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ABSTRACT

In the present communication, we describe how to produce long light distributions in the focal area of a high numerical aperture optical system using a custom modulation function with spiral charge. This analysis expands our previous developments in the field. We analyze the effect of this new element on the behavior of light along the optical axis.

Keywords: Highly focused fields, Optical needles, Angular Momentum of Light

1. INTRODUCTION

Recently, multiple publications on shaping subwavelength needles with long focal length using polarized light have been published.\textsuperscript{1–5} In a previous publication,\textsuperscript{6} we introduced a set of modulation functions able to produce tunable-length vector light needles in the focal domain of a high numerical aperture lenses. These distributions display interesting mathematical properties such as radial symmetry and large derivative values. Moreover, the modulation function has a zero value jump at the entrance pupil of the focusing system. Taking into account the region of the propagation axis that encloses the 75\% of the on-axis irradiance, we derived a formula that provides a fair a-priori estimation of the length of the needle. Our approach was experimentally tested and verified in the laboratory. The modulation distribution was optically implemented using spatial light modulators and digital holography techniques.\textsuperscript{7, 8} A variety of optical needles of different lengths were produced. In particular, we reported optical needles with lengths larger than 80$\lambda$.\textsuperscript{9, 10} In the present communication, we describe how to produce azimuthally polarized long light distributions in the focal area of a high numerical aperture optical system by using our proposed modulation function with spiral charge.

2. THEORETICAL APPROACH

The Richards-Wolf formula describes the propagation and behavior of the electric field at the focal area of a high NA (numerical aperture) objective lens\textsuperscript{11}

\begin{equation}
E(r, \phi, z) \propto \int_0^{\theta_0} \int_0^{2\pi} E_0(\theta, \varphi) e^{ikr \sin \theta \cos(\phi-\varphi)} e^{-ikz \cos \theta} \sin \theta \, d\theta \, d\varphi
\end{equation}

where $k = 2\pi/\lambda$ is the wave number and $(r, \phi, z)$ are the coordinates at the focal area. The numerical aperture (NA) is related to the maximum $\theta$ angle, namely $\theta_0$: NA=$\sin \theta_0$. The plane waves angular spectrum $E_0$ reads

\begin{equation}
E_0 = \sqrt{\cos \theta} \left( (E_s \cdot e_1) e_1 + (E_s \cdot e'_2) e_2 \right).
\end{equation}

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\( \mathbf{E}_0 \) is described in terms of unit vectors \( \mathbf{e}_1 = (-\sin \varphi, \cos \varphi, 0) \), \( \mathbf{e}_2 = (\cos \theta \cos \varphi, \cos \theta \sin \varphi, \sin \theta) \), \( \mathbf{e}'_2 = (\cos \varphi, \sin \varphi, 0) \) and the input field \( \mathbf{E}_s \). An sketch of the optical system with information of the coordinates and variables used can be found elsewhere.\(^{12,13}\)

The objective of this communication is to describe the behavior of light needles in the focal area of a high NA system using an azimuthally polarized input field \( \mathbf{E}_s \) with spiral charge:

\[
\mathbf{E}_s(\theta, \varphi) \propto \left( \frac{2}{f_0 \sin \theta_0} \right)^m \sin^m \theta \exp \left( -\frac{\sin^2 \theta}{f_0^2 \sin^2 \theta_0} \right) e^{im\varphi} h(\theta) \mathbf{p}(\varphi),
\]

where \( f_0 \) is the filling factor, \( m \) is the topological charge, and \( \mathbf{p}(\varphi) = (-\sin \varphi, \cos \varphi, 0) \) is the polarization vector. In order to produce a long focused field, we use the following modulation function \( h(\theta) \):\(^6\)

\[
h(\theta) = N \text{sinc} \left( \frac{2\pi N \cos \theta - (1 + \cos \theta_0) / 2}{1 - \cos \theta_0} \right) \sin \theta;
\]

\( \text{sinc}(x) = \sin(x)/x \) is the unnormalized cardinal sine function. Interestingly, the length of the needle is determined by tuning the value of parameter \( N \). Other interesting mathematical properties associated to this modulating function are discussed in.\(^6\) A detailed description on how holographic needles are degraded as a consequence of the use of actual optical equipment can be found in.\(^9\) Finally, it is worth to point out that the proposed modulation function \( h(\theta) \) is related with the proposed in\(^{14}\) intended to be used in paraxial conditions.

3. RESULTS

Using function \( h(\theta) \) described in Eq. (4), we calculated several needles for \( N = 10 \). Cases \( m = 0, m = 1 \) and \( m = 2 \) are depicted in Figs 1, 2, and 3 respectively. In the three cases considered, we show the following information:

(a) Light distributions \( I(r, z) \). Since \( |\mathbf{E}(r, \phi z)|^2 \) presents circular symmetry with respect to the propagation axis \( z \), \( I(r, z) = |\mathbf{E}(r, \phi = 0, z)|^2 \). Note that the selected value \( \phi \) is arbitrary.

(b) Plot \( I(r, z = 0) \). As expected, for \( m = 0 \) and \( m = 2 \) pipe-like distributions are obtained. Nevertheless, normal needles are generated for \( m = 1 \).

(c) The transverse irradiance at plane \( z = 0 \) (normal to the direction of propagation.)

The length of the generated light distributions (both pipes and needle) are around \( 80\lambda \) long. Since optical needles generated with input beams with topological charge \( m \neq 1 \) produce pipe-like light distributions, we analyzed the dependence of the pipe inner radius as a function of the topological charge \( m \) (see Fig. 4). Interestingly, this curve displays a linear behavior.

4. CONCLUDING REMARKS

In the present communication we discussed the behavior of a modulation function able to produce tunable long light needles in the focal area of a high NA objective lens system. In particular, we analyzed how the needle is modified if the modulation function includes spiral charge. Moreover, we showed how the radius of the pipe inner radius depends on the topological charge \( m \).

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Figure 1. Azimuthal polarization with topological charge $m = 0$: (a) Light distribution in the plane $rz$. Note that the scale of the horizontal and vertical axes is not the same; (b) Irradiance $I(r,z=0)$; (c) Irradiance $I(x, y, 0)$.

Figure 2. Azimuthal polarization with topological charge $m = 1$: (a) Light distribution in the plane $rz$. Note that the scale of the horizontal and vertical axes is not the same; (b) Irradiance $I(r,z=0)$; (c) Irradiance $I(x, y, 0)$.

Figure 3. Azimuthal polarization with topological charge $m = 2$: (a) Light distribution in the plane $rz$. Note that the scale of the horizontal and vertical axes is not the same; (b) Irradiance $I(r,z=0)$; (c) Irradiance $I(x, y, 0)$.

Figure 4. Light-pipe inner radius as a function of the topological charge $m$.

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