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Introducing media in a model of electoral competition with candidate quality

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Abstract: This work proposes and studies a two candidate model of electoral competition with candidate quality and media. The role of media is to inform voters about the quality of each candidate. We assume that there are two non-strategic media outlets, each one with a different ideal policy (there is a leftist media outlet and a rightist one), and that both of them transmit lower quality for a candidate the further from their ideal policy the policy the candidate proposes is. We also assume that the rightist media outlet has greater coverage, in the sense that it informs neutral voters and voters slightly on the left side of the political spectrum. We study the model under the classical assumption of risk-averse voters. Classical results concerning PSNE generally hold with a "media bias". We extend and characterize in our setting the MSNE found in Aragonés and Xefteris (2012), which sometimes fails to exist in our model.

JEL Codes: C72, C82, D72.

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1 Introduction

The aim of this work is to introduce media in a model of electoral competition. We take as a benchmark the Downsian model of electoral competition, as widely done in this strand of literature. The Downsian model, introduced in Downs (1957), builds on the classical Hotelling model of spatial competition (Hotelling, 1929), turning buyers into voters and firms into political parties; in fact, it is sometimes referred to as Hotelling-Downs model. The skeleton of the model is rather simple: roughly speaking, a number of parties propose a policy from a given unidimensional policy space and, afterwards, voters choose the policy that they consider more appropriate (traditionally, every voter has an ideal policy and chooses the proposed one closer to it). However, when going into detail, many different assumptions can be found in the literature. De Donder and Gallego (2017) provides a quite complete summary of the main ones, their motivation and their effect on the model. It is interesting to notice that, in many (if not all) of the aspects that need to be modeled, there is not a "correct" set of assumptions, but different electoral processes might lead to different valid assumptions.

Crucial assumptions of the classical Downsian model have to do with the goal of the candidates and how voters penalize deviations from their ideal policies. Concerning the goal of the candidates, there are three main assumptions in the literature: candidates want to maximize their chances to win [1], candidates want to maximize their expected vote-share or candidates are policy-motivated. Although we do not use it here, some authors, in the spirit of Calvert (1985), allow for mixed motivations via linear combinations of the above. Concerning how voters penalize deviations from their ideal policies, there are two main approaches: voters are risk-neutral or voters are risk-averse. In this paper we consider that voters are risk-averse and that candidates can be either winning chances maximizers or vote-share maximizers. We present the model, some basic results related to the classical theory and we are able to extend in our setting the main result of Aragonès and Xefteris (2012).

In particular, we adopt what in this strand of literature is usually referred to as a "valence model", which takes into account the importance of the non-policy evaluations, or valences, of candidates by the electorate. These evaluations, or valences, are often understood as the quality of a candidate, and they are introduced in the Downsian model based on the seminal papers Stokes (1963, 1992). Papers such as the aforementioned Aragonès and Xefteris (2012), Groseclose (2001) and Hummel (2010) investigate, as we partly do here, the consequences of a difference in candidate quality (with two competing candidates).

[1] Downs (1957) only considers the case of candidates that maximize their chances to win. The other two motives appeared later in the literature.

The introduction of media in such models is a recent issue. Despite the fact that we do not aim to provide an exhaustive picture of the different forms this introduction has adopted in the existing literature, since we follow a remarkably different path, we make a couple of general comments. Although in every "category" different assumptions and details can be found (hence, what follows needs to be taken lightly), there are two main ideas authors have used to introduce media in the Downsian model: i) there are several "states of the nature" and media outlets inform voters on the realized state; ii) media outlets are in charge of informing voters of the policies proposed by the candidates. Interesting examples of the first path are Puglisi (2007) and, specially, Chan and Suen (2009). A key modelling difference between our work and Chan and Suen (2009) is that we consider that voters are non-strategic when it comes to information, in the sense that we assume that they are not aware of the goals of media, which biases the true state of nature in their model, and the true quality of candidates in ours. We take this assumption in light of Pew Research Center (2018), which points towards consumers of a certain media outlet believing that it reflects their own personal views. On the second path, we highlight the recent Miura (2019), which studies the incentives of a biased media outlet to provide voters with ambiguous information on the policies proposed by the candidates. Although this latter study and ours differ significantly on the modelling strategy, when we analyze the case of risk-neutral voters (which is how Miura (2019) considers voters to be), we get a similar conclusion: if media is not too biased, usual properties of the Downsian model are recovered.

Another crucial difference of our study with the aforementioned ones, and with most of the literature on the topic, is that we assume media outlets to be non-strategic. The reason underneath is that it seems rather impossible to introduce in a model of electoral competition all the incentives and constraints that a media outlet has in real life. That is why, based on Pew Research Center (2018), which studies the European case, and Lott and Hassett (2004) and Groseclose and Milyo (2005), which study the USA case, we exogenously simulate how media outlets behave introducing ourselves a certain bias.

We have already justified why we assume that voters and media outlets are non-strategic. Another important assumption of our model has to do with the coverage of each media outlet: we assume that there are two media outlets (a leftist one and a rightist one) and some voters with ideal policies on the left consume the rightist media outlet. First of all, notice that the assumption could be reversed without any change in the analysis of the model. It could be the leftist media outlet the one that attracts voters from the other side of the political spectrum and the conclusions would be derived in the same way and would be symmetrically identical. However, our assumption is based again on Pew Research Center (2018), which highlights the divergence in some European countries of the political stance of the consumers of a particular media outlet and the political position occupied by the outlet [2]. This divergence is particularly accentuated in Spain and Italy for the most consumed media outlets, which appear to be more rightist than the mean of their consumers.

Concerning the information that media outlets transmit, and although the model involves different elements, we proceed similarly as Chan and Suen (2009). As they do, we assume that voters receive the true policies proposed and that media provides some additional information. In their study, it is the true state of nature, and in ours it is the quality of each candidate. As they put it:

> While a media outlet may quote a candidate selectively, on the whole it has little leeway in the reporting of this kind of information. (Chan and Suen, 2009; p.802).

