Universitat de Barcelona

Facultat de Física

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Interpreting the $b \to s\,\mu^+\mu^-$ anomaly in a heavy fermion model with Minimal Flavor Violation

Author:
Clara Hormigos Feliu

Supervisors:
Domènec Espriu Climent
Federico Mescia

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Universitat de
BARCELONA

Abstract

Discrepancies with the Standard Model (SM) have recently been observed in $b \to s$ transitions, which hint at a possible description in terms of four-fermion operators. In this work, we seek to describe the anomaly by introducing new heavy fermions. Firstly we build four-fermion operators out of SM and new fermions and introduce the hypothesis of Minimal Flavor Violation. Next we integrate out the heavy degrees of freedom to form operators of four SM fields and obtain an effective coupling constant. Two of the operators containing left-handed quarks show especial potential to describe the anomaly, and we fit their coefficients to experimental data using open-source code `flavio` to find that non-zero values are preferred. Using the result of adimensional coefficient $\overline{\delta}_+ = 0.49 \pm 0.16$, we perform predictions for observables P_5' and R_K^* and find that they lie closer to experimental data than the SM predictions.

Contents

1. Introduction

The Standard Model (SM) of particle physics has proven a very successful theory, having passed numerous experimental tests. Nonetheless, it is an effective theory, valid up to some cutoff scale Λ. The gauge hierarchy problem, which concerns the stability of the Higgs mass, suggests that this scale be in the TeV region. Therefore, hints of new physics can be expected at the TeV scale, which is currently being probed at LHC [1].

In the quark sector, flavored interactions show potential in terms of unearthing New Physics (NP). Flavor symmetry is broken in the Standard Model by the Yukawa interactions, giving rise to Flavor-Changing Charged Currents at tree level. The probability of transitions between flavor quark states is governed by the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, which is approximately flavor diagonal with small off-diagonal elements [2]. On the other hand, Flavor-Changing Neutral Currents (FCNC) are forbidden at tree level in the SM, and at one-loop they are suppressed by at least one off-diagonal CKM element. For this reason they fall in the category of *rare* processes, and are especially sensitive to new physics contributions. In this study we focus on B meson decays involving $b \to s$ transitions with final lepton pairs, which occur at lowest order at one loop in the SM.

Since flavor is not a good symmetry of the Standard Model, new physics is not expected to have an exact flavor symmetry, but rather a non-generic flavor structure. However, since large flavor-violating contributions are precluded by experimental data, this structure should be mostly flavor-symmetric, with small symmetry-breaking terms [1]. One of the many possibilities of such a flavor structure is the hypothesis of Minimal Flavor-Violation (MFV), which assumes that flavor violation in new physics follows the same structure as in the SM, that is, is governed by the Yukawa couplings. In this study we propose a new physics model involving MFV. It must be noticed that, even though MFV provides a framework to describe flavor interactions beyond the Standard Model, it is not a fundamental theory of flavor, in the sense that it does not provide an explanation for the hierarchical structure of the Yukawas and the CKM elements.

At low energies, weak interactions of quarks can be described using the Operator Product Expansion (OPE), a series of effective local operators multiplied by effective coefficients which can be computed in perturbation theory [3]. The coefficients, called Wilson coefficients, are sensitive to contributions of new physics below the scale Λ. Recently there has been evidence of deviations from the SM in observables related to $b \to s$ transitions, as is the case of observable P_5'. As can be seen in figure (1), measurements from LHCb lie approximately 3σ away from the Standard Model in some bins [2], and the Belle experiment results from 2016 seem to follow the same trend [4]. Evidence suggests that the anomaly affects decays involving muons and not electrons, and a nonvanishing contribution of NP to Wilson coefficients C_9 and C_{10} has been seen to provide an improved description of the data [5].

In this work, we propose the introduction of new heavy fermions which couple to Standard Model quarks and leptons through four-fermion operators involving Minimal Flavor Violation. We aim to test whether these operators in their low-energy limit could give contributions through four SM-fermion operators, which in turn could explain the observed tensions with the Standard Model. A connection could be established to scenarios describing flavor and electroweak symmetries and their breaking, such as technicolor, extended technicolor or composite Higgs models. For the purpose of this study, we do not adhere to any particular model.

First, in section 2 we construct all possible four-fermion operators involving two SM-fermions and two beyond the Standard Model (BSM) fermions. The number of operators to study could be at first glance excessively large, but symmetries allow for a considerable reduction of this number. We also make some choices as of the operators to study, focusing on those which preserve chirality. Next, in section 3 we introduce Minimal Flavor Violation in the bilinears containing SM fields, not assuming any flavor structure for the BSM fermions. In section 4 we build a four-point effective vertex by integrating the heavy degrees of freedom and obtain a list of operators involving four SM quarks or two SM quarks and two SM leptons. At his point, we obtain an effective coupling constant that depends on the

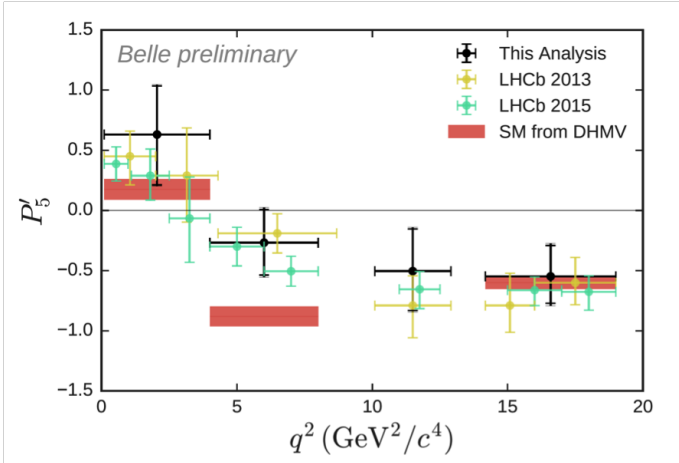

FIG. 1: Experimental values and Standard Model predictions for observable P_5'. Source: ref. [4].

scale of the process, the cutoff scale and the mass of the heavy fermions. Our choice for the mass of the BSM fermions and the cutoff is 1 TeV, and we take the scale of the process to be of order of the mass of the b quark. Finally, in section 5 we relate our operators to those in the OPE and use experimental data to study whether there is evidence of nonvanishing contributions. On the one hand, we compute the coefficients of our operators by relating them to fit results for C_9, C_{10}, C_9' and C_{10}' in the literature. Additionally, we perform our own fits. In order to do this, we use the open-source code `flavio` [6], which implements fitting routines for new physics contributions and their statistical analysis. We restrict this analysis to two of the operators we obtained, which give contributions to the process $b \to s\mu^+\mu^-$. Lastly, we compute the predicted values of observables P_5' and R_K^* in our NP scenario and compare them with the experimental points by LHCb and the SM predictions.

2. Constructing four-fermion operators

The first step is to construct all possible four-fermion operators using the same procedure as in [7]. The group structure assumed for the BSM fermions is that they appear in a $SU(2)_L$ and a $SU(2)_R$ doublet and in the representation 3 of $SU(3)_c$. This allows us to contract their indices with the SM fields, which (without any source of electroweak symmetry breaking) belong to the same representations. However, at the end of this procedure we would like to obtain operators of the type $(\bar{Q}Q)(\bar{q}q)$, so that all indices of the BSM fermions (Q) are contracted with themselves and the same for the SM fermions (q) in such a way that it be in fact irrelevant to which representations of $SU(3)_c$ and $SU(2)_L \otimes SU(2)_R$ the BSM fermions belong. At first the only requirement imposed is that the operators be Lorentz invariant and hermitian, and that the fermions be contracted in group invariants. We start by considering two pairs of fermions, expressed as $\{\psi, \psi, \psi', \psi'\}$, where primes indicate fermions that carry the same indices, and in the end we specify which correspond to the SM fermions and which to the BSM ones. First we consider the possible color structures an then we analyze the Dirac structure and make use of the Fierz identities. Next we look at the $SU(2)$ structure and again use Fierz identities to express the operators in the desired form, and finally we list all the operators we obtain.

2.1. Color structure

There are two possible ways of contracting the color indices, denoted by greek letters, to obtain color singlets. These are

$$\overline{\psi}^{\alpha}\psi^{\alpha}\overline{\psi}^{\beta}\psi^{\beta} = (\overline{\psi}\psi)(\overline{\psi}\psi)\,, \tag{2.1}$$

$$\overline{\psi}_{\alpha}\vec{\lambda}_{\alpha\beta}\psi_{\beta}\,\overline{\psi}_{\delta}\vec{\lambda}_{\delta\gamma}\psi_{\gamma} = (\overline{\psi}\vec{\lambda}\psi)(\overline{\psi}\vec{\lambda}\psi)\,, \tag{2.2}$$

where parentheses indicate contracted color indices, and $\vec{\lambda}$ are the $SU(3)_c$ generators. In the case of two pairs of fermions with different additional indices, each of these cases allows two possibilities of contracting them (denoted by the primes). Making use of the parentheses notation, these are

$$(\overline{\psi}\psi)(\overline{\psi}'\psi')\,, \qquad (\overline{\psi}\psi')(\overline{\psi}'\psi)\,, \tag{2.3}$$

$$(\overline{\psi}\vec{\lambda}\psi)(\overline{\psi}'\vec{\lambda}\psi')\,, \qquad (\overline{\psi}\vec{\lambda}\psi')(\overline{\psi}'\vec{\lambda}\psi)\,. \tag{2.4}$$

However, in order to simplify our analysis and with the aim not to be bound, at the end, to a particular group structure, color indices are assumed to be contracted in the more simple way shown in (2.1). Furthermore, in later steps leptons will be included in our study. When that is the case the same operators will be considered, only taking into account that they are color singlets.

2.2. Dirac structure

In order to obtain all possible Dirac structures the chiral basis is used. This will permit an easy distinction between right-handed and left-handed currents. The elements of the basis are $\Gamma^A = \{P_R, P_L, \gamma^\mu P_L, \gamma^\mu P_R, \sigma^{\mu\nu}\}$ [8], where

$$P_L = \frac{1}{2}(1-\gamma^5)\,, \qquad P_R = \frac{1}{2}(1+\gamma^5)\,, \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\,. \tag{2.5}$$

Seeing that we are building operators of two fermion bilinears, the structures we want to construct are of the type $(\overline{\psi}\Gamma^{\mu_1\cdots\mu_n}\psi)(\overline{\psi}\Gamma'_{\mu_1\ldots\mu_n}\psi)$. The possible Lorentz-invariant tensor products $\Gamma^{\mu_1\cdots\mu_n}\otimes\Gamma'_{\mu_1\ldots\mu_n}$ built from the elements of the chiral basis are

$$\{P_R\otimes P_R, P_R\otimes P_L, P_L\otimes P_R, P_L\otimes P_L, \gamma_\mu P_R\otimes\gamma^\mu P_R, \gamma_\mu P_R\otimes\gamma^\mu P_L, \gamma_\mu P_L\otimes\gamma^\mu P_R, \gamma_\mu P_L\otimes\gamma^\mu P_L, \sigma_{\mu\nu}\otimes\sigma^{\mu\nu}, \epsilon_{\mu\nu\sigma\tau}\sigma^{\mu\nu}\otimes\sigma^{\rho\tau}\}\,. \tag{2.6}$$

The four-fermion operators are constructed by inserting these possibilities in the bilinears. Before proceeding, some useful relations to keep in mind are

$$\begin{aligned} &P_L^2 = P_L\,, \quad P_R^2 = P_R\,, \quad P_L\psi = \psi_L\,, \quad P_R\psi = \psi_R\,, \\ &\overline{\psi}P_L = \overline{\psi}_R\,, \quad \overline{\psi}P_R = \overline{\psi}_L\,, \quad \epsilon_{\mu\nu\rho\tau}\sigma^{\mu\nu} = 2i\gamma^5\sigma_{\rho\tau}\,. \end{aligned} \tag{2.7}$$

Using the above we obtain the following relations which are useful to check the hermiticity and $\mathcal{CP}$ invariance of the operators:

$$\begin{aligned} &(\overline{\psi}_L\psi'_R)^\dagger = \overline{\psi}'_R\psi_L\,, \qquad (\overline{\psi}_L\gamma^\mu\psi'_L)^\dagger = \overline{\psi'}_L\gamma^\mu\psi_L\,, \\ &\mathcal{CP}(\overline{\psi}_L\psi'_R)\mathcal{C}^{-1}\mathcal{P}^{-1} = \overline{\psi}'_R\psi_L\,, \qquad \mathcal{CP}(\overline{\psi}\gamma^\mu P_L\psi')\mathcal{C}^{-1}\mathcal{P}^{-1} = \overline{\psi}'\gamma^\mu P_L\psi\,. \end{aligned} \tag{2.8}$$

