

On the Relation between Modality and Tense

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Abstract: We critically review two extant paradigms for understanding the systematic interaction between modality and tense, as well as their respective modifications designed to do justice to the contingency of time's structure and composition. We show that on either type of theory, as well as their respective modifications, some principles prove logically valid whose truth might sensibly be questioned on metaphysical grounds. These considerations lead us to devise a more general logical framework that allows accommodation of those metaphysical views that its predecessors rule out by fiat.

Keywords: modality, tense, modal logic, temporal logic

Introduction

Once we incorporate operators for metaphysical possibility and necessity into a tensed language that already comes equipped with temporal operators, we can articulate a number of philosophically interesting questions which we would otherwise not be able to systematically discuss (cf. Scott 1970: 161). Is what once was the case still possibly the case at present? Could time have come to an end? Satisfactory answers to these metaphysical questions and others cannot be given without systematic reflection on the ways in which metaphysical modality interacts with tense; and it is the task of a logical theory for the enriched language to codify this interaction.

The present paper proposes such a logical theory. The logical theory emerges from a critical review of two competing logical paradigms: one that originates from the work of Kit Fine (1977) and David Kaplan (1989), and one that originates from the work of Richard Montague (1973). Each paradigm gives rise to a natural model theory for languages involving both standard modal and standard temporal operators.

As we shall argue, each of these two theories validates some principles that would seem metaphysically highly contentious. Amongst these principles are some that both theories alike validate, *viz.* principles that concern the structure of time. Arguably, however, what structure time exhibits is a contingent affair. To accommodate this thought, the two theories need to be modified. However, like the theories they modify, each of these modified versions still rules out metaphysical theories that should not be discarded on purely logical grounds.

These findings lead us to propose a more general model theory that leaves room for those metaphysical theories, as well as for the contingency of time's structure and composition. We can accordingly be understood to define a multi-modal logic for metaphysical modality and tense that is more neutral than its predecessors. It can systematically be strengthened by adding further conditions on the modal accessibility relation, or the temporal orderings, or both, corresponding to particular, substantive metaphysical views about metaphysical modality, tense and their interaction. It can thus be shown that our logic subsumes its predecessors as special cases. To the extent that, in devising a logical theory, we aim to stay as metaphysically neutral as the debate we want to have demands, the proposed logic recommends itself as the one we should adopt when thinking about the relation between metaphysical modality and tense.

The paper is structured as follows. In section 1, we reflect on the distinction between logical theory and metaphysical theory and clarify in what sense, and to what extent, the provision of a logic is answerable to the quest for metaphysical neutrality. Section 2 rehearses the original version of the Fine-Kaplan view and shows that it is at odds with the contingency of time's structure and composition. Although a natural modification of the view fares better in this regard, it fails to go far enough. Section 3 turns to the original version of the Montagovian view and shows that it, too, is at odds with the contingency of time's structure and composition. Although a natural modification of the view avoids these problems, it unduly precludes a number of metaphysical theories whose dismissal requires metaphysical argument and should not, therefore, be simply built into the logic itself. In section 4, we at last propose our preferred logic for the interaction of modality and tense and show that it is not beset by the kinds of problems its predecessors face. We conclude our discussion by comparing our view to the one recently advanced by Cian Dorr and Jeremy Goodman (Dorr and Goodman 2019).

1. Methodological preamble

Logic is traditionally conceived of 'as a neutral arbiter of metaphysical disputes, at least as a framework on which all parties can agree for eliciting the consequences of [...] rival metaphysical theories' (Williamson 2014: 212). According to this conception, if a set of principles is to qualify as a logic, 'we should not build [...] metaphysical prejudices into it. We logicians strive to serve ideologies not to constrain them' (Kaplan 1995: 42).

However, this idea – and with it the absolute and global distinction between logic, on the one hand, and metaphysical theory, on the other – has come under considerable strain. As evidenced by the surge of alternative logics in the 20th century, there would now seem to be 'no core of universally accepted logical principles', as any one of them would seem open to challenge on metaphysical grounds (Williamson 2014: 212). Intuitionists like Brouwer famously reject the law of excluded middle, in application to certain statements of mathematics, on the grounds that it presupposes a controversial conception of the mathematical realm. Dialetheists lift the ban on contradictions by rejecting *ex contradictione quodlibet* and *modus tollendo ponens* because, in their view, the nature of truth and the nature

of change call for true contradictions (Priest 2006). And the list goes on. Even the validity of *modus ponendo ponens* has been doubted (McGee 1985). Whittling down the list of logical principles to those no one might seriously challenge results, if not in nothing at all, at least in nothing worth considering a logic.

