Master of Actuarial and Financial Sciences

MASTER THESIS

Title: Modelling a Pricing Strategy for ADC Finite Risk Reinsurance Treaties with GLMM Approach

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ABSTRACT

Using RBNS (Reported But Not Settled) claims data from an accident business portfolio with 11 accident years and 5 development years, this paper conducts a case study that attempts to establish a comparison of the goodness of fit of Chain Ladder and Generalised Linear Mixed Models made with their mean squared errors once outstanding claim payments are estimated with R software and, afterwards, show a pricing strategy for a quota share, excess and at-the-money adverse development cover (ADC) types of finite risk reinsurance contract. In this thesis, finite risk treaties are disclosed putting the focus on LPT and ADC transactions.

Keywords: traditional reinsurance, ART, finite risk, Chain Ladder, GLMM, LPT, ADC, regulation, reserving, pricing

RESUMEN

Mediante datos RBNS (*Reported But Not Settled*) de una cartera de accidentes con 11 años de ocurrencia y 5 de desarrollo, este artículo realiza un caso práctico que trata de comparar la bondad de ajuste de los modelos Chain Ladder y GLMM con sus errores cuadráticos medios después de estimar los pagos pendientes con el programa R a fin de mostrar una estrategia de tarificación para la modalidad de reaseguro financiero de cobertura de desarrollo adverso (ADC) en forma de cuota parte, exceso de pérdidas y *stop-loss*. Esta tesis también tiene como objetivo divulgar información sobre los contratos de reaseguro financiero, centrándose en las transacciones LPT y ADC.

Palabras clave: reaseguro tradicional, ART, reaseguro financiero, Chain Ladder, GLMM, LPT, ADC, regulación, cálculo de reservas, tarificación

INDEX

0. Introduction

Insurers' reserves could be insufficient to cover risks such as natural disasters (earthquakes, hurricanes, floods…), nuclear accidents or the asbestos consequences to human health. The reinsurer comes up to provide protection against those large possible claims.

A reinsurance is a contract or agreement that allows an insurer (the ceding company) to totally or partially transfer risk in order to reduce greater losses derived from an insurance claim. Pricing a reinsurance is a complex task that can be done following different types of methods. The aim of this dissertation is to provide a pricing strategy for a quota share, an excess and an at-the-money adverse development cover (ADC) with the Generalised Linear Mixed Model's (GLMM) mean squared error (MSE) using RBNS (Reported But Not Settled) claims data. However, RBNS future claim payments are estimated using both Chain Ladder (CL) and GLMM methods in order to see the differences between each model results and discard the method which does not fit properly the claims data (in this case, the CL). Nevertheless, before this, the traditional general framework of reinsurance is exposed. Afterwards, alternative risk transfer (ART) mechanisms are introduced by reviewing its origin and their purpose. Finally, to end the theoretical part, finite risk reinsurance are explained putting the focus on loss portfolio transfer (LPT) and ADC schemes. The literature reviewed for this study is composed by journal articles, books, news and seminars.

The reason why I chose this topic is due to the attractive new shapes reinsurance is adopting and the need to disclose information about retrospective reinsurance practices that occur nowadays in the current world. The ability to make a structured coverage reinsurance product against extra-large losses that already occurred due to natural catastrophes, extraordinary accidents or human ignorance consequences about hazardous materials such as asbestos sheds a light of calmness over the financial markets which are usually closely related to the insurance world.

I would like to express thanks to my tutors, Dr Francisco Javier Sarrasí Vizcarra and Dr Eva Boj del Val, whose implication in this project has been relentless, indispensable and very treasured. On the one hand, Dr Francisco Javier Sarrasí Vizcarra is an expert in the reinsurance field and his extent knowledge has guided me through the lines of traditional reinsurance and innovative ART techniques. On the other hand, Dr Eva Boj del Val possesses the brain of a GLMM modeller who envisions the results interpretation long before she finishes the model, an intrinsic trait of an RBNS claims data dealing mastermind. To both of them, thank you.

1. Traditional general framework of reinsurance

From *Rückversicherung eine Einführung* (Grossmann), the Mapfre Dictionary, the *History of Reinsurance* book (Golding), the German Commercial Code and the law firm Garrigues; the concept of reinsurance has been extensively defined as the contract or instrument aiming to transfer the total risks or part of them from a primary insurer (the ceding company) to the reinsurer in order to homogenise and reduce portfolio risks so as to diminish losses from insurance claims and strengthen the insurer's solvency and its credibility to meet future payments in the original business-to-consumer contract (Sarrasí, 2017). For this reason, the reinsurer's client is an insurer or another reinsurer; but not the general public.

The transfer of risks from the primary insurer to the reinsurer allows the ceding company to embrace higher than usual risks and increase its underwriting capacity, attending clients' risks who in the first instance would be directly rejected. There are different paths and methodologies to transfer risks between these two parties.

Looking at a legal perspective, reinsurance can be grouped into obligatory, facultative or mixed. The obligatory reinsurance targets the entire insurer's portfolio or just a specific line of insurance making both parties bind together to accept and cede the agreed risks, guaranteeing the reinsurer a certain amount of premiums and reducing administrative costs because of the automatically recurring obligation to accept all those future risks taken on by the insurer which obey the agreed reinsurance contract. However, the obligatory reinsurance distances the reinsurer from accepting those risks which go beyond the agreed limits of the contract. In this way, the facultative reinsurance allows both the ceding company and the reinsurer to freely accept an agreement on transferring an individual risk that may hold or may not hold the same original contractual conditions between the insurer and its insured. Examples of enforcing this type of contract are those excluded risks or geographical zones from the obligatory reinsurance or great sums that exceed the limits stipulated in obligatory contracts and that can be individually transferred with a facultative contract. Finally, the mixed or obligatory-facultative contract consists of that case where one party (usually the reinsurer) is bound to accept, within a few limits, those individually-selected risks that the ceding company wants to transfer in order to enhance its underwriting capacity with greater covered sums (Sarrasí, 2017).

Focusing on a technical point of view, reinsurers use two main methods to offer their coverage: the proportional and the non-proportional reinsurance. On one hand, in the proportional reinsurance, which is direct-proportional between ceded risks and premiums, the reinsurer takes a proportion of the sum insured which reveals its liability in case of insurance claims and also finds the percentage of the premium that corresponds to it:

$$\frac{\text{Sum reinsured}}{\text{Total sum insured}} = \frac{\text{Reinsurer's loss}}{\text{Total loss}} = \frac{\text{Reinsurance premium}}{\text{Total insurance premium}}$$

Within this method, a reinsurance commission goes to the ceding company in order to compensate those administrative costs it will continue to incur. There are three subgroups in this method: the quota share, the surplus and the mixed reinsurance (Minzoni, 2009). Below, there are their respective mathematical formalisations (Sarrasí, 2017). In the first one, the reinsurer fixes a percentage on all the ceding company's portfolio risks or just a chosen line of business to assume responsibility for the losses occurring during the contract period. It is easy to manage, saves costs and is adequate for brand-new insurers but its main disadvantage is that it does not homogenise the portfolio, leaving aside those top risks with the highest sums insured and ceding away profitable business (little losses still will cost the same premium percentage compared to larger ones).

Quota share reinsurance mathematical formalisation

$$S = S_c + S_r$$

$$\left.\begin{array}{l} \left.\begin{array}{l} S_c = k \cdot S \\ S_r = (1 - k) \cdot S \end{array}\right\} k = \frac{S_c}{S} \quad ; \quad 1 - k = \frac{S_r}{S} \\ \\ X = X_c + X_r \\ \left.\begin{array}{l} X_c = k \cdot X \\ X_r = (1 - k) \cdot X \end{array}\right\} k = \frac{X_c}{X} \quad ; \quad 1 - k = \frac{X_r}{X} \end{array}\right\} \begin{array}{l} 0 < k < 1 \\ X \leq S \end{array}$$

Legend

S: Policy total sum insured

S_c: Policy sum insured by the ceding company

S_r: Policy sum reinsured by the reinsurer

k: Quota rate that the ceding company retains

$1 – k$: Remaining quota rate taken by the reinsurer

X: Policy total loss

X_c: Policy Insured loss

X_r: Policy reinsured loss

The second method, the surplus reinsurance, consists in fixing a retention (or a net line) on a policy sum insured from a certain line of insurance making the reinsurer responsible for the amount that exceeds that retention within the pre-established contract limits (cases that exceed contractual limits shall be dealt with a facultative reinsurance). Therefore, the quota rate is not constant and depends on the policy sum insured (S) and the ceding company's net line (M), which is the maximum loss quantity the insurer can assume for the policy. This method allows the reinsurer to limit the excessive risk and correctly homogenises its portfolio but it is more difficult to manage (Macedo, 2010).

Surplus reinsurance mathematical formalisation

$S = S_c + S_r \quad ; \quad X = X_c + X_r$

$S_c = k \cdot S \quad ; \quad X_c = k \cdot X$

$$k = \begin{cases} 1 & \text{if} \quad S \leq M \\ \dfrac{M}{S} & \text{if} \quad S > M \end{cases} \qquad S_c = \begin{cases} S & \text{if} \quad S \leq M \\ M & \text{if} \quad S > M \end{cases} \qquad X_c = \begin{cases} X & \text{if} \quad S \leq M \\ \dfrac{M}{S} \cdot X & \text{if} \quad S > M \end{cases}$$

$S_r = k \cdot S \quad ; \quad X_r = k \cdot X$

$$1 - k = \begin{cases} 0 & \text{if} \quad S \leq M \\ 1 - \dfrac{M}{S} & \text{if} \quad S > M \end{cases} \qquad S_r = \begin{cases} 0 & \text{if} \quad S \leq M \\ S - M & \text{if} \quad S > M \end{cases} \qquad X_r = \begin{cases} 0 & \text{if} \quad S \leq M \\ \dfrac{S - M}{S} \cdot X & \text{if} \quad S > M \end{cases}$$

Legend

S: Policy total sum insured

Sc: Policy sum insured by the ceding company

Sr: Policy sum reinsured by the reinsurer

k: Quota rate that the ceding company retains

1 – k: Remaining quota rate taken by the reinsurer

M: Ceding company’s net line

X: Policy total loss

Xc: Policy Insured loss

Xr: Policy reinsured loss

Finally, the mixed reinsurance is a combination of the previous, operating a quota share until it reaches the retention and operating a surplus from then on. This type of contracts are mostly used when the insurer is in solid financial conditions and their advantage relies on the surplus coverage that offers protection and lets property portfolios grow faster (Méndez, 2005).

On the other hand, the non-proportional reinsurance is based on fixing a maximum loss quantity that the ceding insurer will be able to pay leaving the reinsurer what remains, within the pre-established contract limits. Due to the fact this method is not ceding a proportional part of the primary insurer's portfolio but just the excess-loss from the pre-established limit upwards, there is no sense to talk about a reinsurance commission as it does not apply (Minzoni, 2009). According to Macedo (2010), two types of non-proportional contracts can be distinguished: the excess-loss (and its Catastrophe version or Cat XL) and the stop-loss. The first one covers the insurer from those losses that go over the limit, retention or "priority" established by the ceding company. The Cat XL version focuses on the sum of the losses occurred by one identical incident (for example, an earthquake) and establishes the priority on this sum. Finally, in the stop-loss reinsurance the primary insurer fixes a percentage on the total loss that it is able to absorb for a specific line of business in an entire fiscal year (Alegre *et al.*, 2017), leaving what remains to the reinsurer (for example, if the loss ratio priority is 110% and the actual loss ratio has been 130%, the reinsurer will pay those 20 points above the priority).

<u>Excess-loss (XL) reinsurance mathematical formalisation</u>

$$X_c = \begin{cases} X & \text{if} \quad X < M \\ M & \text{if} \quad X \geq M \end{cases} \qquad X_r = \begin{cases} 0 & \text{if} \quad X < M \\ X - M & \text{if} \quad X \geq M \end{cases}$$

Legend

X: Claim amount of a policy

X_c: Policy insured loss

X_r: Policy reinsured loss

M: Priority

<u>Stop-loss reinsurance mathematical formalisation</u>

$$Z_c = \begin{cases} Z & \text{if} \quad Z < M \\ M & \text{if} \quad Z \geq M \end{cases} \qquad Z_r = \begin{cases} 0 & \text{if} \quad Z < M \\ Z - M & \text{if} \quad Z \geq M \end{cases}$$

Legend

Z: Total portfolio loss of a specific line of business in an entire fiscal year

Z_c: Total portfolio insured loss

Z_r: Total portfolio reinsured loss

M: Priority

Ultimately, the traditional reinsurance encompasses all these types of contracts and usually lasts one year. It can be prospective (covering future losses) or retrospective (covering past losses) and it only takes into account the underwriting risk. The general formulation when a reinsurance is introduced to the ceding company is obtained as follows:

$$Z = Z_c + Z_r = \sum_{i=1}^{N} X_i$$

where

$$\left.\begin{array}{ll} Z_c = \sum_{i=1}^{N} X_{c,i}\ . & Z_r = \sum_{i=1}^{N} X_{r,i}\ . \end{array}\right\} \quad (X_1, X_2, \ldots, X_N, N)$$

When investment risk and timing risk are added to the reinsurance layout, it comes up the financial reinsurance, an alternative risk transfer which is gaining popularity over the past few years. Financial reinsurance contracts are multiyear, which is why the moment of time where claims are paid is considered to calculate the premium, contrary to traditional reinsurance contracts which do not give that much importance to this variable as they are short-term annually renewable treaties. Considering the moment of time where claims are expected to be paid and the interest rates of the financial returns, the mathematical formalization of the financial reinsurance would be:

$$\left.\begin{array}{ll} Z_c = \sum_{i=1}^{N} X_{c,i}\, f_c(T_i,0)\ . & Z_r = \sum_{i=1}^{N} X_{r,i}\, f_r(T_i,0)\ . \end{array}\right\} \quad (X_1, X_2, \ldots, X_N, T_1, T_2, \ldots, T_N, N)$$

Legend

Z Total portfolio loss

Z_c Total portfolio insured loss

Z_r Total portfolio reinsured loss

N: Number of losses occurring in the interval [0, t]

T_i: Moment of the *i*th loss payment in years

$X_{c,i}$: Insured loss at the *i*th accident

$X_{r,i}$: Reinsured loss at the *i*th accident

fc (Ti,0) Ceding company's discount factor. The interest rate is based on the financial returns that the ceding company obtains from the retained premium.

fr (Ti,0) Reinsurer's discount factor. The interest rate is based on the financial returns that the reinsurer obtains from the collected premiums.

2. Alternative risk transfer mechanisms

2.1 Origin

Innovative opportunities of cost effective techniques that appear when traditional reinsurance cover becomes expensive due to new emerging risks such as cyber, terrorism and liability risks that change the insurance prices over the course of the cycles are the alternative risk transfer (ART) mechanisms (Sibindi, 2015a).

The first time an ART technique popped up in the market was in the 1960s when organisations began to fully focus on risk management designing systems for loss prevention (Doherty, 2000; Giddy, 2006; Dionne, 2013) such as captive insurance companies and risk retention groups (Schanz, 1999). Captives are self-insurance programs developed in order to reduce transactions costs from the insurance industry (caused by adverse selection, moral hazard, credit risk, basis risk, etc) and to obtain more investment control of premiums (Dionne, 2013). If located in offshore domiciles, they can have tremendous tax-advantages. In 2012, the world's three largest captive domiciles were Bermuda with 856 captives, the Cayman Islands with 741 and Vermont with 586 (Zolkos, 2013). According to Dionne (2013), "risk retention groups (RRG) are a special type of group captive authorized by Congress in response to the liability insurance crisis of the 1980s to provide additional liability insurance capacity to businesses". They account for only a small proportion of the US liability market and their main lines of insurance are liability coverage for professionals, the healthcare industry and educational institutions.

