How farmers shape cultural landscapes. Dealing with information in farm systems (Vallès County, Catalonia, 1860)

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17 Highlights

- New agro-ecosystem energy graph including Farmland, Livestock and Farming
 Community funds is shown.
- The new *information-as-structure* indicator in combination with land uses optimization
 scenarios describes the energy profile of the Vallès County for 1860.
- The information indicator is both useful for understanding past agricultural landscapes
 and as a tool to compare different trends in agrarian systems management.

24 Abstract

25 In this paper we propose an approach to understand how different farmer's goals can contribute 26 to structure cultural landscapes and how the information-as-structure held in energy flows within 27 farm systems can be measured. We start from a historical case study located in a Mediterranean landscape in the Vallès County (Catalonia, 1860) and apply an optimization model by using a 28 29 socio-metabolic approach that responds to three different strategies at farm gate: maximizing 30 population, minimizing labour, and maximizing income. The modelled farm pattern of energy flows, the information indicator and the landscape structure that would be obtained under each 31 32 optimization strategy are then compared with actual historical data. The results obtained confirm 33 that it is the farmers' know-how and culture what allows to manage the energy distribution into 34 the farm system in order to maintain a sustainable management of the territory. We take lessons in terms of socioecological transition analysis, and to offer novel insights on how information-as-35 36 structure driven by farmers' intentionality, knowledge and cultural practices plays a key role in 37 structuring cultural landscapes.

Keywords: Information, Social Metabolism, Farm System Modelling, Mixed farming, LinearOptimization, Cultural Landscapes

40 **1. Introduction**

Farm systems can be seen as the historically changing outcome of the interplay between socio-41 42 metabolic flows (Haberl, 2001), land-use patterns set up by farmers, and their ecological functioning (Wrbka et al. 2004). Despite the recent work carried out on energy analysis of farm 43 systems from a circular multi-EROI approach (Tello et al., 2016; Gingrich et al., 2017) the role 44 played by different farmers' strategies, as one of the main driving forces of contemporary land 45 46 use change, is not yet well-understood (Peterseil et al. 2004). This requires specifying and 47 measuring the pattern of energy flows in a way capable to bring to light the information held in farm systems that contribute to shape cultural landscapes. 48

49 We conceive farm systems as ecosystems modified by human activity in order to get biomass 50 useful for societies under certain goals (Georgescu-Roegen, 1971). This conception of agro-51 ecosystems leads us to account the socio-metabolic pattern of flows set among the different funds 52 regarding the human-nature relations (Marull et al., 2016). By funds we refer to the durable 53 components of agroecosystems that can provide useful flows as long as they are reproduced over time in a sustainable manner. However, this sustainable reproduction can be achieved by different 54 55 fund-flow configurations of the agro-ecosystems according to the information and purposes 56 driven by farmers. Following Passet (1996), there are two types of information relevant for the 57 agro-ecosystem functioning: as-message and as-structure. The pattern of energy flows of an agro-58 ecosystem can be used to account for both kinds of information. The information-as-message 59 expresses the relation among different energy flows taking place in the agricultural landscapes, 60 and can be useful for understanding landscape ecological processes (Marull et al., 2019a). The 61 information-as-structure is linked to the purposely driven fund-flow relations regarding how 62 these flows allow or not for the maintenance of the auto-reproducible funds of the agro-63 ecosystems.

Linear optimization models are suitable tools for studying farming systems under an objective 64 65 purpose or goal (Groot et al., 2012; Rodrigues et al., 2013; Kennedy et al., 2016). We adopt the Sustainable Agro-ecological Farm Reproduction Analysis (SAFRA) to measure how farmers 66 67 organize the energy flows in family farm systems according to different managing strategies 68 (Padró et al., 2019). This methodology carries out optimization analyses of land uses and energy flows by means of a linear programming tool. In this way, SAFRA determines the optimal 69 70 combination of land uses, and the energy flows associated to them (i.e., a flow-fund optimality), 71 which can be sustainable within the farm system boundaries. The current manuscript combines 72 this methodology with information theory, opening a way to measure information-as-structure 73 held in farm systems. Hence, the combined methodology allows us to capture this particular type 74 of information with the aim of approaching which would be the socioecological structure of an 75 agro-ecosystem to reach an objective.

76 By doing so, the indicator of *information-as-structure* proposed in this paper assesses the 77 optimality degree of energy flows distribution at domestic farming unit level in order to maintain 78 the agricultural funds over time. This new indicator measures the information farmers use to 79 distribute the flows of energy carriers in the farm system according to a defined purpose, while

ensuring the sustainability of a farm unit¹. The pattern adopted by these set of flows means losing 80 81 degrees of freedom in a subtle human-nature far-from-thermodynamic equilibrium system, driven by organized information that allows transferring energy while maintaining their complexity over 82 time (Ulanowicz 2003). Sustainability in family farm systems is achieved, then, by keeping the 83 84 complexity of the socio-metabolic cycles, so that internal information increases while entropy 85 decreases. This strategy relies on land use heterogeneity, a long-lasting characteristic of mixed 86 farming that has shaped different bio-cultural landscapes in many parts of the world (Wrbka et 87 al., 2003; Marull et al., 2019b).

Therefore, maximum *information-as-structure* is derived from the flow-fund pattern resulting from SAFRA optimal strategies. We define and account three strategies that farmers might pursue: maximizing population density, minimizing labour, or maximizing income. The strategies of these farmers are expressed with the information indicator that we present, which means that when applied to empirical data it reaches its maximum value when the observed energy flows coincide with the optimal pattern found through the optimization procedure.

94 The evidence obtained by comparing the results of our optimization model with empirical data of 95 current farm systems aims at opening and framing a deliberation among stakeholders about how 96 different optimization goals would lead to different cultural landscapes. Given that we are using 97 a historical example as a first test, the contrast between the empirical data obtained from a past 98 organic farm system and the counterfactual results generated by the model allows us to better 99 understand how farmers had actually oriented their labour and knowledge when they made a 100 choice between several possible options. The whole procedure reveals how agro-ecological 101 landscapes, and the energy flow patterns that imprint them in the territory, might have been shaped like by adopting specific optimization strategies. 102

We start Section 2 with the presentation of the historical case study in a Mediterranean landscape of north-eastern Spain and introduction of the method used to define the indicator of *informationas-structure* and the optimization model for the counterfactual analysis. Then, in Section 3 we show the results of the optimization model. Finally, the results are discussed in Section 4. Section 5 presents the conclusions and further research possibilities opened.

108 2. Materials and methods

In this section we: i) present in 2.1 the case study -an agro-ecosystem of the Vallés County, Spain, circa 1860; ii) introduce the methodological improvements to the representation via graph of an agro-ecosystem's energy flow (section 2.2) and formulate the indicator of *information-as-structure* (section 2.3). iii) In this way, we explain in 2.4 how the energy profile of a farm ecosystem can be optimized to pursue different strategies.

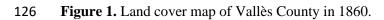
114 2.1 Case study

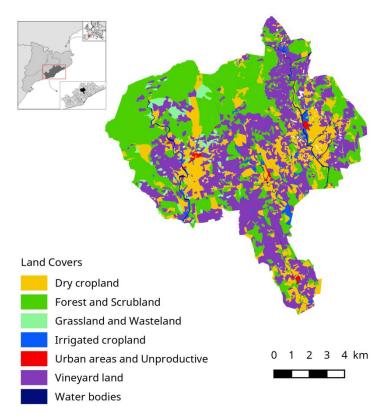
In order to check the usefulness of this new farm system graph and derived indicators, we applied the SAFRA model to a case study located in the Vallès County (Catalonia, Spain), see Figure 1. For long, it has been a test bench for our research on social metabolism, which allows us to account its energy and material flows in mid-19th century (Cussó et al., 2006; Marull et al., 2010;

¹ A farm unit includes as funds the domestic unit, the livestock and the farm surface, The representative domestic unit of five people (the average family type in the area of study c.1860) would comprise two children between 0-5 and 5-10 years old, a woman and a man between 18 and 60 years old, and an adult older than 60.

