Understanding the Cosmic Microwave Background and its relevance in Cosmology

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Abstract: The Cosmic Microwave Background (hereafter CMB) allows the accurate determination of cosmological parameters upon a careful study of its temperature anisotropies. In this work, physical phenomena that give rise to such anisotropies are discussed, along with an exhaustive analysis on the impact of baryon and dark matter density over the power spectrum's shape by using an online simulator. Finally, a sky map, illustrating the spatial distribution of temperature anisotropies, is generated.

I. INTRODUCTION

Photons and baryons were strongly coupled via Thomson scattering in the early Universe. When this plasma cooled down to about 3000 K, the ionisation rate quickly decreased, electrons recombined and photons could freely propagate. In other words, the Universe became transparent. For historical reasons, this period is called recombination. The observed radiation that constitutes the Cosmic Microwave Background (CMB) is, in fact, made up by photons decoupled at recombination.

The CMB radiation follows the blackbody spectrum up to a high degree of precision. Today, its associated temperature is near 2.7 K and it can be detected in any direction. This fact strongly implies that the entire observable Universe was once in thermal equilibrium, and thus in causal contact. However, extremely accurate measurements of the CMB done with COBE revealed that there were small temperature fluctuations, the so called anisotropies[1]. They were imprinted by tiny density perturbations during the epoch that matter and radiation were coupled. It is believed with a high degree of certainty that cosmic structure arises from those density perturbations.

When recombination occurs, photons can travel freely carrying valuable information about the physics of the Universe just before decoupling. Such information can be condensed in the angular power spectrum, a powerful tool that is notably useful to study the CMB anisotropies. Since the shape of the power spectrum depends on global properties of the Universe, cosmological parameters can be estimated with great accuracy using this tool.

In this work, the angular power spectrum (section II) and physical phenomena associated with anisotropies (section III) are discussed. Using CAMB's web interface (an online simulator of CMB anisotropies), graphical results are obtained showing some insight on the power spectrum's shape and its relationship to cosmological parameters, in particular, the baryonic and cold dark matter densities (section IV). Also, a sky map is generated to give a qualitative idea of the scale and intensity of CMB anisotropies (section IV). We end up with the conclusions (section V).

II. ANGULAR POWER SPECTRUM

The quantitative analysis of the statistical properties of the temperature anisotropies requires computing the angular correlation function of the temperature contrast $\Delta T = T - \langle T \rangle$, or rather, its spherical harmonics transform. It yields information about the magnitude of the perturbations and their angular scale.

First, it is necessary to define the normalized temperature Θ in the direction $\hat{n}$ on the celestial sphere as the deviation ΔT from the average: $\Theta(\hat{n}) = \frac{\Delta T}{\langle T \rangle}$. The multipole decomposition of this temperature field in terms of spherical harmonics Y_{lm} is given by the following expression [2]:

$$\Theta_{lm} = \int \Theta(\hat{n}) Y^*_{lm}(\hat{n}) d\Omega \,, \tag{1}$$

where Ω is the solid angle and integration is done over the entire sphere. The asterisk denotes the complex conjugate. The order m describes the angular orientation of a fluctuation mode. The degree (or multipole) l describes its characteristic angular frequency, such that a given multipole corresponds to an angular scale $\theta \sim 180^{\text{o}}/l$. The reciprocal of l is proportional to the so-called fluctuation wavelength.

If the inflationary model is correct, the coefficients Θ_{lm} can be assumed to be uncorrelated random Gaussian variables with zero mean and a variance that is independent of orientation (statistical isotropy). In this way, $\langle \Theta^*_{lm} \Theta_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'}$. The angular power spectrum C_l is then defined by:

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{l} \Theta^*_{lm} \Theta_{l'm'} \,. \tag{2}$$

Only $2l+1$ modes can be used in order to determine C_l. Therefore, when determining the angular power spectrum, there will be an intrinsic statistical uncertainty

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proportional to $\sqrt{\frac{1}{2l+1}}$. This is known as the cosmic variance, which takes the exact form:

$$\frac{\Delta C_l}{C_l} = \sqrt{\frac{2}{2l+1}} \,. \tag{3}$$

As it can be seen in Fig.(1), low multipole values are associated with a high degree of intrinsic uncertainty. It is important to note that the full uncertainty in the power in a given multipole degrades from instrumental noise, finite beam resolution, and observing over a finite fraction of the full sky. Finer technology is needed in order to obtain reliable data for $l > 2000$.