According to Ostaszewski (2006), the field of ART grew out when insurance capacity was not sufficient and in the 1970s through 1990s insurers were driven from looking for traditional coverage to seek alternative ways to buy protection. Most of these techniques permit investors in the capital markets to take a more direct role in providing insurance and reinsurance protection, hence bringing up a convergence of insurance and financial markets. An ART form which reduces moral hazard and passes the risk to the financial market is through securitization such as reinsurance sidecars (financial structures that allow external market investors to take on the risk and benefit from the return of specific insurance or reinsurance active business), swaps and catastrophe bonds. Catastrophe bonds (CAT bonds), linked to a catastrophic-loss index, are insurance-linked securities that appeared first in 1992 on the Chicago Board of Trade ready to be launched (Minzoni, 2009; Dionne, 2013).

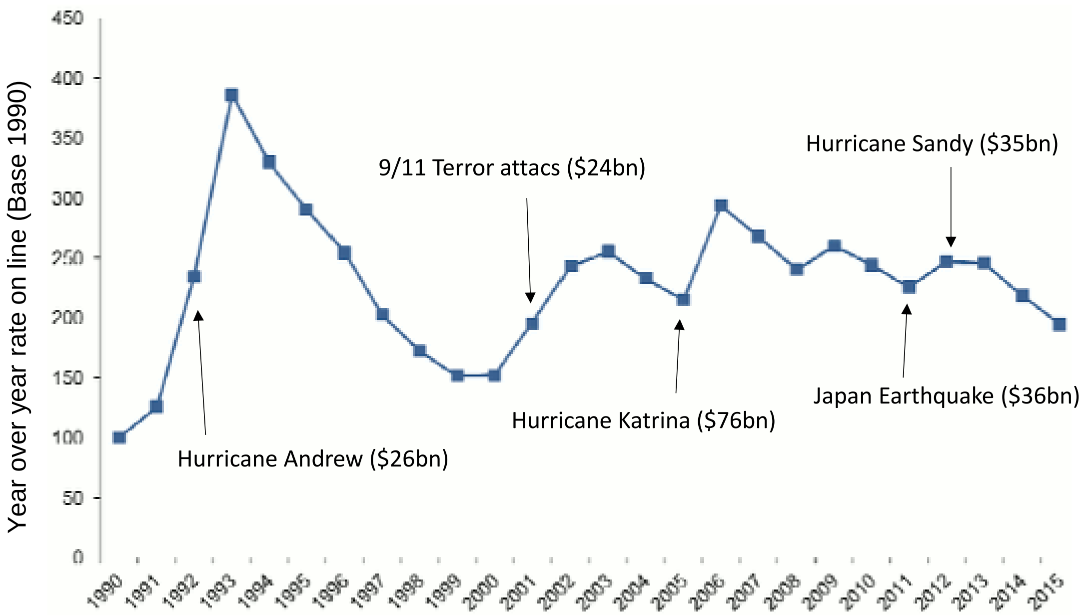
Some key market participants in ART operations are investment banks such as Goldman Sachs and Citibank; insurers such as AIG, Zurich and XL; reinsurers such as Munich Re, Hannover Re, Swiss Re; brokers like AON, Willis Towers Watson or Marsh and consultants such as Deloitte and ABS Consulting (Ostaszewski, 2006). Ultimately, the alternative risk transfer market has grown an average of 6% per year since the mid-1980s, about twice the growth rate in the commercial insurance market (Andre and Sodowsky, 1997).

2.2 Definition and purpose of ART mechanisms

Recently, ART has acquired a broader meaning and blends risk retention and risk transfer at the lowest total cost of risk resulting in mutually aligning the financial interests of both the insurer and the insured (Giddy, 2006). Today, there exist two broad segments to the ART market: risk transfer through alternative carriers, such as self-insurance, pools, captives and RRGs; and risk transfer through alternative products such as insurance-linked securities or CAT bonds, credit securitization, committed capital, weather derivatives and finite risk products, which include loss portfolio transfers (LPT) as a retrospective cover. Schanz (1999) describes ART mechanisms as tailored solutions to specific problems, multi-year and multi-line cover, a diversification and spread of risk over time that allows to insure those traditionally uninsurable risks and, finally, a way to transfer risk to non-(re)insurers (to investors in stock markets). Dionne (2013) states that reinsurance prices are highly volatile over the course of the cycle especially for reinsuring catastrophic losses. Rohe *et al*. (1998) also explains ART activity has developed in the property and casualty industry because of the very cyclic reinsurance marketplace. When a large event occurs, capacity dries up and makes pricing more expensive. Later, as time goes by and capacity recovers, reinsurance pricing becomes more competitive again. Therefore, ART products concentrate on obtaining an efficient form of capital for the insurance industry, soaring shareholders' value and solving specific industry issues such as the above-mentioned shortage of capacity, uninsurable risks, asbestos and environmental reserve increases (Shepley, 2002).

Graph 1 represents the rate on line index evolution year over year as historical events occurred, considering 1990 as the base year. As stated by the International Risk Management Institute, the rate on line is the ratio or percentage derived by dividing the reinsurance premium by the reinsurance limit or loss recoverable and the inverse is known as the payback period. The higher rate on line, the higher reinsurance premium to be paid. In 1992, Hurricane Andrew wiped out Florida escalating the insured losses to astronomical levels, larger than any model had predicted before. The industry experienced losses of $26 billion (Gonzalez and Sparrow, 2012; Artemis, 2015) and the models had predicted that a bad hurricane would cause $2 billion in claims. As everyone was reconsidering their pricing strategy, reinsurance rates became more expensive and insurance companies realized that they needed much more coverage than they ever thought about (Rohe *et al*., 1998). Later, as time passed reinsurance prices became more competitive until another big event took place (i.e. 9/11 terror attacks, Hurricane Katrina, 2011 Japan Earthquake, Hurricane Sandy…) which would trigger another big increase in the rate on line index making reinsurance premiums more expensive and thereby strengthening the cyclic feature of the reinsurance pricing.

Graph 1. Global property catastrophe rate-on-line index evolution.



Source: Guy Carpenter, Morgan Stanley Research, Artemis.bm (Evans, 2015a; Evans, 2015b)

Finite risk reinsurance, as an alternative risk transfer mechanism, can provide capital coverage without these traditional reinsurance pricing market fluctuations that make it difficult for insurers to satisfy their capacity needs and also to reinsurers who do not offer coverage due to lack of capacity and risk-layering (Rohe *et al.*, 1998). Nevertheless, Sibindi (2015b) concludes in his study that ART techniques "must be understood as complements rather than substitutes to traditional insurance products" and the underlying reason for this argument could be their complexity and, sometimes, their controversy over possible misuses.

3. Finite risk reinsurance

3.1 Concept definition and types

One of the several ART tools companies use to increase their capacity and transfer high risks when natural disasters and traditional insurance losses escalate in times of uncertainty (Zolkos, 2003; Minzoni, 2009) is the finite risk reinsurance, which made the first scene in 1960s' insurance marketplace and peaked its popularity in the late 1990s (Wiggins, 2004). Some authors, such as Culp and Heaton (2005) prefer to differentiate finite risk and financial reinsurance. They state that corporate uses of risk finance and risk transfer are finite risk transactions and applications within the reinsurance industry (reinsurance or retrocession transactions for insurance companies) are financial reinsurances. However, not everyone follows this terminology since for example Fitch Ratings published a critical assessment titled Finite Risk Reinsurance despite describing reinsurance applications and not corporate uses. The present work does not apply the different terminologies used by Culp and Heaton (2005) which only differ on whether this ART tool is used by reinsurers for corporations or reinsurers for insurers.

Contrary to traditional reinsurance, as Table 2 shows, finite risk transactions can be defined as multi-line, multiyear contracts where the reinsurer offers the insurer a coverage on a limited amount of risk relative to the aggregated premiums (Dahlen, 2007). This coverage extends to investment risk (i.e. real investment return resulting lower than expected), credit risk (i.e. the ceding company cannot pay the reinsurer the premiums), underwriting risk (uncertainty on the final cost of an insurance claim) and timing risk, which is the uncertainty on the period of time where the insurance claims must be paid off (Méndez, 2005; Minzoni, 2009). These contracts are based on assembling an account or fund, called experience fund, in which the reinsurance premium, which takes into account interest rates, is put in there as well as its investment return and as time goes by, insurance claims and administrative costs from the reinsurer are paid off through the experience fund (Sarrasí, 2017). At the end of the contract, in case there is a positive balance in the fund due to the fact that the reinsurer has obtained an investment return higher than expected since time series of losses occurred slower than expected, the reinsurer usually gives back 100% of the remaining amount (Culp and Heaton, 2005; Sarrasí, 2017).

Table 2. Differences between finite risk and traditional reinsurance

Traditional reinsurance	Finite risk reinsurance
Annual and single line contracts	Multiyear and multiline contracts
Does not take into account interest rates to calculate the reinsurance premium	Takes into account interest rates to calculate the reinsurance premium.
Premium is pre-fixed and is not returned	Premium is usually returned if there are no losses
Covers underwriting risk	Covers underwriting, investment, credit and timing risk
No experience fund	There is an experience fund
Combination of risks	Risks treated individually during a few years
Limited lines of business covered	Covers lines of business that traditional reinsurance cannot
Focuses on the risk transfer and reducing underwriting risk	Focuses on obtaining an investment return, managing liquidity and long-term planning

Source: Sarrasí (2007), Álvarez (1995)

Finite structures can be prospective (deals with future losses), taking the forms of spread loss cover and financial quota share, or retrospective (deals with past losses that have already occurred), in which time and distance policy, loss portfolio transfer (LPT) and adverse development cover (ADC) can be distinguished as the most relevant actual retrospective forms (Greig, 2005; Sarrasí, 2007; Minzoni, 2009).

The spread loss cover is a prospective contract that affects an entire risk portfolio and is based on spreading losses over several earnings reporting periods in exchange of the ceding premium which is put in an experience fund to obtain an investment return (Álvarez, 1995; Minzoni, 2009). It provides financing, liquidity management and long-term planning.

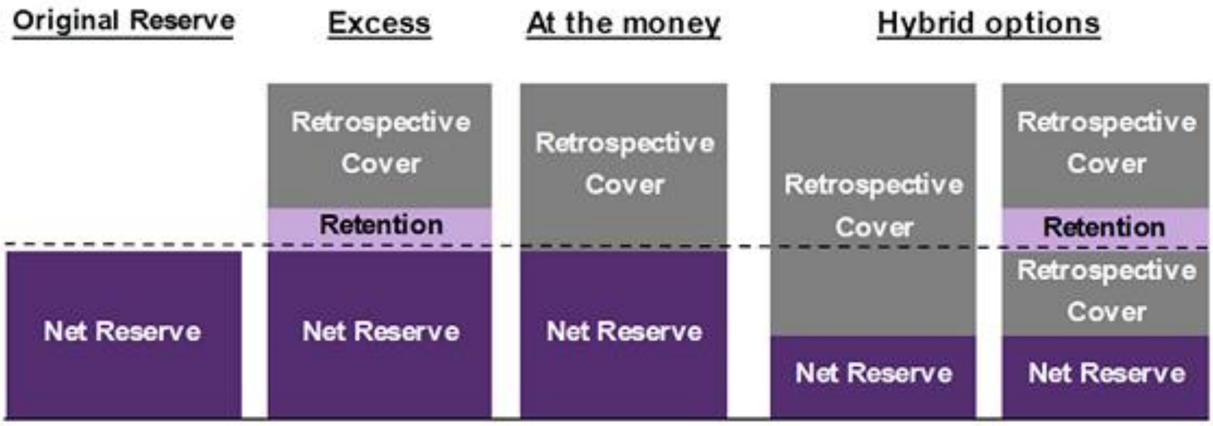
The financial quota share is another prospective contract where the reinsurer agrees to take a percentage of the cedent's premiums, which is the potential loss limit the reinsurer is going to be liable to, in exchange for a ceding commission that enables the cedent to increase its writing capacity as it arises surplus relief from transferring a portion of its risk to the reinsurer (Greig, 2005).

The time and distance policy is a retrospective contract where the reinsurer guarantees the cedent specific payments at pre-established future periods of time which are funded with the initial cedent's premium and the return on capital the reinsurer expects from its investments, therefore, timing risk could be a threat in this case (Minzoni, 2009).

The loss portfolio transfer (LPT) is a retrospective reinsurance transaction in which cedent's remaining unclaimed loss obligations associated with a previously incurred liability are partially or totally transferred to the reinsurer (Culp and Heaton, 2005; Minzoni, 2009; Partlow, 2012; Ingram, 2018). That is to say, reported but not settled (RBNS) liabilities are transferred. When the deal includes incurred but not reported claims (IBNR) or reinsurance above net carried reserve level (future payments cover), it is known as an Adverse Development Cover (ADC) which may work like a stop-loss or excess-loss treaty (Sarrasí, 2007; Loeffler, 2019). The main difference between LPT and ADC is that the first one involves a total or partial cession of a company's reserves to the reinsurer who assumes financial responsibility for the ceded reserves (similar to a quota share treaty) while the second one provides reinsurance cover above net carried reserve level without involving any transfer of existing reserves (Mishra *et al.*, 2005; Loeffler, 2019).

Figure 3 shows the ADC can take the form of "at-the-money" cover, making the reinsurer assume the excess loss above a fixed limit on the cedent's net reserves; "excess" cover, fixing a limit plus a retention on the cedent's net reserves; or "hybrid options", attaching cover within net reserves with the possibility to incorporate additional retentions or loss corridors (Mounty, 2016).

Figure 3. Types of ADC covers

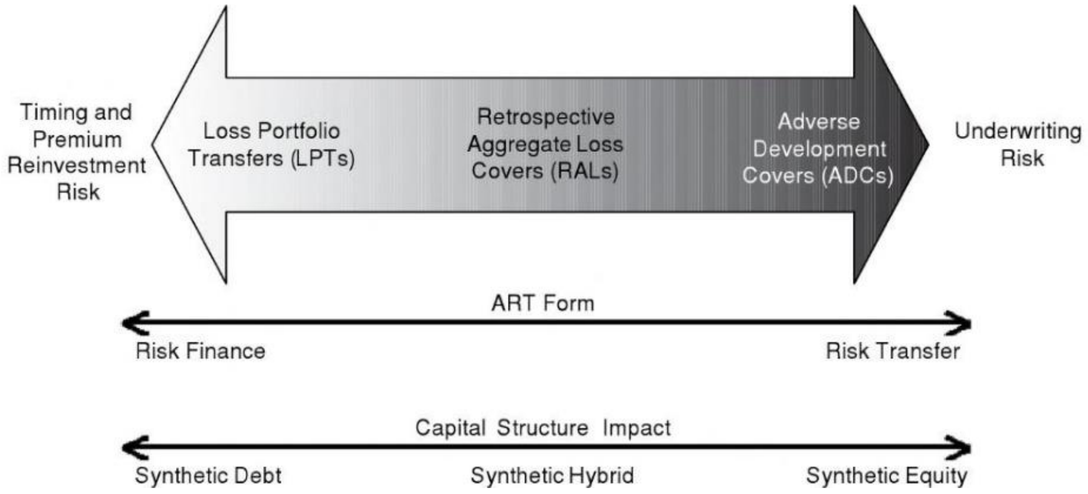


Source: Willis Towers Watson (Mounty, 2016)

3.2 An insight in LPT and ADC

In retrospective finite risk products, apart from paying an arrangement fee to the reinsurer, the ceding insurer also pays a premium where the time value of money or net present value of reserves related to the transferred liabilities is considered plus a premium to compensate the reinsurer for the underwriting and other risks assumed (Koegel, 2003; Culp and Heaton, 2005; Palmer *et al.*, 2015). LPT contracts, designated by Giddy (2006) as the most used finite product, can be used to exit a line of business, to transfer risk, to eliminate long-term liabilities from a company during merger and acquisition activities or transfer historical losses of the parent company to a captive where tax advantages appear in the form of deductible reimbursement programs, making the LPT formal legal transaction require sometimes the approval from the regulator and triggering a public debate on whether the finite risk scheme purpose is to transfer risk or to keep a company afloat by "making-up" its balance sheet losses (Quane *et al.*, 2002; Zolkos, 2003). In any case, used correctly, LPT contracts are frequently combined with ADC defined as a "hybrid LPT-ADC" by WillisRe (2017) or a "Retrospective Aggregate Loss Cover" (RAL) by Culp (2002). This combination allows the time value of money to be embedded in the ceded carried reserves from the LPT contract funding the ADC layer (Loeffler, 2019). LPT treaties bear timing, investment, credit and reserving risks (Culp, 2002; Shepley, 2002; Sarrasí, 2007; Mlej, 2015), while ADCs deal more with underwriting risk than timing risk. According to Culp (2002), the risk premium charged by the reinsurer is higher in an ADC treaty than in an LPT due to greater residual underwriting risk arising from the possibility that the ceding insurer has underestimated its reserves. In Figure 4, Culp (2002) schematises how LPTs and ADCs can be considered as synthetic hybrid debt and equity securities where ADCs have a greater equity component than LPTs. Hybrid options (RALs) are in between these two treaties and the proportion of LPT versus ADC can be chosen by the cedent.