Related to different information on the quality of the candidates among media outlets because of political differences, there are multiple recent examples: Spanish *OK diario* claiming that Pablo Iglesias had received money from the Government of Iran (case recently closed); or, again in Spain, the claims on Pedro Sánchez plagiarizing his PhD thesis or on Albert Rivera being part of a fascist movement when young. These are shocking examples of manipulation (fake news) that attack the quality of a candidate. However, the effect we put into the model can be subtler than that: different media outlets might deliver different information on the quality of the candidates, either manipulating or not (they can highlight different true information, in the spirit of Puglisi (2007), which works with states of nature). The point is that media outlets channel differences with the proposed policies through candidate quality: if they agree with a proposed policy, they help the candidate releasing information supporting his quality, and conversely if they do not. This mechanism can also be supported by means of the 2016 Trump-Clinton electoral race and other non-Spanish related examples. However, I want to point out that we are dealing with media in the classical meaning of the term; we do not pretend to study the effect of social networks such as Twitter. The effect and incentives of social media might differ substantially and are beyond the scope of this work, although they are often also fed by attacks to the quality of a candidate (Lee and Xu, 2018).

The structure of the paper is as follows. In section 2, we introduce the general model that we work on and that we have partly described in this Introduction. Afterwards, in section 3, we derive some basic properties of the model concerning PSNE and, in section 4, we focus on the extension of the main result of Aragonès and Xefteris (2012). At last, in section 5, we address some concluding remarks and comment on further research.

[2] The political position of a particular media outlet is measured taking into account the opinion among all consumers/voters.

2 Description of the game

A *game of electoral competition with valences and media* is defined by the following elements:

i) The policy space: $X = [0, 1]$.

ii) The set of voters, V, each one of whom has a unique ideal policy $v_i \in [0, 1]$.

iii) The set of candidates, $C = \{1, \ldots, n\}$.

iv) A density function f describing the distribution of voters' ideal policy platforms.

v) The utility functions of the voters, $\{u_i\}_{i \in V}$.

vi) The utility functions of the candidates, $\{w_j\}_{j \in C}$.

vii) The maximum transmitted quality of each candidate, $\{k_j\}_{j \in C}$.

viii) The set of media outlets, $M = \{1, \ldots, m\}$.

ix) The media outlet that consumes each voter, defined by a function $\mu : V \to M$.

x) The "media influence functions" of each outlet for each candidate, $\{g_{m,j}\}_{m \in M, j \in C}$. For example, function $g_{1,2}$ reflects how a voter informed through media outlet 1 perceives the quality of candidate 2.

The game consists of two stages: in the first stage each of the candidates chooses a policy $x_j \in [0, 1]$; and in the second stage each voter votes for the candidate that maximizes his utility. However, since voters cannot behave strategically (voting their most preferred candidate weakly-dominates any other strategy), the game can be seen as a one-shot game where candidates choose the policy they propose (the first-stage of the game). With that in mind, a *game of electoral competition with valences and media* is defined by a tuple of the elements just listed above:

$$(X, V, C, f, \{u_i\}_{i \in V}, \{w_j\}_{j \in C}, \{k_j\}_{j \in C}, M, \mu, \{g_{m,j}\}_{m \in M, j \in C}). \quad (2.1)$$

Let us now discuss the assumptions we take in regard to these elements. First of all, as in many studies on electoral competition, we assume that there are two candidates, that is $n = 2$. We assume that there is a continuum of voters and that the distribution defined by f is symmetric with respect to $1/2$. Regarding the media outlets, we assume that there are two of them, which we denote by L (left-wing) and R (right-wing), and that there exists $L < 1/2$ such that voters with ideal policies in $[0, L]$ consume media outlet L and voters with ideal policies in $(L, 1]$ consume media outlet R (noting this threshold L

might look like a slight abuse of notation, but is very intuitive and does not originate any misunderstanding in the analysis, where media outlets are limited to sub-indices). This explanation clearly defines the function μ.

We will take into account two different assumptions for the utilities of the candidates: they can either want to maximize their winning chances or maximize their expected vote-share. The main result assumes that candidates maximize their vote-share. Voters are assumed to be risk-averse, that is, in our model, that the utility a voter with ideal policy $v \in [0,1]$ who observes quality k for a candidate proposing policy x, is $k - (x - v)^2$.

At last, the functions $\{g_{m,j}\}_{m \in M, j \in C}$ will be discussed in the next section, where an example will show the importance of their shape.

Definition 1. From now on, in order to keep statements reasonably long, we will call simply a *game of electoral competition with media* any game of the form (2.1) with 2 media outlets (L and R), 2 candidates, the function μ specified above and risk-averse voters.

Regarding the assumptions on media influence functions, we find it appropriate to devote a little bit of time to justify the shape of the functions we will consider. Let the following subsection be an explanation of it.

2.1 Media functions: shape and role

In this subsection we see that, even if all media functions are parabolic and concave, it may happen that there exists a policy $v \in (L, 1]$ such that voters who have it as their ideal policy vote for the candidate proposing the policy more to the left and that some voters with ideal policies in $[0, L]$ vote for the candidate proposing the policy more to the right. This feels unnatural and we want to avoid such phenomenon.

We see that this phenomenon might take place even when we limit ourselves to parabolic media functions. In fact, it is the product of different "influence abilities" from the media outlets, as we clearly see in the following example.