As long as we do not conflate the logical with what is compelling enough to command universal assent, however, such disagreements do nothing to show that we cannot distinguish, amongst the metaphysical truths in a given area, between the logical truths and the remainder.¹ The disagreements merely confirm that it is often hard to negotiate on which side of the divide a given principle falls or whether it is a truth at all. But even if there is independent argument, of an inevitably highly abstract nature, to the conclusion that the aforementioned distinction collapses, there then still is a distinction that can sensibly be drawn between the logic underlying a given debate and the metaphysical theories whose consequences it helps teasing out in ways appreciable by all parties to that debate. Traditionally, these two distinctions were of course assumed to align. But the latter distinction can survive demolition of the former. The finding that there is widespread disagreement about even the most deeply entrenched principles traditionally conceived of as logical then at best goes to show that the latter distinction is not an absolute and global one.

As long as metaphysicians do not merely parade their preferred theory in front of their opponents and invite them to be impressed by the beauty of its overall gestalt, they will engage in reasoned argument with their opponents in order to win them over or at least convince them to abandon their own view. Such arguments require rules of engagement on which the relevant parties can agree and which the logic underlying their debate is meant to codify. Even if not all of these rules will be acceptable to all customers alike, this merely means that some theories will be set aside for the purposes of the debate in question. That is to say, in the context of a given debate amongst proponents of rival theories, other theories, while still rivals, are treated as bodies of thought for whose rejection one owes no argument. This may of course change once the context shifts and a different audience becomes the target. Yet, for any given target audience of competitors, the logic being used can and should be free of ‘metaphysical prejudices’. For example, consider the position known as *permanentism* according to which always everything always is something, and its opposition – *temporaryism* – according to which sometimes something sometimes is nothing (Williamson 2013: 4). In a debate about time and existence, to which both permanentists and temporaryists are party, assuming classical quantification theory as part of one’s first-order logic prejudices the issue under dispute, as it privileges permanentism. This assumption should therefore not be made, as long as one does not decide to leave that debate behind (Correia and Rosenkranz 2018).

¹ Fine (1994: 9-10) distinguishes, amongst the metaphysically necessary truths, between the logically necessary ones and the remainder by taking the former to be those ‘propositions which are true in virtue of the nature of all logical concepts’. The work by Kaplan (1989) suggests that not all logically true propositions need to be necessary. But we can perhaps still use Fine’s characterisation of truths in virtue of the nature of all logical concepts as a means to isolate the logical truths.

If there is the one ‘true’ logic that includes all and only the logical truths constraining the interaction between modality and tense – where, *ex hypothesi*, these may not exhaust all the metaphysical truths in that area – the system to be proposed in section 4 should be seen as our preferred answer to the question of what that logic is. To this extent, we need not take issue with suggestions, made by other parties, that certain of the principles pronouncing on this interaction, but not included in our logic, after all qualify as metaphysical truths. What we do have to say about such principles rather is that they fail to be logical truths in the relevant sense. To identify this sense is beyond the scope of the present paper, though; and so the best we can do on this score is to offer a number of metaphysical theories, or scenarios, violating those principles and record our conviction that, whether these theories be true or these scenarios possible, they should be taken seriously and their dismissal to require substantive argument.

By contrast, if the assumption of the one ‘true’ logic is foregone – because the very distinction between logical truths and merely metaphysical truths is being abandoned – then our proposal is to be seen in a different light. For in such circumstances, what shape a combined logic for modality and tense should assume cannot properly be assessed independently from which metaphysical theories should be taken seriously in those debates structured by its use. Accordingly, our verdict should then be understood to be that extant logical systems are unnecessarily dismissive of certain metaphysical theories about the interaction between modality and tense. The foregoing already makes plain that this does not then render these systems incorrect in any absolute sense. But they might nonetheless still be accused of ruling out by logical fiat certain views for whose rejection one can be expected to owe an argument, as soon as one engages in systematic reflection on how modality and tense interact. The alternative logical system we set out to devise below is, in this regard, more neutral, if not absolutely neutral, and as such recommends itself as a better means to structure the debate. In this case, too, the best we can do is to record our conviction that those views be taken seriously and invite our readers to join us in this assessment.

2. The Fine-Kaplan view

How should we think of the interaction between metaphysical modality and tense? According to the logical approach to tense made famous by Arthur Prior, tenses can be conceptualised by means of temporal operators, of which H (‘Always in the past’) and G (‘Always in the future’) are paradigmatic examples. It is similarly standard to express metaphysical modalities by means of operators, *viz.* the duals \Box (‘Necessarily’) and \Diamond (‘Possibly’). So a natural first approach to our question is to ask what uses of these operators accomplish.

Sentential operators, as studied by logicians and formal semanticists, have the function to allow shifting of the circumstances of evaluation. If they occur unembedded, they do so by operating on certain parameters, but not others, that help to define the context of utterance. Depending on the expressive resources of the language in question, these parameters may include the time, place, subject and world of evaluation. It is very intuitive to assume that, in a tensed language,

temporal operators such as H and G merely allow shifting of the time of evaluation. It is less straightforward to assess which function is performed by modal operators like \Box .