Figure 4. Retrospective Finite Risk Products and Synthetic Capital



Source: Culp (2002)

Table 5 does a similar description for ADC treaties. Contracting an ADC with a loss retention or "buffer" has less effective coverage and is cheaper (Loeffler, 2019), although Collins (2016) argues that "companies that did this in the past paid the price when losses came in and they lost market capitalisation many multiples of the 'savings' they achieved".

Table 5. AON's ADC Structure Options Summary

	Attachment at Carried	Attachment Above Buffer
Adverse Development Cover	**1** A pure adverse development cover attaching at carried reserve amount ▪ In many ways the simplest cover. Insurer pays a premium and receives coverage for any adverse development above current carried ▪ This option will have good coverage but will have the most immediate cost impact	**2** An adverse development cover with a retained underlying buffer ▪ Less effective coverage due to the additional retained loss but will have the lowest pure economic costs. As with the first option the cost of this option will impact current year income statements
Partial / Full Loss Portfolio Transfer with ADC Limit	**3** A hybrid cover that attaches below carried and involves immediate cession of a portion of reserves and losses. ▪ The coverage beyond carried is paid for by the interest on the ceded assets associated with the ceded reserves ▪ This structure will provide coverage beyond carried and mitigate the current year expense issues. Instead the economic cost is reflected in lower investment income in future years	**4** A hybrid cover that attaches below carried and involves cession of a portion of reserves above an underlying retained buffer. ▪ The coverage beyond carried is paid for by the interest on the ceded assets associated with the ceded reserves ▪ This structure will require a smaller cession of carried reserves since the cedant is retaining a buffer layer before the reinsurer's adverse development layer attaches

Source: AON (Loeffler, 2019)

It is important to remember that with this retrospective reinsurance solution, which does not require the approval of a supervisory authority, the insurer keeps its legal obligations towards the policyholders (who do not detect the reinsurance operation) and, hence, the risk of default entailed by the reinsurer should be considered by hedging and backing it by capital according to Solvency II (Eling and Schaper, 2016). Besides, it is also possible to lift the reinsurance cover in exchange for a settlement, known as commutation and usually available after a couple of years (Mlej, 2015), and does not require the approval of a supervisory authority either. Commutation can reduce run-off portfolios on the contractual level before the final winding up (Eling and Schaper, 2016) and it could be settled, for instance, on the payment of shares in profits equal to 100% of the assets value in the experience fund to the ceding company, releasing the reinsurer from past, present and future liabilities. Culp and Heaton (2005) declare that the principal risk for reinsurers is that losses or claims arrive faster than expected, triggering lower realised returns on investment from reserves (investment risk) and perhaps inadequate amounts of reserves to offset losses (reserving or underwriting risk). If the assets value in the experience fund is negative, commutation will occur with another kind of mutual agreement (Sarrasí, 2017).

Finally, according to Margraf (2017), regarding the designing of an ADC contract it is necessary first to do the estimation of future claim payments to provide a pricing strategy based on future costs for reported claims that have not yet been settled (reported but not settled or RBNS). The following case study deals with RBNS data and, therefore, LPT contracts are considered to be applicable for settled claims but not yet paid and ADC contracts are applicable to open claims that have not yet been settled. Therefore, on one hand, the classic Chain Ladder (CL) framework will be put into practice to estimate the aggregate ultimate amount of claims and set the total future claim payments for a portfolio and on the other hand, a Generalised Linear Mixed Model (GLMM) will be considered to estimate individual ultimate amount of claims, following the steps of Boj and Esquinas (2016), in order to, finally, price three ADC treaties for those not-yet settled claims with the best fitting model between CL and GLMM according to their respective mean squared errors.

4. Case study: Designing a pricing strategy for ADC treaties

In this section, used data is disclosed, presented and structured with R (see Annex 7.1). Then, the loss reserving methods (CL and GLMM) are described and the estimation of ultimate incurred claim payments follows afterwards using R software tools (see Annex 7.2 and 7.3). Finally, the pricing strategy is defined and enforced under either CL or GLMM models because depending on their goodness of fit one of them will be dismissed.

4.1 Data structuring and characteristics

The excel data file used in the case study is anonymous and contains only RBNS accident and liability portfolio claims data. It is divided in columns representing Valuation year, Line of business, Claim ID, Accident date, Reported date, Settlement date, Amount paid to date and Outstanding case estimate. Later, Reported claim cost, Number of claims, Valuation year, Accident year, Reported year, Settlement year, Accident to reporting delay, Reporting to valuation delay, Accident to valuation delay, Reporting to settlement delay and Claim Settled (yes or no) are added. Table 6 summarises the number of claims by reporting year and line of business. After the deaccumulation process of the Amount-paid-to-date variable, the claim occurrence year predictor is built and for RBNS claims this is the same as the Reported year. There are 13,662 accident claims and 2,911 liability claims, therefore, the total sum of reported claims is 16,573 for the 1996-2008 period.

Table 6. Number of claims during 1996-2008

	Number of claims		
Reported year	**Accident**	**Liability**	**Total**
1996	2,309	40	**2,349**
1997	2,340	184	**2,524**
1998	2,059	280	**2,339**
1999	2,155	320	**2,475**
2000	1,529	336	**1,865**
2001	1,168	344	**1,512**
2002	801	280	**1,081**
2003	520	246	**766**
2004	354	241	**595**
2005	287	263	**550**
2006	140	200	**340**
2007	0	134	**134**
2008	0	43	**43**
TOTAL	13,662	2,911	**16,573**

Source: Own elaboration

In this case study, the focus is put on the accident business portfolio. The dataset has an intrinsic characteristic that is worth to stand out. The reported claim quantity is not the same for all the payments of the same group of claims. That is to say, it can be found that the same claim ID which has different claim payments also may have different reported claim quantities, whether it is in a different year or in the same one. One could argue this is an inconsistency but the reason behind this might be due to a constant process that the whole file must incur occasionally, without a specific declared period of time, to update reported claim quantities when the insurance company requires to do so due to judicial claims (i.e. the insured is required to attend a legal trial whose costs will be summed to the previous reported claim, hence, updating the RBNS claim). Therefore, after a linear predictor is built to indicate the Claim ID, the risk factor linear predictor is also built. This risk factor is the reported claim cost in the reported year and not always coincides with the sum of every payment made and, in addition, it can be different for each claim payment as discussed in the previous lines.

Next, unnecessary payment observations recorded as zeroes are deleted as they only indicate there was no payment at that period of time but the claim was still open. Thus, the development period will be 5 years, summing up 2,759 claim payments which have at least one payment in the period 1996-2006, instead of 13,662, as shown in Table 6, which includes zero payments for the whole period 1996-2008. Furthermore, one deliberate assumption is made in order to simplify the outstanding claim payments prediction process. It is assumed that there is only one payment per year for each claim ID in order to end up with a single unique outstanding claim payment every future development year. This is done by aggregating all payments from the same claim ID incurred in the same year.

Finally, by means of run-off triangles, the insurer's data for the number of claims settled every development year is shown in Figure 7 and the settled claim payments evolution is depicted in Figure 8. Each triangle has a run-off period of 10 years, from 1996 to 2006, in which at least one claim is counted (Figure 7) and one claim payment is made (Figure 8).

Figure 7. Run-off triangle representing incremental number of payments

Incremental number of payments		Development year										
		0	1	2	3	4	5	6	7	8	9	10
Accident year	1996	230	47	3	2	0	0	0	0	0	0	0
	1997	287	49	5	1	0	0	0	0	0	0	
	1998	251	45	5	0	0	5	0	0	0		
	1999	310	52	4	0	1	0	0	0			
	2000	305	39	5	0	0	0	0				
	2001	226	45	2	6	3	1					
	2002	172	52	1	0	0						
	2003	167	37	4	0							
	2004	122	36	1								
	2005	135	20									
	2006	83										

Source: Own elaboration

Figure 8. Run-off triangle representing incremental claim payments

Incremental claim payments		Development year										
		0	1	2	3	4	5	6	7	8	9	10
Accident year	1996	28,376,529	4,850,687	1,966,969	255,010	0	0	0	0	0	0	0
	1997	36,800,164	7,218,619	1,173,907	45,766	0	0	0	0	0	0	
	1998	41,167,372	9,890,777	6,242,820	0	0	351,810	0	0	0		
	1999	46,348,883	6,499,309	2,202,034	0	6,000	0	0	0			
	2000	43,066,179	7,282,026	265,030	0	0	0	0				
	2001	24,009,189	4,676,347	855,771	5,071,813	117,373	6,250					
	2002	20,135,397	3,389,023	545	0	0						
	2003	19,920,216	4,545,188	43,144	0							
	2004	21,391,683	3,163,574	1,130								
	2005	15,711,704	1,862,858									
	2006	7,175,067										

Source: Own elaboration

4.2 Loss reserving methods

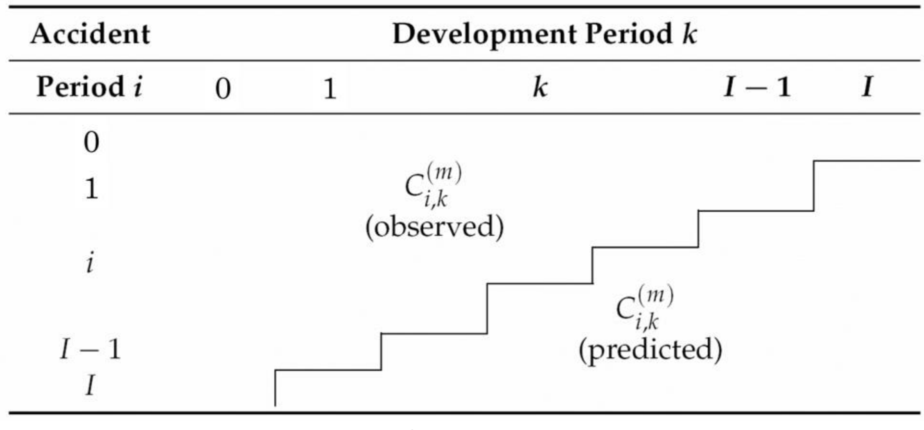
Through the use of two actuarial loss reserving methods, ultimate incurred claims will be estimated. These methods are the Chain Ladder and Generalised Linear Mixed Models.

The Chain Ladder (CL) method is a particular case of the Generalised Linear Models (GLM) reproduced as the maximum likelihood estimation in a Poisson model (Kuang *et al.*, 2009; Boj and Esquinas, 2016). It is a deterministic approach which assumes that claim patterns observed in the past will go on for future payments which will follow the same development pattern by delay year, and suggests that forthcoming settlement-amounts estimates will be more solid if all available data is used in the estimation (Verrall, 1994; Weindorfer, 2012), relying then on the use of all past years with cumulative claims loss settlement data rather than taking into account just the latest claims occurrence year (Schmidt, 2006). In Neuhaus' (2014) words, CL "grosses up" observed claim number in proportion with the estimated delay probabilities (related to development patterns and factors) which may come from estimates from the observed data, expert assessment and/or industry statistics. In CL methodology, development patterns are needed to estimate outstanding claims within run-off triangles where the development of settled claims losses follows the same pattern for every claims occurrence year (Schmidt, 2006) and insurers tend to use the development factors for that purpose (Weindorfer, 2012; Neuhaus, 2014). These development factors (δ_d) or cumulative claims loss settlement factors are defined as the relative increase in the reported proportion from a development period to the next one, that is, the following ratio:

$$\delta_d = \frac{\text{Expected cumulative claims losses settled up to and including the development year}}{\text{Expected cumulative claims losses settled up to and including the previous development year}} .$$

Non-life insurance companies usually divide portfolios into correlated subportfolios with particular satisfied homogeneous properties in order to, thereafter, apply CL method to each one of them and present a single run-off triangle (see Figure 9), ignoring the correlations among these subportfolios (Peremans *et al.*, 2018). CL predictions for an aggregate portfolio (sum of subportfolios) are different from the aggregate sum of CL predictions for each separated subportfolio (Ajne, 1994).

Figure 9. Typical representation of subportfolio m as a run-off triangle



Source: Adapted from Peremans *et al.* (2018)

Figure 9 shows a run-off triangle where i is the accident year of occurrence (going from $0 \leq i \leq I$), m denotes a specific subportfolio ($1 \leq m \leq M$), k is the development year ($0 \leq k \leq K$) and

$C_{i,k}^{(m)}$represents cumulative claims amount of accident period i and development period k of subportfolio *m*. Note that other literature use "j" to represent the development year (Neuhaus, 2014; Boj and Esquinas, 2016). Depending on the size of K, long or short tail business are indicated and but for simplicity it is assumed that $K = I$. Hence, at time I the claims $C_{i,k}^{(m)}$with $i + k - 1 \leq I$ are observed, while the claims $C_{i,k}^{(m)}$with $i + k - 1 > I$ are unobserved and must be predicted. The triangle shows the development of claims for each accident period (usually on a yearly, quarterly or monthly basis) on the rows and the development years on the columns whereas payments in the same calendar year are presented on the diagonal.

The outstanding reserve Q required to be paid in the future is defined as

$$Q = \sum_{m=1}^{M} \sum_{i=1}^{I} (C_{i,I}^{(m)} - C_{i,I-i+1}^{(m)}) .$$

Q depends on the ultimate incurred claim values $C_{i,I}^{(m)}$, which are the sum of past payments on the claim and its outstanding case estimate (Neuhaus, 2014). Claims reserving pursuits to complete the run-off triangles into squares and to that end it is needed to forecast the future claims in the bottom right corner of the run-off triangles in order to estimate the overall outstanding reserves and get the ultimate incurred claims estimate (Peremans *et al.*, 2018).