Example 1. Assume $L = 0.4$, $x_L = 0.2$, $x_R = 0.7$, $x = 0.65$, $y = 0.8$, $k_2 = 0.5$ and $k_1 = 0.3$. Assume that the media functions are as follows:

$$\begin{aligned}
g_{L,2}(y) &= -0.5y^2 + 0.2y + 0.42, \\
g_{L,1}(x) &= g_{L,2}(x) - 0.2, \\
g_{R,2}(y) &= -18y^2 + 25.2y - 8.32, \\
g_{R,1}(x) &= g_{R,2}(x) - 0.2.
\end{aligned}$$

Plotting them:

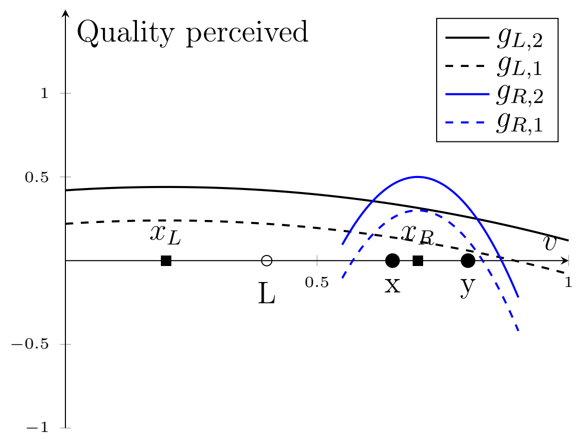

Figure 1: Media functions of Example 1.

Then, just by computing, it is immediate to check that $g_{L,2}(y) - g_{L,1}(x) > (y-0.4)^2 - (x-0.4)^2$ and that, therefore, voters with ideal policy L vote for candidate 2. On the other hand, $g_{R,1}(x) - g_{R,2}(y) > (x-0.5)^2 - (y-0.5)^2$ and the voters with ideal policy $1/2$, vote for candidate 1.

This example has a very clear interpretation. Looking at the plots, it is clear that the rightist media outlet has a greater ability to influence the perceived the quality of the candidates. Since we assume media to be non-strategic, this fact might have several explanations outside of the model. A reasonable explanation seems to be that, in an example such as the one proposed, leftist media outlet consumers trust less what the media says, which causes this media outlet to only be able to mildly manipulate the perceived quality of the candidates in their interest.

This phenomenon clearly cannot take place when the valences are perceived the same by all voters, as in the vast majority of previous literature in the topic (if not all). As it is clear, in particular, it could cause that the candidate supported by neutral voters would not win the election.

In order to avoid the non-regular behavior observed in the above example, we assume the media functions adopt the following form, for some $a > 0$.

$$\begin{aligned} g_{L,1}(x) &= -ax^2 + 2ax_Lx + (k_1 - ax_L^2), \\ g_{L,2}(x) &= -ax^2 + 2ax_Lx + (k_2 - ax_L^2), \\ g_{R,1}(x) &= -ax^2 + 2ax_Rx + (k_1 - ax_R^2), \\ g_{R,2}(x) &= -ax^2 + 2ax_Rx + (k_2 - ax_R^2). \end{aligned} \tag{2.2}$$

As we see next, these media functions, which basically tie the ability to influence of the media outlets to each other, avoid the appearance of what we have just discussed.

3 A first analysis of the model: basic properties and PSNE

Formally, the utility of a voter with ideal policy v of voting candidate j when he proposes x_j takes the form:

$$u_v(x_j) = g_{L,R}(x_j) - (x_j - v)^2.$$
$$\implies u_v(x_j) = (-a-1)x_j^2 + (2ax_{L,R} + 2v)x_j - (k_j - ax_{L,R}^2 - v^2).$$

As in the rest of the document, we assume that candidate 1 proposes x and candidate 2 proposes y. It will be useful to consider:

$$\Delta_v = u_v(x) - u_v(y) = (a+1)(y^2 - x^2) - 2(ax_{R,L} + v)(y - x) + k_1 - k_2.$$

This utility form has also been exploited in the literature, being Aragonès and Xefteris (2012) a particularly pertinent example, and which main result we will put to the test in our model. One of the main technical advantages of this assumption is that when candidates propose different policies, there is at most one policy such that voters who have it as the ideal one are indifferent (which is something that does not happen when voters are risk-neutral). We prove this next.

3.1 Voting patterns

We start by assessing which candidates do the voters vote for, once the candidates have chosen two policy platforms.

Proposition 1. *Let be a game of electoral competition with media and media functions as in (2.2). If $x \neq y$, there is at most one policy such that voters who have it as the most preferred one are indifferent.*

Proof. Assume that voters such that $v \in (L, 1]$ are indifferent between the proposed policies x and y. This happens if, and only if,

$$\Delta_v = (a+1)(y^2 - x^2) - 2(ax_R + v)(y - x) + k_1 - k_2 = 0.$$

Since y and x are fixed (and $y - x \neq 0$), the LHS is a linear function on v and, as a consequence, intersects at most once with the horizontal axis. Furthermore, for any voter such that $v' \in [0, L]$,

$$\Delta_{v'} = (a+1)(y^2 - x^2) - 2(ax_L + v')(y - x) + k_1 - k_2 \gtrless 0,$$

since $x_L < x_R$ and $v' < v$, hence $2(ax_L + v') \gtrless 2(ax_R + v))$. The proof is completely analogous if it is assumed that $v \in [0, L]$. $\square$

Proposition 2. *Let be a game of electoral competition with media and media functions as in (2.2). If $x \neq y$ and there exists a policy $v \in [0,1]$ such that voters who have it as the ideal one are indifferent, then the voters with ideal policy $v' < v$ vote for $\min\{x, y\}$ and voters with ideal policy $v'' > v$ vote for $\max\{x, y\}$.*

Proof. Assume without loss of generality that $x < y$. We have to see that if a voter with ideal policy v votes for x so does a voter with ideal policy $v' < v$. It is clearly enough to see that

$$v' < v \implies \Delta_v < \Delta_{v'};$$

but since $x < y$, this is clear.