According to one characterisation, which might strike one as initially plausible, modal operators like \Box merely allow shifting of the world of evaluation, while leaving the time of evaluation untouched. This is the characterisation given by Fine (1977: 157) and Kaplan (1989: 545). On this type of view, \Box and H do not interact, and neither do \Box and G: they perform their respective functions independently from one another. According to another, competing characterisation, championed by Montague (1973: 231), modal operators like \Box not only operate on the world of evaluation but also on the time of evaluation. On this view, the ways things used to be still are possibilities at present; and similarly, the way things will be already are possibilities at present. This, too, might look initially plausible. However, we should be cautious before opting for either type of view. For, as we shall see shortly, each view turns out to have untoward consequences, once we render precise what it commits us to.

We begin by considering the view attributed to Fine and Kaplan (FK, for short) and its underlying model theory. To this end, we let $At = \{p, q, r \dots\}$ be a set of atomic formulae and define the target language as follows:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \ \& \ \varphi) \mid \Box\varphi \mid H\varphi \mid G\varphi$$

where $p \in At$. \Diamond is defined as $\neg\Box\neg$, P ('Sometimes in the past') as $\neg H\neg$, F ('Sometimes in the future') as $\neg G\neg$, $A\varphi$ ('Always, φ ') as $H\varphi \ \& \ \varphi \ \& \ G\varphi$ and S ('Sometimes') as $\neg A\neg$; we let \top stand for an arbitrary tautology and define \perp as $\neg\top$. According to one straightforward way of rendering FK precise in model-theoretic terms, a model is a tuple $\langle W, T, <, \alpha, \tau, I \rangle$, where W (worlds) and T (times) are non-empty sets, $<$ (precedence) is a strict total order on T , α (the actual world) is a member of W , τ (the present time) is a member of T , and I (interpretation) is a function that takes any atomic formula into a set of world-time pairs. The truth predicate \models is relativised to world-time pairs. A formula is said to be *true in a model* iff it is true at the present time at the actual world of the model, and a formula is said to be *valid* iff it is true in every model. According to FK, the semantic clauses for the basic temporal operators are as follows:

$$\begin{aligned} wt \models H\varphi & \text{ iff for all } t^* \text{ in } T \text{ such that } t^* < t, wt^* \models \varphi \\ wt \models G\varphi & \text{ iff for all } t^* \text{ in } T \text{ such that } t < t^*, wt^* \models \varphi \end{aligned}$$

Consequently, the clause for A reads as follows:

$$wt \models A\varphi \text{ iff for all } t^* \text{ in } T, wt^* \models \varphi$$

The clause for the necessity operator, by contrast, is this:

$$wt \models \Box\varphi \text{ iff for all } w^* \text{ in } W, w^*t \models \varphi$$

Accordingly, \Box and \Diamond operate in strictly symmetric but mutually independent fashion.

To keep matters simple, our language does not contain sentential operators such as NOW and ACTUALLY. By including α and τ in the model structure, we allow for the possibility of expanding the language to one that does. Which clauses are best suited to govern these operators is a vexed issue which need not detain us here.

We will, however, assume that there is a distinguished atomic formula in our language which, on the intended interpretation, means the same as ‘Now exists’, where ‘Now’ is the indexical noun phrase that designates, relative to any given context of use, the time of that context. We will write this formula simply as ‘Now exists’, bearing in mind that from the point of view of the language under consideration it has no logical complexity. Naturally, if the models were endowed with a function *dom* assigning to each world-time pair a set of objects purported to represent the objects that exist at that time in that world, then the truth-clause for ‘Now exists’ would be as follows:

$$wt \models \text{Now exists} \text{ iff } \tau \in \text{dom}(wt)$$

On the same assumption about the models, we would have to require that $\tau \in \text{dom}(w\tau)$ for any $w \in W$, from which it follows that a sufficient condition for ‘Now exists’ to be true at a world-time pair of a model is that the time of this pair be the present time of the model. Accordingly, we will assume that given any model $\langle W, T, <, \alpha, \tau, I \rangle$ of the sort introduced above (i.e. without domain function), the following condition must hold:

$$(+) \quad \text{For all } w \in W, w\tau \models \text{Now exists}$$

Given this assumption, FK not only validates ‘Now exists’, it moreover validates

$$(1) \quad \Box(\text{Now exists})$$

This is an unpalatable result. For, our logic should not be prejudiced against pertinent metaphysical views according to which it is counterfactually possible that time came to an end before today, and so possible that today was never going to exist – in which case, ‘ $\Diamond \Box \neg(\text{Now exists})$ ’ will be true, and hence so will be ‘ $\Diamond \neg(\text{Now exists})$ ’. Even if such metaphysical views are in the end mistaken, it would take metaphysical argument to show that they are. To simply build their rejection into one’s logic is to dodge a dialectical burden.