The advantage of the CL technique is that its predicted claims are highly responsive to changes in the observed claims, there is a positive relationship between observed (past) claim numbers and predicted (future) claim numbers. In her model study, Schiegl (2013) shows that the structure or symmetry of claim payments elects the 2D CL method as an appropriate estimation for reserves, even though the author develops a 3D stochastic model for claim reserving that finds useful for lines of business with long reporting and/or run-off periods.

The main disadvantage of the CL technique is its sensitivity (Neuhaus, 2014), due to misleading CL predictions of unreported claims in long-tail lines of business. These predictions omit possible random fluctuations in the future of the newer reported claims that have usually been recorded small (bottom-left position in the triangle). And a small number will destabilise a heavy tail. Schiegl (2013) examines the correlations among CL reserves and finds out that CL estimates tend to forecast a large (small) reserve when the reserve is really large (small), another revealing proof that CL method does not suit for long-tail business reserves estimation. Besides, the CL model does not contain a priori information such as factors like trends or patterns in development factors, trends across accident years and trends across payment years and assumes that these factors are unrelated, putting them in different levels which according to Zehnwirth (1997) is absurd. The assumption on homogeneity in development patterns or, in other words, unrelated claim frequencies is strongly refused by reviewed contemporaneous authors (Zehnwirth, 1997; Neuhaus, 2014; Peremans *et al.*, 2018) and one of them publicly states that homogenous development patterns where variance is 0 or very close to it do not exist in the real world (Zehnwirth, 1997). The reasons behind why the observed claims loss settlement patterns might vary over time are enumerated by Weindorfer (2012) as changes in product design and conditions; changes in the claims reporting, assessment and settlement processes; changes in the legal environment; and abnormally large or small claim settlement amounts, which in turn lead to overestimation or underestimation of claim reserves predictions. Therefore, CL should only be considered when the company's past experience data is accurate and error-free and the insurer should always look at the fact that its experience does not contradict the assumptions under this technique, otherwise data should be adjusted accordingly or an alternative method should be used (Weindorfer, 2012) such as Generalised Linear Mixed Models.

The Generalised Linear Mixed Model (GLMM) is a multilevel GLM that incorporates fixed effects from systematic variables and random effects from group or temporal variables in the linear predictor (Klinker, 2010; Boj and Esquinas, 2016) used to deal with a wide range of actuarial applications detailed by McCullagh and Nelder (1989), McNail and Wendin (2005), Antonio *et al.* (2005, 2006), Boucher and Denuit (2006), Boucher *et al.* (2008), Frees (2010), Verrall *et al.* (2010), Antonio and Valdez (2012), Kroon (2012), Pigeon *et al.* (2013, 2014), Frees *et al.* (2014), Godecharle and Antonio (2014) and Boj and Esquinas (2016). The fixed effect is a classification variable with some few levels in which we are interested in and random effect is a classification variable with many levels which only some of them appear in the sample dataset. According to Klinker (2010), fixed and random effects can vary among different contexts of study and, for instance, fixed effects of one study can be random in another.

A GLMM is fully specified by the link function (normally, but not necessarily logarithmic in loss reserving), the structure of covariates and the probability distribution, whose parameters are estimated by numerical likelihood maximisation (Booth and Hobert, 1998; Verbeke and Molenberghs, 2000; Molenberghs and Verbeke, 2005; Jiang, 2007). Following the definition scheme from Boj and Esquinas (2016), suppose the random variable $\boldsymbol{Y}_i$ and individual or group-of-similar-individuals observations $i = 1, \dots, N$ gathered by $Y_{i1}, \dots, Y_{in_i}$, the general expression of GLMM is:

$$\boldsymbol{Y}_i = g^{-1}(\boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i + \boldsymbol{\varepsilon}_i), \; i = 1, \dots, N$$

with

$$\boldsymbol{\mu}_i = \mathrm{E}\,[\boldsymbol{Y}_i \mid \boldsymbol{b}_i] = \psi'(\boldsymbol{\theta}_i) \text{ y } \mathrm{Var}\,[\boldsymbol{Y}_i \mid \boldsymbol{b}_i] = \emptyset\, V(\boldsymbol{\mu}_i) = \emptyset\, \psi''(\boldsymbol{\theta}_i).$$

Where:

$\boldsymbol{Y}_i = (y_{i1}, \dots, y_{in_i})$ is the observations vector for the i-th indivual (n_i x 1),

n_i is the observations number for the i-th individual,

N is the total different individuals number of the sample ($1 \le i \le N$),

$\boldsymbol{\beta}$ (p x 1) is the vector of fixed effects which contains the p fixed parameters, unknown and common for all sample individuals,

$\boldsymbol{b}_i$ (q x 1) is the vector of random effects with q random parameters for the i-th individual, and therefore, different for each individual i,

$\boldsymbol{X}_i$ (n_i x p) is the design matrix for fixed effects for each individual i,

$\boldsymbol{Z}_i$ (n_i x q) is the design matrix for random effects for every individual i,

$\boldsymbol{\varepsilon}_i$ (n_i x 1) is the vector that contains the residuals for each individual i,

$g(\cdot)$ is the link monotonous and differentiable function

$g^{-1}(\cdot)$ is the inverse of the linking function,

$\emptyset$ is the dispersion parameter,

$V(\boldsymbol{\mu}_i)$ is the variance function of the freely chosen distribution (usually an exponential family one)

It is worth mentioning that the GLMM linear predictor contains the sum of the systematic component $\boldsymbol{X}_i\boldsymbol{\beta}$ which includes the fixed effects and the random component $\boldsymbol{Z}_i\boldsymbol{b}_i$ which includes the random effects. On this matter, GLMM can be considered as a multilevel GLM that provides a multilevel part (or mixt) with respect to GLM classic approaches.

Similar to Boj and Esquinas (2016), the assumed hypothesis are the following:

1) Given the vector $\boldsymbol{b}_i$, it is assumed that $\boldsymbol{Y}_i$ observations are independent with an exponential-family density:

$$(y_{ij} \mid \boldsymbol{b}_i, \boldsymbol{\beta}, \emptyset) = \exp\left((y_i j\theta_{ij} - (\theta_{ij}))/\emptyset + c(y_{ij}, \emptyset)\right), \; j = 1, \ldots, n_i.$$

The answer is linked with the linear predictor by the link function like in GLM:

$$g(\boldsymbol{\mu}i) = \boldsymbol{X}_i\boldsymbol{\beta} + \boldsymbol{Z}_i\boldsymbol{b}_i \,.$$

2) $\boldsymbol{b}_i$ and $\boldsymbol{\varepsilon}_i$ are also independent and normally distributed with a vector whose mean is 0 and variance and covariances matrixes are $\boldsymbol{D}$ $(q\times q)$ and $\Sigma_{\boldsymbol{i}}$ $(n_i \times n_i)$, respectively. Thus, $\boldsymbol{b}_i \sim N(0, \mathrm{D})$ and $\boldsymbol{\varepsilon}_i \sim N(0, \Sigma_i)$.

Using the quasi-likelihood function defined for the fixed and random effects, the model is estimated. The estimator $\widehat{\boldsymbol{\beta}}$, known as BLUE (Best Linear Unbiased Estimator), for the fixed effects and once maximised the quasi-likelihood function is expressed as:

$$\widehat{\boldsymbol{\beta}} = \left(\sum_{i=1}^{N} \boldsymbol{X}'_i \boldsymbol{V}'^{-1}_i \boldsymbol{X}_i\right)^{-1} \sum_{i=1}^{N} \boldsymbol{X}'_i \boldsymbol{V}'^{-1}_i \boldsymbol{Y}_i \,.$$

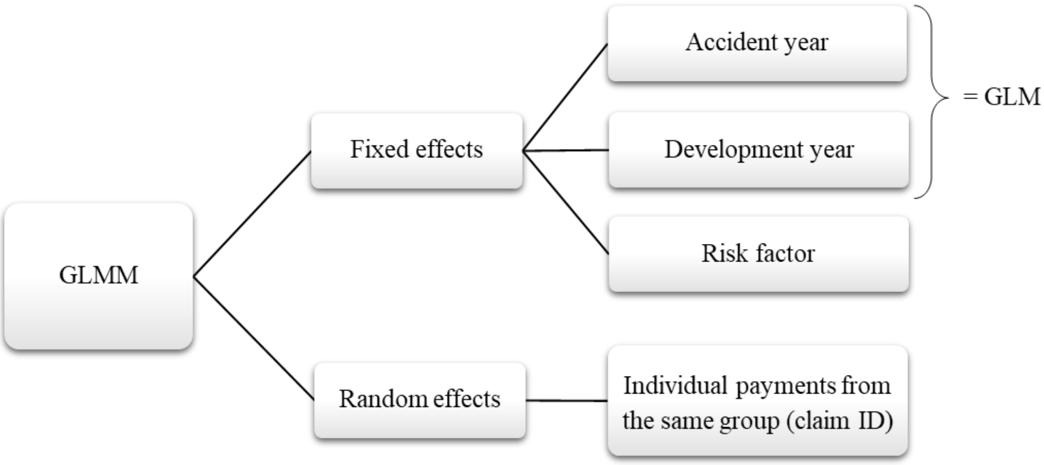
And in the same way, random effects, $\boldsymbol{b}_i$, are estimated with $\widehat{\boldsymbol{b}}$ expressed as:

$$\widehat{\boldsymbol{b}} = \boldsymbol{D}\boldsymbol{Z}_i'\boldsymbol{V}_i^{-1}(\boldsymbol{Y}_i - \boldsymbol{X}\widehat{\boldsymbol{\beta}}) \,.$$

In general, the estimation needs likelihood or numerical integration techniques. In the case study, R package *lme4* is used to estimate and adjust the GLMM with its *glmer* function (Kaas *et al.*, 2008; Bates *et al.*, 2019a). With this package, one can calculate everything, including the *bootstrap*-adjusted forecasting distributions (Boj *et al.*, 2014). For the *bootstrap* process calculations, *lme4*'s *simulate* and *refit* functions are very helpful (Bates *et al.*, 2019b). However, the present case study does not contemplate the *bootstrap* procedure.

The choice of GLMM instead of just GLM would rely on the fact that, as mentioned before, this model has a part of fixed effects constituted by accident years and development years (until here is the same as the linear predictor of a GLM) and, in addition, a risk factor is added to the fixed part (which collects the effect of the expert's report of the claim for every period). After that, a part of random effects is also included in the model. This part defines what payments belong to the same claim using claim IDs. On the whole, the model becomes a fully specified GLMM. Figure 10 portrays a diagram of the GLMM effects.

Figure 10. Diagram of GLMM effects



Source: Own elaboration

4.3 Estimating future incremental claim payments with R

4.3.1 Chain Ladder approach results

Using the *glm* function from R *stats* package (see Annex 7.2) to model a GLM defined with the quasipoisson family which has a logarithmic link function by default and reproduces the Chain Ladder method including two predictor binary variables which refer to occurrence years and development years, the run-off triangle representing incremental claim payments (therefore, non-cumulative) from Figure 8 is completed with the future payment estimates as it is shown in the blue zone from Figure 11.

Figure 11. Run-off triangle representing original and CL estimated incremental claim payments

Observed and estimated incremental claim payments	Development year										
Accident year	0	1	2	3	4	5	6	7	8	9	10
1996	28,376,529	4,850,687	1,966,969	255,010	0	0	0	0	0	0	0
1997	36,800,164	7,218,619	1,173,907	45,766	0	0	0	0	0	0	0
1998	41,167,372	9,890,777	6,242,820	0	0	351,810	0	0	0	0	0
1999	46,348,883	6,499,309	2,202,034	0	6,000	0	0	0	0	0	0
2000	43,066,179	7,282,026	265,030	0	0	0	0	0	0	0	0
2001	24,009,189	4,676,347	855,771	5,071,813	117,373	6,250	0	0	0	0	0
2002	20,135,397	3,389,023	545	0	0	30,257.52	0	0	0	0	0
2003	19,920,216	4,545,188	43,144	0	10,019.19	31,535.48	0	0	0	0	0
2004	21,391,683	3,163,574	1,130	411,095.80	10,206.81	32,126.00	0	0	0	0	0
2005	15,711,704	1,862,858	673,514.90	305,489.10	7,584.77	23,873.13	0	0	0	0	0
2006	7,175,067	1,289,857.00	324,403.60	147,141.20	3,653.26	11,498.68	0	0	0	0	0

Source: Own elaboration

The GLM adjustment formula instruction is set with a "aoglm" defined nominal variable as R factor which contains 11 occurrence years enumerated from 1 to 11 (1996-2006 in triangles), a "adglm" defined nominal variable as R factor which contains 11 development years enumerated from 1 to 11 (0-10 in triangles) and a "cij" variable which contains the original claim payment quantities aggregated in each year (coordinates i and j represent occurrence and development years, respectively), forming the so-called aggregated GLM. The instruction looks as follows:

```
glm1<-glm(formula = cij ~ aoglm + adglm, family = quasipoisson)
```

The resulting model where the first R factor from the occurrence year and the development year is included in the independent term $\hat{\beta}_0$ is:

$$\hat{\mu} = e^{(\hat{\beta}_0 + \hat{\beta}_{ao_2} + \cdots + \hat{\beta}_{ao_{11}} + \hat{\beta}_{ad_2} + \cdots + \hat{\beta}_{ad_{11}})}.$$

This GLM-Poisson model can be found in Annex 7.2, together with its coefficients, whose most significant values are found in the intercept (at 0.1% level), the 11th occurrence year (at 1%) and from the 2nd to the 6th development year. The Mean Squared Error (MSE) is also calculated within the disaggregated (aggregated) GLM and its value is 941,829,257,933 monetary units (1.656742e+12), a very high amount that will be compared to the GLMM's MSE.

4.3.2 GLMM approach results

As mentioned earlier, the GLMM adjustment is done using R package *lme4* and its *glmer* function (Kaas *et al.*, 2008; Bates *et al.*, 2019a). After calling *lme4* package from the library, the instruction to put in R to estimate the GLMM is

```
glmm1 <- glmer(pagos~ao+ad+fr+(1|siniestro), family = poisson)
```

where

pagos = fixed effect predictor representing aggregated payment quantities every year.

ao = fixed effect predictor representing occurrence or accident year.

ad = fixed effect predictor representing development year.

fr = fixed effect predictor performing the risk factor which in turn represents the reported claim quantity in each reported year, as it does not always coincide with the sum of all settled claim payments and besides it can be different for every claim payment.

(1|siniestro) = random effect predictor representing payments that refer to the same claim (in Spanish, siniestro = claim).

The building process of these variables can be found in Annex 7.3.

The resulting model where the first R factor from the occurrence year and the development year is included in the independent term $\hat{\beta}_0$ is:

$$\hat{\mu}_i = e^{(\hat{\beta}_0 + \hat{\beta}_{ao_2} + \cdots + \hat{\beta}_{ao_{11}} + \hat{\beta}_{ad_2} + \cdots + \hat{\beta}_{ad_{11}} + \hat{\beta}_{fr_i} + \hat{b}_{sin_i})}.$$

The estimates from the random part, $\hat{b}_{sin_i}$, are displayed with the ranef(glmm1) instruction. $\hat{b}_{sin_i}$ symbolises the random effect predictor which takes into account what group of claims each payment belongs to and this deviation correction is individual for each payment i.

This GLMM model can be found in Annex 7.3, together with its coefficients, whose most significant values are found in the intercept, the occurrence years 3, 4, 6 and the risk factor (at 0.1% level); at 5th occurrence year (1% level) and at the 2nd occurrence year (10% level). The Mean Squared Error (MSE) is also calculated within the disaggregated (aggregated) GLMM and its value is 79,709,894,341 (3.0734) monetary units (m.u.), 862,119,363,592 m.u. lower than CL's MSE (941,829,257,933 m.u.) and, therefore, GLMM is a lot more accurate than CL classical methodology (see Table 12).