It is analogous to prove that if a voter with ideal policy v votes for y so does a voter with ideal policy $v' > v$ $\square$

Proposition 3. *Let be a game of electoral competition with media and media functions as in (2.2). A voter with ideal policy $v \in [0,1]$ has his utility maximized when the candidate chooses the policy platform $x = (ax_{R,L} + v)/(a+1)$, which clearly belongs to $[0,1]$.*

From now on, we will denote by z^* the policy that a candidate has to propose in order to maximize the utility of neutral voters, which, in light of the last proposition is

$$z^* = \frac{ax_R + 1/2}{a+1}. \tag{3.1}$$

3.2 Pure strategy Nash equilibria

From the results in the previous subsection, we can easily infer some results on the existence and shape of PSNE.

Proposition 4. *Let be a game of electoral competition with media, media functions as in (2.2) and winning chances maximizing candidates. If $k_1 = k_2$, the only PSNE is (z^*, z^*).*

Proof. Assume $k_1 = k_2$ and, without loss of generality, that $x \neq z^*$. Then candidate 2 wins for sure the election by proposing z^*, since all voters with ideal policy the segment $[0, 1/2]$ or $[1/2, 1]$ (the one without x) will vote for him. Hence, since with the profile $(1/2, 1/2)$ both candidates have the same chance of winning, this is the only PSNE. $\square$

Proposition 5. *Let be a game of electoral competition with media, media functions as in (2.2) and winning chances maximizing candidates. If $k_1 \neq k_2$, the candidate with a natural advantage wins in any PSNE by proposing z^*.*

Proposition 6. *Let be a game of electoral competition with media, media functions as in (2.2) and vote-share maximizing candidates. If* $k_1 = k_2$*, the only PSNE is* (z^*, z^*).

Proof. The proof is analogous to the one of Proposition 4. With the same argument we can conclude that any candidate facing z^* sees his expected vote-share maximized at z^*. Furthermore, the same can be used to argue that in any other equilibrium, where z^* was not proposed by anybody, some of the candidates would benefit from deviating to z^*.

□

4 A mixed strategy Nash equilibrium (MSNE)

In this subsection we aim to extend the existence of the MSNE derived in Aragonès and Xefteris (2012) for a model with candidate quality but no media (that is, constant quality perception among voters) to our model including media.

Their model is "simpler" than the one we are dealing with, in the sense that there are no media outlets and, as a consequence, the valences of the candidates are perceived equally by all voters. In their study, it is assumed that there is a finite and odd number of voters, with ideal policies randomly drawn from the uniform distribution in $[0, 1]$. They try to characterize equilibria when candidates maximize their winning chances. As the authors point out in the paper, this is equivalent to studying the behavior of vote-share maximizing candidates when there is a continuum of voters and the distribution of voters' ideal policies follows a symmetric Beta distribution with parameters $a = b = (n + 1)/2$.

Hence, in our analysis, we are doing the latter exercise: study the existence of equilibria for vote-share maximizing candidates and under the presence of media. However, for convenience, we use the intuition behind the model in Aragonès and Xefteris (2012), with and odd number of voters, n. We assume that candidate 1 proposes policy x and candidate 2 proposes policy y (and any policy y with a subindex). Furthermore, assume that candidate 1 has a natural advantage over candidate 2 of $d = k_1 - k_2 > 0$.

We try to reproduce the MSNE derived in Aragonès and Xefteris (2012). In their model, there is only one MSNE, where candidate 1 proposes 1/2 and candidate 2 randomizes with probability 1/2 over two policies symmetric with respect to 1/2. In our model, if the result carries on, we expect to find an equilibrium where candidate 1 proposes z^* and candidate 2 randomizes with probability 1/2 over two policies symmetric with respect to z^*; where z^* is defined by (3.1).

Let $p(x, y)$ denote the probability that candidate 1 (who proposes x) is elected by a voter randomly drawn form the uniform distribution on $[0, 1]$. If $x < y$, a voter with ideal policy $v \in [0,1]$ votes for candidate 1 if, and only if,

$$
\begin{aligned}
& - ax^2 + 2ax_{L,R}x + k_1 - ax_{L,R}^2 - (x-v)^2 > -ay^2 + 2ax_{L,R}y + k_2 - ax_{L,R}^2 - (y-v)^2 \\
& \iff v < \frac{(a+1)(y+x)}{2} + \frac{d}{2(y-x)} - ax_{L,R},
\end{aligned}
$$

where $x_{L,R}$ is x_L if the voter is informed via the leftist media outlet, and it is x_R if it is informed via the rightist one.

Since we will extensively work with this expression, we denote the RHS of the last inequality by $\hat{x}_{L,R}(x,y)$ (which are two expressions, $\hat{x}_L(x,y)$ and $\hat{x}_R(x,y)$, resulting from considering a voter informed by L or by R).

It is immediate that, if $y < x$ the condition translates to $v > \hat{x}_{L,R}$. Furthermore, it is clear that $\hat{x}_L > \hat{x}_R$. Hence, working a little bit on the different cases that arise, we can write in a compact way:

$$
p(x,y) = \begin{cases} \min\{\max\{0,\hat{x}_L\},L\} + \min\{1-L,\max\{\hat{x}_R - L,0\}\} & if \quad x < y \\ 1 & if \quad x = y \\ \max\{L - \max\{0,\hat{x}_L\},0\} + \max\{0,1-\max\{\hat{x}_R,L\}\} & if \quad y < x \end{cases} \tag{4.1}
$$

Let $q(x,y) = 1 - p(x,y)$ be the probability that the voter votes for candidate 2. Then, given n voters (we assume n odd) independently randomly drawn from the uniform distribution over $[0,1]$, we consider the probabilities of winning the election (P refers to candidate 1 and Q to candidate 2):

$$
\begin{aligned}
P_n(x,y) &= \sum_{k=(n+1)/2}^{n} \binom{n}{k} p(x,y)^k (1-p(x,y))^{n-k}, \\
Q_n(x,y) &= \sum_{k=(n+1)/2}^{n} \binom{n}{k} q(x,y)^k (1-q(x,y))^{n-k} = 1 - P_n(x,y).
\end{aligned}
$$

As we have stated, we aim to prove that, for n large enough, there is a MSNE where candidate 1 proposes z^*. Hence, next we study which are the best replies that candidate 2 might have to candidate 1 proposing z^*. As shown in Kirstein and Wagenheim (2010), $Q_n(q)$ is an S-shaped function, increasing in $[0,1]$. Hence, in order to maximize $Q_n(q(z^*,y))$, candidate 2 has to maximize $q(z^*,y)$.