Suppose that, as a matter of fact, sometimes, time comes to an end. This is equivalent to the supposition that, sometimes, it is not the case that sometimes in the future, \top holds, which is in turn equivalent to the supposition that $\text{SG}\perp$ holds. It is easy to see that FK validates

$$(2a) \quad \text{SG}\perp \rightarrow \Box \text{SG}\perp$$

For, if $\text{SG}\perp$ holds at $\alpha\tau$, T has a last member, and then for any $t \in T$, t is either last or precedes a time that is. Given the clause for \Box , $\Box\text{SG}\perp$ follows. Accordingly, if sometimes time comes to an end, it necessarily sometimes does.

Suppose instead that, as a matter of fact, time never comes to an end. This is equivalent to the supposition that, always, sometimes in the future, \top holds, i.e. that AFT holds. It is easy to see that FK validates

(2b) $\text{AFT} \rightarrow \Box\text{AFT}$

For, if AFT holds at $\alpha\tau$, T has no last member, and then for any $t \in T$, t precedes some time. Given the clause for \Box , $\Box\text{AFT}$ follows. Accordingly, if time never ends, it necessarily never ends.

Either way, whether time sometimes ends or never, according to FK, neither is contingent. Replacing G by H in (2a), and replacing F by P in (2b), we can similarly establish that, either way, whether time has ever begun or never, according to FK, neither is contingent. Already from a purely methodological point of view, these are clearly unacceptable consequences: whichever structure time has, that it necessarily has this structure should not be ruled by logical fiat (Thomason 1984: 160n5).

In the light of these latter considerations, we might proceed to amend FK by allowing each world to have its own time-series. We shall label the resultant theory 'FK+'. According to the most straightforward implementation of the present suggestion, FK+ takes a model to be a tuple $\langle W, B, \alpha, \tau, I \rangle$, where W , α and I are specified as before, and B is a function which assigns to each w in W a time-series $B_w = \langle T_w, <_w \rangle$, with T_w a non-empty set (the times of w) and $<_w$ a binary relation on T_w (the temporal ordering of w), and where τ is in T_α .

It might be a natural tendency to think that further conditions ought to be imposed on the temporal orderings of worlds, e.g. that each $<_w$ is at the very least transitive. However, as long as we want to be neutral on the space of possible temporal orderings, we should resist this temptation. For instance, on one conception of circular time, the relevant relation of precedence is not even transitive (see Reynolds 1994). As long as we wish to engage with such a view, as we think we should, our logic must allow for its coherent formulation.

Let us call a world-time pair wt *real* iff $t \in T_w$. In FK+ the truth predicate is relativised to real world-time pairs, and the following semantic clauses are adopted:

$$\begin{aligned} wt \models H\phi & \text{ iff for all } t^* \in T_w \text{ such that } t^* <_w t, wt^* \models \phi \\ wt \models G\phi & \text{ iff for all } t^* \in T_w \text{ such that } t <_w t^*, wt^* \models \phi \\ wt \models \Box\phi & \text{ iff for all real } w^*t, w^*t \models \phi \end{aligned}$$

This is still in the spirit of the original Fine-Kaplan view, to the extent that \Box merely operates on the world of evaluation, and not on the time of evaluation.

On this modified view, (2a) and (2b) prove invalid. Thus, (2a) is invalid because in some model the actual time-series may end, while some other time-series does not; and *mutatis mutandis* for (2b) and the corresponding principles regarding the

past. So this is good news. However, FK+ still validates (1), because (+) remains compelling. As such, even this modification of FK remains unsatisfactory.

3. The Montagovian view

These findings may be taken to suggest that we had better opt for the Montagovian view (M, for short) and its underlying model theory. On one straightforward way of rendering M precise in model-theoretic terms, a model is once again a tuple $\langle W, T, <, \alpha, \tau, I \rangle$ whose elements are defined as in the original Fine-Kaplan view FK. Validity is defined as before, the clauses for H and G stay the way they were according to FK, but the clause for the necessity operator is replaced by

$$wt \models \Box \phi \text{ iff for all } w^* \text{ in } W \text{ and for all } t^* \text{ in } T, w^*t^* \models \phi$$

Accordingly, as advertised, on the Montagovian view, \Box operates on both the world and the time of evaluation. Let us enquire how M fares with respect to (1), (2a) and (2b).

Since, so far, nothing has been said about what it takes for ‘Now exists’ to be true at a given world-time pair, there is as yet no way to construct a counter-model invalidating (1). Indeed, given specific further assumptions about the existence of times, and of τ in particular, M can be seen to after all validate (1).

Thus consider the necessitation of permanentism, i.e. the necessitation of the thesis that always everything always exists. Implementing this view in the present semantic framework involves taking the following principle to hold for any world-time pair wt of any model:

$$\text{If } wt \models \text{Now exists, then for all } t^* \text{ in } T, wt^* \models \text{Now exists}$$

Given (+), it then follows that (1) is valid according to M.