2.2.1. Chiral Fierz Identities

Some of the bilinears can be expressed as a function of others through the Fierz identities. These are usually expressed in the Dirac basis but they can be found as well in the chiral basis. Their use allows to reduce the number of independent operators that we need to study. In order to simplify the expressions we introduce the Takahashi notation, in which parentheses and brackets are used to indicate in which way the color indices of the spinors at each side of the chiral basis matrices are contracted. Each parenthesis (bracket) represents a color index that is contracted with the other parenthesis (bracket). As an example we can write

$$\overline{\psi}^{\alpha} P_L\, \psi^{\alpha} \overline{\psi}^{\beta} P_L\, \psi^{\beta} = (P_L)[P_L]\,, \qquad \overline{\psi}^{\alpha} P_L \psi^{\beta} \overline{\psi}^{\beta} P_L \psi^{\alpha} = (P_L][P_L)\,. \tag{2.9}$$

The orthogonality property relating the elements Γ^A of the chiral basis reads $\mathrm{Tr}[\Gamma^A \Gamma_B] = 2\delta^A_B$, and from it the chiral Fierz identities are derived [8]. They read

$$(\Gamma^A)[\Gamma^B] = \frac{1}{4}\mathrm{Tr}[\Gamma^A \Gamma_C \Gamma^B \Gamma_D](\Gamma^D][\Gamma^C)\,. \tag{2.10}$$

These identities have been evaluated in [9] for all possible Γ^A, Γ^B, and the relations they establish between the different products show that not all operators obtained by introducing the different tensor products are independent. In the next sections, the identities will allow us to reduce the number of independent operators. As an example, from (2.10) stem the following (see [9] for a complete list):

$$\begin{aligned} -2(P_R)[P_L] &= (\gamma^\mu P_L][\gamma_\mu P_R)\,, & -2(P_L)[P_R] &= (\gamma^\mu P_R][\gamma_\mu P_L)\,, \\ (\gamma^\mu P_R)[\gamma_\mu P_R] &= -(\gamma^\mu P_R][\gamma_\mu P_R)\,, & (\gamma^\mu P_L)[\gamma_\mu P_L] &= -(\gamma^\mu P_L][\gamma_\mu P_L)\,. \end{aligned} \tag{2.11}$$

2.2.2. Operators with Dirac Structure

To obtain the possible Dirac structures, the elements in (2.6) are systematically introduced in the color structures of (2.3). The following operators are obtained:

$$\begin{aligned}
P_R \otimes P_R &\to (\overline{\psi}_L \psi_R)(\overline{\psi'}_L \psi'_R)\,, (\overline{\psi}_L \psi'_R)(\overline{\psi}'_L \psi_R)\,, \\
P_R \otimes P_L &\to (\overline{\psi}_L \psi_R)(\overline{\psi'}_R \psi'_L)\,, (\overline{\psi}_L \psi'_R)(\overline{\psi}'_R \psi_L)\,, \\
P_L \otimes P_R &\to (\overline{\psi}_R \psi_L)(\overline{\psi'}_L \psi'_R)\,, (\overline{\psi}_R \psi'_L)(\overline{\psi}'_L \psi_R)\,, \\
P_L \otimes P_L &\to (\overline{\psi}_R \psi_L)(\overline{\psi'}_R \psi'_L)\,, (\overline{\psi}_R \psi'_L)(\overline{\psi}'_R \psi_L)\,, \\
\gamma^\mu P_L \otimes \gamma_\mu P_L &\to (\overline{\psi}_L \gamma^\mu \psi_L)(\overline{\psi'}_L \gamma_\mu \psi'_L)\,, (\overline{\psi}_L \gamma^\mu \psi'_L)(\overline{\psi}'_L \gamma_\mu \psi_L)\,, \\
\gamma^\mu P_L \otimes \gamma_\mu P_R &\to (\overline{\psi}_L \gamma^\mu \psi_L)(\overline{\psi'}_R \gamma_\mu \psi'_R)\,, (\overline{\psi}_L \gamma^\mu \psi'_L)(\overline{\psi}'_R \gamma_\mu \psi_R)\,, \\
\gamma^\mu P_R \otimes \gamma_\mu P_L &\to (\overline{\psi}_R \gamma^\mu \psi_R)(\overline{\psi'}_L \gamma_\mu \psi'_L)\,, (\overline{\psi}_R \gamma^\mu \psi'_R)(\overline{\psi}'_L \gamma_\mu \psi_L)\,, \\
\gamma^\mu P_R \otimes \gamma_\mu P_R &\to (\overline{\psi}_R \gamma^\mu \psi_R)(\overline{\psi'}_R \gamma_\mu \psi'_R)\,, (\overline{\psi}_R \gamma^\mu \psi'_R)(\overline{\psi}'_R \gamma_\mu \psi_R)\,, \\
\sigma^{\mu\nu} \otimes \sigma_{\mu\nu} &\to (\overline{\psi} \sigma^{\mu\nu} \psi)(\overline{\psi}' \sigma_{\mu\nu} \psi')\,, (\overline{\psi} \sigma^{\mu\nu} \psi')(\overline{\psi}' \sigma_{\mu\nu} \psi)\,, \\
\epsilon_{\mu\nu\sigma\tau} \sigma^{\mu\nu} \otimes \sigma^{\rho\tau} &\to \epsilon_{\mu\nu\rho\tau} (\overline{\psi} \sigma^{\mu\nu} \psi)(\overline{\psi}' \sigma^{\rho\tau} \psi')\,, \epsilon_{\mu\nu\rho\tau} (\overline{\psi} \sigma^{\mu\nu} \psi')(\overline{\psi}' \sigma^{\rho\tau} \psi)\,.
\end{aligned} \tag{2.12}$$

At this point, the fields between parentheses have the same color and Dirac indices, and the primes indicate how other indices (e.g. $SU(2)$) might be contracted. Using the Fierz relations found in [9] we see that only 10 out of the operators in (2.12) are linearly independent. Our choice is

$$(\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi'}_L\gamma_\mu\psi'_L)\,, \tag{2.13}$$
$$(\overline{\psi}_R\gamma^\mu\psi_R)(\overline{\psi'}_R\gamma_\mu\psi'_R)\,, \tag{2.14}$$
$$(\overline{\psi}_R\psi_L)(\overline{\psi'}_R\psi'_L)\,, \tag{2.15}$$
$$(\overline{\psi}_L\psi_R)(\overline{\psi'}_L\psi'_R)\,, \tag{2.16}$$
$$(\overline{\psi}_R\psi'_L)(\overline{\psi'}_R\psi_L)\,, \tag{2.17}$$
$$(\overline{\psi}_L\psi'_R)(\overline{\psi'}_L\psi_R)\,, \tag{2.18}$$
$$(\overline{\psi}_L\gamma^\mu\psi'_L)(\overline{\psi'}_L\gamma_\mu\psi_L)\,, \tag{2.19}$$
$$(\overline{\psi}_R\gamma^\mu\psi'_R)(\overline{\psi'}_R\gamma_\mu\psi_R)\,, \tag{2.20}$$
$$(\overline{\psi}_R\gamma^\mu\psi_R)(\overline{\psi'}_L\gamma_\mu\psi'_L) \tag{2.21}$$
$$(\overline{\psi}'_R\gamma^\mu\psi'_R)(\overline{\psi}_L\gamma_\mu\psi_L)\,. \tag{2.22}$$

This list can be reduced by imposing that all operators be hermitian. To this end (2.15) and (2.17) must be paired with their hermitian conjugates, which are precisely (2.16) and (2.18) respectively. Moreover, (2.21) and (2.22) are redundant, since the primes only indicate the same additional indices. This yields the following Lorentz-invariant, hermitian operators:

$$(\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi'}_L\gamma_\mu\psi'_L)\,, \tag{2.23}$$
$$(\overline{\psi}_L\gamma^\mu\psi'_L)(\overline{\psi'}_L\gamma_\mu\psi_L)\,, \tag{2.24}$$
$$(\overline{\psi}_R\gamma^\mu\psi_R)(\overline{\psi'}_R\gamma_\mu\psi'_R)\,, \tag{2.25}$$
$$(\overline{\psi}_R\gamma^\mu\psi'_R)(\overline{\psi'}_R\gamma_\mu\psi_R)\,, \tag{2.26}$$
$$(\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi}'_R\gamma_\mu\psi'_R)\,, \tag{2.27}$$
$$(\overline{\psi}_L\psi_R)(\overline{\psi'}_L\psi'_R) + h.c.\,, \tag{2.28}$$
$$(\overline{\psi}_L\psi'_R)(\overline{\psi'}_L\psi_R) + h.c.\,. \tag{2.29}$$

2.3. $SU(2)$ structure

Operating with the chiral basis has led us to write the operators in terms of left-handed and right-handed fields. We now promote these to $SU(2)_L$ and $SU(2)_R$ doublets respectively and, since no source of $SU(2)_L \otimes SU(2)_R$ breaking has yet been introduced, we require that the operators be $SU(2)_L \otimes SU(2)_R$ invariant. This means that they should remain invariant under

$$\begin{aligned} \psi_L &\to e^{i\vec{\theta}_L\vec{\tau}}\psi_L\,, \qquad \overline{\psi}_L \to \overline{\psi}_L e^{-i\vec{\theta}_L\vec{\tau}}\,, \\ \psi_R &\to e^{i\vec{\theta}_R\vec{\tau}}\psi_R\,, \qquad \overline{\psi}_R \to \overline{\psi}_R e^{-i\vec{\theta}_R\vec{\tau}}\,, \end{aligned} \tag{2.30}$$

where τ^i are the $SU(2)$ generators in the fundamental representation. Since $U_L = e^{i\vec{\theta}_L\vec{\tau}}$, $U_R = e^{i\vec{\theta}_R\vec{\tau}}$ are unitary, operators containing only left-handed or right-handed fields are trivially $SU(2)_L \otimes SU(2)_R$ invariant. This is so because we are constructing bilinears with two doublets and two hermitian conjugate doublets. For the mentioned operators (which correspond to (2.23)-(2.26)), there are only four independent operators when $SU(2)$ indices are contracted. Since no further symmetries are assumed, we drop the primes and obtain the following:

$$\begin{aligned}
&(\overline{\psi}^i_L\gamma^\mu\psi^i_L)(\overline{\psi}^j_L\gamma_\mu\psi^j_L) = (\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi}_L\gamma_\mu\psi_L)\,, \qquad &&(\overline{\psi}^i_L\gamma^\mu\psi^j_L)(\overline{\psi}^j_L\gamma_\mu\psi^i_L)\,,\\
&(\overline{\psi}^i_R\gamma^\mu\psi^i_R)(\overline{\psi}^j_R\gamma_\mu\psi^j_R) = (\overline{\psi}_R\gamma^\mu\psi_R)(\overline{\psi}_R\gamma_\mu\psi_R)\,, \qquad &&(\overline{\psi}^i_R\gamma^\mu\psi^j_R)(\overline{\psi}^j_R\gamma_\mu\psi^i_R)\,,
\end{aligned} \tag{2.31}$$

When no indices are explicitly written all indices are contracted for the fields between parentheses. We will assume that flavor indices are contracted within the parentheses as well when we promote the fields to vectors in flavor space later on. In the case of operator (2.27), the contraction gives

$$(\overline{\psi}^i_L\gamma^\mu\psi^i_L)(\overline{\psi}^j_R\gamma_\mu\psi^j_R) = (\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi}_R\gamma_\mu\psi_R)\,. \tag{2.32}$$

The operators involving fields with both chiralities, (2.28) and (2.29), need to be handled more carefully. First we notice that contracting the indices naively, for example as in $(\overline{\psi}^i_L\psi^j_R)(\overline{\psi}^i_L\psi^j_R)$, does not give an $SU(2)$ invariant. Nonetheless, an invariant structure can be obtained by introducing the-two dimensional antisymmetric tensor (we do not write the γ^0 matrices as they are not relevant for the $SU(2)$ analysis):

$$\psi^{\dagger i}_L\epsilon_{ij}\psi^{\dagger j}_L = \begin{pmatrix} u^\dagger_L & d^\dagger_L \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} u^\dagger_L \\ d^\dagger_L \end{pmatrix} = \begin{pmatrix} u^\dagger_L & d^\dagger_L \end{pmatrix}\begin{pmatrix} d^\dagger_L \\ -u^\dagger_L \end{pmatrix} = \psi^\dagger_L\psi^c_L\,. \tag{2.33}$$