Proposition 7. *Let be a game of electoral competition with media, media functions as in (2.2) and vote-share maximizing candidates. If candidate 1 proposes z^* and $L \leq 1/2 - \sqrt{d(a+1)}$, then candidate 2 has two best replies: $y_L^* = z^* - \sqrt{d/(a+1)}$ and $y_R^* = z^* + \sqrt{d/(a+1)}$. Furthermore, $q(z^*,y_L^*) = q(z^*,y_R^*) = 1/2 - \sqrt{d(a+1)}$.*

Proof. In the first place, notice that the condition $1/2 - \sqrt{d(a+1)} > L$ implies both $1/2 + \sqrt{d(a+1)} < 1$ and $z^* + \sqrt{d/(a+1)} \leq 1$. The first implication is obvious and the second one is purely technical and can be easily checked by taking into account that $z^* = \frac{1/2 + a x_R}{a+1}$.

As we have said, we need to maximize $q(z^*, y)$. First of all, consider the domain $z^* < y$. In this case it is clear that, in particular, voters with ideal policy $v = 1/2$ vote for candidate 1, so $\hat{x}_L > \hat{x}_R > L$. Hence, in this case,

$$q(x, y) = 1 - p(x, y) = 1 - \min\{1, \hat{x}_R\}.$$

Hence, candidate 2 wants to minimize $\hat{x}_R$ (if he can make it less than 1). Derivating $\hat{x}_R(z^*, y)$ and equating to 0, we get the solution: $y_R^* = z^* + \sqrt{d/(a+1)}$, which we assume to be in $[0, 1]$. The second derivative is trivially positive and we have found the minimum of $\hat{x}_R(z^*, y)$ in the desired domain. However, in order for it to be the maximum of $q(z^*, y)$ in the domain, we need to check that $\hat{x}_R(z^*, y_R^*) < 1$. And we have that $\hat{x}_R(z^*, y_R^*) = 1/2 + \sqrt{d(a+1)} < 1$, which once again, we assume to fulfill the technical requirement.

Let us move on to the case $y < z^*$. Now, taking into account that, if $y < z^*$, $\hat{x}_R(z^*, y) < 1/2$, since neutral voters vote for candidate 1, it can be derived from the general expressions shown of $p(x, y)$ that:

$$q(x, y) = \begin{cases} 0 & if \quad \hat{x}_L \leq 0 \\ \hat{x}_L & if \quad 0 < \hat{x}_L \leq L \\ L & if \quad L < \hat{x}_L \text{ and } \hat{x}_R \leq L \\ \max\{1, \hat{x}_R\} & if \quad L < \hat{x}_R \end{cases}$$

First of all, we try to maximize $\hat{x}_R(y, z^*)$ for $y < z^*$, and then we check whether $L < \hat{x}_R$ or not; if the latter condition holds, we are done. The maximization easily yields: $y_L^* = z^* - \sqrt{d/(a+1)}$, which implies $\hat{x}_R(z^*, y_L^*) = 1/2 - \sqrt{d(a+1)}$, and taken into account our assumptions, $\hat{x}_R(z^*, y_L^*) > L$.

□

Now we need to check if proposing z^* is a best reply for candidate 1 to the mixed strategy consisting in randomizing between y_L^* and y_R^* with probability 1/2. From now on we denote this mixed strategy σ_2^*.

Lemma 1. *The function* $\rho(x)$*, defined as*

$$\rho(x) = \frac{p(x, y_L^*)(1 - p(x, y_L^*))}{p(x, y_R^*)(1 - p(x, y_R^*)},$$

is strictly increasing in all the domain it is properly defined, that is $[y_L^*, \psi_0)$*, where* ψ_0 *is* *such that* $p(\psi_0, y_R^*) = 1$ *and* $p(x, y_R^*) < 1$ *for any* $x \in [y_L^*, \psi_0)$.

Proof. It is fundamental to have a deeper understanding of the functions $p(x, y_L^*)$ and $p(x, y_R^*)$ in the interval of interest, that is $[y_L^*, y_R^*]$. By taking a close look to (4.1), we can conclude that:

i) $p(x, y_L^*) = 1$ if $x \in [y_L^*, \theta_0]$, is strictly decreasing if $x \in [\theta_0, \theta_l)$, $p(x, y_L^*) = 1 - L$ if $x \in [\theta_l, \theta_r]$ and is strictly decreasing if $x \in [\theta_r, y_R^*]$; where $\theta_0 < \theta_l < \theta_r < z^*$. Furthermore, for all $x \in [y_L^*, y_R^*]$, $p(x, y_L^*) > 1/2$.