It is to be expected that *a priori* arguments for theses like permanentism at once establish their truth and their necessity. However, there is no evident reason why misgivings about a claim of *de re* necessity like (1) should *eo ipso* extend to the necessitation of permanentism, which is a *de dicto* modal claim.

As far as (2a) and (2b) are concerned, the lack of progress is even more evident. For, M straightforwardly validates both these principles, since, as in the case of FK, all the worlds of each model share their time-series. The possibility that time extends infinitely into the future cannot be accommodated by M, unless time does in fact extend infinitely into the future; similarly, the possibility that time will end cannot be accommodated by M, unless time will in fact end.

Both types of problems can be solved at once, however, if we modify M along the very same lines along which we modified FK to arrive at FK+. Thus, according to the modified Montagovian view M+, a model is a tuple $\langle W, B, \alpha, \tau, I \rangle$ whose elements are defined as in FK+. Validity is defined as before, the clauses for H and G stay the way they were according to FK+, but the clause for the necessity operator is replaced by

$wt \models \Box\phi$ iff for all real w^*t^* , $w^*t^* \models \phi$

The reasons why (2a) and (2b) prove invalid on M_+ are the same reasons why they prove invalid on FK_+ . But unlike FK_+ , M_+ fails to validate (1); and unlike M , M_+ can moreover be taken to *invalidate* (1). It is indeed plausible that for any real wt , ‘Now exists’ at wt , only if τ belongs to the time-series of w , i.e. that

(++) For any real wt , $wt \models$ Now exists, only if $\tau \in T_w$

Given (++)), there are M_+ models that invalidate (1). Thus let $\langle W, B, \alpha, \tau, I \rangle$ be such that τ does not belong to T_w for some w in W . *Modulo* (++)), for any w of that kind and any t in T_w , we have: $wt \not\models$ Now exists, and so we have: $\alpha\tau \not\models \Box(\text{Now exists})$.

Once different time-series have been introduced, permanentism can be secured in the model theory by imposing the condition that for any real world-time pairs αt and αt^* , what exists at αt likewise exists at αt^* . Correspondingly, the necessitation of permanentism can be secured by imposing the stronger condition that for any real world-time pairs wt and wt^* , what exists at wt likewise exists at wt^* . It is easy to see that imposing this stronger condition in no way precludes models of the aforementioned kind that invalidate (++)), and hence (1).

Accordingly, it would seem that, on balance, M_+ is more acceptable than FK_+ . However, note that M_+ (like unmodified M) immediately validates

(3) $\Box\phi \rightarrow A\phi$

By contrast, it can easily be shown that FK and FK_+ invalidate (3).

Proof. Consider an atomic formula ψ and an FK model $\langle W, T, <, \alpha, \tau, I \rangle$ such that (i) for all $w \in W$, $w\tau \in I(\psi)$, and (ii) for some $t \in T$ distinct from τ , $\alpha t \notin I(\psi)$. We then have $\alpha\tau \models \Box\psi$ but $\alpha\tau \not\models A\psi$. Consequently, FK invalidates (3). But then so does FK_+ : the FK model in question can be seen as an FK_+ model where all the worlds are assigned the same temporal structure by function B . ■

Cian Dorr and Jeremy Goodman regard (3) as ‘obvious’ (Dorr and Goodman 2019: 1). They seek to defend (3) by showing that certain arguments against (3) fail. These arguments are arguments in favour of a principle incompatible with (3), *viz.* the principle according to which every truth is necessitated by a permanent truth (i.e. a proposition that is always true). We agree that this principle is very plausibly false. But this does nothing to show that we should accept (3).

Unlike these authors, we regard (3) as far from being compelling. At the very least, we insist that it should not be considered as part of one’s logic – for the reasons we detail below. Note that our misgivings about (3) do not rest on any conflation between metaphysical and so-called *historical necessity*, as the latter notion is commonly understood. Thus, for example, below we nowhere assume that to the extent that (3) fails, $P\psi \rightarrow \Box P\psi$ should be true for all atomic formulae

ψ , even if, as is typically held, for such ψ , $P\psi$ entails that ψ 's past truth is inevitable and so historically necessary (Prior 1967: 122-27; cf. also Thomason 1970: 275-76; Dorr and Goodman 2019: 2).

According to Aristotle, essential properties of a given thing are those of its properties it cannot lose without ceasing to exist, and cannot have acquired having existed before.² But these two theses are logically independent; and so one might consistently accept the first without accepting the second. Indeed, some authors suggest that this combination of ideas can be rendered plausible by looking at real-life cases (for the following, see Bird 2009: 498, 501).