Olarieta et al. 2006; Rodriguez Valle, 2003; Tello et al., 2004, 2008). The time point analysed was long before the Green Revolution, which allows considering organic reproducibility of the agro-ecosystem funds with any non-renewable inputs, or only very few. The case was experiencing a widespread winegrowing specialization c.1860, but maintaining a significant level of self-subsistence through poly-cultural farm management and a complex landscape mosaic (Garrabou et al., 2007; Planas, 2015).

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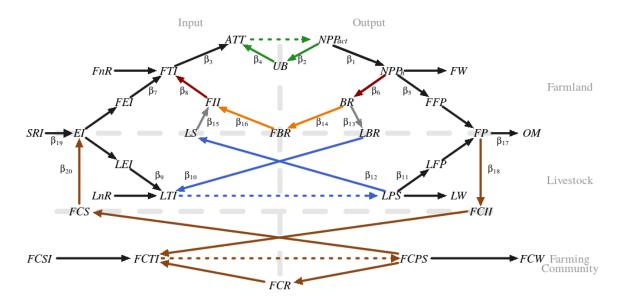
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128 Source: Our own from historical cadastral maps.

129 2.2 The farm system energy graph

130 A graph is a mathematical model that can be used to study several kinds of systems and processes. 131 In order to represent the set of socio-metabolic relations underlying a cultural landscape, we treat the pattern of energy flows in a farm system as a graph where the energy carriers are represented 132 133 as nodes (Figure 2), while the associated outgoing arrows account for the decisions that farmers 134 take with respect to incoming energy flows: they can either choose to make them go inflowing 135 within each (sub)system or drive these energy flows out of these (sub)systems, in different 136 proportions. The whole graph represents the set of farm system's processes that occur when any 137 energy carrier splits into two or when two energy flows are joined into one. Therefore, each 138 process is composed by three nodes and two arrows, except for those cases where waste or non-139 renewable inputs are present. The graph applied to agro-ecosystems flows, originally introduced 140 in Marull et al. (2016) and composed of three subsystems, is improved here by introducing a 141 fourth loop. So that, the farming community is considered because its maintenance is a relevant 142 characteristic for the system reproducibility (Figure 2).

144



145 Actual Net Primary Production (NPP_{act}); Unharvested Biomass (UB); Harvested Net Primary Production (NPP_h); 146 Biomass Reused (BR); Farmland Biomass Reused (FBR); Livestock Biomass Reused (LBR); Farmland Waste (FW); 147 Farmland Final Produce (FFP); External Input (EI); Farmland External Input (FEI); Livestock External Input (LEI); 148 Livestock Total Input (LTI); Livestock Produce and Services (LPS); Livestock Final Produce (LFP); Livestock Services 149 (LS); Livestock Waste (LW); Final Produce (FP); Agro-ecosystem Total Turnover (ATT); Farmland Total Input (FTI); 150 Farmland Internal Input (FII); Output Marketed (OM); Farm Community Internal Input (FCII); Farm Community 151 Produce and Services (FCPS); Societal Renewable Inputs (SRI); Farm non-Renewable Inputs (FnR); Livestock non-152 Renewable Inputs (LnR); Farming Community Societal Inputs (FCSI); Farming Community Total Inputs (FCTI); 153 Farming Community Services (FCS); Farming Community Reproduction (FCR); Farming Community Waste (FCW). 154 β_i 's are the incoming-outgoing flow proportions (see Section 3).

The graph represents how the farmers' activity yearly distributes all the energy flows moving into agro-ecosystems in the form of biomass and work, and it is used to analyse how each subsystem behaves in relation to the system as a whole. Farmers organize farm systems with the information embedded in the labour they carry out, both performing work and conducting livestock, as well as deciding crops and land distribution. So that, any decision farmers do on the landscape impact the energy flows.

We build this graph to represent the energy flows related to the maintenance of the three abovementioned funds which are explicitly mentioned on the right side underlying the three piled sections in Figure 2: the farmland, the livestock and the farming community that interact within the boundaries of the farm system considered. In doing so we adopt a family farm system's reproducibility standpoint, considering that the farming community maintenance is a relevant characteristic for the system reproducibility.

167 The farm system graph we propose is made of four loops (Figure 2). In the first loop we can 168 differentiate the 'unharvested subsystem' within a farm system. This is defined by three variables: 169 the actual Net Primary Production (NPP_{act}) , the Unharvested Biomass (UB) and the Agro-170 ecosystem Total Turnover (ATT). This subsystem can work as an independent system, as it 171 happens in ecosystems i.e., with minimum (or none) human intervention.

172 In the second loop, we identify the labour done to maintain soil fertility and to provide good 173 cropping. This is composed by the harvested Net Primary Production (NPP_h) that splits into the 174 Farmland Final Produce (*FFP*) and the Biomass Reused (*BR*). In turn, this *BR* splits into Farmland BR, FBR, which together with the Farmland External Input (FEI) joins into the Farmland Total
Input (FTI). This loop, together with the first loop originates the Farmland subsystem (Figure 2).

The third loop belongs to the 'livestock subsystem', addressed to feed the domesticated animals.
It is composed by Livestock External Input (*LEI*) and Livestock Biomass Reused (*LBR*), which

sum into the Livestock Total Input (*LTI*). The Livestock Produce and Services (*LPS*) is obtained

after the energy spent in animal bioconversion, then *LPS* is split into Livestock Services (*LS*) such

181 as draught power and manure, and Livestock Final Produce (*LFP*). On the other hand, the part of

182 BR that remains in the farmland subsystem is called Farmland Biomass Reused (FBR), which

183 together with LS forms the Farmland Internal Inputs (FII). Furthermore, we consider External

- 184 Inputs (*EI*) as the sum of *FEI* and *LEI*.
- 185 This is the graph presented Marull et al. (2016). Now we introduce the novel elements of the fourth loop, the 'farming community subsystem' whose relevance as an agroecosystem fund is 186 justified in Padró et al., (2018 and 2019). It starts from a decomposition of the final produce (FP) 187 188 into a part that flows to the market (output marketed, OM), and another which is recycled back 189 into the farm system because it is required for the maintenance of the farming community (internal 190 input, FCII), conceptually expressed in terms of food, fuel, fibre and timber. Of course, the 191 maintenance of the farming community fund might require also external inputs which would come 192 from outside their farms, which is the farming community societal inflow $(FCSI)^2$.
- 193 While during the traditional organic agricultural metabolism the external requirements for the maintenance of the farming community were minimal (Padró et al., 2017), this fraction has been 194 195 largely increased throughout industrialization of agriculture. In the same vein, there will be a part 196 of this energy output that after the dissipative process flows back into the agrarian funds, as 197 farming community services (FCS). This is the reproductive part, that is labour, humanure and 198 farmers' domestic residues which are already considered in other works (Tello et al, 2016). When 199 a fraction of them is not recycled back into the farm system, and it becomes a form of waste, it is 200 considered Farming Community Waste (FCW). The other way round, FCS contributes to EI as 201 farm system internal input, together with the societal renewable inputs (SRI). Last but not least, there is a part of the total labour done by the farming community that is reinvested within it. It is 202 203 considered as the reproductive fraction (FCR) that includes all farmers' activities that are not 204 directly required for the land and livestock productivity but that, nonetheless, constitute the 205 fundamental conditions for the reproduction of the farm community fund-i.e., physiological 206 overhead, household chores and care activities (Marco et al., under review).

In order to add the Farming Community Subsystem to the graph, we take the two more 'external' energy flows of the system (*FP* and *EI*) and we divide each one following the idea of inward/outward energy movements. This idea can be extended to other levels. For example, taking *OM* and *SRI* and considering that part of the society that lives in the farm system borders and do not contribute to its maintenance (artisans, traders, etc.). However, such considerations are beyond the aim of this work.

 $^{^2}$ We consider here the whole inputs without discerning about its renewable or non-renewable character. This is because we focus our analysis on the functioning of the agricultural metabolism, and most part of the impacts derived from the use of non-renewable inputs by the farming community affects non-agricultural areas.