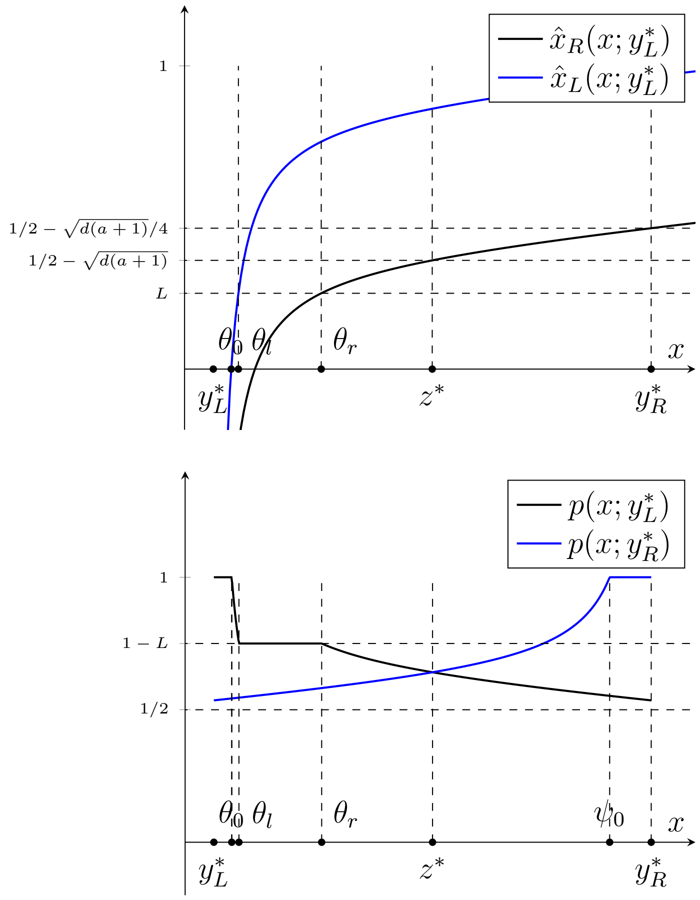

Figure 2: Plots of $\hat{x}_L(x, y_L^*)$, $\hat{x}_R(x, y_L^*)$ and $p(x, y_L^*)$ on the interval $[y_L^*, y_R^*]$, with parameters that satisfy: $1/2 - \sqrt{d(a+1)} > L$. In particular, the plots haven been obtained by using a=1, d=0.01, $x_L = 0.25$, $x_R = 0.75$ and $L = 0.25$.

ii) $p(x, y_R^*) = 1$ if $x \in [\psi_0, y_R^*]$, and is strictly increasing if $x \in [y_L^*, \psi_0)$; where $z^* < \psi_0$. Furthermore, for all $x \in [y_L^*, y_R^*]$, $p(x, y_R^*) > 1/2$.

On the other hand, considering the function $f(p) = p(1-p)$, it is easily checked that, if $p > 1/2$, it is decreasing in p. Hence, with the description provided of the functions $p(x, y_L^*)$ and $p(x, y_R^*)$ in the interval $[y_L^*, y_R^*]$, it is already clear that $\rho(x)$ is strictly increasing in $[y_L^*, \psi_0)$ and $\rho(x) = +\infty$ if $x \in [\theta_r, y_R^*]$.

□

The technical lemma that follows will prove useful in the proof of Proposition 8, which follows immediately afterwards.

Lemma 2. *The following holds:*

$$\lim_{x\to x^{*+}} \frac{\partial \frac{-\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}}{\partial x} = \frac{4\sqrt{d}}{(a+1)^2\left(\sqrt{a+1}+\sqrt{d}\right)}. \tag{4.2}$$

Proof. If $x \in (\theta^-, z^*)$ with $\theta_r < \theta^-$, then $p(x, y_R^*) = \hat{x}_R(x, y_R^*)$ and $p(x, y_L^*) = 1-\hat{x}_R(x, y_L^*)$. As a consequence:

$$\lim_{x\to x^{*+}} \frac{\partial \frac{-\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}}{\partial x} = \lim_{x\to x^{*+}} \frac{\partial \frac{-\frac{\partial \hat{x}_R(x,y_R^*)}{\partial x}}{\frac{\partial (1-\hat{x}_R(x,y_L^*))}{\partial x}}}{\partial x}.$$

We have that,

$$\frac{-\frac{\partial \hat{x}_R(x,y_R^*)}{\partial x}}{\frac{\partial (1-\hat{x}_R(x,y_L^*))}{\partial x}} = \frac{\frac{a+1}{2}+\frac{d}{2(y_R^*-x)^2}}{\frac{a+1}{2}+\frac{d}{2(y_L^*-x)^2}}$$

$$\implies \frac{\partial \frac{-\frac{\partial \hat{x}_R(x,y_R^*)}{\partial x}}{\frac{\partial (1-\hat{x}_R(x,y_L^*))}{\partial x}}}{\partial x} = \frac{\left(\frac{a+1}{2}+\frac{d}{2(y_L^*-x)^2}\right)\frac{d}{(y_R^*-x)^3} - \left(\frac{a+1}{2}+\frac{d}{2(y_R^*-x)^2}\right)\frac{d}{(y_L^*-x)^3}}{\left(\frac{a+1}{2}+\frac{d}{2(y_L^*-x)^2}\right)^2}.$$

Substituting $x = z^*$, the expression can be simplified, since $(y_R^* - z^*)^2 = (y_L^* - z^*)^2$ and $(y_R^* - z^*)^3 = -(y_L^* - z^*)^3$. Hence,

$$\left.\frac{\partial \frac{-\frac{\partial \hat{x}_R(x,y_R^*)}{\partial x}}{\frac{\partial (1-\hat{x}_R(x,y_L^*))}{\partial x}}}{\partial x}\right|_{x=z^*} = \frac{\frac{4d}{(y_R^*-z^*)^3}}{a+1+\frac{d}{(y_L^*-z^*)^2}} = \frac{4\sqrt{d}}{(a+1)^2\left(\sqrt{a+1}+\sqrt{d}\right)}.$$

□

Proposition 8. *Let be a game of electoral competition with media, media functions as in (2.2) and vote-share maximizing candidates. If candidate 2 uses the mixed strategy σ_2^*, proposing z^* is the only best reply of candidate 1 for n large enough.*

Proof. First of all, notice that any mixed strategy of candidate 1 only involves policies in the interval $[\theta_0, \psi_0]$. If he proposed a policy outside of this interval, proposing θ_0 or ψ_0 would clearly strictly dominate it. (The key is that if $x < y$, then $\hat{x}_{L,R}(x, y)$ are strictly increasing functions of x in the domain; and if $y < x$, $\hat{x}_{L,R}(x, y)$ are as well).