Consider a caterpillar that turns into a butterfly. Arguably, once it has turned into a butterfly, it is essentially a butterfly, and so, necessarily, it will always in the future be, if anything, a butterfly. But it was not always the case that it would always in the future be, if anything, a butterfly. There was a past time t at which it was a caterpillar and at which, for some past time t' later than t , at t' , it still was a caterpillar and existed.³

In other words, according to the metaphysical view that emerges, we have

$$\begin{aligned} &\Box G(a \text{ exists} \rightarrow a \text{ is a butterfly}) \\ &\neg \Box G(a \text{ exists} \rightarrow a \text{ is a butterfly}) \end{aligned}$$

where ' a ' names the caterpillar turned butterfly. Consequently, (3) precludes such a view. But if this is not the kind of view we should engage with when exploring the interaction between modality and tense, then what is? We do not stand to gain anything from designing our logic in such a way as to preclude such a view from the very debate that the logic is meant to structure; and whatever its alethic status, the view itself seems perfectly coherent so as to merit an argument against it, should it be rejected in the end.

The metaphysical view just presented is not the only one in the ballpark that conflicts with (3). Thus, on certain conceptions of *haecceities*, such individual essences come to exist when the individuals do whose essences they are, and then cannot go out of existence ever after, surviving those individuals as soon as the latter cease to be; and yet, it was not necessary that those individuals would be going to exist, before they eventually did, and hence neither that so would their *haecceities*.⁴ On Brouwer's view of the creating subject, mathematical objects are created by the human mind and, once created, necessarily persist, where there is nothing that guarantees that such objects will be created, before they are in fact created (cf. Niekus 2010: 37-39). Similarly, on one debatable variant of

² In Aristotle's *Topics* (*Top.* IV 5, 125b37-39, Ross 1958), it says, literally: '[I]t is indeed impossible for the same thing still to persist, if it completely moves out of the species [*ek tou eidous*], as for instance it is impossible for the same animal sometimes to be a man, sometimes not'. The first half of the quote can be seen as an articulation of the first thesis identified above, while the second half makes clear that the second thesis is likewise intended. Thanks to Paolo Natali for the translation.

³ Alexander Bird (*ibid.*) would seem to suggest that this kind of view still is Aristotelian; however, see previous footnote.

⁴ The envisaged theory is a modally strengthened version of the type of theory of *haecceities* defended by Adams (1986) and Ingram (2016).

creationism about fictional characters, such characters are *abstracta* that come into being by a creative act, and from then on cannot cease to be, populating the realm of abstract things forever after; and yet, the creative act in question was not necessarily going to occur before it occurred, and so at some past time the fictional characters might never have come to exist in the first place.⁵

Lastly, consider the following scenario, which should not be discarded as impossible on purely logical grounds:

The universe is such that it cannot last longer than eight million years from now, while it is contingent whether it lasts even that long. Suppose that, as a matter of contingent fact, it indeed last until d , where d is the time eight million years from now (the time of doom). At d , the universe necessarily ends, not having ended any earlier.

According to this scenario, since at d , the universe necessarily ends, at d , time necessarily ends. So, at d , $\Box G \perp$ is true. But since time did not end before d , at d , $AG \perp$ is false.

What the foregoing metaphysical reflections show is that the quest for metaphysical neutrality, as characterised in section 1, provides us with a reason for abandoning both $M+$ and $FK+$. Instead, we should search for a more general framework that is neutral enough neither to validate any of (1), (3) and (2a) and (2b), nor other rigidity principles concerning time's structure similar to the latter. We should indeed search for a framework that does justice to the fact that it is not incoherent to hold at once that (1) is false, that (3) has false instances – in particular of the kind we discussed above – and that there are various metaphysically possible temporal orderings which diverge both in structure and content.

4. A better view

We suggest modifying the Montagovian theory even further by expanding its model structure so as to include an accessibility relation between real world-time pairs (i.e. between those w such that $t \in T_w$). We call this theory 'NM' (for 'neutral Montagovian view'). According to NM, a model is a tuple $\langle W, B, R, \alpha, \tau, I \rangle$, where W, B, α, τ and I are specified as before, and where R is a binary reflexive relation between real world-time pairs. Validity is defined as before, the clauses for H and G stay the way they were according to $FK+$ and $M+$, while the clause for the necessity operator is replaced by

⁵ We do not pretend that this view has been defended anywhere in print. Extant versions of creationism about fictional characters rather hold that the continued existence of such *abstracta*, once created, depends on contingent factors: 'Once created, clearly a fictional character can go on existing without its author or her creative acts. Nonetheless, a fictional character can fall out of existence with the stories of a culture. [...] [C]haracters depend on the creative acts of their authors in order to come into existence, and depend on stories in order to remain in existence' (Thomasson 1996: 303). Even if this might ultimately be the most sensible thing for a creationist about fictional characters to say, what matters for our present purposes is that the two dependencies Thomasson mentions are themselves independent, and that a view that posits the first but not the second is not unintelligible (even if false).