Given that the conjugate doublet transforms like the doublet, $\psi^c_L \to e^{i\vec{\theta}_L\vec{\tau}}\psi^c_L$, the product $\psi^\dagger_L\psi^c_L$ is $SU(2)_L$ invariant. Hence for (2.28) and (2.29) we write the following operator:

$$(\overline{\psi}^i_L\psi^j_R)(\overline{\psi}^k_L\psi^l_R)\epsilon_{ik}\epsilon_{jl} + h.c.\,, \tag{2.34}$$

which is $SU(2)_L \otimes SU(2)_R$ invariant. Putting everything together, and dropping the $SU(2)$ indices when they are contracted in the same way as the color indices, we obtain the following list of Lorentz-invariant, $SU(2)_L \otimes SU(2)_R$, hermitian operators:

$$\begin{aligned}
&(\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi}_L\gamma_\mu\psi_L)\,, \quad &&(\overline{\psi}^i_L\gamma^\mu\psi^j_L)(\overline{\psi}^j_L\gamma_\mu\psi^i_L)\,,\\
&(\overline{\psi}_R\gamma^\mu\psi_R)(\overline{\psi}_R\gamma_\mu\psi_R)\,, \quad &&(\overline{\psi}^i_R\gamma^\mu\psi^j_R)(\overline{\psi}^j_R\gamma_\mu\psi^i_R)\,,\\
&(\overline{\psi}_L\gamma^\mu\psi_L)(\overline{\psi}_R\gamma_\mu\psi_R)\,, \quad &&(\overline{\psi}^i_L\psi^j_R)(\overline{\psi}^k_L\psi^l_R)\epsilon_{ik}\epsilon_{jl} + h.c.\,.
\end{aligned} \tag{2.35}$$

Using the relations of (2.8), we see that they are as well $\mathcal{CP}$ invariant. Now we are ready to obtain the operators with two Q and two q fermions. All possible combinations of $\psi = \{\overline{Q}, Q, \overline{q}, q\}$ are inserted in the list above. A distinction is made between chirality-preserving operators (in which each pair of fermions has the same chirality) and chirality-violating operators (when it is not the same). The operators we obtain are

- Chirality preserving:

$$\begin{aligned}
&L^2 = (\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_L\gamma_\mu q_L)\,, \quad &&a = (\overline{Q}_L\gamma^\mu q_L)(\overline{q}_L\gamma_\mu Q_L)\,,\\
&R^2 = (\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_R\gamma_\mu q_R)\,, \quad &&b = (\overline{Q}_R\gamma^\mu q_R)(\overline{q}_R\gamma_\mu Q_R)\,,\\
&c = (\overline{Q}^i_L\gamma^\mu Q^j_L)(\overline{q}^j_L\gamma_\mu q^i_L)\,, \quad &&d = (\overline{Q}^i_L\gamma^\mu q^j_L)(\overline{q}^j_L\gamma_\mu Q^i_L)\,,\\
&e = (\overline{Q}^i_R\gamma^\mu Q^j_R)(\overline{q}^j_R\gamma_\mu q^i_R)\,, \quad &&f = (\overline{Q}^i_R\gamma^\mu q^j_R)(\overline{q}^j_R\gamma_\mu Q^i_R)\,,\\
&LR = (\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_R\gamma_\mu q_R)\,, \quad &&RL = (\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_L\gamma_\mu q_L)\,.
\end{aligned} \tag{2.36}$$

- Chirality violating:

$$\begin{aligned} LR' &= (\overline{Q}_L\gamma^\mu q_L)(\overline{q}_R\gamma_\mu Q_R) + h.c.\,, \\ A &= (\overline{Q}^i_L Q^j_R)(\overline{q}^k_L q^l_R)\epsilon_{ik}\epsilon_{jl} + h.c.\,, \\ B &= (\overline{Q}^i_R Q^j_L)(\overline{q}^k_R q^l_L)\epsilon_{ik}\epsilon_{jl} + h.c.\,, \\ C &= (\overline{Q}^i_L q^j_R)(\overline{q}^k_L q^l_R)\epsilon_{ik}\epsilon_{jl} + h.c.\,, \\ D &= (\overline{Q}^i_R q^j_L)(\overline{q}^k_R Q^l_L)\epsilon_{ik}\epsilon_{jl} + h.c.\,. \end{aligned} \tag{2.37}$$

As stated before, it is desirable to obtain operators of the form $(\bar{Q}Q)(\bar{q}q)$. For the operators which are still not in this form this can be achieved through the Fierz identities for $SU(2)$ and $SU(3)$ [8], which read, respectively,

$$\delta_{ij}\delta_{kl} + (\tau_m)_{ij}(\tau_m)_{kl} = 2\delta_{il}\delta_{kj}\,, \quad \frac{1}{3}\delta_{ij}\delta_{kl} + \frac{1}{2}(\lambda_a)_{ij}(\lambda_a)_{kl} = \delta_{il}\delta_{kj}\,, \tag{2.38}$$

or expressing it in terms of $SU(2)$ vectors χ_i and $SU(3)$ vectors η_i,

$$(\bar{\chi}_1\chi_2)(\bar{\chi}_3\chi_4) + (\bar{\chi}_1\vec{\tau}\chi_2)(\bar{\chi}_3\vec{\tau}\chi_4) = 2(\bar{\chi}_1\chi_4)(\bar{\chi}_2\chi_3)\,, \qquad \frac{1}{3}(\bar{\eta}_1\eta_2)(\bar{\eta}_3\eta_4) + \frac{1}{2}(\bar{\eta}_1\vec{\tau}\eta_2)(\bar{\eta}_3\vec{\tau}\eta_4) = (\bar{\eta}_1\eta_4)(\bar{\eta}_2\chi_3)\,. \tag{2.39}$$

Using these, it is shown in [7] that $a - f$ can be written in terms of

$$\begin{aligned} \vec{L}^2 &= (\overline{Q}_L\vec{\tau}\gamma^\mu Q_L)(\overline{q}_L\vec{\tau}\gamma_\mu q_L)\,, & \vec{R}^2 &= (\overline{Q}_R\vec{\tau}\gamma^\mu Q_R)(\overline{q}_R\vec{\tau}\gamma_\mu q_R)\,, \\ \vec{l}^{\,2} &= (\overline{Q}_L\vec{\lambda}\vec{\tau}\gamma^\mu Q_L)(\overline{q}_L\vec{\lambda}\vec{\tau}\gamma_\mu q_L)\,, & \vec{r}^{\,2} &= (\overline{Q}_R\vec{\lambda}\vec{\tau}\gamma^\mu Q_R)(\overline{q}_R\vec{\lambda}\vec{\tau}\gamma_\mu q_R)\,, \\ l^2 &= (\overline{Q}_L\vec{\lambda}\gamma^\mu Q_L)(\overline{q}_L\vec{\lambda}\gamma_\mu q_L)\,, & r^2 &= (\overline{Q}_R\vec{\lambda}\gamma^\mu Q_R)(\overline{q}_R\vec{\lambda}\gamma_\mu q_R)\,. \end{aligned} \tag{2.40}$$

Although keeping a complete basis of operators would require including operators with $SU(3)$ generators (λ), we restrict our study to the operators without them for reasons already stated. This means that from the list above we only keep operators $\vec{L}^2$ and $\vec{R}^2$. We also restrict our study to chirality-preserving operators, which are the ones that show more potential from the experimental side. This allows to work with a reduced number of operators, which are the following:

$$\begin{aligned} \vec{L}^2 &= (\overline{Q}_L\vec{\tau}\gamma^\mu Q_L)(\overline{q}_L\vec{\tau}\gamma_\mu q_L)\,, & \vec{R}^2 &= (\overline{Q}_R\vec{\tau}\gamma^\mu Q_R)(\overline{q}_R\vec{\tau}\gamma_\mu q_R)\,, \\ L^2 &= (\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_L\gamma_\mu q_L)\,, & R^2 &= (\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_R\gamma_\mu q_R)\,, \\ LR &= (\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_R\gamma_\mu q_R)\,, & RL &= (\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_L\gamma_\mu q_L)\,. \end{aligned} \tag{2.41}$$

In addition to not having to assume any particular color representation, in the operators above it need not be assumed any particular representation of $SU(2)$ for the BSM fermions either. If they appeared in a representation other than the fundamental the operators could be simply modified by exchanging the τ matrices ($SU(2)$ generators in the fundamental representation) for the generators in the appropriate representation.

3. Minimal Flavor Violation

So far we have obtained a list of six four-fermion hermitian operators with a simple color structure which are $SU(2)_L \otimes SU(2)_R \otimes SU(3)_c$, Lorentz, and $\mathcal{CP}$ invariant and preserve chirality. In order to introduce in the operators a source

of $\mathcal{CP}$ violation and quark mixing we turn to the hypothesis of Minimal Flavor Violation. Its starting point is the Yukawa sector of the Standard Model Lagrangian containing quarks, which reads

$$\mathcal{L}_{\mathrm{Y,quarks}} = \overline{q}_L Y_u u_R \phi + \overline{q}_L Y_d d_R \tilde{\phi} + h.c.\,, \tag{3.1}$$

where the doublet q_L and the singlets u_R and d_R are quark fields which correspond to 3-component vectors in generation space, ϕ is the Higgs doublet (with $\tilde{\phi} = i\tau_2\phi^*$) and $Y_{u,d}$ are the Yukawa matrices, 3×3 matrices in generation space. The Lagrangian (3.1) is invariant under the changes

$$q_L \to V_Q q_L\,, \quad u_R \to V_u u_R\,, \qquad d_R \to V_d d_R\,, \tag{3.2}$$

where V_i are as well 3×3 matrices in generation space. This is so if the Yukawa matrices transform as

$$Y_u \to V_Q Y_u V_u^\dagger\,, \quad Y_d \to V_Q Y_d V_d^\dagger\,. \tag{3.3}$$

We choose to diagonalize Y_d, then

$$Y_d = \lambda_d, \qquad Y_u = V^\dagger \lambda_u\,, \tag{3.4}$$

where $\lambda_{u,d}$ are diagonal matrices (with its diagonal components being the usual Yukawa couplings) and V is the CKM matrix. Minimal Flavor Violation proposes that the dynamics of flavor violation in all operators arise from the same structures as in the Standard Model, that is, that it be determined by the structure of the Yukawa couplings. This also implies that $\mathcal{CP}$ violation is originated only by the phase in the CKM matrix [7]. Our procedure will be to introduce suitable combinations of Yukawa matrices in our operators in such a way that they remain invariant under the changes (3.2) and (3.3). Before proceeding it is useful to define the flavor-violating parts of certain combinations of Yukawa matrices, such as

$$Y_u Y_u^\dagger \simeq y_t^2 V_{3i}^* V_{3j} \;\to \lambda_{F1} = y_t^2 V_{3i}^* V_{3j}\,, (i \neq j)\,, \tag{3.5}$$

$$Y_d^\dagger Y_u = (\lambda_d V^\dagger \lambda_u)_{ij} \;\to \lambda_{F2} = (\lambda_d V^\dagger \lambda_u)_{ij}\,, (i \neq j)\,. \tag{3.6}$$

In the case of λ_{F1} the fact that the top Yukawa coupling y_t provides the largest contribution has been used, since $y_t \propto \mathcal{O}(1)$ and the other y_i are much smaller. Constructing the possible MFV bilinears and extracting the flavor-changing part gives

$$\begin{aligned} &\overline{q}_L Y_u Y_u^\dagger q_L \to \overline{q}_L \lambda_{F1} q_L\,, && \overline{d}_R Y_d^\dagger Y_u Y_u^\dagger Y_d d_R \to \overline{d}_R \lambda_d \lambda_{F1} \lambda_d d_R\,, \\ &\overline{d}_R Y_d^\dagger Y_u u_R = \overline{d}_R \lambda_d V^\dagger \lambda_u u_R \to \overline{d}_R \lambda_{F2} u_R \;(+h.c.). \end{aligned} \tag{3.7}$$

Structures involving two right-handed up-type quarks do not produce MFV since $\overline{u}_R Y_u^\dagger Y_u Y_u^\dagger Y_u u_R = \overline{u}_R u_R$. Those involving two right-handed down-type quarks are suppressed by λ_d^2, but we include them in our analysis even so. The matrix λ_{F2} is not hermitian, but except when expanding a bilinear explicitly we will simply write λ_{F2} in the understanding that its hermitian conjugate must be taken when the term is of the type $\overline{u}_R \lambda_{F2}^\dagger d_R$. The next step is to introduce the MFV structures in (3.7) in the operators of (2.41). It must be noticed that the Lagrangian (3.1) does not respect $SU(2)_R$ symmetry, and that u and d-type R fields have different transformation properties under

the MFV hypothesis. This forces us to break down our operators and express them in terms of right-handed singlets. To his end it is useful to expand the operators which contain an $SU(2)$ generator as

$$\overline{q}\vec{\tau}q = \frac{1}{2}\big(\overline{u}d + \overline{d}u, -i\overline{u}d + i\overline{d}u, \overline{u}u - \overline{d}d\big)\,. \tag{3.8}$$

The MFV structures are inserted only in the bilinears containing SM quarks, since we have not assumed any flavor transformation properties for the BSM fermions. Promoting the fields in (2.41) to three-column vectors in family space, the following $\Delta F = 1$ operators are obtained:

$$\begin{aligned}
\vec{L}^2 &\to (\overline{Q}_L\vec{\tau}\gamma^\mu Q_L)(\overline{q}_L\vec{\tau}\gamma_\mu\lambda_{F1}q_L)\,,\\
L^2 &\to (\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_L\gamma_\mu\lambda_{F1}q_L)\,,\\
RL &\to (\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_L\gamma_\mu\lambda_{F1}q_L)\,,\\
R^2 &\to (\overline{Q}_R\gamma^\mu Q_R)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,,\\
LR &\to (\overline{Q}_L\gamma_\mu Q_L)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,.
\end{aligned} \tag{3.9}$$