214 2.2.1 Introducing the Farming Community subsystem as a fund

215 When talking about information, we consider that it is important to include farmers in the analysis 216 because it sets a difference that leads to a relevant change in the way we account for complexity 217 in the system managed by them. There is no farm system without farmers, and their intrinsic 218 elements and characteristics differing from an ecosystem can only be maintained with the spatial-219 explicit allocation of flows that farming divert through labour. This, in turn, entails the recognition that the farming population fund needs to be maintained as well with a set of relevant energy 220 221 flows addressed to satisfy the necessary conditions for their production and reproduction. 222 Accordingly, an important step forward of this socio-metabolic analysis is to consider as an agro-223 ecosystem fund the Farming Community that hold the farm system. So, we have added this fourth loop to the original formulation (Tello et al., 2016; Marull et al., 2016). Notwithstanding, we will 224 225 only account for the flows that emerge from the farm system towards the farming community, 226 which are needed for the maintenance and reproduction of it. Therefore, we do not consider the other flows involved in the farming community, shown in Figure 2 as FCSI, FCTI, FCR, FCPS, 227 228 and FCW, as part of the agrarian metabolism.

In the same vein, while from a farmer's standpoint the farming community services (e.g. labour, humanure or domestic residues) can be considered as external inputs (as in Tello et al., 2016), from an agro-ecosystem perspective they are (ontologically) internal as long as they are associated to the territory under analysis. They cannot be mixed with energy flows that come from outside the borders of the family farm system, e.g. imported feed, replacement animals, fossil fuels or

234 machinery, that are provided by agents out of the farm system.

Of course, farmers do more than producing the energy flows associated to their maintenance. However, here we are just studying the internal processes of the family farm system, so we only need to consider those parts of the societal energy flows which works on the farm system as modellers of an agro-ecological landscape.

239 2.2.2 The role of biomass reused and non-renewable fluxes

Following Guzmán and González de Molina (2016), we have split the biomass reused (*BR*) which loop inside the system by distinguishing those flows that go into the farmland soils from those devoted to feed and bed the livestock (i.e., autotrophic and heterotrophic loops). As well, we distinguish in *FII* flows coming from farmland, as *FBR*, as those from livestock, *LS*. These four arcs represent two autotrophic cycles and two heterotrophic ones.

- We have highlighted as well the totally different nature of non-biomass energy flows, such as those of non-renewable character. We have considered relevant to distinguish the nature of the External Inputs, both for Farmland and Livestock systems. This addition reinforces the possibility that some amount of the incoming energy flow would end up being transformed into farmland waste (*FW*) and livestock waste (*LW*)—i.e., in Odum's terms (Odum, 1993), resources out of
- 250 place and in excess of the agro-ecosystem's carrying capacity³.

³ That is, a flow that cannot be 'digested' by farm systems because exceeds the carrying capacity or is not correctly disposed by human activity to be useful for other funds. There are many ways to use biomass. Some are more beneficial than others. For example, the leftover of wine pruning can either be buried or burnt, with the former being more beneficial for soils. However, there are ways in which the opportunity costs of certain ways to use biomass are larger than the benefits they generate. In this case we also consider them as waste flows.

251 Similarly, we have to acknowledge that there are external flows of non-renewable nature which 252 are particularly relevant in current agriculture. These ought to be distinguished from other types of Societal Renewable Inputs (SRI) of organic nature, be they of endosomatic (humanure, 253 labour⁴), local (domestic residues) or external origin (seeds, feed, replacement animals, manure, 254 255 litter, etc.). As Guzmán and González de Molina (2015: 209) state, the fund elements of agro-256 ecosystems cannot be sustained by oil or coal or their fuel derivatives. The only thing that can be 257 done is to replace some ecosystem functions (e.g. fertilization, pest control or pollination) by 258 external inputs, which leads to an increasing dependence on anthropogenic inputs (Gliessmann, 259 1998).

In order to keep them separate from the renewable biomass flows, these non-renewable entries can be added to each subsystem of the graph, and then they will be accounted in the Total Inputs of such subsystem. As can be seen in Figure 2, in addition to the Societal Renewable Inputs there are other inflows from outside the system that directly enter to some cycles: in the farmland loop the entrances are the Farmland non-Renewable Inputs (*FnRI*), while for livestock maintenance there are some Livestock non-Renewable Inputs (*LnRI*).

266 **2.3. Information indicators**

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267 2.3.1 Coefficients of the graph

We observe in Figure 2 that each process bears, at least, two incident flows (arcs of the graph), either incoming or outgoing, and three nodes. One of them, labelled by a β with an odd index, points outward the system, and the other one, with an even index, points inward.

Each β is a coefficient representing the proportion of energy that enters or leave the node through that arc⁵. Specifically, we have the formulae:

273
$$\beta_1 = \frac{NPP_h}{NPP_{act}}, \beta_2 = \frac{UB}{NPP_{act}}, \beta_3 = \frac{FTI}{ATT}, \beta_4 = \frac{UB}{ATT}, \beta_5 = \frac{FFP}{NPP_h}, \beta_5 = \frac{FFP}{NP_h}, \beta_$$

274
$$\beta_6 = \frac{BR}{NPP_h}, \beta_7 = \frac{FEI}{FTI}, \beta_8 = \frac{FII}{FTI}, \beta_9 = \frac{LEI}{LTI}, \beta_{10} = \frac{LBR}{LTI},$$

275
$$\beta_{11} = \frac{LFP}{LPS}, \beta_{12} = \frac{LS}{LPS}, \beta_{13} = \frac{LBR}{BR}, \beta_{14} = \frac{FBR}{BR}, \beta_{15} = \frac{LS}{FII}$$

276
$$\beta_{16} = \frac{FBR}{FII}, \beta_{17} = \frac{OM}{FP}, \beta_{18} = \frac{FCII}{FP}, \beta_{19} = \frac{SRI}{EI}, \beta_{20} = \frac{FCS}{EI}$$

277 Note that there are four cases for which the sum of the pair of betas can be less than one. This is 278 due to the presence of waste of resources (*FW* and *LW*) and of non-Renewable inputs (*FnR* and 279 *LnR*). In these cases, we have

$$NPP_{h} = BR + FP - FW$$
 and $LPS = LS + LP + LW$.

⁴Labour is not an organic flow but mechanical. Yet it can also be considered a result of food's consumption, as it is accounted in social metabolism (Tello et al., 2015).

⁵ We do not include here the flows for the composition of the farming community total inputs (*FCTI*) neither of farming community products and services (*FCPS*) because, as stated in Section 2.1, we consider that this part is not accounting for the agrarian metabolism.

From the above equations, we see that waste is only involved in these split processes. The same is applied for the case of non-Renewable inputs.

283 2.3.2 The *information-as-message* indicator

In a similar but less complex graph, Marull et al. (2016) measured what is called the *informationas-message* (*I*) carried by the energy flows of the farm system as an average of the Shannon entropy index applied to pairs of betas, with some corrections when the pair's sum is less than one. More precisely, consider a pair of betas (β_{2i-1}, β_{2i}) and denote by

288
$$H(\beta_{2i-1},\beta_{2i}) = -\beta_{2i-1}\log_2(\beta_{2i-1}) - \beta_{2i}\log_2(\beta_{2i})$$

which is exactly the usual Shannon entropy index applied to the pairs if its sum is equal to one.Then the mean of these entropies over all pairs is

291
$$I = \left(\frac{1}{10} \sum_{i=1}^{10} H(\beta_{2i-1}, \beta_{2i})\right).$$

In the case that the sum of any pair of betas is strictly less than 1, due to the waste of resources, Marull et al. (2016) used a correction factor accounting for the information loss it caused. This kind of factors are bounded by one, and guaranty that the maximum value of *I* is never greater than one (for details, see Appendix A). Concretely, in Marull et al. (2016), a factor accounting for waste in resources in farmland and livestock is used:

297
$$\gamma_{FW} = \frac{BR + FFP}{NPP_h}, \quad \gamma_{LW} = \frac{LS + LFP}{LPS}, \text{ and } \gamma_w = \frac{\gamma_{FW} + \gamma_{LW}}{2}.$$

Following the same idea, we introduce another factor γ_{nR} accounting for the use of non-renewable energies (Marull et al. 2019a, 2019b):

300
$$\gamma_{FnR} = \frac{FEI + FII}{FTI}$$
, $\gamma_{LnR} = \frac{LEI + LBR}{LTI}$, and $\gamma_{nR} = \frac{(\gamma_{FnR} + \gamma_{LnR})}{2}$.