The quantity that is usually plotted against the multipole l, sometimes termed the TT (temperature-temperature correlation) spectrum, is

$$\Delta T^2 \equiv T^2_{CMB} \frac{l(l+1)}{2\pi} C_l \,, \tag{4}$$

where $T_{CMB} = 2.7K$ is the blackbody temperature of the CMB.

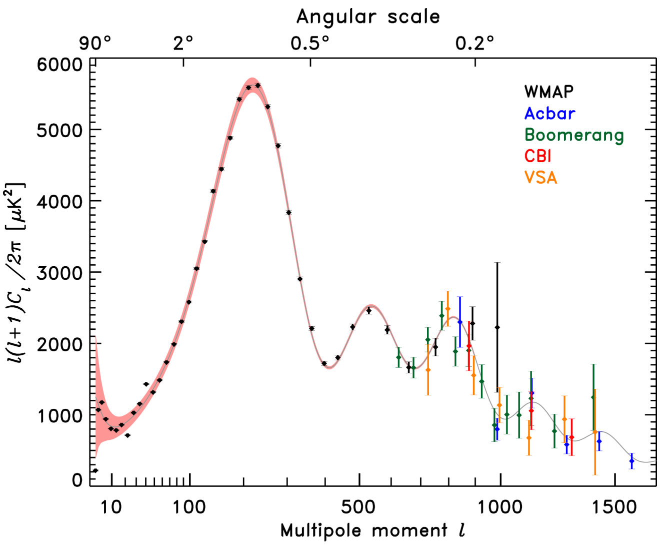

FIG. 1: The TT power spectrum. The red curve represents the best-fit cosmological model to the WMAP data (including cosmic variance). It also includes high-l data from other experiments. Note the multipole scale on the bottom and the angular scale on the top. From [2].

III. PHYSICAL PROCESSES

When discussing the mechanisms of generation of the primordial CMB anisotropies, the three most important physical processes are: the gravitational shift in the frequency that photons experience (the Sachs–Wolfe effect), the acoustic oscillations propagating through plasma due to gravity and radiation pressure and the temperature contrast damping on small scales caused by photon diffusion [4].

A. Sachs-Wolfe effect

For large scales ($l < 100$), the main contributor in shaping the power spectrum is the so called Sachs-Wolfe effect. It may be divided into the ordinary Sachs-Wolfe effect and the integrated Sachs-Wolfe effect (ISW). The former one refers to the energy shift that CMB photons experience when exposed to the gravitational effect of density perturbations at the time of recombination. The energy gained or lost, depending on the sign of the gravitational potential ψ, will induce a temperature contrast which can be observed in the CMB [3].

ISW is the same physical effect, but in this case the time elapsed becomes relevant. It describes the energy shift that photons undergo after recombination when they travel through large massive structures until they reach the earth. This situation is quantitatively different from the ordinary Sachs-Wolfe effect since the potential well presented to the photon is localized in a single region along the path. The blueshift when entering into the potential well would normally be compensated by the redshift when leaving, but large structures evolve with time, meaning that the process is not symmetric. In order to compute the temperature anisotropy due to this effect, an integral over the time variation of the gravitational potential must be done. Thus, the name "integrated" Sachs-Wolfe effect.

This kind of effect is mostly relevant at large scales (or low l) because at smaller scales, photons must traverse many density perturbations and the fluctuations tend to cancel out. The large-scale region in the power spectrum is known as the "Sachs-Wolfe Plateau" (not noticeable in Fig. (1) owing to the logarithmic scale) for its shape.

B. Acoustic peaks

There are two main factors that cause acoustic peaks: gravity and radiation pressure. While the gravitational pull caused by ordinary and dark matter tends to generate high-density baryon zones and enhances collapse, photonic pressure has the opposite effect. This leads to acoustic oscillations in all scales, that is, the photon-baryon fluid has compressing and expanding regions which means that baryon density oscillates constituting sound waves. At t_{rec}, these fluctuations are captured in the CMB and can be detected as anisotropies on the sky. Hot spots in a sky map represent high-density regions that are compressing whereas cold spots represent low-density expanding regions.

The first peak of the power spectrum is the peak with lowest l (or highest wavelength) and it corresponds to a maximum compression phase at decoupling, thus leading to a high temperature contrast in the CMB. The second peak is, in turn, associated to regions that first compressed and then reached maximum rarefaction (maximum expansion) due to radiation pressure. Generalising, odd peaks are related to the compression phase whereas even peaks are related to the rarefaction phase. Since odd peaks are associated with how much the plasma compresses, they are enhanced if the concentration of baryons is increased [5]. This idea will be further developed in section IV.