Operator $\vec{R}^2$ needs to be handled more carefully because of its $\overline{q}_R\vec{\tau}\gamma^\mu q_R$ bilinear, which mixes up and down-type right-handed quarks. Expanding it according to (3.8), we find that it can accommodate the following MFV structures:

$$\vec{R}^2 \to (\overline{Q}_R\vec{\tau}\gamma^\mu Q_R)(\overline{u}_R\gamma_\mu\lambda^\dagger_{F2}d_R + h.c.,\ -i\overline{u}_R\gamma_\mu\lambda^\dagger_{F2}d_R + h.c.,\ \overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,. \tag{3.10}$$

Taking into account that the operators have mass dimension 6, in the Lagrangian they must be suppressed by a power of 2 of some cutoff scale Λ, which later on we will associate with the mass of the heavy fermions. Making this explicit and taking c_i to be adimensional couplings, we write the following list of operators $\mathcal{Q}_i$, involving two BSM fermions and two SM quarks:

$$\begin{aligned}
\mathcal{Q}_1 &= \frac{c_1}{\Lambda^2}(\overline{Q}_L\gamma^\mu Q_L)(\overline{q}_L\gamma_\mu\lambda_{F1}q_L)\,, & \mathcal{Q}_2 &= \frac{c_2}{\Lambda^2}(\overline{Q}_R\gamma^\mu Q_R)(\overline{q}_L\gamma_\mu\lambda_{F1}q_L)\,,\\
\mathcal{Q}_3 &= \frac{c_3}{\Lambda^2}(\overline{Q}_L\gamma_\mu Q_L)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,, & \mathcal{Q}_4 &= \frac{c_4}{\Lambda^2}(\overline{Q}_R\gamma^\mu Q_R)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,,\\
\mathcal{Q}_5 &= \frac{c_5}{\Lambda^2}(\overline{Q}_L\vec{\tau}\gamma^\mu Q_L)(\overline{q}_L\vec{\tau}\gamma_\mu\lambda_{F1}q_L)\,, & \mathcal{Q}_6 &= \frac{c_6}{\Lambda^2}(\overline{Q}_R\tau_1\gamma^\mu Q_R)(\overline{q}_R\gamma_\mu\tau_1\lambda_{F2}q_R)\,,\\
\mathcal{Q}_7 &= \frac{c_7}{\Lambda^2}(\overline{Q}_R\tau_2\gamma^\mu Q_R)(\overline{q}_R\gamma_\mu\tau_2\lambda_{F2}q_R)\,, & \mathcal{Q}_8 &= \frac{c_8}{\Lambda^2}(\overline{Q}_R\tau_3\gamma^\mu Q_R)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,,
\end{aligned} \tag{3.11}$$

where we have broken up the operator of the form $\vec{R}^2$ in its three components because, as we have seen, they allow different MFV structures (operators $\mathcal{Q}_{6-8}$). In addition to the operators involving quarks, we can also construct operators involving SM leptons. In this case flavor-violating structures are not introduced, since the MFV hypothesis we have adopted does not allow for family mixing in the lepton sector. This entails that it is free from $SU(2)_R$-breaking sources, and therefore we may group the leptons in a right-handed and a left-handed doublet as follows:

$$E_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}\,, \qquad E_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}\,, \tag{3.12}$$

with each doublet being a three-column vector in family space. The following operators involving leptons, $\mathcal{Q}_{il}$, are obtained:

$$
\begin{aligned}
\mathcal{Q}_{1l} &= \frac{c_{1l}}{\Lambda^2}(\overline{Q}_L\gamma^\mu Q_L)(\overline{E}_L\gamma_\mu E_L)\,, & \mathcal{Q}_{2l} &= \frac{c_{2l}}{\Lambda^2}(\overline{Q}_R\gamma^\mu Q_R)(\overline{E}_L\gamma_\mu E_L)\,,\\
\mathcal{Q}_{3l} &= \frac{c_{3l}}{\Lambda^2}(\overline{Q}_L\gamma_\mu Q_L)(\overline{E}_R\gamma_\mu E_R)\,, & \mathcal{Q}_{4l} &= \frac{c_{4l}}{\Lambda^2}(\overline{Q}_R\gamma_\mu Q_R)(\overline{E}_R\gamma_\mu E_R)\,,\\
\mathcal{Q}_{5l} &= \frac{c_{5l}}{\Lambda^2}(\overline{Q}_L\vec{\tau}\gamma^\mu Q_L)(\overline{E}_L\vec{\tau}\gamma_\mu E_L)\,, & \mathcal{Q}_{6l} &= \frac{c_{6l}}{\Lambda^2}(\overline{Q}_R\vec{\tau}\gamma^\mu Q_R)(\overline{E}_R\vec{\tau}\gamma_\mu E_R)\,.
\end{aligned}
\tag{3.13}
$$

4. Effective theory

To this point we have obtained a list of four-fermion operators which involve Minimal Flavor Violation, some containing SM quarks and some SM leptons, and all containing a pair of BSM fermions. Now we would like to quantify the contribution of these operators to processes which can be observed in current experiments such as LHCb, so that it can be determined whether such contributions are allowed from the experimental side. Let's consider diagrams of second order in the operators $\mathcal{Q}$ which involve four fermion external sates, as shown in figure (2), and assume that the mass of the BSM fermions is $m \propto 1$ TeV. Contributions from diagrams of this type at energies much lower than m may be accounted for by an effective four-point fermion interaction as shown in figure (3) once the heavy degrees of freedom have been integrated out, in the same way that Fermi theory is able describe electroweak interactions at scales lower than the mass of the weak bosons.

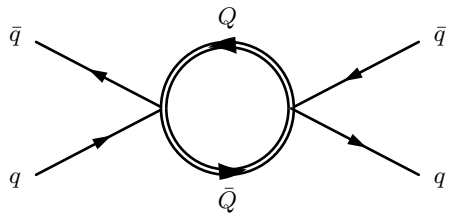

FIG. 2: Four-point fermion interaction mediated by heavy fermions. Double lines indicate heavy fermions.

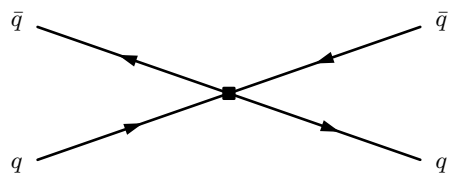

FIG. 3: Effective four-fermion interaction after integration of heavy fermions.

4.1. Integration of heavy degrees of freedom

We proceed to compute the (amputated) diagram of figure (2). Since the diagram is clearly divergent (a quick analysis shows that the superficial degree of divergence is $d^4k/k^2 \propto k^2$), it must be regularized in order to extract its finite contribution. In spite of the presence of definite-chirality currents, which involve γ^5, we use naive dimensional regularization, which should not be inconsistent in the absence of triangular-type diagrams. To start, we explicitly expand the BSM fermion bilinears, which gives

$$
\begin{aligned}
(\overline{Q}_L\gamma^\mu Q_L) &= (\overline{U}_L\gamma^\mu U_L + \overline{D}_L\gamma^\mu D_L)\,, \qquad (\overline{Q}_R\gamma^\mu Q_R) = (\overline{U}_R\gamma^\mu U_R + \overline{D}_R\gamma^\mu D_R)\,,\\
(\overline{Q}_L\gamma^\mu\vec{\tau}Q_L) &= \frac{1}{4}\big(\overline{U}_L\gamma^\mu D_L + \overline{D}_L\gamma^\mu U_L,\, -i\overline{U}_L\gamma^\mu D_L + i\overline{D}_L\gamma^\mu U_L,\, \overline{U}_L\gamma^\mu U_L - \overline{D}_L\gamma^\mu D_L\big)\,,\\
(\overline{Q}_R\gamma^\mu\vec{\tau}Q_R) &= \frac{1}{4}\big(\overline{U}_R\gamma^\mu D_R + \overline{D}_R\gamma^\mu U_R,\, -i\overline{U}_R\gamma^\mu D_R + i\overline{D}_R\gamma^\mu U_R,\, \overline{U}_R\gamma^\mu U_R - \overline{D}_R\gamma^\mu D_R\big)\,.
\end{aligned}
\tag{4.1}
$$

Since the ultimate goal is to study the process $b \to s\mu^+\mu^-$, it suffices to study the case in which the heavy-fermion loop produces a neutral current. The only products that give rise to such currents are

$$
\begin{aligned}
&(\overline{U}_L\gamma^\mu U_L)_x(\overline{U}_L\gamma^\nu U_L)_y, \quad (\overline{U}_R\gamma^\mu U_R)_x(\overline{U}_L\gamma^\nu U_L)_y, \quad (\overline{U}_R\gamma^\mu U_R)_x(\overline{U}_R\gamma^\nu U_R)_y,\\
&(\overline{D}_L\gamma^\mu D_L)_x(\overline{D}_L\gamma^\nu D_L)_y, \quad (\overline{D}_R\gamma^\mu D_R)_x(\overline{D}_L\gamma^\nu D_L)_y, \quad (\overline{D}_R\gamma^\mu D_R)_x(\overline{D}_R\gamma^\nu U_R)_y.
\end{aligned} \tag{4.2}
$$

Let's focus on the term with two left-handed up-type currents. For propagators with four-momenta k and p, Wick's theorem gives

$$
\begin{aligned}
&i\int d^4x\,\langle 0|\,\mathrm{T}\Big[(\overline{U}_L\gamma^\mu U_L)_x(\overline{U}_L\gamma^\nu U_L)_y\Big]\,|0\rangle\\
&= -\frac{i}{4}\int d^4x \frac{e^{-i(k+p)x}}{[k^2-m^2+i\epsilon][p^2-m^2+i\epsilon]}\mathrm{Tr}\big[(\not{k}-m)\gamma^\mu(1-\gamma^5)(\not{p}+m)\gamma^\nu(1-\gamma^5)\big]\,.
\end{aligned} \tag{4.3}
$$

Taking the sum of the incoming momenta of the two quarks as $p_1+p_2=q$ and the loop momenta as k and changing to momentum space the integral becomes

$$
I^{\mu\nu} = -\frac{i}{4}\int d^4k\,\frac{\mathrm{Tr}\big[(\not{k}-m)\gamma^\mu(1-\gamma^5)(\not{k}+\not{q}+m)\gamma^\nu(1-\gamma^5)\big]}{[k^2-m^2+i\epsilon][(k+q)^2-m^2+i\epsilon]}\,. \tag{4.4}
$$

Evaluating the trace gives

$$
\begin{aligned}
&\frac{1}{4}\mathrm{Tr}\big[(\not{k}-m)\gamma^\mu(1-\gamma^5)(\not{k}+\not{q}+m)\gamma^\nu(1-\gamma^5)\big] =\\
&2\Big(-k^2g^{\mu\nu}-k\cdot q\,g^{\mu\nu}+2k^\mu k^\nu+k^\nu q^\nu+k^\mu q^\nu-i\epsilon^{\mu\nu\alpha\beta}k^\alpha q^\beta\Big).
\end{aligned} \tag{4.5}
$$

In order to obtain the denominator as a function of the integrated momentum squared we perform the change $k=l-qx$, so that the new variable of integration is l. This means that the part of the trace proportional to $\epsilon^{\mu\nu\alpha\beta}$ does not contribute: either it is proportional to even powers of l (then it vanishes under the integral) or it is proportional to the totally symmetric tensor $l^\alpha l^\beta$, which vanishes when contracted with $\epsilon^{\mu\nu\alpha\beta}$. This leads to the same integral for the products of two right-handed or two left-handed currents, and as well for up-type and down-type BSM fermions. We also introduce a Feynman parameter x, using the relation

$$
\frac{1}{AB} = \int_0^1 \frac{dx}{[Ax+B(1-x)]^2}\,. \tag{4.6}
$$

The denominator then becomes

$$
\begin{aligned}
\frac{1}{[k^2-m^2][(k+q)^2-m^2]} &= \int_0^1 \frac{dx}{[k^2+2xkq+xq^2-m^2]^2}\\
&= \int_0^1 \frac{dx}{[l^2-(x-1)xq^2-m^2]} = \int_0^1 \frac{dx}{l^2-\Delta}\,,
\end{aligned} \tag{4.7}
$$

and the numerator

$$
-(k^2+k\cdot q)\,g^{\mu\nu}+2k^\mu k^\nu+k^\nu q^\nu+k^\mu q^\nu = (2/d-1)l^2g^{\mu\nu}+x(x-1)(2q^\mu q^\nu-q^2g^{\mu\nu})+\text{ terms linear in } l\,, \tag{4.8}
$$

where we have used that under the integral $l^\mu l^\nu = l^2g^{\mu\nu}/d$, where d is a generic dimension. To evaluate the integral we use the following relations [10]:

$$\begin{aligned}\int \frac{d^d l}{(2\pi)^2}\frac{1}{(l^2-\Delta)^n} &= i\frac{(-1)^n}{(4\pi)^d}\frac{\Gamma(n-d/2)}{\Gamma(n)}\frac{1}{\Delta^{n-d/2}}\,,\\ \int \frac{d^d l}{(2\pi)^2}\frac{l^2}{(l^2-\Delta)^n} &= i\frac{(-1)^{n-1}}{(4\pi)^d}\frac{d}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\frac{1}{\Delta^{n-d/2-1}}\,.\end{aligned} \tag{4.9}$$