Note that both γ_w and γ_{nR} can be written in terms of betas (see Appendix). Then, the *information-as-message* indicator that we propose is

$$I = \left(\frac{1}{10} \sum_{i=1}^{10} H(\beta_{2i-1}, \beta_{2i})\right) \gamma_W \gamma_{nR}.$$
 (1)

303 2.3.3 The *information-as-structure* indicator

We want to reflect the knowledge and wisdom of farmers who, as agents, purposely orient the farm system energy flows. Hence, what we are trying to account for is the so-called *information-as-structure* (Passet, 1996). According to this idea, we are going to connect Information Theory (Shannon & Weaver, 1949) with an optimization model focused on the maintenance and reproducibility of the three main funds that can be measured through our methodology based on energy flows: soil chemical fertility, livestock and the farming
 community⁶.

311 It is well known from Information Theory that the Shannon index reaches its maximum value 312 when all coefficients are equal. Consequently, the maximum *information-as-message I* is 313 obtained for $\beta_i = 0.5$, for all *i*. However, from a farm system reproducibility standpoint, this is 314 not necessarily the best option. Distinct farm systems can establish different compositions of 315 funds, affecting the energy profiles (Marco et al., 2017). Therefore, we need an indicator sensitive 316 to the different relevance of each flow according to the farmers-driven information that structures 317 the fund-flow pattern of a family farm system.

- Specifically, if it is known that the optimal value for a pair of flows is achieved at $(\beta_{2i-1}, \beta_{2i})$, we want to modify *H* in such a way that the maximum is attained precisely there.
- We seek a transformation of the interval [0,1] in such a way that the maximum of the Shannon index is taken at a given arbitrary point $a \in (0,1)$ instead of a = 0.5 (Marull and Font, 2017). This can be achieved with a piecewise linear transformation that map [0,1] onto itself; consider, for each x in [0,1],

324
$$T_a(x) = \begin{cases} \frac{0.5}{a}x, & x < a\\ 0.5 + \frac{0.5}{(1-a)}(x-a), & x \ge a. \end{cases}$$

This function is represented in Figure 3a for a = 0.8. Geometrically, one piece of the unit interval is stretched, and the remaining piece is contracted. Now we define a modified entropy, H_a , for a given $a \in (0,1)$, applied to any pair (x, y) in (0,1) such that $x + y \le 1$:

$$H_a(x, y) = H(T_a(x), T_{1-a}(y))$$
(2)

In the particular case y = 1 - x, $H_a(x, 1 - x)$ is depicted in Figure 3b, for a = 0.8. The maximum value of the entropy is shifted from a = 0.5 to a = 0.8 while keeping the essential shape of the curve. The modified (non-symmetric) curve increases more slowly and decreases faster (for a > 0.5). It possesses the desirable property that H_a values for x < 0.5 are smaller than the corresponding $H_{0.5}$ values, reflecting the fact that they are farther away from the maximum, in the horizontal axis. Similarly, for points x > a the H_a value is higher than the $H_{0.5}$ values, since they are closer to the maximum.

Then we apply H_a defined in eq. (2) to an arbitrary pair of betas (β_{2i-1}, β_{2i}) and write

$$H_a(\beta_{2i-1}, \beta_{2i}) = -T_a(\beta_{2i-1})\log_2(T_a(\beta_{2i-1})) - T_{1-a}(\beta_{2i})\log_2(T_{1-a}(\beta_{2i}))$$
(3)

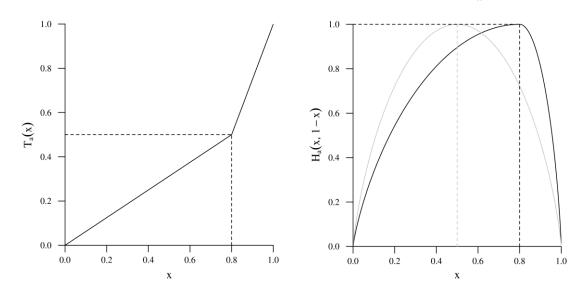
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⁶ The agrarian landscape functional structure is also an important fund: its maintenance depends on the energy reinvested, redistributed and 'imprinted' in the land-matrix by the farmers' knowledge and labor (Marull et al., 2018, 2019a).

338 **Figure 3.** Lineal change with a = 0.8:

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a) Transformation of x to T_a(x)
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b) Shannon index after the linear transformation T. In grey, the original Shannon index. $H_a(x, 1-x)$



Finally, we define the index I^* , departing on $A^* = (a_1, ..., a_{10})$, which we assume that $a_i \in (0,1)$ 339 340 are given for all i :

$$I^* = \left(\frac{1}{10} \sum_{i=1}^{10} H_{a_i}(\beta_{2i-1}, \beta_{2i})\right) \gamma_W^* \gamma_{nR}^*, \tag{4}$$

341 where

342
$$\gamma_W^* = \frac{1}{2} \Big(T_{a_5}(\beta_5) + T_{a_6}(\beta_6) + T_{a_{11}}(\beta_{11}) + T_{a_{12}}(\beta_{12}) \Big)$$

343 and

344
$$\gamma_{nR}^* = \frac{1}{2} \Big(T_{a_7}(\beta_7) + T_{a_8}(\beta_8) + T_{a_9}(\beta_9) + T_{a_{10}}(\beta_{10}) \Big).$$

Notice that I^* defined on eq. (3) applies to the betas in the graph and depends on a hypothetical 345 346 optimal distribution of energy flows A^* . We call I^* the information-as-structure index. Notice 347 that, taking $a_i = 0.5$, $\forall i$, we recover the former index I. Some other properties of T_a , H_a and I^* 348 are stated and proved in the Appendix.

349 Finding suitable values for a_i is, in fact, a big deal. The new formulation opens the way to count 350 with expert criteria based on a deep knowledge on farm systems and their patterns of energy flows. Yet, we can propose values for the energy flows that ensure the reproducibility of the farm 351 system funds while optimising some quantity of interest. This is explained in the next section. 352

353 2.4. Land use optimisation

354 Once defined the new indicator for the *information-as-structure* (I^*) , the next step is to identify 355 to which extent the energy flows of the farm system's graph are supposed to resemble an optimal

distribution. It can be assumed that the structure of the agrarian metabolism set up among the 356

different funds of the system is dependent on the site-specific social intentionality of their managers. From a farm unit standpoint, the intentionality comes from the family goals and priorities. From an aggregated societal perspective, this is in turn defined by the interests of the specific historical dominant class and can be altered from time to time by social struggles.

We seek the sustainable reproducibility of the family farm system. This does not exclude the possibility to search for optimal land uses in order to obtain a maximum economic benefit in a short period, but it requires ensuring that the main funds of the system are reproduced over time.

To this aim, we propose to model the family farm system conditions and its possible different goals through linear optimisation using the SAFRA methodology (Padró et al., 2019). This allows us to obtain the optimal land use pattern for each goal, as well as the energy and material flows devoted to maintaining the three funds. The details of this methodology are further explained in Padró et al. (2019).

369 The linear optimization problem has the form

370 Minimize $\sum_{i=1}^{n} a_i x_i$,

371 Subject to: $\sum_i b_{ij} x_i \le d_j$, j = 1, ..., m,

372 $\sum_{i} c_{ij} x_i \ge e_j, j = 1, \dots, t,$

373
$$x_i \ge 0$$
, for all i .

Where the decision variables x_i , i = 1, ..., n, are the surface area corresponding to each land use 374 375 *i*. For easier model construction, some variables x_i , with i > n, with a direct interpretation in 376 terms of products or by-products per unit of land use, may appear in the restrictions linked to the 377 variables surface) main (the land uses (the full code can be found at 378 https://github.com/cfontm/SAFRA).