$wt \models \Box\phi$ iff for all w^*t^* such that $wt R w^*t^*$, $w^*t^* \models \phi$

NM invalidates (2a) and (2b), and principles like them, for the same kind of reasons as did FK+ and M+: worlds may come with distinct time-series, and there is nothing in the characterisation of R that would preclude that world-time pairs access others whose world has a distinct time-series. But in contrast to FK+ and M+, there are models of NM that at once invalidate (1) and invalidate instances of (3).

Proof. We build a model $\langle W, B, R, \alpha, \tau, I \rangle$ as follows. W has two elements α and w , B assigns to α the non-positive integers $0, -1, -2, \dots$ together with their usual ordering, and to w an isomorphic structure that does not overlap the structure associated with α . We let τ be 0. We stipulate that $v\tau R v^*t^*$ iff either ($v = v^*$ and $t = t^*$) or ($v \neq v^*$ and t is the element corresponding to t^* according to the isomorphism). Finally, we let I be any arbitrary interpretation function. One can then establish the following three facts:

- (I) For all wt such that $\alpha\tau R wt$, t is the last time in B_w
- (II) There is a time in B_α that is not the last time in B_α
- (III) There is a wt such that both $\alpha\tau R wt$ and $\tau \notin T_w$

Given (I) we have

$$\alpha\tau \models \Box G\perp$$

and given (II) we have

$$\alpha\tau \not\models AG\perp$$

Hence, an instance of (3) is false in the model. Recall the earlier assumption (++) according to which for any real wt , ‘Now exists’ is true at wt , only if τ belongs to T_w . Given this assumption and (III), $\alpha\tau \not\models \Box(\text{Now exists})$, and so (1) is false in the model. ■

We are in principle free to impose further conditions on R besides reflexivity. That is to say, nothing stops us from demanding that R be an equivalence relation, in which event \Box behaves as it does in S5. The following two candidate conditions, which indeed make R an equivalence relation, are of particular interest:

- (4) For any real wt and w^*t^* , $wt R w^*t^*$ iff $t = t^*$
- (5) For any real wt and wt^* , $wt R wt^*$

Truth in all FK+ models coincides with truth in all NM models satisfying (4), and truth in all M+ models coincides with truth in all NM models satisfying (5).

Proof. To each FK+ model $\langle W, B, \alpha, \tau, I \rangle$ associate the NM model $\langle W, B, R, \alpha, \tau, I \rangle$ with R defined according to (4). It is then easy to show that for any formula φ , φ is true in $\langle W, B, \alpha, \tau, I \rangle$ iff φ is true in $\langle W, B, R, \alpha, \tau, I \rangle$. This shows that if a formula is true in all NM models satisfying (4), it is true in all FK+ models. For the converse, note that given any NM model $\langle W, B, R, \alpha, \tau, I \rangle$ satisfying (4) and any formula φ , φ is true in $\langle W, B, R, \alpha, \tau, I \rangle$ iff φ is true in the FK+ model $\langle W, B, \alpha, \tau, I \rangle$. This shows that if a formula is true in all FK+ models, it is true in all NM models satisfying (4). Thus, truth in all FK+ models coincides with truth in all NM models satisfying (4). The proof that truth in all M+ models coincides with truth in all NM models satisfying (5) proceeds in the same way. ■

This shows that FK+ and M+ are special cases of NM. (4) and (5) jointly imply

(6) For any real w and t , $t = t^*$

The condition that a structure $\langle W, B, R, \alpha, \tau \rangle$ satisfies (6) is equivalent to the condition that, at all world-time pairs of all models based on that structure, all instances of the schema $\varphi \rightarrow A\varphi$ are true.

Proof. Clearly, if $\langle W, B, R, \alpha, \tau \rangle$ satisfies (6), then all the instances of $\varphi \rightarrow A\varphi$ are true at all world-time pairs of $\langle W, B, R, \alpha, \tau, I \rangle$ for any interpretation function I . For the converse, consider a structure $\langle W, B, R, \alpha, \tau \rangle$ that does not satisfy (6). Then for some w in W , there are t and t^* in T_w with $t \neq t^*$. Let ψ be an atom and define I in such a way that ψ is true at w but not at w . Then $\psi \rightarrow A\psi$ is false at w . ■

The assumption that, at all world-time pairs of all models based on that structure, all instances of the schema $\varphi \rightarrow A\varphi$ are true, is at odds with *temporalism*, i.e. the view that some propositions are true without being always true. As Dorr and Goodman note, rejection of temporalism trivialises (3) and deprives much of the discussion of its interest (Dorr and Goodman 2019: 1).

Accordingly, if we assume temporalism, and find neither (1) nor (3) compelling, we should opt for the neutral account, i.e. NM without any of the two additional constraints (4) and (5). We should anyway opt for NM without those constraints on R , in order to stay as metaphysically neutral as possible.