Putting everything together and changing to the integral in d dimensions we obtain

$$\begin{aligned}I^{\mu\nu} &= -2i\int_0^1 dx\int \frac{d^d l}{(2\pi)^d}\mu^{4-d}\frac{(2/d-1)l^2 g^{\mu\nu}+x(x-1)(2q^\mu q^\nu - q^2 g^{\mu\nu})}{(l^2-\Delta)^2}\\ &= -2i\mu^{4-d}\int_0^1 dx\,\frac{i}{(4\pi)^{d/2}}\left(\frac{1}{\Delta}\right)^{2-d/2}\left[-g^{\mu\nu}(2/d-1)\frac{d}{2}\Gamma(1-d/2)\Delta + 2x(x-1)(q^\mu q^\nu - q^2 g^{\mu\nu})\Gamma(2-d/2)\right]\\ &= 2\int_0^1 dx\,\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}}\left(\frac{\mu^2}{\Delta}\right)^{2-d/2}\left[-g^{\mu\nu}m^2 + 2x(x-1)(q^\mu q^\nu - q^2 g^{\mu\nu})\right],\end{aligned} \tag{4.10}$$

where we have used that $(1-d/2)\Gamma(1-d/2) = \Gamma(2-d/2)$. Taking the limit $d \to 4-\epsilon$ gives

$$I^{\mu\nu} \simeq \quad \simeq \frac{1}{8\pi^2}\int_0^1 dx\Big(-\frac{2}{\epsilon}+\ln\frac{\mu^2}{\Delta}+\gamma-\ln 4\pi\Big)\Big(-g^{\mu\nu}m^2+2x(x-1)(q^\mu q^\nu - q^2 g^{\mu\nu})\Big)\,, \tag{4.11}$$

Working in the $\overline{MS}$ scheme, we only keep as a finite contribution the term proportional to the logarithm. Evaluating the different pieces of the integral over x we obtain

$$\begin{aligned}I_1 &= \int_0^1 dx\ln\frac{\mu^2}{\Delta} = \ln\frac{\mu^2}{m^2} - \frac{2}{q}\sqrt{4m^2-q^2}\tan^{-1}\frac{q}{\sqrt{4m^2-q^2}}\,,\\ I_2 &= \int_0^1 dx\ln\frac{\mu^2}{\Delta}x(x-1) = \frac{1}{18q^3\sqrt{4m^2-q^2}}\Bigg[6(8m^4+2m^2q^2-q^4)\tan^{-1}\frac{q}{\sqrt{4m^2-q^2}}\\ &\quad - q\sqrt{4m^4-q^2}\Big(3q^2\ln\frac{\mu^2}{m^2}+12m^2+5q^2\Big)\Bigg]\,,\\ I^{\mu\nu} &= \frac{i}{8\pi^2}\Bigg[\Big(-\frac{2}{\epsilon}+\gamma-\ln 4\pi\Big)\Big(-g^{\mu\nu}m^2+-\frac{1}{3}(q^\mu q^\nu - q^2 g^{\mu\nu})\Big) - g^{\mu\nu}m^2 I_1 + 2(q^\mu q^\nu - q^2 g^{\mu\nu})I_2\Bigg]\,.\end{aligned} \tag{4.12}$$

The finite part turns out to be

$$I^{\mu\nu} = \frac{1}{8\pi^2}\Bigg[-g^{\mu\nu}m^2 I_1 + 2(q^\mu q^\nu - q^2 g^{\mu\nu})I_2\Bigg]\,. \tag{4.13}$$

The terms proportional to $q^\mu q^\nu$ and q^2 correspond to terms with two derivatives in position space, which give rise to a dimension 8 operator, even more suppressed that the dimension 6 ones. Thus the main contribution to $I^{\mu\nu}$ comes from I_1. This can also be understood simply by noticing that I_1 is proportional to m^2, and I_2 to q^2, leaving the former to dominate at $q << m$. Taking the regularization scale μ to be of the order of q and approximating I_1 for $q/m << 1$ we obtain

$$\begin{aligned}I_1 &\simeq 2\ln\frac{q}{m} + \frac{1}{6}\frac{q^2}{m^2} + \mathcal{O}(q^3/m^3)\,,\\ I^{\mu\nu} &\simeq -\frac{1}{8\pi^2}g^{\mu\nu}\Big(2m^2+\frac{q^2}{6}\Big)\ln\frac{q}{m} \simeq -\frac{m^2}{4\pi^2}g^{\mu\nu}\ln\frac{q}{m}\,.\end{aligned} \tag{4.14}$$

To this point we have evaluated the amputated four-point diagram in figure (2) for two currents of the same chirality. We study now the contribution from the terms

$$(\overline{U}_L\gamma^\mu U_L)_x(\overline{U}_R\gamma^\nu U_R)_y\,, \quad (\overline{D}_L\gamma^\mu D_L)_x(\overline{D}_R\gamma^\nu D_R)_y\,. \tag{4.15}$$

Wick's theorem yields the trace

$$\frac{1}{4}\mathrm{Tr}\big[(\not{k}-m)\gamma^\mu(1-\gamma^5)(\not{k}+\not{q}+m)\gamma^\nu(1+\gamma^5)\big] = -2m^2 g^{\mu\nu}\,. \tag{4.16}$$

The loop integral is

$$\begin{aligned} I^{\mu\nu}_{LR} &= -2m^2 g^{\mu\nu}\mu^{4-d}\int_0^1 dx\int\frac{d^dl}{(2\pi)^d}\frac{1}{(l^2-\Delta)^2} = -2m^2 g^{\mu\nu}\mu^{4-d}\int_0^1 dx\frac{1}{(4\pi)^d}\frac{\Gamma(2-d/2)}{\Gamma(2)}\frac{1}{\Delta^{2-d/2}} \\ &\simeq -\frac{2m^2}{(4\pi)^2}g^{\mu\nu}\int_0^1 dx\Big(-\frac{2}{\epsilon}+\gamma-\ln 4\pi+\ln\frac{\mu^2}{\Delta}\Big)\,, \end{aligned} \tag{4.17}$$

and the finite part in the $\overline{MS}$ scheme is

$$I^{\mu\nu}_{LR} = -2\frac{m^2}{(4\pi)^2}g^{\mu\nu}\int_0^1 dx\ \ln\frac{\mu^2}{\Delta} = -\frac{m^2}{8\pi^2}g^{\mu\nu}I_1 \simeq -\frac{m^2}{4\pi^2}g^{\mu\nu}\ln\frac{q}{m}\,, \tag{4.18}$$

where in the last equality we have taken the limit $q << m$. As it can be seen, the result in this limit is the same that we obtain when the two bilinears have the same chirality. Although we are mainly interested in neutral currents, looking at the loop integral of charged currents, namely

$$(\overline{U}_L\gamma^\mu D_L)_x(\overline{D}_R\gamma^\nu U_R)_y\,, \qquad (\overline{U}_R\gamma^\mu D_R)_x(\overline{D}_R\gamma^\nu U_R)_y\,, \qquad (\overline{U}_L\gamma^\mu D_L)_x(\overline{D}_L\gamma^\nu U_L)_y\,, \tag{4.19}$$

gives the same result.

4.2. Effective operators

The analysis of the previous section has shown that the tensor structure of the loop integral in the approximation $q << m$ is $g^{\mu\nu}$. The four-fermion effective operators constructed from the product of two of the operators in (3.11) and (3.13), expected to provide a good description at energies below m, are then of the type

$$\mathcal{O}_{\text{eff}} \propto I^{\mu\nu}\frac{c_i c_{i'}}{\Lambda^4}\big(\overline{q}\,\gamma^\mu q\big)\big(\overline{q}\,\gamma^\nu q\big) = -\frac{m^2}{4\pi^2}\ln(q/m)\frac{c_i c_{i'}}{\Lambda^4}\big(\overline{q}\,\gamma^\mu q\big)\big(\overline{q}\,\gamma_\mu q\big)\,, \tag{4.20}$$

Taking $\Lambda \propto m$, it is useful to define

$$\alpha_f = -\frac{1}{\Lambda^4}\frac{m^2}{4\pi^2}\ln(q/m) \simeq -\frac{1}{4\pi^2 m^2}\ln(q/m)\,. \tag{4.21}$$

The quantity α_f can be understood as the effective coupling of the four SM fermion interaction, a coupling which runs logarithmically with the energy of the process and has dimensions $[m]^{-2}$. At low energies, α_f is always positive.

The choice of the mass m is of course arbitrary, and it must be taken into account that for a given scale q the results depend on this choice.
When writing the effective operators we must analyze the $SU(2)$ structure of the heavy fermions. As an example we explicitly write the integration of two left-handed bilinears:

$$\begin{aligned}\int \langle 0|\,\mathrm{T}(\overline{Q}_L\gamma^\mu Q_L)_x(\overline{Q}_L\gamma^\nu Q_L)_y\,|0\rangle &= \int \langle 0|\,\mathrm{T}\Big[(\overline{U}_L\gamma^\mu U_L)_x + (\overline{D}_L\gamma^\mu D_L)_x\Big]\Big[(\overline{U}_L\gamma^\nu U_L)_y + (\overline{D}_L\gamma^\nu D_L)_y\Big]\,|0\rangle \\ &= \int \langle 0|\,\mathrm{T}\Big[(\overline{U}_L\gamma^\mu U_L)_x(\overline{U}_L\gamma^\nu U_L)_y + (\overline{D}_L\gamma^\mu D_L)_x(\overline{D}_L\gamma^\nu D_L)_y\Big]\,|0\rangle = 2I^{\mu\nu}\,.\end{aligned} \tag{4.22}$$

The same result is obtained for $(\overline{Q}_L\gamma^\mu Q_L)_x(\overline{Q}_R\gamma^\nu Q_R)_y$ and $(\overline{Q}_R\gamma^\mu Q_R)_x(\overline{Q}_R\gamma^\nu Q_R)_y$ and for the terms including the $SU(2)$ matrices τ_i, for instance

$$\begin{aligned}\int \langle 0|\,\mathrm{T}(\overline{Q}_L\tau_1\gamma^\mu Q_L)_x(\overline{Q}_L\tau_1\gamma^\nu Q_L)_y\,|0\rangle &= \int \langle 0|\,\mathrm{T}\Big[(\overline{U}_L\gamma^\mu D_L)_x + (\overline{D}_L\gamma^\mu U_L)_x\Big]\Big[(\overline{U}_L\gamma^\nu D_L)_y + (\overline{D}_L\gamma^\nu U_L)_y\Big]\,|0\rangle \\ &= \int \langle 0|\,\mathrm{T}\Big[(\overline{U}_L\gamma^\mu D_L)_x(\overline{D}_L\gamma^\nu U_L)_y + (\overline{D}_L\gamma^\mu U_L)_x(\overline{U}_L\gamma^\nu D_L)_y\Big]\,|0\rangle = 2I^{\mu\nu}\,.\end{aligned} \tag{4.23}$$