379 The restrictions encode the reproducibility of the three most relevant funds: farming community, 380 soil fertility, and livestock. In order to ensure their reproducibility, one must consider the 381 investments required, as well as the maximum amount of services they can provide. Taking a 382 representative domestic farming unit as the minimum functional unit, we have to account for the 383 subsistence of the people who make up the community (providing enough food for a specific diet, 384 and sufficient fuel) as well as for the labour requirements throughout the year. Likewise, livestock maintenance requires enough products and by-products to feed the animals with a proper diet, 385 386 materials for stall bedding, and a sufficient supply of draft power and meat for the farming 387 community. Finally, a set of restrictions are needed to ensure the maintenance of soil fertility, 388 which entail a balance keeping a sustainable extraction and replenishment of nutrients, a properly 389 distribution of uses in respect to soil quality, and the ability to irrigate. All these calculations are 390 made taking into account the cultural rotations of the region for a given historical period. They 391 are always site specific.

Intentionality is defined by the coefficients a_i of the objective function. It is obvious that defining a specific aim (i.e. how farmers are supposed to use the land and any other natural resource) is a subjective decision. The prevailing social values will drive the farm system towards one direction or another. Thus, labour is nothing more, but also nothing less, than a farmers' allocation of the available set of material and energy flows in order to obtain a socially-constructed farm system according to a purpose.

398 **3. Results**

399 **3.1.** Actual and counterfactual land uses

Following Padró et al. (2019), we have studied three different optimisation profiles in which farm system funds can be reproduced (as a sustainable management). The three strategies modelled are: *i*) maximizing population density; *ii*) minimizing labour effort; and *iii*) maximizing sustainable winegrowing specialization while maintaining population density in order to increase market income. We will refer to them in the sequel as intensive strategy, extensive strategy and income strategy.

The result of these models can be seen in Tables 1 (land uses) and 2 (energy balances), where three different ways of optimizing the family farm system are presented according to the restrictions explained above.

			Land use (%)	
	Vallès 1860	Intensive strategy	Extensive strategy	Income strategy
Total surface [*]	12 ha	4.3 ha	6.1 ha	7.6 ha
Forest and Scrubland	36.4%	39.0%	43.7%	5.8%
Grassland and Wasteland	7.6%	0.0%	12.1%	8.4%
Dry cereal cropland	17.6%	54.8%	31.4%	17.3%
Irrigated cropland	2.6%	4.2%	3.3%	2.9%
Vineyard land	35.8%	2.1%	9.4%	65.6%
Shannon Index	0.8	0.7	0.8	0.6

Table 1. Land uses for the Vallès case study according to the three optimization strategies.

410 * Surface of historical case study, Vallès 1860; and for each strategy, the minimum surfaces required to ensure

412 Source: Our own from the sources given in the text.

413

⁴¹¹ reproducibility of the three funds considered.

Energy flows (MJ/ha)	Vallès 1860	Intensive strategy	Extensive strategy	Income strategy
FEI	534	1050	685	755
UB	21625	14717	15563	17451
FW	0	0	0	0
FnR	0	0	0	0
FBR	15033	13884	14134	27593
LBR	11364	15939	15558	13191
FFP	16410	20722	13766	8731
LEI	274	489	341	286
LW	0	0	0	0
LnR	0	0	0	0
LS	1968	1918	1250	1685
LFP	111	596	416	334
FCII	7785	13099	9106	6064
FCS	645	1539	1026	1041

Table 2. Energy flows for the Vallès case study according to three optimization strategies.

Variables: Unharvested Biomass (UB); Farmland Biomass Reused (FBR); Livestock Biomass Reused (LBR); Farmland
Waste (FW); Farmland non-Renewable Input (FnR); Farmland Final Produce (FFP); Farmland External Input (FEI);
Livestock External Input (LEI); Livestock Total Input (LTI); Livestock Produce and Services (LPS); Livestock Final
Produce (LFP); Livestock Services (LS); Livestock Waste (LW); Livestock non-Renewable Input (LnR); Farmland
Internal Input (FII); Farm Community Internal Input (FCII); Farming Community Services (FCS). Source: Our own
from the sources given in the text.

422 **3.2.** Information indicators behind the intentionality of these organic farm systems

The new values found of I^* are higher than those of the previous indicator *I*. The three different optimal distribution of energy flows range from a *I* value of 0.682 to the *I** scores between 0.916 and 0.944 for each purpose-oriented strategy (Table 3). That means that the values obtained by the *information-as-structure* (*I**) are much higher than those of the *information-as-message* (*I*). Indeed, the corresponding *I** values of each SAFRA optimization strategies allow assessing in which strategy the observed profile have values closer to the maximum value attained by the optimal flows' distributions.

430 Table 3. Information indicators (*information-as-message I* and *information-as-structure I*^{*})

accounted in the case study and under each optimization strategy.

	Vallès 1860
Ι	0.682
<i>I</i> [*] (intensive)	0.916
I* (extensive)	0.933
I* (income)	0.944

Source: Our own from the sources given in the text.

434 **3.3.** Values of biomass inflow⁷ for sustaining the farm system funds

Pairwise comparisons of I^* values in Table 3 show short differences between the optimized models and the historical case. Another approach to analyse how the funds would had been sustained in each strategy simulation is to focus only on biomass inflows within the four subsystems. To do so we compare the share of NPP_{act} flows (UB+FBR+LBR+FCII) that goes into each subsystem according to the prevailing strategy and in the actual historical case, and measure the subsystems' contribution to the total energy throughput by:

441
$$k_1 = \frac{UB}{UB + FBR + LBR + FCII}, k_2 = \frac{FBR}{UB + FBR + LBR + FCII}$$

442
$$k_3 = \frac{LBR}{UB + FBR + FBR + FCII}, k_4 = \frac{FCII}{UB + FBR + LBR + FCII}$$

These values indicate the share of biomass inflows going towards the 'unharvested' subsystem – which contributes to the fund that sets the material basis of farmland associated biodiversity (k_1) ; towards the 'farmland' subsystem –which refers to the fund of soil fertility (k_2) ; towards the 'livestock' subsystem–referring to the livestock fund (k_3) ; and towards the 'farming community' subsystem–referring to the farming population fund (k_4) . Results are presented in Table 4.

448 **Table 4.** Subsystems contribution to total energy throughput.

	Vallès 1860	Intensive strategy	Extensive strategy	Income strategy
k_1	0.387	0.255	0.286	0.271
<i>k</i> ₂	0.269	0.241	0.260	0.429
k_3	0.204	0.277	0.286	0.205
k_4	0.139	0.227	0.168	0.094
	a 0	C 1		

449

450 **3.4.** Comparing energy flows and land uses

451 How similar were the energy flows and land uses in the actual historical case with respect to the 452 optimised strategies? To make the comparison, we calculate the Euclidean distance between the vectors corresponding to each scenario (other distances could be used as well, we choose the 453 Euclidean distance because it is a well-known and easy to understand measure). The total surface 454 455 is different for each scenario (see Table 1), hence we work with the proportion of land covers and 456 energy flows, respectively. Table 5 shows the Euclidean distances among the optimized strategies and the historical case with respect to energy flows and to land uses. The values in Table 5 are 457 458 Euclidean distances divided by the square root of two, that is, the distance between two polarized cases (e.g. (1,0,0,0)), so that the values range from 0 to 1. 459

Source: Our own from the sources given in the text.

⁷ We refer as inflow the part of the energy flow that is reused into the agro-ecosystem.

461 **Table 5.** Euclidean distances in energy flows and land uses for the Vallès case study and the

462 optimization strategies.

		Vallès 1860	Intensive strategy	Extensive strategy	Income strategy
sw	Valles 1860	-	0.10	0.07	0.14
/ flows	Intensive	0.10	-	0.06	0.18
Energy	Extensive	0.07	0.06	-	0.13
E	Income	0.14	0.18	0.13	-
s	Valles 1860	-	0.37	0.20	0.25
use	Intensive	0.37	-	0.25	0.55
Land uses	Extensive	0.20	0.25	-	0.44
	Income	0.25	0.55	0.44	-

463

Source: Our own from the sources given in the text.