Assuming adiabatic perturbations, the wavelength of the first peak can be found using the relation $t_{rec}c_s = \lambda_1$, where c_s is the sound speed in the photon-baryon fluid. The angular size of this wavelength viewed from earth is

$$\theta_1 \simeq \frac{\lambda_1}{D_A(t_{rec})}, \quad l_1 \simeq \frac{\pi}{\theta_1}\,, \tag{5}$$

where the angular diameter distance D_A depends on the assumed cosmology of the universe and the scale factor. The position of the first peak computed via Eq. (5) assuming a flat universe yields $\theta_1 \approx 1^{\circ}$ [6]. It agrees with experimental data and provides strong evidence for null curvature. About one degree is, in fact, the angular scale of the particle horizon at time t_{rec} (the distance sound can travel before recombination). Given the high degree of isotropy that the CMB exhibits on all scales and the fact that no causal process could explain that isotropy level beyond 1°, this naturally leads to the "horizon problem". Cosmic inflation offers a remarkable answer to that problem.

C. Silk damping

Silk damping, also referred as diffusion damping, is a physical process that made the CMB more uniform. During recombination, diffusing photons travelled from regions with high temperature to regions with lower temperature, thus reducing anisotropies. The strength of this effect depends on the distance photons travel before being scattered, that is, its diffusion length. A higher diffusion length will ultimately lead to a more damped spectra. This characteristic length, in turn, behaves as $\lambda_D \propto (x_e n_b)^{-1}$, where x_e is the electronic ionisation fraction (nearly 1 at recombination) and n_b is the baryon number density. It is important to note that the more ionised baryons are, the shorter the photons could travel before encountering one and being scattered.

The damping factor, defined as $\mathcal{D} \sim \exp\left[-\left(\frac{\lambda_D}{\lambda}\right)^2\right]$ (where λ is the wavelength of the acoustic mode being suppressed) is a reliable indicator on how "damped" the fluctuations are, with maximum damping at $\mathcal{D} = 0$. Therefore, the higher the value $\left(\frac{\lambda_D}{\lambda}\right)$, the lower the damping factor. Since $\lambda \propto l^{-1}$, the high-multipole part of the power spectrum is severely damped. In particular, Silk damping exponentially decreases anisotropies in the CMB on a scale $l > 800$ known as Silk scale.

IV. ANISOTROPIES AND COSMOLOGY

The influence of cosmological parameters over anisotropies and ultimately, over the power spectrum, will be discussed in this section. Variations of these parameters lead to shape, position and height changes on the peaks.

The Hubble rate is defined as $H \equiv \frac{\dot{a}}{a}$ (where a is the scale factor) and its current value is $H_0 = 100h$ km s^{-1} Mpc^{-1}, where $h = 0.678 \pm 0.009$ is the Hubble parameter [7]. It is linked to the contents of the Universe through the equation:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{\kappa}{a^2}; \quad 1 = \Omega_m + \Omega_\Lambda + \Omega_\kappa\,, \tag{6}$$

where the matter-energy density ρ, the cosmological constant Λ and curvature κ are introduced. The right hand equation is the result of dividing by H^2, thus a dimensionless form is obtained. Ω_m includes baryons (ordinary matter made by protons, neutrons and electrons, denoted by Ω_b) and also dark matter (we will assume the cold type, denoted by Ω_{cd}). Ω_Λ and Ω_κ are respectively related to dark energy and the curvature of the universe. High quality experimental data allows the determination of these parameters: $\Omega_b h^2 = 0.02226 \pm 0.00023$, $\Omega_{cd}h^2 = 0.1186 \pm 0.0020$, $\Omega_\Lambda = 0.692 \pm 0.012$ [7]. In a flat universe model $\Omega_k = 0$.

The CAMB or Code for Anisotropies in the Microwave Background is a Fortran 90 and Python application which computes CMB spectra given several cosmological parameters inputs. It was developed by Antony Lewis and Anthony Challinor in collaboration with NASA's LAMBDA team (`https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm`). Its latest version (April, 2014) was released alongside a web interface that optimizes the computing process and minimizes the user mandatory requirements. Whereas this is the tool used to obtain sky map and power spectra data, gnuplot is the software that allowed its graphical representation. More details about CAMB code can be found at `http://camb.info/`.