Furthermore, we find that when two different τ matrices appear the integral vanishes. This implies that, when computing the product of two operators involving $\vec{\tau}$, the effective operator can be written as $2I_{\mu\nu}(\overline{q}\,\vec{\tau}\gamma^\mu q)(\overline{q}\,\vec{\tau}\gamma^\nu q)$ times some constants. Bearing this in mind we find that all the possible effective operators involving quarks, which we name $\mathcal{O}_i$, are

$$\begin{aligned}
&\mathcal{O}_1 = \delta_1(\overline{q}_L\gamma^\mu\lambda_{F1}q_L)(\overline{q}_L\gamma_\mu\lambda_{F1}q_L)\,, && \mathcal{O}_2 = \delta_2(\overline{q}_L\gamma^\mu\lambda_{F1}q_L)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,,\\
&\mathcal{O}_3 = \delta_3(\overline{d}_R\gamma^\mu\lambda_d\lambda_{F1}\lambda_d d_R)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,, && \mathcal{O}_4 = \delta_4(\overline{q}_L\vec{\tau}\gamma^\mu\lambda_{F1}q_L)(\overline{q}_L\vec{\tau}\gamma_\mu\lambda_{F1}q_L)\,,\\
&\mathcal{O}_5 = \delta_5(\overline{q}_R\tau_1\gamma^\mu\lambda_{F2}q_R)(\overline{q}_R\tau_1\gamma_\mu\lambda_{F2}q_R)\,, && \mathcal{O}_6 = \delta_6(\overline{q}_R\tau_2\gamma^\mu\lambda_{F2}q_R)(\overline{q}_R\tau_2\gamma_\mu\lambda_{F2}q_R)\,,\\
&\mathcal{O}_7 = \delta_7(\overline{q}_L\tau_1\gamma^\mu\lambda_{F1}q_L)(\overline{q}_R\tau_1\gamma_\mu\lambda_{F2}q_R)\,, && \mathcal{O}_8 = \delta_8(\overline{q}_L\tau_2\gamma^\mu\lambda_{F1}q_L)(\overline{q}_R\tau_2\gamma_\mu\lambda_{F2}q_R)\,,\\
&\mathcal{O}_9 = \delta_9(\overline{q}_L\tau_3\gamma^\mu\lambda_{F1}q_L)(\overline{d}_R\gamma_\mu\lambda_d\lambda_{F1}\lambda_d d_R)\,,
\end{aligned} \tag{4.24}$$

where the δ_i are some combination of the more fundamental c_i, which account for the fact that an operator $\mathcal{O}_i$ is produced from different products of $\mathcal{Q}_i$. The δ_i are found to be

$$\begin{aligned}
&\delta_1 = 2\alpha_f(c_1^2 + c_1c_2 + c_2^2)\,, && \delta_2 = 2\alpha_f(c_1c_3 + c_1c_4 + c_2c_4 + c_2c_3)\,,\\
&\delta_3 = 2\alpha_f(c_3^2 + c_3c_4 + c_4^2 + c_8^2)\,, && \delta_4 = 2\alpha_f c_5^2\,,\\
&\delta_5 = 2\alpha_f c_6^2\,, && \delta_6 = 2\alpha_f c_7^2\,,\\
&\delta_7 = 2\alpha_f c_5c_6\,, && \delta_8 = 2\alpha_f c_5c_7\,,\\
&\delta_9 = 2\alpha_f c_5c_8\,.
\end{aligned} \tag{4.25}$$

The operators involving two SM leptons and two SM quarks, obtained from the product of one operator from (3.11) and one from (3.13), are

$$
\begin{aligned}
&\mathcal{O}_{1l} = \delta_{1l}(\overline{q}_L\gamma^\mu\lambda_{F1}q_L)(\overline{E}_L\gamma_\mu E_L)\,, && \mathcal{O}_{2l} = \delta_{2l}(\overline{q}_L\gamma^\mu\lambda_{F1}q_L)(\overline{E}_R\gamma_\mu E_R)\,,\\
&\mathcal{O}_{3l} = \delta_{3l}(\overline{d}_R\gamma\mu\lambda_d\lambda_{F1}\lambda_d d_R)(\overline{E}_L\gamma_\mu E_L)\,, && \mathcal{O}_{4l} = \delta_{4l}(\overline{d}_R\gamma^\mu\lambda_d\lambda_{F1}\lambda_d d_R)(\overline{E}_R\gamma_\mu E_R)\,,\\
&\mathcal{O}_{5l} = \delta_{5l}(\overline{q}_L\vec{\tau}\gamma^\mu\lambda_{F1}q_L)(\overline{E}_L\vec{\tau}\gamma_\mu E_L)\,, && \mathcal{O}_{6l} = \delta_{6l}(\overline{q}_R\tau_1\gamma^\mu\lambda_{F2}q_R)(\overline{E}_L\tau_1\gamma_\mu E_L)\,,\\
&\mathcal{O}_{7l} = \delta_{7l}(\overline{q}_R\tau_2\gamma^\mu\lambda_{F2}q_R)(\overline{E}_L\tau_2\gamma_\mu E_L)\,, && \mathcal{O}_{8l} = \delta_{8l}(\overline{d}_R\gamma^\mu\lambda_d\lambda_{F1}\lambda_d d_R)(\overline{E}_L\tau_3\gamma_\mu E_L)\,,\\
&\mathcal{O}_{9l} = \delta_{9l}(\overline{q}_R\tau_1\gamma^\mu\lambda_{F2}q_R)(\overline{E}_R\tau_1\gamma_\mu E_R)\,, && \mathcal{O}_{10l} = \delta_{10l}(\overline{q}_R\tau_2\gamma^\mu\lambda_{F2}q_R)(\overline{E}_R\tau_2\gamma_\mu E_R)\,,\\
&\mathcal{O}_{11l} = \delta_{11l}(\overline{d}_R\gamma^\mu\lambda_d\lambda_{F1}\lambda_d d_R)(\overline{E}_R\tau_3\gamma_\mu E_R)\,,
\end{aligned}
\tag{4.26}
$$

with the coefficients

$$
\begin{aligned}
&\delta_{1l} = 2\alpha_f(c_1c_{1l} + c_{1l}c_2 + c_{2l}c_2 + c_{2l}c_1)\,, && \delta_{2l} = 2\alpha_f(c_1c_{3l} + c_{3l}c_2 + c_{4l}c_2 + c_{4l}c_1)\,,\\
&\delta_{3l} = 2\alpha_f(c_{1l}c_3 + c_{1l}c_4 + c_{2l}c_4 + c_{2l}c_3)\,, && \delta_{4l} = 2\alpha_f(c_3c_{3l} + c_4c_{3l} + c_4c_{4l} + c_{4l}c_3)\,,\\
&\delta_{5l} = 2\alpha_f c_{5l}c_5\,, && \delta_{6l} = 2\alpha_f c_{5l}c_6\,,\\
&\delta_{7l} = 2\alpha_f c_{5l}c_7\,, && \delta_{8l} = 2\alpha_f c_{5l}c_8\,,\\
&\delta_{9l} = 2\alpha_f c_{6l}c_6\,, && \delta_{10l} = 2\alpha_f c_{6l}c_7\,,\\
&\delta_{11l} = 2\alpha_f c_{6l}c_8\,.
\end{aligned}
\tag{4.27}
$$

We notice that $\mathcal{O}_{1-3}$, $\mathcal{O}_{1l-4l}$ and $\mathcal{O}_{8l,11l}$ give rise to purely neutral currents, while the other operators involve charged currents as well as a consequence of the additional $SU(2)$ structure. Moreover, the operators involving the flavor-changing structure $\lambda_d\lambda_{F1}\lambda_d$ ($\mathcal{O}_{2,3,9,3l,4l,8l,11l}$) are suppressed by a power of two of the down-type Yukawa couplings. Operators involving MFV matrix λ_{F2} are as well suppressed by at least one power in λ_d; this is the case of $\mathcal{O}_{5-8,7l-8l}$. This considerations could reduce the relevant operators to $\mathcal{O}_{1,4,1l,2l,5l}$.

4.3. Relation to Operator Product Expansion

A standard technique to study weak decays is the Operator Product Expansion (OPE), which is of interest to this study because a connection can be established between our operators and those in the OPE. In this approach, short-range interactions mediated by W bosons are approximated as a sum over products of local operators times some coefficients C_i, the Wilson coefficients [3]. The products of operators ($\mathcal{O}_i$) have dimension 6, and higher-order operators are neglected. The OPE provides a good description for low-energy processes, neglecting contributions of order μ^2/M_W^2, with μ being the typical momentum of the process. For $b \to s$ transitions, the effective Hamiltonian can be written as [5]

$$
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{ts}^*V_{tb}\frac{e^2}{16\pi^2}\sum_i C_i(\mu)\mathcal{O}_i(\mu)\,. \tag{4.28}
$$

The coefficients C_i are non-vanishing in the Standard Model. However, they are small for operators describing FCNC, since they occur only beyond tree level. For this reason this coefficients are particularly sensitive to new physics contributions. Some of the operators in (4.28) are the following

$$
\begin{aligned}
&\mathcal{O}_9 = (\overline{s}_L\gamma^\mu b_L)(\bar{l}\gamma_\mu l)\,, && \mathcal{O}_9' = (\overline{s}_R\gamma^\mu b_R)(\bar{l}\gamma_\mu l)\,,\\
&\mathcal{O}_{10} = (\overline{s}_L\gamma^\mu b_L)(\bar{l}\gamma_\mu\gamma^5 l)\,, && \mathcal{O}_{10}' = (\overline{s}_R\gamma^\mu b_R)(\bar{l}\gamma_\mu\gamma^5 l)\,.
\end{aligned}
\tag{4.29}
$$

They can be related to some of the operators in (4.26) if some relations amongst the coefficients are assumed. We will make this explicit in next sections when trying to impose some constraints on our coefficients.

4.4. Scale hierarchy

The introduction of the heavy fermions can be related to the masses of the Standard Model fermions. In fact, the masses of all SM fermions could be generated from condensates of heavy fermions, as is claimed in technicolor models. In this case the masses of the fermions are given by condensation of chirality-flipping operators (such as those in (2.37)), which give

$$m_f \simeq \frac{g^2}{\Lambda^2}\langle \bar{Q}Q\rangle \,, \tag{4.30}$$

where g is a coupling of $\mathcal{O}(1)$, as argued in [7]. Since the value of $\langle \bar{Q}Q\rangle$ is universal, there is a scale Λ associated to each fermion. Taking $\langle \bar{Q}Q\rangle = v^2 m$, where m is the mass of the heavy fermions and v the Higgs vacuum expectation value, one can determine the scale Λ_f associated to each fermion. For $m = 1$ TeV, the scales regarding leptons are shown in table (I). Although these are rough estimates, it can be seen that the scales involving lighter particles are higher, being ~ 350 TeV for the electrons, ~ 20 TeV for the muons and ~ 5 TeV for the tau leptons.
If Λ_f is taken to be the scale associated to operators involving each kind of fermion, this provides an explanation why processes involving electrons which arise from interactions with heavy fermions are further suppressed than those involving muons and even more taus. However, it can also mean that taking $\Lambda = 1$ TeV as a generic scale, which we do in the following sections, leads us to overestimate the contribution from operators involving muons.

	e	μ	τ	ν
Λ_f (TeV)	344	24	6	$1.7 \cdot 10^5$

TABLE I: Scale associated to leptons at $m = 1$ TeV and coefficients of order 1.

5. Experimental evidence

In this section we use some of the available experimental data in the literature to test the effective interaction we have constructed. In order to do this, we use measurements of some observables related to the $b \to s\, l^+ l^-$ transition to fit certain coefficients of the semileptonic operators in (4.26). From now on we will refer to α_f as the effective coupling at $q = m_b$ and $m = 1$ TeV, with value

$$\alpha_f = -\frac{1}{4\pi^2 m^2}\ln\frac{m_b}{m} = 1.387 \cdot 10^{-7}\ \text{GeV}^{-2}. \tag{5.1}$$

It is also useful to define the adimensional coefficients $\overline{\delta}_i = \delta_i/\alpha_f$. Before going on to analyze recent experimental evidence, it is useful to look at the operator

$$\frac{1}{2}(\overline{q}_L Y^u Y^{u\dagger} q_L)(\overline{q}_L Y^u Y^{u\dagger} q_L)\,, \tag{5.2}$$

which has a well-reviewed bound on its scale, $\Lambda = 5.9$ TeV [1, 11]. This operator can be directly related to operator $\mathcal{O}_1$ in (4.24). Using the mentioned bound yields $\overline{\delta}_1 < 0.1$. Recalling that $\overline{\delta_1} = 2(c_1^2 + c_1 c_2 + c_2^2)$, we find that the - more fundamental - coefficients of the SM quark-BSM fermion operators have the bound $c_{1,2}^2 < 0.05$.
Recent results show tensions with the Standard Model arising from operators $\mathcal{O}_9$, $\mathcal{O}_{10}$, $\mathcal{O}'_9$ and $\mathcal{O}'_{10}$ in the OPE (shown in equation (4.29)), which correspond to $b \to s\, l^+ l^-$ transitions [5]. In particular, experimental data are better described to different extents when new physics contributions to coefficients C_9, C_{10}, C'_9 and C'_{10} are switched on for decays involving muons while keeping $b \to s\, e^+ e^-$ transitions as in the SM. For instance, this pattern is manifest in the analysis of observables R_K and R_K^* in [12]. As for tau leptons, their fast decay into hadrons still makes accurate

measurements difficult. For these reasons, we study only the effects of allowing nonvanishing contributions to the process $b \to s\,\mu^+\mu^-$, while keeping only SM contributions for decays involving electrons and not studying observables involving taus. On the other hand, we have seen that a theoretical argument on the masses of the leptons yields a higher suppression for operators involving electrons. This could explain why the deviation from the SM is not observed in this case. For the tau leptons, the scale is lower than for electrons and muons, so one would expect that when measurements of $b \to s\,\tau^+\tau^-$ are accessible an even larger deviation be observed.
Now we turn again to our basis of operators and find that $\mathcal{O}_{1l-4l}$ involve the mentioned $b \to s\,l^+l^-$ transitions. Particularizing them for an initial b and a final s quark and explicitly writing the MFV matrices in terms of CKM elements and Yukawa couplings we obtain

$$\begin{aligned} \mathcal{O}_{1l}^{bs} &= \delta_{1l} y_t^2 V_{ts}^* V_{tb} (\overline{s}_L \gamma^\mu b_L)(\overline{E}_L \gamma_\mu E_L)\,, & \mathcal{O}_{2l}^{bs} &= \delta_{2l} y_t^2 V_{ts}^* V_{tb} (\overline{s}_L \gamma^\mu b_L)(\overline{E}_R \gamma_\mu E_R)\,, \\ \mathcal{O}_{3l}^{bs} &= \delta_{3l} y_s y_b y_t^2\, V_{ts}^* V_{tb} (\overline{s}_R \gamma^\mu b_R)(\overline{E}_L \gamma_\mu E_L)\,, & \mathcal{O}_{4l}^{bs} &= \delta_{4l} y_s y_b y_t^2\, V_{ts}^* V_{tb} (\overline{s}_R \gamma^\mu b_R)(\overline{E}_R \gamma_\mu E_R)\,. \end{aligned} \tag{5.3}$$