...

464 **3.5.** Making the results spatially explicit: cells resemblance to the optimization models

We also want to see how sample cells are spatially distributed according to the models'intentionality. To this aim we proceed as follows.

467 First of all, in order to have a suitable area to compare the real data with those arising from the
468 model's results, and given that the total area required for each optimization model ranges from 4
469 to almost 8 ha, we have split the whole area into a grid of 300x300m sample cells.

470 Then, for each sample cell we considered the vector of land cover proportions, $p = (p_1, ..., p_n)$ 471 where p_i is the proportion of land cover *i* and *n* is the total number of land covers⁸. Once we have 472 settled this, we take the Euclidean distance between the vector *p* and the homonymous from each 473 of the optimization models. Finally, a category is assigned to each cell depending on the minimum

475 of the optimization models. I many, a category is assigned to each cent dependence of the optimization models

distance the vector *p* reaches with respect to the optimization models.

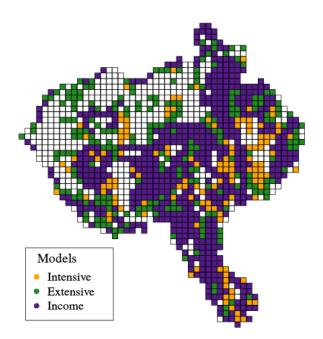
As a dissimilarity criterion, we have established that when the minimum distance between the sample cell and all the optimization strategies is higher than 0.35, that is the maximum distance)

then the cell doesn't resemble any model and appear as 'no category' cells (white cells in Figure

478 4). This threshold is arbitrary, and other criteria could be considered.

⁸ Note that here we use land covers instead to land uses. This is because in order to make spatially explicit the land distributions that correspond to each of these strategies, we are constrained by the land covers defined by the cadastral maps. Therefore, here we merge the land use categories 'herbaceous crop rotations' and 'olive tree rotations' under the land cover of dry cropland. This is not for functional resemblance but because of the limitations set by the available historical sources.

480 Figure 4. Vallès county cell map, with colours according to model resemblance.



481

482 Source: Our own from the sources given in the text.

483 **4. Discussion of the results of counterfactual analysis**

484 The purpose of the first counterfactual strategy was to minimize the area required for sustaining with an appropriate diet an average family of five members while reproducing the other farm 485 486 system funds. This is called intensive strategy and responds to a strategy of land use 487 intensification. We obtained an agro-forestry mosaic of 4.26 ha (Table 1) per typical household 488 composition with close to 55% of the area devoted to dry cereal cropland. In this case vineyard would be required for only 2.1% of the area because, in terms of intensity of cash, area olive trees 489 490 are a superior strategy to get income entries to face payments for taxes, housing and clothes. 491 Finally, close to one third of the farmland area would had been forest for firewood and grazing.

492 The results of the optimisation following what we call an extensive land use strategy have led to 493 a total amount of counterfactual land of 6.10 happer household, where the less-intensive land uses 494 (forest and pasture) would had been close to 56% of the total area (see Table 1). The aim of this 495 second strategy was to minimize the total amount of labour required to fill the demands of the 496 family farm system. As a result, the pursuit of feeding the livestock in these extensive uncultivated 497 parts strongly reduces the demand for cultivated land. In the same vein, the proportion of irrigated 498 land is reduced because of its high demand on intensive practices, whereas vineyards cover up to 499 10% of the area.

500 The third counterfactual strategy, that we call the income strategy, maximized the total monetary 501 gain obtained from the farmland output taking advantage of the biogeographic and economic 502 suitability for growing vineyards in the region at that time. In turn, it considers the option of using 503 the already developed markets for importing grains from inner Spain. The results reduce the need 504 to grow grains within the farm system but require the freedom for importing labour from outside 505 in the peak months of harvesting and pruning as well. In terms of land uses it is clearly different 506 from the others. In this case the counterfactual area obtained is assumed to be the amount that 507 rests from dividing the total area among the existing population, which was 7.60 ha (as shown in Table 1). Here, the dominant use in the resulting counterfactual agroforest mosaic would have been vineyards that amounted 66% of the total area, followed far behind by a 15% of olive trees associated to a cereal intercrop rotation, while forest and pasture were reduced to 6 and 8% respectively.

512 In order to asses which of those counterfactual scenarios are closer to the historical one we first 513 look at the values of the information indicators that express the different goals behind them. 514 According to the results shown in Table 3, the actual case study was closer to the maximum specialization on vineyards seeking an income strategy to than the other ones $(I^* = 0.944)$, 515 followed by the extensive model $(I^* = 0.933)$ because of its similarity regarding the share of 516 cropland extensive uses also adopted by the large farmsteads. Finally, the most dissimilar one is 517 the strategy of maximizing population density ($I^* = 0.916$) which did not seem to have been the 518 519 main driver c.1860. Indeed, these results clearly fit with what agricultural historians know about 520 this farming community led by a group of rich landowners that followed a cropland extensive 521 strategy in their better lands to minimize hiring external labour, while they set tenancy contracts 522 to a larger group of landless smallholders to grow vines in the worst soils by paying a third of the 523 vintage to them (Garrabou, Tello & Cussó, 2010). This explains why the actual pattern was 524 somewhere in between an income maximization and an extensive strategy.

525 Secondly, given the relatively short differences in terms of information values found between the 526 optimized scenarios and the historical case, it is worth supplementing them by looking at the 527 structural differences in their internal biomass inflows. This, in turn, helps to infer some insights 528 about their likely environmental impacts either aboveground and belowground the farmland.

529 With respect to k_1 (biomass left in the farm system for potentially feeding the associated 530 biodiversity), the three values from the models appear in the 25-30% range while in the historical 531 case it was 39%. Was an NPP_{act} inflow of less than 30% enough to sustain farmland associated 532 biodiversity? Our model cannot ensure the sustainability of the 'unharvested' subsystem, hence 533 these values must be taken with caution and require a specific enquiry. Being lower than the actual 534 historical ones we cannot ensure that this NPP_{act} flow would be enough to feed the heterotrophic 535 chains of the entire associated biodiversity without endangering any species: in fact, 536 undomesticated species do not depend only on agro-ecosystem energy flows but also on landscape 537 ecology parameters (Tscharntke et al., 2012) not addressed in our model (Marull et al., 2019a has 538 recently tested the links between landscape structure, energy and information flows driven by 539 farming and biodiversity).

We can also infer some clues on that issue from the value of the Shannon index of land cover equi-diversity (Table 1), which can be used as a proxy of a set of differentiated habitats. The index is higher under the extensive strategy, while the income strategy would imply a polarized landscape probably less capable to host biodiversity. The intensive strategy would also show a relatively low level of landscape heterogeneity (mainly due to the disappearance of pastureland) whereas the historical case shows a relatively high value of land cover richness.

In the 'farmland' subsystem (which refers to the reproducibility of soil fertility) income maximization is the model devoting the greatest share of biomass (with 43% of NPP_{act} inflows towards this sub-system), while the more extensive model dedicates only 26% of the harvested biomass to restore fertility. This is because the share of cropped area is lower. In turn, sustaining the livestock subsystem requires an inflow of NPP_{act} between 20% and 29% depending, again, on the intensity of the land use management. Finally, for sustaining the 'farming community' subsystem a great variability in the share of NPP_{act} inflows is observed (k_4) . Under the more intensive strategy 23% of NPP_{act} is invested as fuel and food, whereas only 9% is dedicated to the cash-crop specialization (mainly because 45% of the diet is brought from outside the farm system). The income specialization strategy is the most similar to the historical case $(k_4$ close to 14%). This is because food imports to the Vallès area were already relevant but also due to the historical lesser intensity of land uses practised by wealthy landowners.

Therefore, while both in the intensive and the extensive strategies the biomass inflows are more evenly distributed, in the income strategy inflows appear more skewed towards maintaining soil fertility and less towards maintaining the farming community. The historical case was more similar to this last strategy in which trade played a relevant role.