The effect of slight baryon density variations can be seen in Fig. (2). Baryon loading enhances odd peaks the same way adding mass in a spring enhances the distance travelled by such mass [8]. Because radiation pressure must overcome deeper potential wells, even peaks are lowered. It can be said that the oscillation presents asymmetry in the sense that the extrema that constitute compressions are increased over those that represent expansion or rarefactions. Because the exact form of higher order peaks depends on several other parameters, the previously mentioned effect can be seen in a concise way in the height difference between the first and second peak. As the baryon number density is increased, the distance becomes higher.

There are two more issues to be considered: oscillation's period and damping. Fig. (2) shows how less baryons imply lower oscillation periods, as well as more damping. The former fact can be explained again using the spring analogy: adding mass will lower the frequency of the oscillations, thus increasing the period. Interestingly, temperature contrast enhancement in odd peaks and lower oscillation frequency have the same cause. Additional damping is explained by the photonic diffusion length dependence on the number density of baryons: $\lambda_D \propto (x_e n_b)^{-1}$. A lower n_b will induce an increased damping effect by enlarging the diffusion length.

The fact that baryons are related to at least the three distinct considered cases taking part in shaping the power spectrum (distance between 1st and 2nd peaks, oscillation period and damping) leads to high precision measurements of Ω_b.

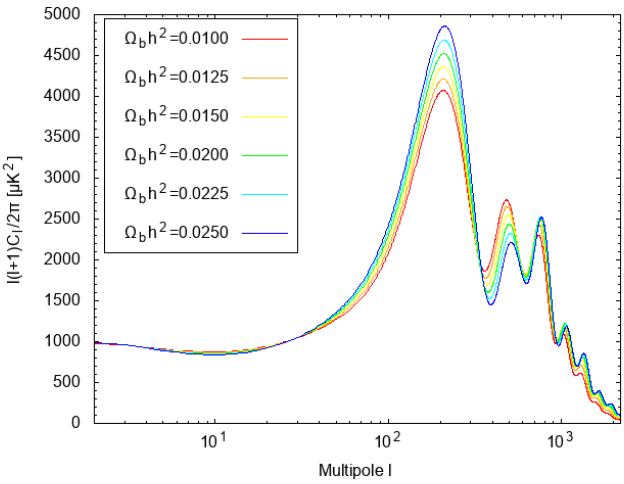

FIG. 2: Angular power spectrum for different baryon densities Ω_b. Cold dark matter density and curvature are fixed, such that $\Omega_{cd}h^2 = 0.11$, $\Omega_\kappa = 0$. Baryonic matter enhances odd peaks and increase the distance between first and second peak.

The most obvious effect of introducing cold dark matter density variations is to reduce or to enhance the overall amplitude of the peaks as it can be seen in Fig. (3). The amount of dark matter plays a relevant role in determining at which time the universe transitioned from a radiation-dominated era to a matter-dominated era. The higher the dark matter density, the earlier it will happen. Gravitational potential wells cannot be considered as fixed at the radiation era. Indeed, the potential decays in such a manner that drives the amplitude of the oscillations up. This happens because the main source of the potential is radiation itself. The complete decay happens at maximum compression, when pressure eventually stops radiation from any further compression. Because the baryon-photon fluid do not experience a gravitational potential anymore, it bounces back with an enhanced amplitude. This is the so called driving effect, and it does not take place when the universe is matter-dominated because the main source generating the potential is not radiation. Because dark matter reduces the radiation-dominated time, an increase in such kind of matter will reduce the driving effect thus reducing peak's amplitude. The regions on the power spectrum that are more sensitive to this effect are the high-multipole regions, since lower wavelengths oscillate earlier and have more exposure to the radiation era.

Because the first and second peaks are highly dependent on gravitational effects caused by photons and baryons, the third peak is the most reliable indicator of dark matter, or, more formally, the dark matter to radiation ratio. A third peak large enough to be higher than the second peak is a strong sign pointing out dark matter domination [9].