Table 12. CL and GLMM's disaggregated and aggregated MSE comparative table

Chain Ladder		GLMM	
Disaggregated	Aggregated	Disaggregated	Aggregated
941,829,257,933.00	1.66E+18	79,709,894,341.00	3.0734

Source: Own elaboration

The run-off triangle representing incremental claim payments (therefore, non-cumulative) from Figure 8 is completed with the future payment GLMM estimates as it is shown in the blue zone from Figure 13.

Figure 13. Run-off triangle representing original and GLMM estimated incremental claim payments

Observed and estimated incremental claim payments		Development year										
		0	1	2	3	4	5	6	7	8	9	10
Accident year	1996	28,376,529	4,850,687	1,966,969	255,010	0	0	0	0	0	0	0
	1997	36,800,164	7,218,619	1,173,907	45,766	0	0	0	0	0	0	0
	1998	41,167,372	9,890,777	6,242,820	0	0	351,810	0	0	0	0	0
	1999	46,348,883	6,499,309	2,202,034	0	6,000	0	0	0	0	0	0
	2000	43,066,179	7,282,026	265,030	0	0	0	0	0	0	0	0
	2001	24,009,189	4,676,347	855,771	5,071,813	117,373	6,250	0	0	0	0	0
	2002	20,135,397	3,389,023	545	0	0	17,355,896.18	0	0	0	0	0
	2003	19,920,216	4,545,188	43,144	0	13,862,587.70	18,193,416.61	0	0	0	0	0
	2004	21,391,683	3,163,574	1,130	56,415,734.86	13,778,069.00	18,082,493.32	0	0	0	0	0
	2005	15,711,704	1,862,858	11,234,605.71	40,258,887.02	9,832,181.12	12,903,865.51	0	0	0	0	0
	2006	7,175,067	6,362,869.86	4,525,458.57	16,216,850.88	3,960,542.05	5,197,860.11	0	0	0	0	0

Source: Own elaboration

Thus, the CL model will be rejected in the pricing study but it is still going to be visible in the present value estimation of future payments to see by how much the reinsurer would be underestimating future payments.

4.4 Estimating present value future payments

Due to the fact that it is unknown the exact time when claims are going to be paid, beginning in 2006 (ao = 11), the cedent's future payments (C_{11j}) are supposedly valued at the middle of the year (assuming a uniform distribution of the claims), hence, it is calculated, considering the UK's risk-free rate (r) of 2%, simplified from 2.1% (Cherowbrier, 2019), the present value (PV) of the future payments (R), a variable that the reinsurer will use for pricing:

$$R = \sum_{j=1}^{5} C_{11j} * (1 + 0.02)^{-j+\frac{1}{2}}.$$

Table 14 depicts the estimated incremental claim payments from development year 1 to 5, as for both models from the 6th to the 10th development year zero-quantity incremental payments are recorded. The total PV of future payments for CL and GLMM is visible at the bottom of the table.

Table 14. Present value of estimated future payments under Chain Ladder and GLMM methods

	Chain ladder		**GLMM**	
Development year	Estimated future claim payments	Present Value with r = 2%	Estimated future claim payments	Present Value with r = 2%
1	1,289,857.00	1,277,148.74	6,362,869.86	6,300,179.96
2	324,403.60	314,909.24	4,525,458.57	4,393,011.46
3	147,141.20	140,034.12	16,216,850.88	15,433,559.26
4	3,653.26	3,408.63	3,960,542.05	3,695,336.81
5	11,498.68	10,518.34	5,197,860.11	4,754,707.60
Total PV of future payments		**1,746,019.08**		**34,576,795.09**

Source: Own elaboration

The Chain Ladder's PV of future payments is very low compared to GLMM's. Looking at both models' MSE, choosing Chain Ladder would mean incurring a high risk of underestimating the PV of future payments by 32,830,776.01 m.u.. These results would make substantial differences on the pricing results under each model and it would be very risky to use Chain Ladder as an option to price reinsurance policies so it is completely discarded.

4.5 Pricing ADC treaties using GLMM's MSE

The pricing strategy for this case study is based on relating the safety loading factor with the Mean Squared Error (MSE) by means of fluctuation bands, in other words, the total reinsurance premium will depend on the MSE and, in consequence, on the applied model to estimate future payments (in this case, GLMM). However, Mlej (2015) suggests conducting a bootstrapping of triangles procedure to estimate variability of reserves but in the present case this is not contemplated. It is neither taken into consideration the tax implication, cost of allocated capital, the quality of data and Bornhuetter Fergusson model as a method to look for "best estimates" liabilities.

The MSE measures the average squared error of the predictions calculating for each point the square difference between estimates and targets and then dividing by the total number of observations, hence, making the average of those squared-differenced values. In Zhu's *et al*. (2019) words, the MSE "measures the forecasting power of the models" or "assesses the in-sample prediction ability". This indicator is generally defined by:

$$\mathrm{MSE} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 \ .$$

Nonetheless, in this case study the formula varies to the same one as Boj and Esquinas (2016) apply in theirs:

$$\text{MSE} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(Y_{ij}-\hat{\mu}_{ij}\right)^2}{\sum_{i=1}^{N} n_i} \, .$$

This is because occurrence years (i) and development years (j) are taken into account. Nevertheless, the interpretation of this indicator does not vary: The lower the MSE value, the better fitting the model is. For a perfect fitting model, MSE = 0. MSE is never negative due to the squared difference. Sometimes, analysts study the Root Mean Squared Error (RMSE) to make the scale of the errors equal to the scale of the actual quantity values (Drakos, 2018) and it is the distance of a data point from the fitted line, comparable to the observed variation in measurements of an actual point (Vernier, 2018). Its interpretation is the same as the MSE as the square root is a non-decreasing function and the analyst will look for the lowest possible value and Vernier (2018) affirms it might be a better measure of goodness of fit than a correlation coefficient. An estimation of the pure premium using RMSE in Property and Casualty reinsurance modelling can be seen at Chasseray *et al.* (2017).

MSE is considered to be a simple metric for regression evaluation and this is its main advantage. However, the disadvantage of MSE is that it is too sensitive to individual predictions and, hence, a single extremely bad prediction will worsen the error and the model's badness will be overestimated due to an undesired skewness of the metric (Drakos, 2018). With noisy data it may be difficult to measure the fitness of a model. And, on the other hand, if all errors are small or smaller than 1 there could be an underestimation of the model's badness.

Part of the definition of the loading factor could be considered as the main reason to choose to equate the MSE to it. The reinsurance loading factor depends on the individual circumstances of each business case and, according to Carter (1983), it can be broken down into three components:

(1) a fee to cover the reinsurer's acquisition and administrative costs;
(2) a safety loading to cover the risk accepted by the reinsurer (the fluctuations in claim costs); and
(3) a contribution to profit.

The following study only focuses on the second component, the safety loading, as it is the one that justifies the relationship between the disaggregated MSE with the whole premium loading factor. Fluctuation bands based on the value of the CL and GLMM's MSE are assumed in order to provide cover to fluctuations in claim costs (see Table 15). In this case, GLMM has a safety loading equal to 0.2 whilst CL's is equal to 0.8, making the reinsurance more expensive as the reinsurer incurs much more risk if it considers the CL's estimated future payments.

Table 15. Assumed fluctuation bands for the value of disaggregated MSE

Fluctuation bands	Safety loading
MSE < 100 x 10^9	0.2
100 x 10^9 < MSE < 500 x 10^9	0.5
MSE > 500 x 10^9	0.8

Source: Own elaboration

The reinsurance premium calculation is managed without considering expenses loading for the expected total administrative costs of the treaty (1), the reinsurer's gross net premium income derived by the contribution to profit (3), inflation, volatility charge (Mlej, 2015), exchange rate risk and the so-called "superimposed inflation", a concept proposed by Benktander (1974) and defined by Carter (1983) as the combination disproportionate increases in a reinsurer's liability caused not only by inflation. Examples of causes for these increases are higher compensations for personal injury related to earnings during expansionary economic times even yet prices remain stable, the worldwide tendency for courts to be more generous towards claimants at the expense of the (re)insurance system and the increasing costs of medical treatments that keep alive more seriously injured individuals. Carter (1983) stands out that inflation, changes in law and other developments need to be accounted for when adjusting premiums accordingly. Excess-loss reinsurers should also consider this, but it is difficult enough to apply such parameters for direct insurers and even more for reinsurers (Belloy and Gabus, 1976). The problem gets worse for long-tail business as claims remain outstanding for five or more years increasing the probability that potential costs rise through inflation. Carter (1983) states that reinsurers prefer to adjust the treaty limits according to the inflation in an annual basis, hence, indexing their retention limit to the annual inflation rate. Should the reader be interested in the index clause, defined by Schiffer (2010) as the distribution of the effects of inflation on reinsured claims costs frequently developed in excess-loss reinsurance contracts (XL contracts) covering long-tail risks, Carter (1983) shows an example of indexing the reinsurer's retention at p. 207.

All in all, in this study, the technical reinsurance premium which is equal to the pure premium (amount related to claim losses excluding any profit margin or administrative expenses) plus the GLMM's safety loading is computed in order to set an example of a reinsurance pricing strategy with the GLMM's MSE. Due to the fact this case study tackles RBNS claims data, three ADC treaties (quota share, excess-loss and at-the-money types) issued on 31/12/2006 are enforced to provide three types of retrospective reinsurance cover to possible incremental future payments coming from those reported claims that have not been settled yet to the contract's issue date. Finally, it is assumed that the reinsurer sets an upper limit of loss coverage for every contract defined as the maximum amount of estimated future payments for every development year, and that the technical interest rate assumed by the reinsurer and used to calculate the present value of future payments (R) is equal to the UK's risk-free rate (r) of 2%, simplified from 2.1% (Cherowbrier, 2019).

4.5.1 Pricing a quota share ADC

The reinsurance technical premium for the ADC in the form of quota share loss portfolio arrangement can be formulated similarly to the classic quota share reinsurance.

Firstly, the reinsurance pure premium (π_p^R) must be obtained exposing the quota share rate (k) of the present value (PV) of the future claim payment amounts (R) that the insurance ceding company will retain:

$$\pi_p^R = (1 - k) \cdot R \,.$$

The PV of future claim payments R is that one obtained for the GLMM in Table 14 from section 4.4 with the formula:

$$\mathrm{R} = \sum_{j=1}^{5} C_{11j} * (1+0.02)^{-j+\frac{1}{2}} = \ 34{,}576{,}795.09 \text{ m.u.}$$

Eventually, the loading factor –from now on "the safety loading" (γ^R) due to the assumptions made before– is added to the pure premium to get the reinsurance technical premium (π_t^R) which will be charged to the cedent (in monetary units or m.u.):

$$\pi_t^R = \pi_p^R \cdot (1+\gamma^R)\,.$$

Putting it into numbers, suppose the ceding company wants to retain 40% of its accident business future payment liabilities. The quota share contract will cover the remaining 60%. Therefore, the reinsurer will be liable to 60% of those future incremental payments that are estimated to occur in the run-off period. So, $k = 0.4$ and $\gamma^R = 0.2$ (as GLMM's MSE are lower than 100 x 10^9)

Under the GLMM model, the pure and technical premium are:

$$\pi_p^R = (1-k)\cdot R = (1-0.4)\cdot\ 34{,}576{,}795.09 = 20{,}746{,}077.05 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1+\gamma^R) = 20{,}746{,}077.05 \cdot (1+0.2) = 24{,}895{,}292.47 \text{ m.u.}$$

The results, portrayed in Table 16, show that in order to cover a maximum of 34,576,795.09 of possible future payments, the insurer will have to pay 24,895,292.47 m.u., from which 4,149,215.41 m.u. account for the reinsurer's safety loading.

Table 16. Recalculated PV of estimated future payments and technical premium under GLMM model for the quota share ADC

	GLMM	
Development year	Estimated future claim payments	Present Value with r = 2%
1	6,362,869.86	6,300,179.96
2	4,525,458.57	4,393,011.46
3	16,216,850.88	15,433,559.26
4	3,960,542.05	3,695,336.81
5	5,197,860.11	4,754,707.60
Total PV of future payments		**34,576,795.09**
Technical premium		**24,895,292.47**

Source: Own elaboration

4.5.2 Pricing an excess ADC

In order to price an excess ADC, the total present value of future payments must be recalculated (R') as the aim of this contract is to cover future payments subtracting a retention (M) on each of the cedent's n-numbered estimated incremental claim payments (C_{11j}^{n}) in each development year, from the first (j = 1) to the fifth (j = 5), *caeteris paribus* (r = 2% and MSE fluctuation bands kept equal):

$$\mathrm{R}' = \sum_{n=1}^{83}\sum_{j=1}^{5}(C_{11j}^{n} - M_j) * (1+0.02)^{-j+\frac{1}{2}}.$$

Remembering Sarrasí (2017), the mathematical formalisation for the policy insured (C_c^n) and reinsured (C_r^n) loss in an excess-loss treaty goes as follows:

$$C_c^n = \begin{cases} C^n & \text{if} \quad C^n < \mathrm{M} \\ \mathrm{M} & \text{if} \quad C^n \geq \mathrm{M} \end{cases} \qquad C_r^n = \begin{cases} 0 & \text{if} \quad C^n < \mathrm{M} \\ C^n - \mathrm{M} & \text{if} \quad C^n \geq \mathrm{M} \end{cases}$$

Therefore, if $C_{11j}^{n} - M_j < 0$, then $\mathrm{C_r} = C_{11j}^{n} - M_j = 0$. The reinsurance will cover the excess above M of the individual estimated future payments for each development year.

In this case, the amount of retention (M) is chosen to be the average of all 83 individual incremental claim estimates (maximum n = 83) for each development year after accident year 2006 (ao = 11) from the GLMM model estimation. Table 17 depicts each M based on the average of incremental claim estimates for each development year under GLMM approach. Annex 7.4 shows all individual claim estimates for every development year and claims from accident year 2006 (ao = 11) computed under GLMM method.

Table 17. Estimated future payments and retention limits on each development year under GLMM

GLMM	ad1	ad2	ad3	ad4	ad5
Total estimated Claim payment	6,362,869.86	4,525,458.57	16,216,850.88	3,960,542.05	5,197,860.11
Retention (M)	76,661.08	54,523.60	195,383.75	47,717.37	62,624.82

Source: Own elaboration

Under the GLMM model, the recalculated PV of future payments amount (R') and the ADC's pure and technical premium are:

$$\mathrm{R}' = \sum_{n=1}^{83}\sum_{j=1}^{5}\left(C_{11j}^{n} - M_j\right) * (1+0.02)^{-j+\frac{1}{2}} = 16{,}793{,}665.62 \text{ m.u.}$$

$$\pi_p^{R'} = \mathrm{R}' = 16{,}793{,}665.62 \text{ m.u.}$$

$$\pi_t^{R'} = \pi_p^{R'} \cdot (1+\gamma^R) = 16{,}793{,}665.62 \cdot (1+0.2) = 20{,}152{,}398.74 \text{ m.u.}$$

These results, displayed in Table 18, show that the technical premium of the excess ADC treaty is a lower than quota share one due to lower total present value of future claim payments that the reinsurer will be entitled to once the different retentions are applied.