- Despite the differences found in the pattern of energy flows, the differences in land uses are even more relevant, as shown by Euclidean distances in Table 5. Overall, the differences in energy flows expressed by this distance range between 0.06 and 0.18, whereas differences in land uses range from 0.20 and 0.55. In particular, the historical case was more similar to the extensive strategy than to any other⁹, both for the energy flows (distance = 0.07) and for the land uses (distance 0.20). This is because of the relevance of forestland among rich landowners and despite the high share of vineyards intensively grown by many smallholders in this historical case.
- 570 Therefore, the results show that while in terms of land use distribution the actual intentionality 571 strongly differs from the optimized composition among them, in terms of internal energy flows 572 differences are less important (see Table 2). This means that, the structural configuration of each 573 fund in relation to the others within the farm system was strongly defined by the unavoidable 574 links between them under the constraints of the organic farming c.1860. The sole exception was 575 the farming community that could allocate their internal resources with greater flexibility, since 576 the most important incoming flows depended on that.

577 By making pairwise comparisons we also observe that the income strategy is the most distant to 578 the others, followed by the intensive strategy and then the real historical case and the extensive 579 strategy which stand in between the others. While the models tend to polarize family farm system 580 management towards maximizing only one strategy –in particular population density or income 581 revenue- in the historical case a plurality of actors with multiple interests and different forms of 582 managing the farm system had the effect of standing in between the energy profiles of these 583 models.

Note that the indicator I^* has shown the historical case to be closer to the income and extensive strategies, while the distances show other proximities. This is because the comparison patterns differ. On one hand, indicator I^* relates the historical case with each one of the counterfactuals in terms of modified Shannon indices based on the whole set of energy flows entering and outcoming in the subsystems. On the other hand, the Euclidean distances rely on biomass relative flows and land uses distributions, respectively, and other similarities are described on the basis of these particular features.

⁹ To compare the relative values of energy flows and the land-use distribution through a modified Shannon index we use here a different measure from the *information-as-structure*. So, despite some similarities, they have different interpretations by each unit and type of measurement.

591 Finally, in order to find out what kind of dynamics was present in the Vallès case study we can 592 compare maps in Figures 1 and 4, i.e. land cover map and sample cells' similarities. We observe 593 that close to urban areas of the towns, isolated farmhouses, watercourses and flat irrigable lands, 594 cells tend to the intensification strategy. Farther away steeper and poorer soils appear to be closer to the income strategy. Thirdly, cells resembling the extensive strategy appear across forestland 595 596 and pastureland areas where pressure over natural resources was lower. By making the 597 optimization model spatially explicit we can see that while just above a third of the total number 598 of cells have land uses with no particular similarity to any of the optimization strategies, in the 599 remaining cells we can see a clear resemblance to the income strategy model 59% of the cases, 600 followed by a 26% of land uses in cells that resemble the extensive strategy, leaving only 15% of 601 the cells to land uses that can be associated to the intensive strategy.

602 From a landscape ecology perspective, the functional structure obtained from the Shannon index 603 of the land distribution among six different covers (irrigated gardens, dry herbaceous cropland, 604 vineyard, olives groves, pasture and shrub, and woodland; see Table 1) shows that the prevalence 605 of vineyards in the territory not only responded to the income strategy. It also implied that, given 606 the relatively high population density in the case study area, the landscape looked less 607 heterogeneous in case fewer vineyards were implanted disregarding other land uses (e.g. other 608 crops, pasture or forestland areas). From Table 1 we also observe that the strategy that would 609 allow the highest Shannon index is the extensive one (that potentially means more habitats for 610 non-domesticated species). Instead, by maximizing population or income goals the index would 611 decrease. This happens because by pursuing either a population or an income optimization land 612 uses would be polarized towards those particular ones that best fit these strategies, eliminating or 613 minimizing land uses that would not be required in this specialization (pastureland and dry annual 614 crops respectively). Finally, in the real historical case the index is lower than in the case of the 615 extensive strategy and higher than the rest. We understand this result, once more, as a situation in which a plurality of strategies was pursued by the farming community in which, however, the 616 ruling class of wealthy landowners prevailed. They possessed most of the land and controlled the 617 618 access to it from the rest of smallholders through tenancy contracts. While they tended to follow 619 a poly-cultural extensive strategy in their farmsteads, the leases they offered to the smallholder 620 families who lived in the towns forced them to pursue a more intensive specialization in vineyards 621 (Marco et al., 2017; Tello et al. 2008).

622 In summary, the real case stood between the various strategies considered in the model. In 623 particular it seems to move between the extensive and the intensive ones according to the land 624 endowment of different families, combining both with a partial commercial specialization, mainly 625 vineyards. Yet, in general, the actual situation was closer to the income strategy-an outcome of 626 our SAFRA modelling that is coherent with the drivers that can statistically explain this vineyard 627 specialization in the whole Barcelona province at that time (Badia-Miró & Tello, 2014). Each driving force explains a part of vineyard spreading, but only in conjunction with the others: e.g. 628 629 population density increase only mattered up to the point of exhaustion of the 'inner frontier' of 630 land use intensification that landowners were eager to offer to winegrower tenants; and the greater 631 market profitability of growing vines, instead of grains or keeping forestry and pasture uses, 632 tightly depended on the quality and location of soils. The adoption of this partial winegrowing 633 specialization strategy did not imply that the overall farming population attained higher standards 634 of living. There existed limits in the access to land due to social inequalities (Marco et al., 2017). 635 The study of this very important dimension goes beyond the scope of this paper, and it might be

worth to examine in further researches that use the SAFRA modelling to bring to light therelationship between social inequalities and their imprint on the farming landscape.

638 5. Conclusions

639 In order to understand the relationship between the energy reinvested and redistributed in a family 640 farm system, and its impression on the land matrix as land-use and livestock optimization, we 641 have developed a methodology linking Information Theory with a Sustainable Agro-ecological 642 Farm Reproductive Analysis. The results obtained in a Mediterranean organic agricultural system (Vallès County, Catalonia, 1860) can be interpreted in the sense that it is the farmers' know-how 643 644 and culture (the information passed down from generation to generation), what allows to manage 645 the energy entering to the farm system in the most efficient way in order to maintain a sustainable 646 exploitation of the agro-ecological territory, always within the main goals adopted by the ruling 647 class that controlled the access to natural resources.

648 According to Marull et al. (2019), the information-driven redistribution of energy flows within 649 agroecosystems appears to be a major factor behind biodiversity patterns in Mediterranean 650 cultural landscapes. In this paper we have departed from the use of Shannon Index through Information Theory directly applied to the energy profile of energy fluxes driven within farm 651 systems, and we move towards assessing farmers' structuring information by assuming a 652 653 maximum value of I derived from three different strategies of land use optimization (I^*) . By doing 654 so, we observe that in the historical case the value of I^* associated to any optimization strategy reaches a very high level (in the three cases was over 0.9). This means that in the real case, the 655 656 complexity of the interwoven pattern of energy flows in the graph could differ according to each 657 optimization strategy adopted by farmers. We claim that the new indicator I^* expresses the actual capacity of farmers (and their site-specific endowment of resources and local knowledge) to shape 658 659 landscapes in a fairly sustainable way, under the assumption that sustainability was the capacity 660 to reproduce the different funds of the farm. This is relevant for understanding past agricultural 661 landscapes. But it can be a useful tool in order to get information on the current trends and aims 662 in which agrarian systems are being managed at present as well.

663 Farmers were managing c.1860 the Vallès agro-ecosystem studied close to optimal conditions in 664 terms of the relative magnitudes of biophysical flows needed to reproduce the farming 665 community, their livestock, and soil fertility, always under the technical and social settings which 666 prevailed at that time. Moreover, independently from which optimization strategy this local 667 population might have decided to pursue, the actual patterns show that there existed a set of 668 incoming and outgoing pairs of energy flows which were always close to the general optimum 669 needed for keeping the family farm system reproducibly. Put it bluntly, the actual path adopted 670 was not the only possible one that might have been compatible with this sustainability criterion.