If we assume that new physics contributions arise only from our operators $\mathcal{O}_{il}^{bs}$, we can impose that the new physics part of the OPE effective Hamiltonian be equal to the sum of our operators. In order to do this, we redefine the Wilson coefficients to contain only new physics contributions: $C_i \equiv C_i^{\rm NP}$. Then we can switch from the OPE operators $\mathcal{O}_i$ to the $\mathcal{O}_{il}^{bs}$ in (5.3) with a straightforward change of basis, which in terms of the different coefficients reads

$$\begin{aligned} -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9 &= \delta_{1l} + \delta_{2l} = \delta_+\,, & -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9' &= (\delta_{3l} + \delta_{4l}) y_s y_b\,, \\ -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_{10} &= -\delta_{1l} + \delta_{2l} = \delta_-\,, & -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_{10}' &= (-\delta_{3l} + \delta_{4l}) y_s y_b\,, \end{aligned} \tag{5.4}$$

where we have defined the $\delta_\pm$ for later convenience. In the special cases where $C_9 = \pm C_{10}$ and $C_9' = \pm C_{10}'$ these are directly related to δ_{1l}, δ_{2l} and δ_{3l}, δ_{4l} as follows:

$$\begin{aligned} C_9 = -C_{10} &\to -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9 = \delta_{1l}\,, & C_9' = -C_{10}' &\to -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9' = \delta_{3l} y_s y_b\,, \\ C_9 = C_{10} &\to -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9 = \delta_{2l}\,, & C_9' = C_{10}' &\to -\frac{\alpha_{em} G_F}{\pi\sqrt{2}} 2C_9' = \delta_{4l} y_s y_b\,. \end{aligned} \tag{5.5}$$

No CKM elements are involved in the rotation, since the suppression is the same in the OPE expansion and the MFV hypothesis. When right-handed quarks are involved, the MFV operators are further suppressed by the Yukawa couplings of the b and s quarks, a suppression that is not present in the OPE operators. The authors of ref. [5] use a wealth of experimental data to obtain the best fit values of the new physics contributions to C_9, C_{10}, C_9' and C_{10}'. We use their results to obtain the values of our coefficients, which are shown in table II. For the reasons stated before, the results are exclusively for decays involving muons ($b \to s\,\mu^+\mu^-$).
The best fit values of coefficients $\overline{\delta}_{3l}$ and $\overline{\delta}_{4l}$, which correspond to operators with right-handed quarks, are of $\mathcal{O}(10^4)$. Since they are suppressed by $y_s\, y_b$, evidence for a small contribution from these operators calls for a high value of their coefficients. However, the evidence for nonvanishing $\overline{\delta}_{3l}$ and $\overline{\delta}_{4l}$ is much weaker than for $\overline{\delta}_{1l}$ and $\overline{\delta}_{2l}$, given that for the former the the errors are large and the 1σ interval is compatible with the SM (vanishing coefficients). In this sense, the right-handed contributions are disfavored with respect to the left-handed ones, and MFV provides a natural explanation why. For the left-handed currents, all best fit values and errors are of $\mathcal{O}(1)$ and in the cases of $\overline{\delta}_{1l}$ and $\overline{\delta}_+$ they are further than 1σ away from the SM, thus they are the strongest candidates to describe the anomaly. The scale Λ of the operator assuming coefficients $\overline{\delta} = 1$ has been also computed for left-handed currents, and we find that it is of $\mathcal{O}(1\text{TeV})$. The results are shown in table II.

	Best fit	1σ range	Λ if $\bar{\delta} = 1$ (TeV)
$\bar{\delta}_{1l}$	0.24	[0.30,0.18]	1.3
$\bar{\delta}_{2l}$	0.13	[0.28, 0.04]	2.0
$\bar{\delta}_{+}$	0.48	[0.56,0.39]	1.3
$\bar{\delta}_{-}$	-0.27	[-0.17, -0.37]	2.0
$\bar{\delta}_{3l}$	-4.9 $\cdot 10^4$	[1.2, -10.1] $\cdot 10^4$	-
$\bar{\delta}_{4l}$	$-4.2 \cdot 10^4$	$[1.2, -1.9] \cdot 10^5$	-

TABLE II: Best fit value of the $\bar{\delta}$ coefficients obtained from the relations in (5.5) and the results in [5].

5.1. Fitting the coefficients

With the above results we gather that $\bar{\delta}_{1l}$ and $\bar{\delta}_{+}$ are good candidates to explain possible new physics contributions to different observables related to $b \to s\,\mu^+\mu^-$. Using the open-source code `flavio` we perform the following fits on these coefficients using experimental data:

- Two-coefficient fit allowing $\bar{\delta}_{1l}$ and $\bar{\delta}_{2l}$ to vary independently.
- One-coefficient fit with $\bar{\delta}_{1l}$, taking $C_9 = -C_{10}$, and with $\bar{\delta}_{2l}$, taking $C_9 = C_{10}$.
- One-coefficient fit with $\bar{\delta}_{+}$ ($\propto C_9$) and with $\bar{\delta}_{-}$ ($\propto C_{10}$).

Although a wealth of data is available, with the aim to reduce computational time we choose a limited number of observables that constrain the coefficients in study. All the measurements we include are from the LHCb collaboration. The observables we use, together with the year of publishing, are the following:

- Bin-averaged angular observables P_5' and F_L for $B_0 \to K^{*0}\mu^+\mu^-$ (2014).
- Bin-averaged differential branching ratio of $B^+ \to K^{*+}\mu^+\mu^-$ (2015), $B_s \to \phi\mu^+\mu^-$ (2015) and $B_0 \to K^{*0}\mu^+\mu^-$ (2016).
- Bin-averaged ovservables R_K (2014) and R_K^* (2017).

Lepton flavor universality ratios R_K and R_K^* deserve a comment. They are defined as the following ratios of branching ratios involving different leptons:

$$R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}\,, \qquad R_K^* = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)}\,. \tag{5.6}$$

They prove especially clean observables, due to the fact that hadronic uncertainties of each branching ratio cancel, leaving theoretical uncertainties of $\mathcal{O}(1\%)$. The Standard Model predicts $R_K^* = 1$ for a broad range of q^2, but measurements by LHCb show discrepancies at the level of 2.5σ [12]. The 2017 result for R_K^* was not included in [5], but we take it into account in our analysis to see whether it is compatible with the deviations in other observables when allowing for new physics contributions only in the decays involving muons. Observable P_5', proposed in [13], presents some cancellation of uncertainties as well.
Since J/ψ and ψ resonances occur in the region of squared center of mass energy (q^2) between 6 an 15 GeV^2, we do not use data from this range (an example of this is seen in figure (1) in the introduction). Although in our model the coefficients $\bar{\delta}_{il}$ are purely real, when fitting them we allow for an imaginary part to check whether it improves the fit. In all cases we find that an imaginary part is not favored, giving central values close to zero in all cases. The results we obtain fitting only one real coefficient at a time for $\bar{\delta}_{1l}, \bar{\delta}_{2l}, \bar{\delta}_{+}$ and $\bar{\delta}_{-}$ are shown in table (III). The results of the

	Best fit	1σ range	Best fit from [5]
$\overline{\delta}_{1l}$ (1c)	0.26	[0.34,0.18]	0.24
$\overline{\delta}_{2l}$ (1c)	-0.05	[0.11, -0.22]	0.13
$\overline{\delta}_{+}$ (1c)	0.49	[0.65,0.33]	0.48
$\overline{\delta}_{-}$ (1c)	-0.41	[-0.29, -0.56]	-0.27
$\overline{\delta}_{1l}$ (2c)	0.25	[0.33,0.18]	-
$\overline{\delta}_{2l}$ (2c)	-0.06	[0.10, -0.22]	-

TABLE III: Best fit value of the $\overline{\delta}$ coefficients obtained using `flavio`. The best fit results from table II (Best fit from [5]) are quoted again for comparison. 1c: one-coefficient fit; 2c: two-coefficient fit.

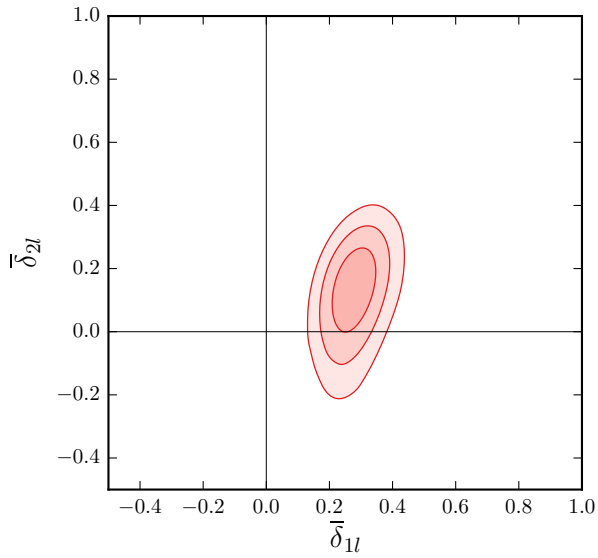

FIG. 4: 1, 2 and 3σ contours of the coefficients $\overline{\delta}_1$ and $\overline{\delta}_2$ in a 2-coefficient fit.

2-coefficient fit with $\overline{\delta}_1$ and $\overline{\delta}_2$ are also shown in table (III). Due to the fact that a reduced number of observables has been used in comparison to [5], our uncertainties are larger than the ones shown in table (II). For coefficients $\overline{\delta}_{2l}$ and $\overline{\delta}_-$ the results of columns 1 and 3 in table III present some difference, but in both cases the 1σ intervals overlap. On the other hand the results for $\overline{\delta}_{1l}$ and $\overline{\delta}_+$ are in agreement to a great extent, and in both cases the pull from the Standard Model is larger than 1σ.

In figures (7)-(10) in Appendix A we present the likelihood normalized to its maximum value, with the 1σ range highlighted, that results from the one-coefficient fits in $\overline{\delta}_{1l}$, $\overline{\delta}_{2l}$, $\overline{\delta}_+$ and $\overline{\delta}_-$. In general the functions are well behaved except for a small fluctuation in $\overline{\delta}_{2l}$. In figure (4) we present the joint 1, 2 and 3σ contours obtained when fitting coefficients $\overline{\delta}_{1l}$ and $\overline{\delta}_{2l}$.

A comment must be made regarding the scale Λ suppressing the operators. Here we have worked with $\Lambda = 1$ TeV, but in section 4.4 we found that in fact higher scales should be considered for operators involving muons. Taking into account the dependence of α_f on Λ, we find that the coefficients $\overline{\delta}$ scale as

$$\overline{\delta} = \frac{\delta}{\alpha_f} = -\frac{\Lambda^4}{m^2}\frac{\delta}{\ln(q/m)} = \frac{\Lambda^4}{m^4}\,\overline{\delta}(\Lambda = m)\,. \tag{5.7}$$

Given that we have found coefficients of $\mathcal{O}(1)$ at $\Lambda = 1$ TeV, using the above we would find, for instance, coefficients of $\mathcal{O}(10^4)$ at $\Lambda = 10$ TeV. In further work it should be studied if such large coefficients have an impact on other observables that may rule out their viability.

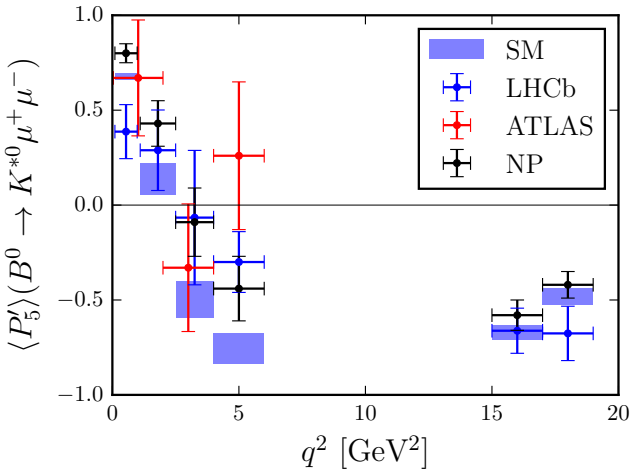

FIG. 5: Predictions for observable P_5' when including a contribution from $\overline{\delta}_+$ (NP), Standard Model prediction by `flavio` (SM) and measurements from LHCb (LHCb) and ATLAS (ATLAS).