Table 18. Recalculated PV of estimated future payments and technical premium under GLMM model for the excess ADC

	GLMM	
Development year	Estimated future claim payments	Present Value with r = 2%
1	3,090,393.67	3,059,945.70
2	2,197,978.09	2,133,649.61
3	7,876,391.39	7,495,953.08
4	1,923,602.77	1,794,794.76
5	2,524,558.00	2,309,322.46
Total PV of future payments		**16,793,665.62**
Technical premium		**20,152,398.74**

Source: Own elaboration

4.5.3 Pricing an at-the-money ADC

As for excess ADC, the total PV of future payments for an at-the-money adverse development cover must be recalculated (R′′) once again. An at-the-money ADC works like a stop loss contract. All claims in each development year are added up ($C_{11j} = \sum_{n=1}^{83} C_{11j}^{n}$ where C is the total portfolio loss of a specific line of business in an entire development year) and then a retention (M) is subtracted for each development year. In this particular case, there will be exposed five different present values of future payments, that is, five different reinsurance pure and technical premiums with five different retentions (x = 1, 2, 3, 4 and 5), *caeteris paribus*:

$$R_x'' = \sum_{j=1}^{5} (C_{11j} - M_x) * (1 + 0.02)^{-j+\frac{1}{2}}.$$

Remembering Sarrasí (2017), the mathematical formalisation for the total portfolio insured (C_c) and reinsured (C_r) loss in a stop-loss treaty goes as follows:

$$C_c = \begin{cases} C & \text{if} \quad C < M \\ M & \text{if} \quad C \geq M \end{cases} \qquad C_r = \begin{cases} 0 & \text{if} \quad C < M \\ C - M & \text{if} \quad C \geq M \end{cases}$$

Therefore, if $C_{11j} - M_x < 0$, then $C_r = C_{11j} - M_x = 0$. The relationship between the excess and the at-the-money (or stop loss) ADC is that $C_{11j} = \sum_{n=1}^{83} C_{11j}^{n}$. If M is subtracted from the total portfolio insured loss (C_{11j}), an at-the-money ADC treaty is taking place. If M is subtracted from each policy insured loss (C_{11j}^{n}), an excess ADC is being enforced.

The random amounts of M_x are:

$M_1 = 20{,}000 \quad M_2 = 40{,}000 \quad M_3 = 60{,}000 \quad M_4 = 80{,}000 \quad M_5 = 200{,}000$

Under the GLMM model, the recalculated PV of estimated future payments (R''_x) and the ADC's pure and technical premium using five different retention schemes are as follows:

$$R''_1 = \sum_{j=1}^{5}(C_{11j} - 20{,}000) * (1 + 0.02)^{-j+\frac{1}{2}} = 34{,}481{,}587.88$$

$$\pi_p^R = \mathrm{R''}_1 = 34{,}481{,}587.88 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1 + \gamma^R) = 34{,}481{,}587.88 \cdot (1 + 0.2) = 41{,}377{,}905.45 \text{ m.u.}$$

$$R''_2 = \sum_{j=1}^{5}(C_{11j} - 40{,}000) * (1 + 0.02)^{-j+\frac{1}{2}} = 34{,}386{,}380.66$$

$$\pi_p^R = \mathrm{R''}_2 = 34{,}386{,}380.66 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1 + \gamma^R) = 34{,}386{,}380.66 \cdot (1 + 0.2) = 41{,}263{,}656.79 \text{ m.u.}$$

$$R''_3 = \sum_{j=1}^{5}(C_{11j} - 60{,}000) * (1 + 0.02)^{-j+\frac{1}{2}} = 34{,}291{,}173.45$$

$$\pi_p^R = \mathrm{R''}_3 = 34{,}291{,}173.45 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1 + \gamma^R) = 34{,}291{,}173.45 \cdot (1 + 0.2) = 41{,}149{,}408.13 \text{ m.u.}$$

$$R''_4 = \sum_{j=1}^{5}(C_{11j} - 80{,}000) * (1 + 0.02)^{-j+\frac{1}{2}} = 34{,}195{,}966.23$$

$$\pi_p^R = \mathrm{R''}_4 = 34{,}195{,}966.23 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1 + \gamma^R) = 34{,}195{,}966.23 \cdot (1 + 0.2) = 41{,}035{,}159.48 \text{ m.u.}$$

$$R''_5 = \sum_{j=1}^{5}(C_{11j} - 200{,}000) * (1 + 0.02)^{-j+\frac{1}{2}} = 33{,}624{,}722.94$$

$$\pi_p^R = \mathrm{R''}_5 = 33{,}624{,}722.94 \text{ m.u.}$$

$$\pi_t^R = \pi_p^R \cdot (1 + \gamma^R) = 33{,}624{,}722.94 \cdot (1 + 0.2) = 40{,}349{,}667.53 \text{ m.u.}$$

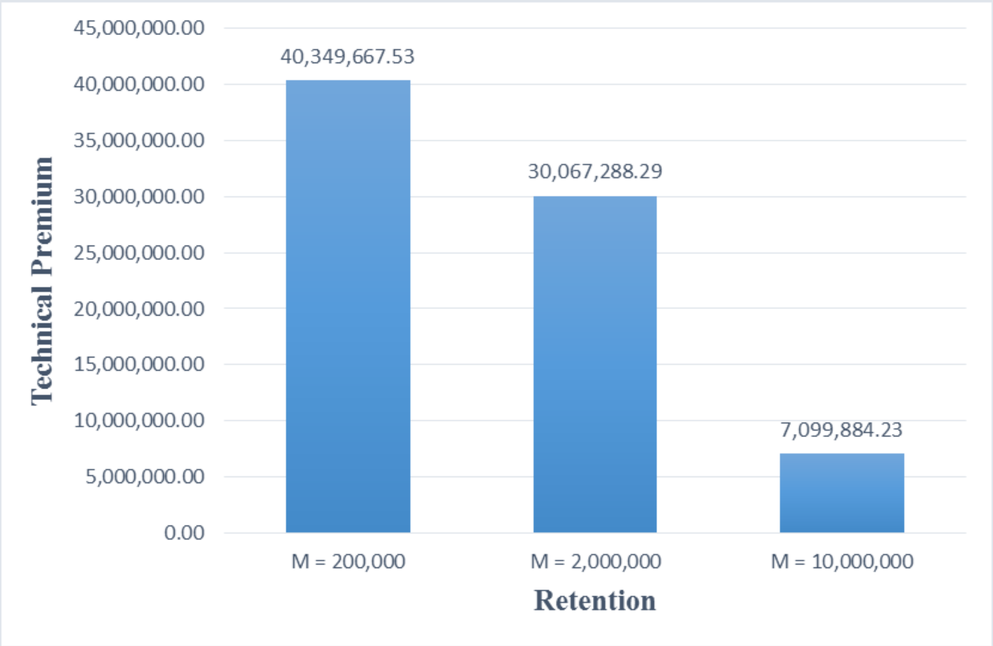
As it is observed in the results exhibited in Table 19, the technical premium of an at-the-money ADC treaty is the most expensive compared to the previous two treaties' premiums. This confirms Loeffler's (2019) statement saying that "this option will have good coverage but will have the most immediate cost impact”. Besides, the premium decreases as M increases. By adding up all claim costs and subtracting a retention or priority from the total sum of future payments (which is going to be the amount the ceding company will cover), the reinsurer is bearing a higher underwriting risk than if it had decided to apply an excess ADC treaty. Besides, it is noticeable that increasing 10 times the retention M (from 20,000 to 200,000), does not make a big difference on the technical premium, which is just 2.48% cheaper, because the reinsured future payments still amount to a very large quantity. Graph 20 portrays, within the GLMM framework, the different technical premiums the reinsurer could offer to the cedent when increasing M ten times 200,000 up to 2,000,000; and this last quantity five times more (up to 10,000,000). As it can be seen, the technical premium diminishes as the retention increases.

Table 19. Recalculated PVs of estimated future payments and technical premiums under GLMM model for the at-the-money ADC

	GLMM				
	Recalculated present value of reserves with r = 2% and M as…				
Development year	**M = 20,000**	**M = 40,000**	**M = 60,000**	**M = 80,000**	**M = 200,000**
1	6,280,377.01	6,260,574.06	6,240,771.11	6,220,968.16	6,102,150.45
2	4,373,596.80	4,354,182.14	4,334,767.49	4,315,352.83	4,198,864.88
3	15,414,525.29	15,395,491.31	15,376,457.33	15,357,423.35	15,243,219.48
4	3,676,676.05	3,658,015.28	3,639,354.52	3,620,693.76	3,508,729.18
5	4,736,412.73	4,718,117.87	4,699,823.00	4,681,528.14	4,571,758.94
Total PV of future payments	**34,481,587.88**	**34,386,380.66**	**34,291,173.45**	**34,195,966.23**	**33,624,722.94**
Technical premium	**41,377,905.45**	**41,263,656.79**	**41,149,408.13**	**41,035,159.48**	**40,349,667.53**

Source: Own elaboration

Graph 20. Technical premiums of GLMM-based at-the-money ADCs for different M values



Source: Own elaboration

5. Conclusion

The present thesis has defined the traditional reinsurance concept expressing it with words and mathematical formulation. On one hand, the theoretical part surrounding the topic about alternative risk transfer (ART) mechanisms has been introduced to finally break down the finite risk reinsurance schemes focusing on loss portfolio transfer (LPT) and adverse development cover (ADC) transactions. LPT is the one of the most finite risk reinsurance transactions and involves a total or partial cession of a company's reserves to the reinsurer who assumes financial responsibility for the ceded reserves (similar to a quota share treaty). These contracts bear timing, investment, credit and reserving risks. ADC treaties provide reinsurance cover above net carried reserve level without involving any transfer of existing reserves and have greater underwriting risk. In order to go further with the case study aiming to price retrospective finite risk transactions possessing only RBNS (Reported But Not Settled) claims data, a new definition assumption of LPT and ADC has been introduced. This assumption dictates that LPT is applicable to settled yet not totally paid claims and ADC is applicable to unsettled claims. Therefore, the case study has focused on three types of ADC transactions (quota share, excess and at-the-money ADC) that estimate those future payments from reported unsettled past claims within 5 years' time.

After using R software to structure the available data and estimate future payments under Chain Ladder (CL) and Generalised Linear Mixed Models (GLMM) approaches, the mean squared errors (MSE) of both methodologies have been calculated and show that CL must be strongly rejected and GLMM has a robust accuracy which strengthens its goodness of fit to the data. When the individual claim amount is known and identified to a specific claim ID, the GLMM takes on special relevance as actuaries can incorporate individual information in the form of risk factors to the adjusted model and enhance its efficiency. Examples of risk factors are the insured's age, gender, residence, etc; but the project's original data did not have these variables. The only risk factor added to the GLMM was the one representing the reported claim quantity in each reported year because it varies for every claim payment and this fact makes the sum of all settled claim payments not to be equal to the reported claim quantity. All in all, with RBNS data it is possible to know the individual claims it contains, unlike IBNR (Incurred But Not Reported) data in which no information is disclosed about the future individual claim amount.

When pricing the three contracts after estimating the present value of future payments, at-the-money ADC contracts' prices are consistent to what literature argues (they are more expensive than other types of ADCs). However, the limitations of this paper are that important variables such as expenses and profit loading for the expected total administrative costs, inflation, volatility charge and the exchange rate risk have been omitted. More research on other models such as Bornhuetter Ferguson or Benktander's should be done in order to compare their MSEs to CL's and analogous GLMM's using other error distributions, link functions or risk factors to increase the accuracy of the MSE indicator. Additionally, more investigation into hybrid LPT-ADC reinsurance options would be recommended as they are increasing their popularity year over year because of its competitive combination of price and coverage of RBNS and IBNR claims, as this work has not been able to do it due to lacking of IBNR data. Lastly, despite its simplicity, relating the MSE with the safety loading factor might lead to some big overestimated or underestimated pricing effects that could affect the reinsurance business, that is why further research should be done in other types of pricing strategies which include tax implications, cost of allocated capital, analyses on the quality of data and bootstrapping of triangles to get the "best estimate liabilities" (BEL).

In a world where climate change is affecting the metrics of reinsurance modelling and threatening the insurance industry with protection capacity shortages, finite risk transactions appear to retaliate against unexpected volatility within the reinsurance prices marketplace covering startling catastrophic events such as earthquakes, hurricanes or floods even when claims have already been reported in the past waiting to be completely settled and paid, determining their possible run-offs and adjusting reinsurance pricing procedures. However, ART techniques seem to be still acting as complements rather than substitutes due to their complexity and widespread distrust. For this reason, reinsurers should place great importance on the understanding and justification of the price for their offered coverages.

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7. Annex

7.1 Data reading and assembling process

 # Reading the data

```
datos <- read.csv("Datos.csv",header=TRUE,dec=".",sep = ";")
names(datos)

> names(datos)
[1] "Valuation.date"              "Line.of.business"             "Claim.ID"
[4] "Accident.date"               "Reported.date"                "Settlement.date"
[7] "Paid.to.date"                "Outstanding.case.estimate"    "Reported.claim.cost"
[10] "Number.of.claims"           "Valuation.year"               "Accident.year"
[13] "Reported.year"              "Settlement.year"              "Accident.to.reporting.delay"
[16] "Reporting.to.valuation.delay"  "Accident.to.valuation.delay"
"Reporting.to.settlement.delay"
[19] "Settled"                    "Last.column.for.pivot..table"

table(datos$Number.of.claims)
> table(datos$Number.of.claims)
1
16573

table(datos$Line.of.business)
> table(datos$Line.of.business)
       Accident   Liability
48962     13662      2911

table(datos$Reported.year)
> table(datos$Reported.year)

1 996 1 997 1 998 1 999 2 000 2 001 2 002 2 003 2 004 2 005 2 006 2 007 2 008
 2349  2524  2339  2475  1865  1512  1081   766   595   550   340   134    43

table(datos$Reported.year[datos$Line.of.business=="Accident"])
> table(datos$Reported.year[datos$Line.of.business=="Accident"])

1 996   1 997   1 998   1 999   2 000   2 001   2 002   2 003   2 004   2 005   2 006   2 007   2
008
2309    2340    2059    2155    1529    1168     801     520     354     287     140       0       0
```

```
datosAccident <- subset(datos,datos$Line.of.business=="Accident")
as.numeric(datosAccident$Claim.ID)
min(datosAccident$Claim.ID)
max(datosAccident$Claim.ID)
dim(datosAccident)

> min(datosAccident$Claim.ID)
[1] 5023
> max(datosAccident$Claim.ID)
[1] 30627
> dim(datosAccident)
[1] 13662    20
```

 # Deaccumulation of Paid.to.date variable to a new "pagos" (payments) variable

```
pagos <- datosAccident$Paid.to.date
for (i in 5023:30627) {
ini <- 0
for (j in 1:13662) {
if (datosAccident$Claim.ID[j]==i) {pagos[j] <- pagos[j] - ini;
  ini <- datosAccident$Paid.to.date[j]}
}
}

pagos <- as.numeric(pagos)
sum(is.na(pagos))

> sum(is.na(pagos))
[1] 0
```

 # Building the occurrence year (ao) predictor (which for RBNS claims is the reported year)

```
table(datosAccident$Reported.year)
> table(datosAccident$Reported.year)

1 996 1 997 1 998 1 999 2 000 2 001 2 002 2 003 2 004 2 005 2 006 2 007 2 008
0  2309  2340  2059  2155  1529  1168   801   520   354   287   140     0     0

sum(datosAccident$Reported.year=="1 996")
> sum(datosAccident$Reported.year=="1 996")
```

```
[1] 2309
sum(datosAccident$Reported.year=="2 006")
> sum(datosAccident$Reported.year=="2 006")
[1] 140

ao <- as.numeric(datosAccident$Reported.year)
table(ao)

> table(ao)
ao
2    3    4    5    6    7    8    9   10   11   12 # Empezará en 1996
2309 2340 2059 2155 1529 1168  801  520  354  287  140

ao <- ao -1 # Empezará en 1996 hasta 2006
table(ao)

> table(ao)
ao
1    2    3    4    5    6    7    8    9   10   11
2309 2340 2059 2155 1529 1168  801  520  354  287  140
```