It is clear that the derivative of $p(x, y_L^*)$ is not defined at θ_0 and that the derivative of

$p(x,y_R^*)$ is not defined at ψ_0. However, we can assume that the derivative is defined as the right and left limit, respectively.

When candidate 1 maximizes $P_n(x,\sigma_2^*)$, let us focus first on the interval $[\theta_0,\psi_0]$, as just argued. By Kirstein and Wagenheim (2010) we know that:

$$\begin{aligned}\frac{\partial P_n(x,\sigma_2^*)}{\partial x} =& \frac{n}{2}\binom{n-1}{(n-1)/2}\left((p(x,y_L^*)(1-p(x,y_L^*)))^{(n-1)/2}\frac{\partial p(x,y_L^*)}{\partial x}\right)\\ &+\frac{n}{2}\binom{n-1}{(n-1)/2}\left((p(x,y_R^*)(1-p(x,y_R^*)))^{(n-1)/2}\frac{\partial p(x,y_R^*)}{\partial x}\right)\end{aligned} \tag{4.3}$$

With this expression, it is immediate to check that $\left.\frac{\partial P_n(x,\sigma_2^*)}{\partial x}\right|_{x=z^*} = 0$, since, in a neighbourhood of z^*, $p(x,y_L^*) = 1-\hat{x}_R(x,y_L^*)$ and $p(x,y_R^*) = \hat{x}_R(x,y_R^*)$, and the derivatives are straightforward.

We want to prove that for n large enough, z^* is a global maximum of $P_n(x,\sigma_2^*)$. We split the proof in two parts:

i) We see that, for any $\theta^- \in (\theta_r, z^*)$ and any $\theta^+ \in (z^*,\psi_0)$, there exists n large enough such that $\frac{\partial P_n(x,y_L^*)}{\partial x} > 0$ for all $x \in (y_L^*, \theta^-]$ and $\frac{\partial P_n(x,y_L^*)}{\partial x} < 0$ for all $x \in [\theta^+, y_R^*)$.

ii) We see that that there exist $\theta^- \in (\theta_r, z^*)$, $\theta^+ \in (z^*,\psi_0)$ and n large enough such that, for all $n' \geq n$, $\frac{\partial P_n(x,y_L^*)}{\partial x} > 0$ for all $x \in (\theta^-, z^*)$ and $\frac{\partial P_n(x,y_L^*)}{\partial x} < 0$ for all $x \in (z^*, \theta^+)$.

Since it is clear that $p(x,y_R^*)(1-p(x,y_R^*)) \neq 0$ for any $x \in [\theta_0, z^*)$, working a little bit on equation (4.3), we get that $\frac{\partial P_n(x,y_L^*)}{\partial x} > 0$ if, and only if,

$$\left(\frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)}\right)^{(n-1)/2}\frac{\partial p(x,y_L^*)}{\partial x} + \frac{\partial p(x,y_R^*)}{\partial x} > 0. \tag{4.4}$$

By means of Lemma 1, since $p(z^*,y_L^*) = p(z^*,y_R^*)$ and $1-p(z^*,y_L^*) = 1-p(z^*,y_R^*)$, it is immediate that for all $x \in [\theta_0, z^*)$,

$$\frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)} < 1.$$

At this point it is interesting to notice that, with the notation introduced in Lemma 1, proposing any policy in $[\theta_l,\theta_r)$ cannot be optimal for candidate 1, since the probability provided by any such policy is strictly dominated by the probability provided by proposing θ_r; strict domination being a consequence of $p(x,y_L^*)$ being strictly increasing in this domain. Hence, it is sufficient to see that $\frac{\partial P_n(x,y_L^*)}{\partial x} > 0$ for $x \in [\theta_0,\theta_l) \cup [\theta_r, z^*)$. Furthermore, in this domain, we have that $\frac{\partial p(x,y_R^*)}{\partial x} > 0$ and that $\frac{\partial p(x,y_L^*)}{\partial x} < 0$ is bounded. Hence, taking into account Lemma 1 , given $\theta^- \in (\theta_r, z^*)$, there exists $n_1 > 0$ large enough such that (4.4) holds for all $x \in [y_L^*, \theta^-]$.

The proof of $\frac{\partial P_n(x,\sigma_2^*)}{\partial x} < 0$ if $x \in [\theta^+, y_R^*]$ is analogous for n large enough, and follows in a similar way (actually slightly less complicated way, since θ_l and θ_r do not play a role) after dividing (4.4) by $\rho(x)^{(n-1)/2}$. This exercise provides n_2.

In order to prove $ii)$ we should see that there exist $\theta^- \in (\theta_r, z^*)$ and n large enough such that $\frac{\partial P_n(x,y_L^*)}{\partial x} > 0$ for all $x \in (\theta^-, z^*)$. Once again, this condition is given by (4.4), which can be rewritten as (taking into account the signs of the expressions under logarithms)

$$n > 2\frac{\ln\left(-\frac{\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}\right)}{\ln\left(\frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)}\right)} + 1.$$

It is fundamental to notice that the RHS is a continuous function in $(\theta^-, z^*) \cup (z^*, \theta^+)$ (in other words, it only has a discontinuity in z^* in the domain of interest). Once again, we focus on the left side of z^* and later argue that the equivalent study for the other side is completely analogous. Hence, we take the limit of the RHS when x approaches z^* from the left.

