyshev polynomials. The conclusions proved to be stable to the assumed parameterisation for $w(z)$.

In the CPL parameterisation, from the combination of late-Universe probes alone we find $w_0 = -0.9 \pm 0.18$ and $w_a = -0.5 \pm 1.7$, and when combining also CMB measurements from Planck15 we obtain $w_0 = -0.98 \pm 0.11$ and $w_a = -0.30 \pm 0.4$. We compared these constraints with a set of theoretical dark energy models, finding that quintessence is disfavoured by the data at high significance, and providing strong constraints on the allowed families of models. We also tightly constrain the geometry of the Universe, obtaining a density parameter for curvature $\Omega_k = 0.003 \pm 0.003$.

Finally, we explored the power of cosmic chronometers in breaking degeneracies between parameters, and hence to constrain quantities that do not directly affects the expansion history of the Universe, namely the number of relativistic species and the total mass of neutrinos. Our measurements provide a value $N_{\text{eff}} = 3.17 \pm 0.15$ that excludes at more than 5σ the presence of an extra sterile neutrino, putting also a limit on the sum of neutrino masses, $\Sigma m_\nu < 0.27$ eV at 95% confidence level.

It is interesting to note that, even if below two sigma, the $w(z)$ obtained from the Chebyshev expansion appears to slightly deviate from a cosmological constant at high redshift. This deviation is, however, not significant with the current data; future surveys, such as Euclid [44], WFIRST [45], DESI [46] and LSST [47], will therefore represent the next significant step, providing sub-percent measurements of the dark energy EoS, and giving fundamental insights for our comprehension of the elusive nature of dark energy.

Acknowledgments

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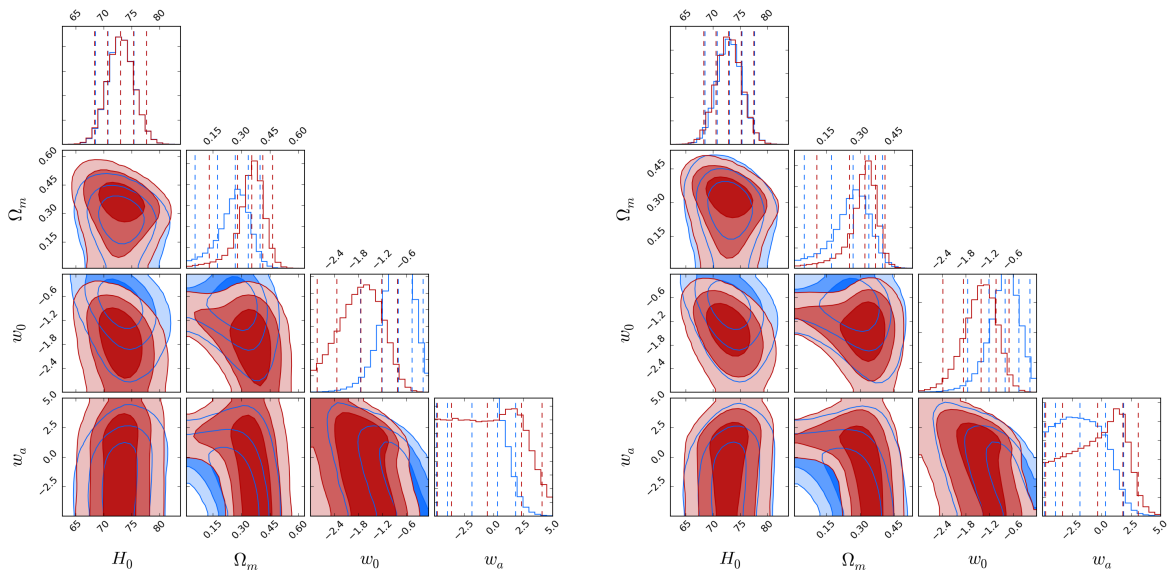


Figure 10. Comparison of the results obtained fitting CC with BC03 (in red) and M11 models (in blue). In the left panel are shown the constraints using measurements taken only from refs. [2, 3, 6], while in the right panel are also considered additional measurements obtained with BC03 model from refs. [11–13]. For this constraints, we assumed a Gaussian prior on $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39].

to thank the anonymous referee for the useful comments and suggestions, which helped to improve the paper.

A Dependence of the results on the assumed evolutionary stellar population synthesis model

To measure the Hubble parameter $H(z)$ within the CC approach, an evolutionary stellar population synthesis (EPS) model has to be assumed, to calibrate the relation between age and age-related observables used in our analysis. Amongst the models available in literature, so far BC03 [18] and M11 [19] have been used, encompassing significant differences in stellar phases considered, methods implemented and models adopted; for a more detailed discussion, we refer to ref. [3]. In the following, we explore the dependence of our results on the assumed EPS model.

As a reference, we consider the fw_0w_a CDM model, and fitted CC data obtained assuming BC03 and M11 models. We performed two different test: firstly, we consider only the limited dataset for which both BC03 and M11 measurements are available [2, 3, 6], and then we compare the result obtained for the full BC03 sample, i.e. by considering also the measurements from refs. [11–13] (for these measurements, only BC03 constraints are available). We also explored how the constraints change by adding the local measurements of H_0 , as a pivot value for the $H(z)$ relation.

The results are shown in figure 10, and reported in table 6. We find no significant difference between the two constraints, with only M11 measurements pointing to a value of H_0 at higher odds than BC03 with respect to state-of-art measurement. The measurements are still well compatible also when a Gaussian prior on H_0 is assumed, with the full

MARGINALIZED 1D CONSTRAINTS				
$f w_0 w_a$ CDM model				
	H_0	Ω_m	w_0	w_a
BC03 [2, 3, 6]	$74.2^{+20.1}_{-15.8}$	$0.33^{+0.26}_{-0.19}$	$-1.86^{+1.61}_{-1.1}$	$-0.8^{+5.2}_{-4}$
M11 [2, 3, 6]	$78.3^{+19.6}_{-15.2}$	$0.23^{+0.18}_{-0.18}$	$-1.19^{+1.1}_{-1.5}$	$-1.1^{+4.7}_{-3.7}$
BC03 [2, 3, 6, 11–13]	$73.4^{+19.2}_{-13.4}$	$0.3^{+0.18}_{-0.2}$	$-1.46^{+1.27}_{-1.4}$	$-0.5^{+4.5}_{-4.2}$
BC03 [2, 3, 6]+ H_0	—	$0.36^{+0.11}_{-0.23}$	$-1.73^{+0.9}_{-1.1}$	$-0.5^{+4.6}_{-4.2}$
M11 [2, 3, 6]+ H_0	—	$0.27^{+0.13}_{-0.21}$	$-0.81^{+0.6}_{-0.9}$	$-1.8^{+3.7}_{-3}$
BC03 [2, 3, 6, 11–13]+ H_0	—	$0.32^{+0.09}_{-0.21}$	$-1.4^{+0.7}_{-1}$	$-0.3^{+3.4}_{-4.4}$

Table 6. Comparison of the constraints obtained by fitting CC assuming different EPS (BC03 and M11) on H_0 , Ω_m , w_0 and w_a at 95% confidence level obtained for a flat Λ CDM cosmology with a CPL parameterisation for the dark energy EoS. In the lower part of the table, we assumed a Gaussian prior on $H_0 = 73 \pm 2.4$ km/s/Mpc from refs. [30, 31, 39], and hence do not provide the constraints for H_0 .

BC03 dataset providing tighter constraints on cosmological parameters, as expected given the higher number of data available.

We therefore conclude that our measurements are robust, and do not significantly depend on the assumed EPS model.

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