 # Building the development year (ad) predictor

```
table(datosAccident$Valuation.year)

> table(datosAccident$Valuation.year)

1 996 1 997 1 998 1 999 2 000 2 001 2 002 2 003 2 004 2 005 2 006 2 007 2 008
0   207   411   656   929  1170  1344  1510  1660  1787  1929  2059     0     0

ad <- as.numeric(datosAccident$Valuation.year)
table(ad)

> table(ad)
ad
2    3    4    5    6    7    8    9   10   11   12
207  411  656  929 1170 1344 1510 1660 1787 1929 2059

ad <- ad -1  # Empezará en 1996 hasta 2006
table(ad)
```

```
> table(ad)
ad
1    2    3    4    5    6    7    8    9   10   11
207  411  656  929 1170 1344 1510 1660 1787 1929 2059

for (j in 1:13662) {
  for (i in 2:11) {
    if (ao[j]==i) ad[j] <- (ad[j] - (i-1))
  }}

table(ad)

> table(ad)
ad
1    2    3    4    5    6    7    8    9   10   11
1927 1919 1784 1671 1540 1372 1167  948  677  447  210
```

 # Building the claim ID indication predictor (siniestro= claim)

```
siniestro <- datosAccident$Claim.ID
```

 # Building the risk factor (fr), representing the reported claim quantity in each reported year because it does not always coincide with the sum of all settled claim payments and besides it can be different for every claim payment.

```
fr <- as.numeric(datosAccident$Reported.claim.cost)
```

 # Deleting unnecessary observations (zeroes indicating that the claim is not closed yet)

```
ao <- ao[pagos>0]
ad <- ad[pagos>0]
siniestro <- siniestro[pagos>0]
fr <- fr[pagos>0]
pagos <- pagos[pagos>0]
length(pagos)
>  length(pagos)
[1] 2759

table(ao)
> table(ao)
ao
```

```
1   2   3   4   5   6   7   8   9  10  11
282 342 306 367 349 283 225 208 156 157  84

table(ad)
> table(ad)
ad
1    2    3    4    5    6    7    8    9   10
1657  651  179  119   65   38   23   13    9    5
```

 # Aggregating claim quantities inside each year from the same claim ID

```
datosAccidenttotal <- cbind(pagos,ao,ad,siniestro,fr)
length(pagos) # 2759

> siniestro[1]
[1] 5023
1997 Levels: 5023 5024 5025 5026 5027 5028 5029 5030 5032 5033 5035 5036 5038 5039
5040 5041 5042 5043 5044 5045 ... 30626

datosAccidenttotalintermedio <- datosAccidenttotal
a<-1
for (i in 1:11) {print(i);
  for (j in 1:11){
    for (k in 5023:30626){
      if (sum((siniestro == k)&(ao==i)&(ad==j))>=1)
      {datosAccidenttotalintermedio[a,] <-
c(pagos[siniestro==k][1],ao[siniestro==k][1],ad[siniestro==k][1],siniestro[siniestro==k][1],fr[
siniestro==k][1]);
      datosAccidenttotalintermedio[a,1] <- sum(pagos[((siniestro == k)&(ao==i)&(ad==j))])
      a<-a+1}

    }
  }
}

datosAccidenttotalintermedio <- datosAccidenttotalintermedio[1:(a-1),]
dim(datosAccidenttotalintermedio) # [1] 2759    5
pagos <- datosAccidenttotalintermedio[,1]
ao <- datosAccidenttotalintermedio[,2]
ao <- as.factor(ao)
```

```
ad <- datosAccidenttotalintermedio[,3]
ad <- as.factor(ad)
table(ao,ad)
```

 # Obtaining the following number of claims

```
> table(ao,ad)
ad
ao    1   2   3   4   5   6
1    230  47   3   2   0   0
2    287  49   5   1   0   0
3    251  45   5   0   0   5
4    310  52   4   0   1   0
5    305  39   5   0   0   0
6    226  45   2   6   3   1
7    172  52   1   0   0   0
8    167  37   4   0   0   0
9    122  36   1   0   0   0
10   135  20   0   0   0   0
11    83   0   0   0   0   0
siniestro <- datosAccidenttotalintermedio[,4]
siniestro <- as.factor(siniestro)
fr <- datosAccidenttotalintermedio[,5]
```

 # Building the claim payments run-off triangle to sum up the case study information

```
triangle <- matrix (0, nrow =11, ncol =11)
for (i in 1:11) {
  for (j in 1:11) {
    triangle[i,j]=sum(pagos[(ao==i)&(ad==j)])
  }
}
triangle
> triangle
         [,1]    [,2]    [,3]    [,4]    [,5]   [,6] [,7] [,8] [,9] [,10] [,11]
[1,] 28376529 4850687 1966969  255010       0      0    0    0    0     0     0
[2,] 36800164 7218619 1173907   45766       0      0    0    0    0     0     0
[3,] 41167372 9890777 6242820       0       0 351810    0    0    0     0     0
[4,] 46348883 6499309 2202034       0    6000      0    0    0    0     0     0
[5,] 43066179 7282026  265030       0       0      0    0    0    0     0     0
[6,] 24009189 4676347  855771 5071813  117373   6250    0    0    0     0     0
```

```
[7,] 20135397 3389023     545       0      0      0    0    0    0     0     0
[8,] 19920216 4545188   43144       0      0      0    0    0    0     0     0
[9,] 21391683 3163574    1130       0      0      0    0    0    0     0     0
[10,] 15711704 1862858      0      0      0      0    0    0    0     0     0
[11,]  7175067       0      0      0      0      0    0    0    0     0     0
```

7.2 Modeling a Chain Ladder with *glm* function from *stats* R package

 # GLM estimation

```
cij <- rep(0, 66); cij
a=0
for (i in 1:11) {
  for ( j in 1:(11-i+1) ) {
    a=a+1
    cij[a]=triangle[i,j]
  }
}
cij

aoglm <- rep(1:11, times = 11:1)
adglm <- c(1:11,1:10,1:9,1:8,1:7,1:6,1:5,1:4,1:3,1:2,1)
aoglm <- as.factor(aoglm)
adglm <- as.factor(adglm)
```

 # family=quasipoisson has logarithmic link function by default and reproduces Chain Ladder

```
glm1<-glm(cij~aoglm+adglm,fam = quasipoisson)
summary(glm1)
anova(glm1)
> summary(glm1)

Call:
  glm(formula = cij ~ aoglm + adglm, family = quasipoisson)

Deviance Residuals:
  Min       1Q   Median       3Q      Max
-1453.6   -380.2     -3.4      0.0   3626.7

Coefficients:
  Estimate Std. Error t value Pr(>|t|)
(Intercept)    17.1624     0.1923  89.249  < 2e-16 ***
  aoglm2         0.2439     0.2538   0.961  0.34174
aoglm3         0.4863     0.2415   2.014  0.05000 .
aoglm4         0.4402     0.2436   1.807  0.07744 .
aoglm5         0.3561     0.2478   1.437  0.15758
aoglm6        -0.0203     0.2701  -0.075  0.94041
```

```
aoglm7        -0.4088     0.3009  -1.359  0.18105
aoglm8        -0.3674     0.2972  -1.236  0.22284
aoglm9        -0.3488     0.2972  -1.174  0.24661
aoglm10       -0.6457     0.3304  -1.955  0.05687 .
aoglm11       -1.3763     0.4641  -2.966  0.00482 **
  adglm2       -1.7161     0.1682 -10.203 2.76e-13 ***
  adglm3       -3.0964     0.3239  -9.561 2.08e-12 ***
  adglm4       -3.8870     0.4930  -7.885 5.07e-10 ***
  adglm5       -7.5827     3.2217  -2.354  0.02302 *
  adglm6       -6.4361     1.8921  -3.402  0.00142 **
  adglm7      -26.7674 31859.4539  -0.001  0.99933
adglm8       -26.7546 35558.7851  -0.001  0.99940
adglm9       -26.7046 41005.7044  -0.001  0.99948
adglm10      -26.5845 50562.1863  -0.001  0.99958
adglm11      -26.4650 71858.5353   0.000  0.99971
---
  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for quasipoisson family taken to be 1279941)

Null deviance: 1001475914  on 65  degrees of freedom
Residual deviance:   38929340  on 45  degrees of freedom
AIC: NA

Number of Fisher Scoring iterations: 7

> anova(glm1)
Analysis of Deviance Table

Model: quasipoisson, link: log

Response: cij

Terms added sequentially (first to last)


Df  Deviance Resid. Df Resid. Dev
NULL                    65 1001475914
aoglm 10  29062830        55  972413084
adglm 10 933483743        45   38929340
```

 # MSE calculation for the “aggregated” GLM

```
cij
cijest<-predict(glm1,type="response"); cijest
sum((cij-cijest)^2)/66
```

```
> sum((cij-cijest)^2)/66
[1] 1.656742e+12
```

 # Total run-off triangle (original data + future estimates)

```
coefs<-exp(as.numeric(coef(glm1)))
alpha<-c(1,coefs[2:11])*coefs[1]
beta<-c(1,coefs[(11+1):(2*11-1)])
orig.fits<-alpha%*%t(beta)
future<-row(orig.fits)+col(orig.fits)-1>11
triangleglm <- triangle
triangleglm[future] <- orig.fits[future]; triangleglm
```

```
[,1]    [,2]      [,3]      [,4]       [,5]      [,6]        [,7]        [,8]        [,9]       [,10]
[1,] 28376529 4850687 1966969.0  255010.0      0.000      0.00 0.000000e+00 0.000000e+00
    0.000000e+00 0.000000e+00
[2,] 36800164 7218619 1173907.0   45766.0      0.000      0.00 0.000000e+00 0.000000e+00
    0.000000e+00 0.000000e+00
[3,] 41167372 9890777 6242820.0       0.0      0.000 351810.00 0.000000e+00 0.000000e+00
    0.000000e+00 1.316003e-04
[4,] 46348883 6499309 2202034.0       0.0   6000.000      0.00 0.000000e+00 0.000000e+00
    1.114459e-04 1.256733e-04
[5,] 43066179 7282026  265030.0       0.0      0.000      0.00 0.000000e+00 9.745781e-05
    1.024523e-04 1.155316e-04
[6,] 24009189 4676347  855771.0 5071813.0 117373.000   6250.00 6.603898e-05 6.688699e-
    05 7.031480e-05 7.929133e-05
[7,] 20135397 3389023     545.0       0.0      0.000  30257.52 4.478148e-05 4.535652e-05
    4.768095e-05 5.376799e-05
[8,] 19920216 4545188   43144.0       0.0  10019.190  31535.48 4.667288e-05 4.727221e-05
    4.969480e-05 5.603894e-05
[9,] 21391683 3163574    1130.0  411095.8  10206.805  32126.00 4.754685e-05 4.815740e-
    05 5.062536e-05 5.708830e-05
[10,] 15711704 1862858  673514.9  305489.1   7584.770  23873.13 3.533250e-05 3.578621e-
    05 3.762017e-05 4.242284e-05
[11,]  7175067 1289857  324403.6  147141.2   3653.263  11498.68 1.701817e-05 1.723670e-
    05 1.812005e-05 2.043329e-05
[,11]
[1,] 0.000000e+00
[2,] 1.163697e-04
[3,] 1.483039e-04
[4,] 1.416246e-04
[5,] 1.301956e-04
[6,] 8.935551e-05
[7,] 6.059258e-05
[8,] 6.315178e-05
```

```
[9,] 6.433432e-05
[10,] 4.780743e-05
[11,] 2.302682e-05
```

 # Original run-off triangle (only original data)

```
triangleestglmorig <- orig.fits
triangleestglmorig[future] <- 0; triangleestglmorig
triangleestglmorigind <- triangleestglmorig; triangleestglmorigind

> triangleestglmorigind <- triangleestglmorig; triangleestglmorigind
[,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
[1,] 28413889 5107944 1284666.7 582691.8 14467.240 45535.76 6.739344e-05 6.825885e-05 7.175696e-05 8.091759e-05
[2,] 36260357 6518498 1639426.1 743601.6 18462.354 58110.41 8.600408e-05 8.710846e-05 9.157258e-05 1.032629e-04
[3,] 46210913 8307302 2089316.9 947660.5 23528.788 74057.05 1.096053e-04 1.110127e-04 1.167019e-04 0.000000e+00
[4,] 44129677 7933160 1995218.7 904980.0 22469.104 70721.69 1.046689e-04 1.060130e-04 0.000000e+00 0.000000e+00
[5,] 40568449 7292961 1834206.3 831948.8 20655.866 65014.51 9.622222e-05 0.000000e+00 0.000000e+00 0.000000e+00
[6,] 27842832 5005286 1258847.6 570981.0 14176.480 44620.59 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[7,] 18880414 3394118 853633.2 387186.1 9613.168 0.00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[8,] 19677849 3537473 889687.3 403539.3 0.000 0.00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[9,] 20046326 3603714 906347.1 0.0 0.000 0.00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[10,] 14896609 2677953 0.0 0.0 0.000 0.00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[11,] 7175067 0 0.0 0.0 0.000 0.00 0.000000e+00 0.000000e+00 0.000000e+00 0.000000e+00
[,11]
[1,] 9.11882e-05
[2,] 0.00000e+00
[3,] 0.00000e+00
[4,] 0.00000e+00
[5,] 0.00000e+00
[6,] 0.00000e+00
[7,] 0.00000e+00
[8,] 0.00000e+00
[9,] 0.00000e+00
[10,] 0.00000e+00
[11,] 0.00000e+00
```

 # Chain Ladder's "disaggregated" MSE calculation with observed claim numbers

```
triangleestglmorigindmed <- triangleestglmorig[,1:6]/table(ao,ad)
triangleestglmorigindmed

> triangleestglmorigindmed
ad
ao          1          2          3          4          5          6
1  123538.649 108679.665 428222.222 291345.898        Inf        Inf
2  126342.709 133030.574 327885.225 743601.573        Inf        Inf
3  184107.225 184606.719 417863.386        Inf        Inf  14811.410
4  142353.795 152560.768 498804.685        Inf  22469.104        Inf
5  133011.308 186998.990 366841.254        Inf        Inf        Inf
6  123198.370 111228.570 629423.824  95163.492   4725.493  44620.586
7  109769.850  65271.507 853633.183        Inf        Inf
8  117831.430  95607.367 222421.825        Inf
9  164314.150 100103.154 906347.136
10 110345.254 133897.632
11  86446.590

glmestindividual <- matrix (0, nrow =2759, ncol =6)
for (i in 1:2759) { print(i);
  for (j in 1:6) {
    glmestindividual[i]<-triangleestglmorigindmed [ao[i],ad[i]]
  }
}


sum((pagos-glmestindividual)^2)/2759

> sum((pagos-glmestindividual)^2)/2759
[1] 941829257933
```