First of all, we immediately notice that the substituting z^* in the above expression we get an indeterminacy that allows us to use L'Hôpital's rule. Derivating both the numerator and the denominator we get:

$$\lim_{x\to x^{*+}} \frac{\ln\left(-\frac{\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}\right)}{\ln\left(\frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)}\right)} + 1 = \lim_{x\to x^{*+}} \frac{\left(-\frac{\frac{\partial p(x,y_L^*)}{\partial x}}{\frac{\partial p(x,y_R^*)}{\partial x}}\right)\frac{\partial\frac{\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}}{\partial x}}{\left(\frac{p(x,y_R^*)(1-p(x,y_R^*)}{p(x,y_L^*)(1-p(x,y_L^*))}\right)\frac{\partial\frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)}}{\partial x}} + 1.$$

Trivially (that is why we had an indeterminacy before),

$$\lim_{x\to x^{*+}} -\frac{\frac{\partial p(x,y_L^*)}{\partial x}}{\frac{\partial p(x,y_R^*)}{\partial x}} = \lim_{x\to x^{*+}} \frac{p(x,y_R^*)(1-p(x,y_R^*)}{p(x,y_L^*)(1-p(x,y_L^*))} = 1.$$

Moving on, in Lemma 2 we have proved that:

$$\lim_{x\to x^{*+}} \frac{\partial\frac{-\frac{\partial p(x,y_R^*)}{\partial x}}{\frac{\partial p(x,y_L^*)}{\partial x}}}{\partial x} = \frac{4\sqrt{d}}{(a+1)^2\left(\sqrt{a+1}+\sqrt{d}\right)}. \tag{4.5}$$

And at this point, if we prove that

$$\lim_{x \to x^{*+}} \frac{\partial \frac{p(x,y_L^*)(1-p(x,y_L^*))}{p(x,y_R^*)(1-p(x,y_R^*)}}{\partial x} > 0,$$

we can already assert that the limit of interest is finite. But this is something that we know from lemma 3 (notice that close to z^*, $\rho(x)$ is continuous and derivable).

Hence, given n greater than the finite limit just shown, there exists $\theta^- < z^*$ such that $\frac{\partial P_n(x,\sigma_2^*)}{\partial x} > 0$ if $x \in (\theta^-, z^*)$.

In a completely analogous way, it can be proved the corresponding result for the right side. In fact, working on the opposite inequality of (4.4), that is what we want to proof in the case being discussed, it is immediate that the limit to solve is the same, hence it can be solved in the exact same way. □

A very natural question to ask ourselves at this point is: what happens if $1/2 - \sqrt{d(a+1)} < L$? Let the following result be an answer (we present before a technical lemma and an straightforward corollary).

Lemma 3. *If* $\epsilon \in (0, 1 - z^*]$, *then the following holds:*

$$\hat{x}_R(z^*, z^* - \epsilon) = 1 - \hat{x}_R(z^*, z^* + \epsilon).$$

Corollary 1. *If* $0 < q(z^*, z^* - \epsilon) = \hat{x}_L(z^*, z^* - \epsilon) \leq L$, *then* $q(z^*, z^* + \epsilon) < q(z^*, z^* - \epsilon)$.

The last corollary is just the materialization of a very intuitive concept in the setting we are working on: it is easier to attract consumers of the leftist media outlet by deviating to the left rather than attracting consumers of the rightist media by deviating to the right. However, this effect is offset when the deviation to the left is also able to attract consumers of the rightist media; which is why we have an equilibrium in that case.

Proposition 9. *Let be a game of electoral competition with media, media functions as in (2.2) and vote-share maximizing candidates. If* $1/2 - \sqrt{d(a+1)} < L$, *the profile* (z^*, σ_2^*) *is a MSNE if, and only if, candidate 2 has no chance to get a positive vote-share when facing* z^*.

Proof. If candidate 2 has no chance to get a positive vote-share when facing z^*, then is is clear that any feasible profile involving z^* by candidate 1 is a MSNE. Let us prove the reverse.

It is useful to consider several cases. Notice that $y_L^* < 0 \iff 1/2 - \sqrt{d(a+1)} < -ax_R$ and $1 < y_R^* \iff 1/2 - \sqrt{d(a+1)} < -a(1 - x_R)$; using these conditions we can separate the proof in several cases:

i) Case 1: $y_L^* < 0$. Then, considering $y < x$, the best candidate 2 can do is to propose 0 (clear by looking at the second derivative of $\hat{x}_{L,R}(z^*, y)$). Since $\hat{x}_L = \hat{x}_R + a(x_R - x_L)$ and $\hat{x}_R(z^*, y_L^*) = 1/2 + \sqrt{d(a+1)}$, we have that

$$\hat{x}_L(z^*.y_L^*) < -ax_R + a(x_R - x_L) = -ax_L.$$

Hence, $q(z^*, 0) \leq q(z^*.y_L^*) = 0$. As we have commented, then $q(z, 1) \leq q(z^*.y_R^*) = 0$ as well.

To conclude, in this case candidate 2 has no chance of getting a positive vote-share, hence proposing any $y \in [0, 1]$ and any mixed strategy yields the same expected outcome.

ii) Case 2: $0 \leq y_L^*$ and $1 < y_R^*$. If $1/2 - \sqrt{d(a+1)} + a(x_R - x_L) > 0$ (which can happen), then proposing y_L^* weakly dominates any other policy. Otherwise, if $1/2 - \sqrt{d(a+1)} + a(x_R - x_L) \leq 0$, the choice is irrelevant and it is impossible for candidate 2 to get a positive vote-share.

iii) Case 3: $0 \leq y_L^*$, $y_R^* \leq 1$ and $1/2 - \sqrt{d(a+1)} < L$. So

$$a(x_R - 1) \leq 1/2 - \sqrt{d(a+1)} < L.$$

Once again, if $1/2 - \sqrt{d(a+1)} + a(x_R - x_L) > 0$ (which can happen), then proposing y_L^* weakly dominates any other policy and if not, there is no relevant choice.

□

We can wrap up all the results we have gathered up to this point in this section in the form of the following theorem:

Teorema 1. *Let be a game of electoral competition with media, media functions as in (2.2) and vote-share maximizing candidates. There exists n large enough such that the profile of mixed strategies* (z^*, σ_2^*) *is a MSNE if* $1/2 - \sqrt{d(a+1)} \geq L$. *For any such n, if* $1/2 - \sqrt{d(a+1)} < L$, *then* (z^*, σ_2^*) *is not a MSNE unless candidate 2 has no chance to get a positive vote-share when facing* z^*.

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