671 Although the actual land uses c.1860 greatly differed from the optimal SAFRA models, the 672 distribution of the associated energy flows did not differ that much. Indeed, the pattern of energy 673 linkages between funds could not diverge so sharply between the optimization models considered. 674 We interpret this as a sustainability imperative: land uses could be very different according to the 675 prevailing strategy, but the sustainability of the energy flows stemming in and out of the agro-676 ecosystem entailed that the intensity of these flows (in energy per surface units) could only vary 677 within the limited range that an organic farm system might then assume. Indeed, the different 678 funds required similar investments no matter the main intentionality of the farming community

- was. Whichever the land use distribution is, an organic farm system can only redistribute flowswithin its underlying structure of funds along a restricted range of values.
- 681 Finally, the spatially explicit analysis carried out yielded a modelled farm landscape that 682 resembled a lot one close to an income optimization, which brings to light the socioeconomic 683 ruling forces behind the real historical landscape studied c.1860. We know from the studies 684 carried out in the same case study (Marco et al. 2017 and under review) that the unequal land and 685 livestock distribution among the farming population played a key role in driving winegrowing 686 specialization as the main cash crop, and shaping that cultural landscape. This opens the way to use the new SAFRA modelling developed in this article for a further research on the impacts 687 688 social inequalities may have on landscape agroecology.

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843 APPENDIX A

In this appendix we show that the indices *I* and *I*^{*} defined in Section 3.2 and 3.3 take values in the interval [0,1]. These are the claims of Lemma 1 and Lemma 2 below. Notice first that an equivalent definition of the quantities γ_W and γ_{nR} is:

847
$$\gamma_W = \frac{1}{2}(\beta_5 + \beta_6 + \beta_{11} + \beta_{12}), \gamma_{nR} = \frac{1}{2}(\beta_7 + \beta_8 + \beta_9 + \beta_{10}).$$

848 <u>Lemma 1</u>

849 1. $0 \le H(\beta_{2i-1}, \beta_{2i})\gamma_W\gamma_{nR} \le 1, i = 1, ..., 10.$

- 850 2. The index *I* defined by formula (1) satisfies $0 \le I \le 1$.
- 851 <u>Lemma 2</u>

852	1.	If $a = 0.5$, then $T_{0.5}(x) = x$, $\forall x \in (0,1)$
853	2.	$\forall a \in (0,1): T_{1-a}(y) = 1 - T_a (1-y)$
854	3.	$\forall a \in (0,1), \text{ if } x + y = 1, \text{ then } H_a(x,y) = H(T_a(x), 1 - T_a(x)).$

855 <u>Lemma 3</u>

856 For any $a \in (0,1)$ and any pair (x, y) in [0,1] such that $x + y \le 1$: $0 \le T_a(x) + T_{1-a}(y) \le 1$.

- 857 <u>Lemma 4</u>
- 858 1. $0 \le H_a(\beta_{2i-1}, \beta_{2i})\gamma_W^*\gamma_{nR}^* \le 1$ 859 2. The index *I** defined by formula (3) satisfies $0 \le I^* \le 1$, for any $A^* = (a_1, ..., a_{10})$, 860 with $a_i \in (0,1)$.
- 861
- 862 <u>Proof of lemma 1</u>
- First notice that γ_W and γ_{nR} are both arithmetic means of proportions, and therefore take values in [0,1]. Also, the quantities $H(\beta_{2i-1}, \beta_{2i})$ are always non-negative.
- In the case when $\beta_{2i-1} + \beta_{2i} = 1$, we know that the entropy satisfies $H(\beta_{2i-1}, \beta_{2i}) \le 1$, and we are done with claim 1. This is not necessary true if $\beta_{2i-1} + \beta_{2i} \le 1$.
- For i = 3,4,5,6, we can only say $\beta_{2i-1} + \beta_{2i} \le 1$. Assume i = 3 to simplify notation. The other cases are identical. We want to prove that

869
$$H(\beta_5,\beta_6)\gamma_W\gamma_{nR} \le 1.$$

870 Clearly, $H(\beta_5, \beta_6)\gamma_W\gamma_{nR} \le f(\beta_5, \beta_6)$, where

$$f(\beta_5, \beta_6) \coloneqq (-\beta_5 \log_2 \beta_5 - \beta_6 \log_2 \beta_6) \frac{(\beta_5 + \beta_6 + 1)}{2}.$$
 (A1)

By symmetry, if the maximum of this function on the triangle $\{\beta_5 + \beta_6 \le 1, \beta_5 \ge 0, \beta_6 \ge 0\}$ is

- achieved at some point (β'_5, β'_6) , then (β'_6, β'_5) is also a maximal point. Both points lie on a certain
- 873 line $\beta_5' + \beta_6' = k$. Restricting f to this line, it is easily seen by elementary calculus that there is a

unique maximal point and $\beta_5' = \beta_6'$. Therefore, we only need to check that the function of one variable,

876
$$g(\beta) = (-2\beta \log_2 \beta) \frac{(2\beta + 1)}{2},$$

877 is bounded by 1 for $0 \le \beta \le 0.5$.

Again, using elementary calculus, it can be seen that g is increasing in [0, 0.5]. Hence, $f(\beta_5, \beta_6) \le g(0.5) = 1$, and claim 1 is verified.

Claim 2 follows immediately, since *I* is the arithmetic mean of quantities belonging to the interval[0,1].

882

883 <u>Proof of lemma 2</u>

The first claim is immediate, and the third is directly implied by the second and the definition of H_a . To prove the second claim:

y

886 If y < 1 - a, then 1 - y > a and we have

887
$$T_{1-a}(y) = \frac{0.5}{1-a}$$

888 and

889
$$1 - T_a(1 - y) = 1 - \left(0.5 + \frac{0.5}{1 - a}(1 - y - a)\right) = \frac{0.5}{1 - a}y.$$

890 If $y \ge 1 - a$, then $1 - y \le a$ and, analogously,

891
$$T_{1-a}(y) = 0.5 + \frac{0.5}{a}(y - (1-a)) = 1 - \frac{0.5}{a}(1-y)$$
892 and

$$1 - T_a(1 - y) = 1 - \frac{0.5}{a}(1 - y).$$

894

895Proof of lemma 3896Take (x, y) in [0,1] such that $x + y \le 1$ and consider three cases:8971)x < a and y < 1 - a8982)x < a and $y \ge 1 - a$ 8993) $x \ge a$ and y < 1 - a

900 In the case 1),

901
$$T_a(x) + T_{1-a}(y) = \frac{0.5}{a}x + \frac{0.5}{1-a}y,$$

902 which is less or equal than 1, under the constraints in 1). In the case 2),

903
$$T_a(x) + T_{1-a}(y) = \frac{0.5}{a}x + 0.5 + \frac{0.5}{a}(y - (1 - a)) = 1 + \frac{0.5}{a}(x + y - 1),$$

904 that clearly is less or equal than 1, because $x + y - 1 \le 0$. In the case 3),

905
$$T_a(x) + T_{1-a}(y) = 0.5 + \frac{0.5}{1-a}(x-a) + \frac{0.5}{1-a}y = 0.5 + \frac{0.5}{1-a}(x+y-a),$$

906 and this is less or equal than 1 because $x + y \le 1$.

907

908 <u>Proof of lemma 4</u>

909 By definition, $H_a(\beta_{2i-1}, \beta_{2i}) = H(T_a(\beta_{2i-1}), T_{1-a}(\beta_{2i}))$, and $T_a(\beta_{2i-1}) + T_{1-a}(\beta_{2i}) \le 1$ by 910 lemma 3. Now, notice that γ_W^* and γ_{nR}^* are exactly the γ_W and γ_{nR} corresponding to the pairs 911 $(T_a(\beta_{2i-1}), T_{1-a}(\beta_{2i}))$. Then, by lemma 1, each term satisfies $H_a(\beta_{2i-1}, \beta_{2i}) \gamma_W^* \gamma_{nR}^* \le 1$, and 912 therefore also I^* .

913