7.3 Modeling a GLMM with *glmer* function from *lme4* R package

```
library(lme4)
glmm1 <- glmer(pagos~ao+ad+fr+(1|siniestro),family = poisson)

summary(glmm1)

> summary(glmm1)
Generalized linear mixed model fit by maximum likelihood (Laplace Approximation)
   ['glmerMod']
Family: poisson  ( log )
Formula: pagos ~ ao + ad + fr + (1 | siniestro)
```

```
AIC       BIC     logLik   deviance   df.resid
307809680  307809787 -153904822  307809644       2741

Scaled residuals:
  Min       1Q   Median      3Q      Max
-1471.64    -0.46     0.00     0.00  3145.01

Random effects:
  Groups    Name        Variance Std.Dev.
siniestro (Intercept) 4.177    2.044
Number of obs: 2759, groups:  siniestro, 1997

Fixed effects:
  Estimate Std. Error z value Pr(>|z|)
(Intercept)  9.373e+00  1.436e-01  65.264  < 2e-16 ***
  ao2         3.690e-01  1.936e-01   1.906 0.056610 .
ao3         7.166e-01  1.954e-01   3.668 0.000245 ***
  ao4         7.261e-01  1.882e-01   3.859 0.000114 ***
  ao5         5.972e-01  1.972e-01   3.028 0.002466 **
  ao6         8.316e-01  2.017e-01   4.124 3.73e-05 ***
  ao7         4.903e-01  2.125e-01   2.307 0.021037 *
  ao8         3.176e-01  2.267e-01   1.401 0.161128
ao9        -8.687e-03  2.372e-01  -0.037 0.970782
ao10       -5.173e-01  2.306e-01  -2.243 0.024901 *
  ao11       -1.528e-01  2.667e-01  -0.573 0.566597
ad2        -1.201e-01  1.264e-01  -0.951 0.341795
ad3        -4.609e-01  5.338e-01  -0.863 0.387901
ad4         8.154e-01  9.183e-01   0.888 0.374557
ad5        -5.942e-01  1.185e+00  -0.502 0.616009
ad6        -3.224e-01  1.185e+00  -0.272 0.785527
fr          2.468e-06  1.644e-07  15.012  < 2e-16 ***
  ---
  Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Correlation matrix not shown by default, as p = 17 > 12.
Use print(x, correlation=TRUE)  or
vcov(x)        if you need it

fit warnings:
  Some predictor variables are on very different scales: consider rescaling
convergence code: 0
unable to evaluate scaled gradient
Model failed to converge: degenerate  Hessian with 2 negative eigenvalues


print(glmm1)
```

```
> print(glmm1)
Generalized linear mixed model fit by maximum likelihood (Laplace Approximation)
  ['glmerMod']
Family: poisson  ( log )
Formula: pagos ~ ao + ad + fr + (1 | siniestro)
AIC       BIC     logLik   deviance   df.resid
307809680  307809787 -153904822  307809644      2741
Random effects:
 Groups    Name        Std.Dev.
siniestro (Intercept) 2.044
Number of obs: 2759, groups:  siniestro, 1997
Fixed Effects:
 (Intercept)        ao2         ao3         ao4         ao5         ao6         ao7         ao8         ao9
9.373e+00   3.690e-01   7.166e-01   7.261e-01   5.972e-01   8.316e-01   4.903e-01
  3.176e-01  -8.687e-03
ao10       ao11        ad2         ad3         ad4         ad5         ad6         fr
-5.173e-01  -1.528e-01  -1.201e-01  -4.609e-01   8.154e-01  -5.942e-01  -3.224e-01
  2.468e-06
fit warnings:
 Some predictor variables are on very different scales: consider rescaling
convergence code 0; 2 optimizer warnings; 0 lme4 warnings

names(ranef(glmm1))
ranef(glmm1)$siniestro[1:3,]

> names(ranef(glmm1))
[1] "siniestro"
> ranef(glmm1)$siniestro[1:3,]
[1] 2.3535320 0.3375364 1.8264290
```

 # Estimations for ad2, ad3, ad4, ad5, ad6 under GLMM as of ao7 (2002)

```
glmmestfuture <- matrix (0, nrow =2759, ncol =5)
for (i in 1:2759) { print(i);
 for (j in 1:5) {
   aof<-ao[i]
   aon<-as.numeric(ao[i])
   aof<-as.factor(aof)
   adf<-j+1
   adf<-as.factor(adf)
   siniestrof<-siniestro[i]
   siniestrof<-as.factor(siniestrof)
   frf<-fr[i]
   dades <- data.frame(ao=aof,ad=adf,siniestro=siniestrof,fr=frf)
   if (((aon>=7)&((aon+j+1-1)>11))=="TRUE") glmmestfuture[i,j]<-
    predict(glmm1,dades,type="response")
   }
```

```
}
```

 # MSE Calculation under "disaggregated" GLMM

```
glmm1est <- predict(glmm1,type="response")
sum((pagos-glmm1est)^2)/2759

> sum((pagos-glmm1est)^2)/2759
[1] 79709894341
```

 # Save the results

```
write.csv2(siniestro, file = "siniestro.csv",row.names = FALSE)
Resultado <- cbind(siniestro,fr,ao,ad,pagos,glmmestfuture)
write.csv2(Resultado, file = "Resultado.csv",row.names = FALSE)

trianglepredglmm <- matrix(0,nrow =11, ncol =11)
for (i in 1:2759) { print(i);
  for (j in 1:5) {
    trianglepredglmm[ao[i],j+1]<-trianglepredglmm[ao[i],j+1]+glmmestfuture[i,j]
   }
}
trianglepredglmm

[,1]   [,2]     [,3]     [,4]     [,5]     [,6] [,7] [,8] [,9] [,10] [,11]
[1,]    0       0        0        0        0        0    0    0    0     0     0
[2,]    0       0        0        0        0        0    0    0    0     0     0
[3,]    0       0        0        0        0        0    0    0    0     0     0
[4,]    0       0        0        0        0        0    0    0    0     0     0
[5,]    0       0        0        0        0        0    0    0    0     0     0
[6,]    0       0        0        0        0        0    0    0    0     0     0
[7,]    0       0        0        0        0 17355896    0    0    0     0     0
[8,]    0       0        0        0 13862588 18193417    0    0    0     0     0
[9,]    0       0        0 56415735 13778069 18082493    0    0    0     0     0
[10,]   0       0 11234606 40258887  9832181 12903866    0    0    0     0     0
[11,]   0 6362870  4525459 16216851  3960542  5197860    0    0    0     0     0


triangleglmm <- triangle
triangleglmm[future] <- trianglepredglmm[future]; triangleglmm

> triangleglmm[future] <- trianglepredglmm[future]; triangleglmm
[,1]   [,2]     [,3]     [,4]     [,5]     [,6] [,7] [,8] [,9] [,10] [,11]
[1,] 28376529 4850687  1966969   255010        0        0    0    0    0     0     0
[2,] 36800164 7218619  1173907    45766        0        0    0    0    0     0     0
[3,] 41167372 9890777  6242820        0        0   351810    0    0    0     0     0
[4,] 46348883 6499309  2202034        0     6000        0    0    0    0     0     0
```

```
[5,] 43066179 7282026   265030        0        0        0    0    0    0     0     0
[6,] 24009189 4676347   855771  5071813   117373     6250    0    0    0     0     0
[7,] 20135397 3389023      545        0        0 17355896    0    0    0     0     0
[8,] 19920216 4545188    43144        0 13862588 18193417    0    0    0     0     0
[9,] 21391683 3163574     1130 56415735 13778069 18082493    0    0    0     0     0
[10,] 15711704 1862858 11234606 40258887  9832181 12903866    0    0    0     0     0
[11,]  7175067 6362870  4525459 16216851  3960542  5197860    0    0    0     0     0
```

 # MSE Calculation under "aggregated" GLMM

```
trianglepredglmmorig <- matrix(0,nrow =11, ncol =11)
for (i in 1:2759) { print(i);
   trianglepredglmmorig[ao[i],ad[i]]<-trianglepredglmmorig[ao[i],ad[i]]+glmm1est[i]
  }
trianglepredglmmorig
sum((trianglepredglmmorig-triangle)^2)/66

> trianglepredglmmorig
[,1]    [,2]         [,3]       [,4]       [,5]       [,6] [,7] [,8] [,9] [,10] [,11]
[1,] 28376532 4850683 1966969.4862  255010.58      0.000      0.000    0    0    0     0     0
[2,] 36800167 7218616 1173907.0534   45765.99      0.000      0.000    0    0    0     0     0
[3,] 41167377 9890773 6242818.7278       0.00      0.000 351809.593    0    0    0     0     0
[4,] 46348885 6499308 2202033.1508       0.00   6000.196      0.000    0    0    0     0     0
[5,] 43066180 7282025  265029.8743       0.00      0.000      0.000    0    0    0     0     0
[6,] 24009188 4676348  855770.4773 5071812.43 117372.803   6250.407    0    0    0     0     0
[7,] 20135390 3389029     545.7424       0.00      0.000      0.000    0    0    0     0     0
[8,] 19920214 4545189   43144.4264       0.00      0.000      0.000    0    0    0     0     0
[9,] 21391680 3163576    1131.0598       0.00      0.000      0.000    0    0    0     0     0
[10,] 15711702 1862860      0.0000       0.00      0.000      0.000    0    0    0     0     0
[11,]  7175067       0      0.0000       0.00      0.000      0.000    0    0    0     0     0

> sum((trianglepredglmmorig-triangle)^2)/66
[1] 3.073374
```

7.4 Individual incremental claim estimates from accident year 11 under GLMM

n	Claim ID	fr	Pagos ao11	ad1	ad2	ad3	ad4	ad5
1	30247	220155	155	138.4549	98.47315	352.8757	86.18067	113.1045
2	30250	990900	900	799.1547	568.3821	2036.781	497.4305	652.8334
3	30254	1430655	655	582.1857	414.0674	1483.799	362.3791	475.5904
4	30339	220450	450	399.8367	284.3755	1019.052	248.8767	326.6286
5	30348	660605	605	537.459	382.2565	1369.805	334.5391	439.0529
6	30352	1285	1285	1139.98	810.7869	2905.432	709.5758	931.2552
7	30362	13013	13013	11539.92	8207.526	29411.44	7182.975	9427.017
8	30370	42660	42660	37830.73	26906.32	96418.01	23547.58	30904.11
9	30415	74305	74305	65893.5	46865.38	167940.7	41015.14	53828.73
10	30429	660740	740	657.1348	467.3734	1674.819	409.0308	536.8167
11	30433	36727	36727	32569.35	23164.27	83008.51	20272.66	26606.07
12	30437	20574	20574	18244.94	12976.33	46500.32	11356.49	14904.38
13	30443	89333	89333	79220.34	56343.82	201906.5	49310.38	64715.49
14	30455	106829	106829	94735.82	67378.88	241450.3	58967.92	77390.16
15	30456	101630	79630	70615.73	50223.96	179976.1	43954.47	57686.34
16	30457	165948	110948	98388.58	69976.83	250760	61241.57	80374.12
17	30459	522460	522460	463318.5	329525.6	1180846	288390.7	378487.2
18	30464	43043	43043	38170.37	27147.88	97283.65	23758.99	31181.57
19	30481	660605	605	537.459	382.2565	1369.805	334.5391	439.0529
20	30483	157843	157843	139975.1	99554.39	356750.3	87126.94	114346.4
21	30487	21132	21132	18739.77	13328.27	47761.48	11664.49	15308.61
22	30492	245	245	218.0555	155.0874	555.7513	135.7277	178.1306
23	30495	605	605	537.1133	382.0107	1368.924	334.324	438.7706

24	30496	11258	11258	9983.609	7100.634	25444.92	6214.256	8155.66
25	30497	25305	3305	2931.134	2084.708	7470.49	1824.472	2394.458
26	30498	225063	5063	4490.147	3193.524	11443.9	2794.874	3668.024
27	30505	88680	88680	78641.26	55931.96	200430.6	48949.93	64242.44
28	30510	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
29	30511	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
30	30515	1147779	738981	655330.1	466089.9	1670220	407907.5	535342.5
31	30518	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
32	30524	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
33	30526	32309	32309	28651.48	20377.77	73023.15	17834	23405.54
34	30533	127347	127347	112931.2	80319.97	287824.3	70293.57	92254.07
35	30534	226573	226573	200925	142903.7	512091.5	125065	164136.7
36	30535	292036	72036	63881.47	45434.36	162812.7	39762.75	52185.09
37	30540	552976	805	714.7027	508.3174	1821.541	444.8637	583.8442
38	30542	16080	16080	14259.7	10141.91	36343.26	8875.891	11648.82
39	30543	52292	52292	46372.37	32981.38	118187.8	28864.29	37881.82
40	30544	89940	89940	79758.63	56726.66	203278.4	49645.43	65155.22
41	30546	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
42	30548	32386	32386	28719.77	20426.34	73197.19	17876.5	23461.32
43	30549	168806	168806	149697.1	106469	381528.5	93178.36	122288.3
44	30550	397880	177880	157744.1	112192.2	402037.5	98187.16	128861.9
45	30551	183319	183319	162567.3	115622.6	414330.3	101189.3	132802
46	30552	26433	26433	23440.67	16671.69	59742.51	14590.55	19148.8
47	30554	216058	216058	191600.3	136271.7	488325.8	119260.8	156519.3
48	30555	62872	62872	55754.71	39654.38	142100.3	34704.29	45546.3
49	30556	93933	93933	83299.63	59245.12	212303.2	51849.51	68047.88

50	30558	26561	26561	23554.18	16752.42	60031.81	14661.2	19241.53
51	30559	221055	1055	936.1722	665.8329	2385.993	582.7165	764.7637
52	30561	253006	253006	224365.9	159575.6	571834.5	139655.6	183285.6
53	30563	272688	272688	241819.9	171989.4	616319	150519.8	197543.9
54	30565	62304	62304	55251.01	39296.13	140816.6	34390.76	45134.83
55	30566	8067	8067	7153.89	5088.055	18232.9	4452.91	5844.049
56	30568	59055	59055	52369.8	37246.93	133473.3	32597.36	42781.15
57	30569	125151	125151	110983.8	78934.91	282861	69081.4	90663.22
58	30571	262343	262343	232646	165464.6	592937.6	144809.5	190049.6
59	30572	220070	70	63.24294	44.98022	161.1853	39.36531	51.66347
60	30573	288396	288396	255749.8	181896.7	651821.7	159190.4	208923.3
61	30574	69181	69181	61349.54	43633.58	156359.7	38186.76	50116.74
62	30576	65450	65450	58040.88	41280.37	147927	36127.31	47413.89
63	30577	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
64	30584	25555	25555	22662.06	16117.92	57758.1	14105.91	18512.75
65	30585	221055	221055	196031.7	139423.4	499619.9	122019.1	160139.2
66	30591	166705	166705	147834	105143.8	376779.9	92018.64	120766.3
67	30592	73031	73031	64763.72	46061.84	165061.3	40311.91	52905.8
68	30593	111214	111214	98624.44	70144.58	251361.1	61388.38	80566.8
69	30594	273805	273805	242810.5	172693.9	618843.7	151136.4	198353.1
70	30601	64104	64104	56847.25	40431.42	144884.8	35384.34	46438.8
71	30602	290349	70349	62385.43	44370.34	158999.8	38831.56	50962.97
72	30604	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
73	30605	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
74	30606	220680	680	603.714	429.379	1538.667	375.7793	493.177
75	30610	72437	72437	64236.96	45687.2	163718.8	39984.03	52475.49

76	30612	477841	477841	423750.2	301383.5	1079999	263761.6	346163.6
77	30613	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
78	30617	220605	605	537.2286	382.0926	1369.218	334.3957	438.8647
79	30618	120162	120162	106559.5	75788.25	271585	66327.55	87049.02
80	30620	220605	12954	11487.71	8170.392	29278.37	7150.476	9384.365
81	30623	97035	97035	86050.49	61201.61	219314.2	53561.77	70295.07
82	30624	22605	605	537.1248	382.0189	1368.954	334.3311	438.78
83	30626	137060	137060	121544.7	86446.15	309777.3	75655.01	99290.49