In their 2019 paper, and in contrast to a previous version that had been circulated as forthcoming for quite some time, Dorr and Goodman offer a model theory that, like NM, is neutral on whether (1), (2a), (2b) and (3) hold (Dorr and Goodman 2019: 6 ff., 30n44). However, they do not explicitly advertise that neutral theory as the strongest we should adopt for the purposes of structuring metaphysical debates in this area; and there are also important differences between that theory and NM.

The authors define model structures – *DG structures*, for short – as tuples $\langle J, <, \approx, \iota \rangle$, where J is a non-empty set of points (intuitively, real world-time pairs), $<$ and \approx are binary relations between points, and ι is a designated point (intuitively, the actual-world-present-time pair). $<$ is understood to be a temporal earlier-later

relation and \approx to be a modal accessibility relation, so that the following semantic clauses are adopted given any structure $\langle J, <, \approx, \iota \rangle$ supplemented by an interpretation function:

$$\begin{aligned} j \models \text{H}\varphi & \text{ iff for all } j^* \text{ in } J \text{ such that } j^* < j, j^* \models \varphi \\ j \models \text{G}\varphi & \text{ iff for all } j^* \text{ in } J \text{ such that } j < j^*, j^* \models \varphi \\ j \models \Box\varphi & \text{ iff for all } j^* \text{ in } J \text{ such that } j \approx j^*, j^* \models \varphi \end{aligned}$$

Dorr and Goodman assume \approx to be symmetric and transitive – instead of merely requiring reflexivity as NM does of R . They also assume $<$ to be both asymmetric and transitive – instead of leaving the formal properties of temporal orderings unspecified as NM does. But none of this is compulsory. Just like NM structures, DG structures do allow us to make sense of different possible worlds having different temporal structures.

Indeed it is possible to associate an NM structure with each DG structure in a natural way as follows. The intuitive meaning of ‘ $j < j^*$ ’ is that (i) the world of j is the world of j^* , and (ii) the time of j precedes the time of j^* . This suggests a way of constructing worlds and their temporal orderings out of any DG structure. Let us say that points j and j^* are *world-mates* iff either $j = j^*$ or j and j^* are the extreme elements of a $<$ -chain, where a $<$ -chain is a sequence $j_1 \dots j_n$, with $n \geq 2$, where any two adjacent members are connected by $<$. The relation so defined is an equivalence relation. Given any $j \in J$, let *the world of j* – $[j]$ – be the class of all points that are world-mates of j , and let W_j be the set of all $[j]$ for $j \in J$. We now define a temporal ordering $B_w = \langle T_w, <_w \rangle$ for each $w \in W_j$ by laying down: $T_w =_{\text{df}} w$, and: $j <_w j^*$ iff_{df} both $j \in w$ and $j < j^*$. Finally, we define the binary relation R between elements of $W_j \times J$ as follows: $wj R w^*j^*$ iff_{df} $j \in w, j^* \in w^*$ and $j \approx j^*$. The tuple $\langle W_j, B, R, [\iota], \iota \rangle$ is an NM structure that naturally corresponds to the base DG structure $\langle J, <, \approx, \iota \rangle$.

However, this construction has two important limitations. First, in the constructed structures, any two distinct times of any world are connected via a finite chain of times related by the earlier-later relation. Consequently, the idea of a world consisting of a multiverse of disconnected time-lines is ruled out from the start. It is unclear what we stand to gain methodologically from a logical theory that treats such worlds as logically impossible, if there is a readily available alternative, i.e. NM, that does not. Secondly and more importantly, the times of a constructed structure are world-bound. Yet it should at least be deemed logically possible that numerically the same time-line constitutes the temporal ordering of distinct worlds. Unlike the logic offered by Dorr and Goodman, NM allows for this possibility and is therefore to be preferred.⁶

Conclusion

⁶ We should add that we conceived of NM independently; our paper already contained NM’s model theory when it was first drafted in October 2017, while we had access to the contents of a draft of the final version of Dorr and Goodman’s paper no earlier than March 2018.

To clarify the logical relations between modality and tense, a multi-modal logic is needed. As a tool for framing pertinent metaphysical debates about their interaction, such a logic should not, from the onset, discard metaphysical theories that are party to those very debates – that is, it should not already build in contentious metaphysical assumptions that contradict those theories and thus render a sensible discussion of their relative merits and shortcomings impossible, while, at the same time, skirting the responsibility to provide arguments for their dismissal. We have argued that two logical paradigms for thinking about modality and tense violate this methodological constraint. That the actual structure of time is necessary is just as contentious a claim, deserving of critical and open-minded discussion, as that some time necessarily exists, or that what is necessary at one time is true at any other. And yet, the extant paradigms logically validate one or the other of these claims. This diagnosis leads to the call for a more neutral logic that not only allows for the contingency of time’s structure and composition, but likewise accommodates the thought that something might become necessary that was not true before. We have devised a logic that meets these desiderata. We conclude that we should use this logic rather than any of its predecessors when we reason in a language with both modal and temporal operators.

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