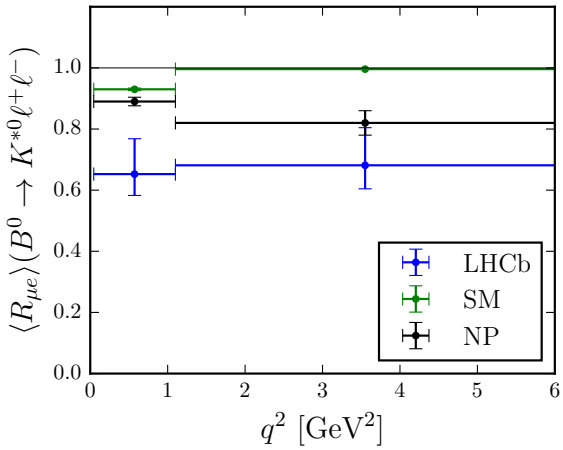

FIG. 6: Predictions for observable R_K^* when including a contribution from $\overline{\delta}_+$ (NP), Standard Model prediction by `flavio` (SM) and measurements from LHCb (LHCb).

5.2. Observable predictions

In view of these results we may conclude that the best fit values of $\overline{\delta}_{1l}$ and $\overline{\delta}_+$ provide an improved description of the data in comparison to the Standard Model. In order to analyze the impact of introducing these coefficients on the predictions of observables, we compute, for the case of $\overline{\delta}_+ = 0.49 \pm 0.16$, the predicted bin-averaged values of observables P_5' and R_K^*. The computations are performed within the bins used by LHCb, in pursuance of a better comparison with experimental points. The results are shown in figures (5) and (6) under the label NP together with the Standard Model predictions performed by `flavio` and experimental points from LHCb (and ATLAS in the case of P_5'). The error bars correspond to 1σ ranges, and in the case of our fit they correspond to the combined error of the coefficient fit and the observable prediction.

In the case of P_5', the NP points are shifted upwards with respect to the SM predictions for all bins. In the low-q^2 region this places them mostly closer to the experimental central values, while in the high-q^2 region the SM prediction is closer to the experimental points. Nonetheless, in the latter region the SM and NP predictions do not fall far apart, in fact their 1σ intervals overlap. Regarding observable R_K^*, we find that the NP scenario prediction lies definitely closer to the measurements in comparison to the SM. Although the NP predictions are still further than 1σ away from the LHCb measurements in the low-q^2 bin, they do reduce the tension, especially in the high-q^2 bin.

Considering the results obtained in this section, we can conclude that the introduction of nonvanishing coefficient $\overline{\delta}_+$ reduces the tensions in some observables related to the process $b \to s\,\mu^+\mu^-$. This implies that the anomaly can be explained through local operators involving two left-handed quarks and two leptons (with the left-handed option being more favored as well for the leptons). These, in turn, can arise from four-fermion operators involving massive fermionic particles with masses around 1 TeV. Furthermore, the introduction of Minimal Flavor Violation provides a natural explanation why the left-handed currents are promoted with respect to the right-handed ones, which inherit a strong suppression from Yukawa couplings y_s and y_b. Moreover, the $\overline{\delta}$ coefficients that we have fitted involve products of coefficients c_i of operators involving heavy fermions and quarks and coefficients c_{il} of operators involving heavy fermions and leptons. While the c_i should remain small in order for the bound on the operator in (5.2) to be fulfilled, the c_{il} should be sufficiently large to provide the correct contributions to the $\overline{\delta}_i$. In this case, the coupling of the

heavy fermions to SM leptons would be stronger than to SM quarks.

6. Conclusions

The main aim of this work has been to assess if the introduction of new heavy fermions could, through four-fermion operators involving Minimal Flavor Violation, reduce the tensions with the Standard Model arising from the $b \to s\,\mu^+\mu^-$ anomaly. To this end, in the first part of the study we have constructed four-fermion operators involving two new heavy fermions and two Standard Model fermions, in such a way that the operators are $SU(2)_L \otimes SU(2)_R \otimes SU(3)_c$ invariant and preserve Lorentz symmetry. Even if the number of operators is a priori large, symmetries and especially Fierz identities allow to reduce considerably this number. Because in the long run we are interested in operators which can contribute to the process $b \to s\,\mu^+\mu^-$, we restrict ourselves to chirality-preserving operators with a simple color structure, and obtain the list of six operators shown in equation (2.41). Then we bring MFV in the bilinears containing SM quarks, which allow only a limited number of MFV matrices. The procedure results in eight operators involving quarks and six operators involving leptons, listed in equations (4.24) and (4.26) respectively. At this point $SU(2)_R$ symmetry is broken in the quark sector, and operators involving right-handed quarks acquire a much stronger suppression that those involving left-handed quarks.

Next we employ the operators involving heavy fermions to construct effective four-point operators of SM particles only. In order to do this, we integrate out the heavy degrees of freedom through a loop integral using dimensional regularization. In the low energy limit, this integral gives rise to an effective coefficient, α_f, which evolves logarithmically and takes the value $\alpha_f = 1.387 \cdot 10^{-7}$ GeV^{-2} at scale $q = m_b$ and cutoff $\Lambda = m = 1$ TeV, with m being the mass of the heavy fermions. These values for q, m and Λ are our choice for this work, and other scales could be explored by obtaining the appropriate value of α_f. The resulting effective operators $\mathcal{O}_i$ carry a power of α_f and an adimensional coefficient $\overline{\delta}_i$, which is built from the more fundamental coefficients attached to the operators containing heavy fermions. In addition, the case is studied where the masses of the fermions are generated by condensates of heavy quarks. This allows to find an estimate of the scale suppressing operators with each type of lepton, and we find that this scale is of $\mathcal{O}$(TeV) for the tau letpons and progressively higher for the other leptons. Although we did not consider this in this work, it could also be argued that the flavor structure of the SM arises from condensates of heavy quarks. In this case, one would need to consider different flavors for the heavy quarks, so that the condensates $\langle \overline{Q_i} Q_j \rangle$ (for flavors i, j) gave rise to the correct Yukawa couplings. An extension of this work could be attempted in this direction.

On the other hand, our effective operators can be related to the Operator Product Expansion when assuming that new physics contributions to the OPE come exclusively from our list of operators. Redefining the Wilson coefficients as their NP part only, the relations between C_9, C_{10}, C_9' and C_{10}' and our coefficients δ_{1l}, δ_{2l}, δ_{3l} and δ_{4l} are obtained. This permits us to use the fits obtained in [5] to compute the best fit values of our coefficients in view of recent experimental results, and we find that a non zero value is especially favored for coefficients $\overline{\delta}_{1l}$ and $\overline{\delta}_+$, as shown in table II. The NP contributions are considered in decays with final muons only. This choice is motivated by experimental evidence and can also be related to our prediction of the scales suppressing operators with leptons, since the scale of the electrons ($\Lambda_e \sim 350$ TeV) is higher than that of the muons ($\Lambda_\mu \sim 20$ TeV). Following this reasoning, a larger deviation from the SM would be expected for decays with tau leptons, since $\Lambda_\tau \sim 5$ TeV. On the other hand, this could mean that our general assumption $\Lambda = 1$ TeV is underestimating the suppression scale of the operators involving muons. Further studies could explore whether the anomaly could still be described when employing a higher scale.

Furthermore, we use `flavio` to perform our own fits using a limited number of observables. The results are in agreement with the previous calculation to a high degree for coefficients $\overline{\delta}_{1l}$ and $\overline{\delta}_+$, but generally have a larger error

due to the reduced number of observables in our fit. Since the value $\overline{\delta}_+ = 0.49 \pm 0.16$ shows better description of the fitted data in comparison to the SM, we use it to provide NP predictions for observables P_5' and R_K^*, and find that in most bins the NP scenario is closer to the experimental points than the SM case.
Bearing all this in mind, one can say that four-fermion operators improve the description of the $b \to s\,\mu^+\mu^-$ anomaly. Furthermore, we have shown that these operators can arise from more fundamental four-fermion operators containing heavy fermions with masses around 1 TeV and which involve Minimal Flavor Violation in the Standard Model fields. In comparison to studying directly the coefficients C_i in the OPE, the analysis of this work in terms of the $\overline{\delta}_i$ gives information as to which couplings with heavy fermions could be favored thanks to the dependence of the $\overline{\delta}_i$ on the c_i, the coefficients of the operators involving heavy quarks. We have seen that the c_i of operators involving two heavy fermions and two SM quarks should remain small in order to fulfill the bounds in the literature, while the coefficients c_{il} form operators containing two heavy fermions and two muons should be larger in order to account for the anomaly. From the point of view of my contribution, I was able to reproduce in detail the building of four-fermion operators and to introduce in them the Minimal Flavor Violation hypothesis, and I worked out how to implement the theoretical framework in `flavio` to perform fits and predictions. Apart from the possibilities mentioned above, this study could be further expanded by considering different operators with heavy fields, such as as a dipole operator containing the electromagnetic field strength, in order to study their effect in other rare decays.

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Appendix A

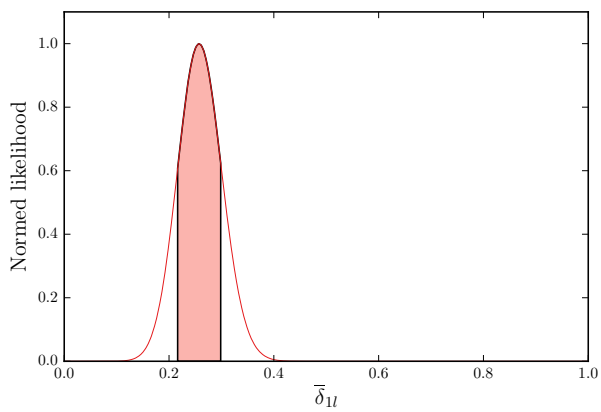

FIG. 7: Likelihood distribution for the real part of coefficient $\overline{\delta}_1$.

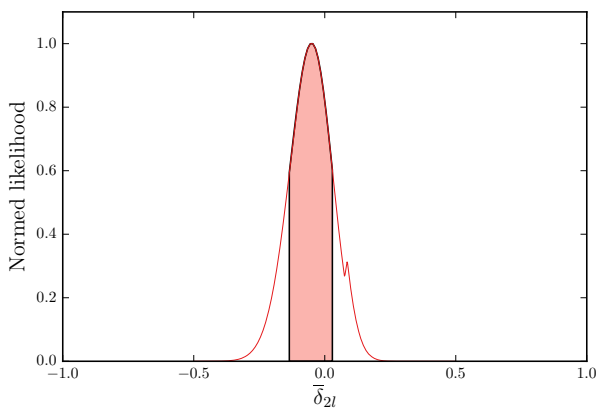

FIG. 8: Likelihood distribution for the real part of coefficient $\overline{\delta}_2$.

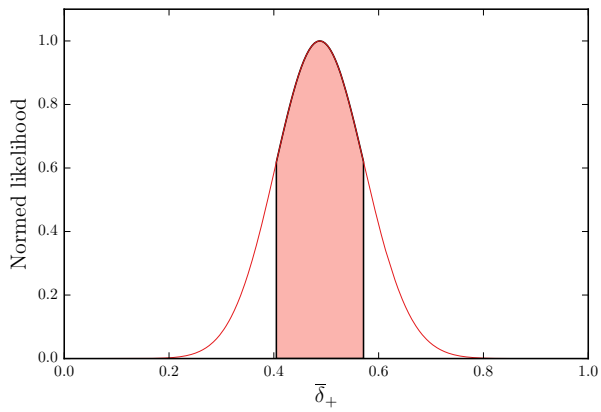

FIG. 9: Likelihood distribution for the real part of coefficient $\overline{\delta}_+$.

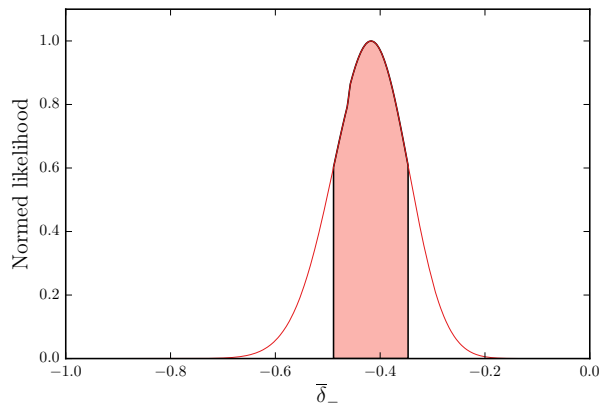

FIG. 10: Likelihood distribution for the real part of coefficient $\overline{\delta}_-$.

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