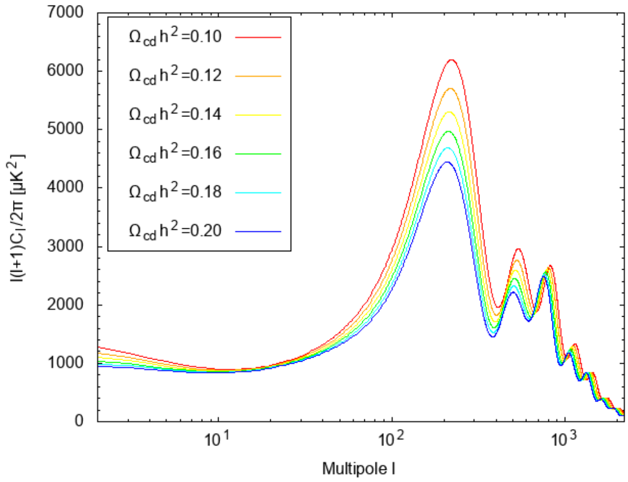

FIG. 3: Angular power spectrum for different cold dark matter densities Ω_{cd}. Baryon density and curvature are fixed, such that $\Omega_b h^2 = 0.02$, $\Omega_\kappa = 0$. Dark matter has an overall suppressing effect on the peaks. For high values of dark matter density the third peak becomes larger than the second one.

In order to obtain sky maps, CAMB uses the Hierarchical Equal Area isoLatitude Pixelisation (or HEALPix) software. Its function is to map a 2-sphere (the surface of a sphere in a three-dimensional space) and to partition the resulting region. This last process is called pixelasing, and it consists in dividing the sphere into 12 base pixels and increase the resolution by the division of each pixel into four new ones.

The output generated by HEALPix is a .fits file (flexible image transport system). Since they have a large combination of variants, reading this kind of files can be complicated if the exact process of their creation is unknown. After several tries, the software that best fitted the purpose of depicting a sky map turned out to be the *fv* software, a FITS file viewer and editor developed at the High Energy Astrophysics Science Archive Research Center (HEASARC) at NASA/GSFC. Given that target users of this software are cosmologists and NASA employees, its interface is rather complex. Adding the fact that the .fits file contained much more than the basic information to generate a sky map, the filtering process was laborious. Eventually, it was possible to plot Fig.(4), where one may graphically see the spatial distribution and scale of hot and cold spots. The universe plotted have $\Omega_\kappa = 0.2$, a non-zero curvature that enlarges the scale of the anisotropies to make them even more visible.

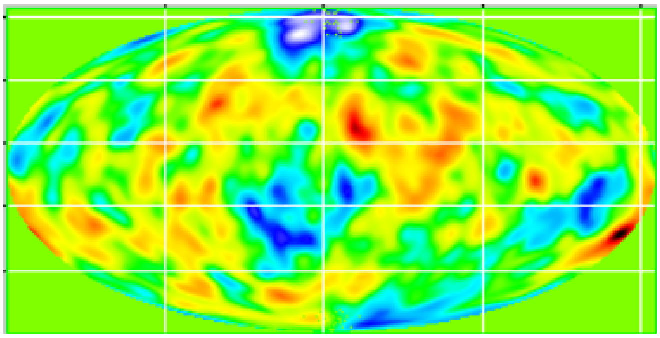

FIG. 4: HEALPix generated sky map plotted using fv software. Red color is associated with cold spots and blue is associated with hot spots. While baryon and dark matter densities have their usual values $\Omega_b h^2 = 0.02$, $\Omega_{cd} h^2 = 0.11$, non-zero curvature is introduced with $\Omega_\kappa = 0.2$, enhancing the scale of anisotropies.

V. CONCLUSIONS

- The CMB constitutes one of the richest sources of information for cosmologists. There are a considerable amount of interesting physical phenomena associated with CMB's anisotropies, the most relevant of which are the Sachs-Wolfe effect, acoustic oscillations and Silk damping. Upon a careful analysis of its power spectrum, a tool that stands out for its clarity, the previously mentioned processes may be identified within the characteristic spectrum's shape and cosmological parameters can be estimated accurately.

- The reverse process, that is, generating several power spectra given different sets of cosmological parameters is done using CAMB web interface. In this way, the underlying physics linked with the variation of parameters are exposed and, ultimately, a better understanding of them is acquired. While baryon loading increases oscillations' period, Silk damping and the height of odd peaks, adding cold dark matter has an overall suppressing effect over peaks' amplitude.

- Given the reduced mandatory extension for this project and the broadness of the studied field, some issues have to be left out. Future work on this topic could include, for example, the effect over the power spectrum of dark energy and curvature variations in addition to further research on the Sachs-Wolfe plateau's shape implications. Sky maps corresponding to universes with different cosmological parameters could be compared and analysed simultaneously with its associated power spectra.

Acknowledgments

Foremost, I would like to thank my advisor Alberto Manrique for his support and because he provided valuable resources and perspective when most needed. My university colleagues fueled the much required motivation with kind words, and I appreciate them. Lastly, the thoughtfullness of my family made possible a work-friendly environment, which is not to take for granted.

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