Methods to detect primordial black holes in astrophysics

Martí Circuns i Duxans
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.[*]

Advisor: Jordi Miralda Escudé

Abstract: Primordial black holes may account for at least part of the dark matter present in the Universe. We study how primordial black holes might be observed through their impact on observable properties of a star formed at redshift $z = 20$ in a low-mass dark matter halo. An analytical expression is presented for the reshaping of primordial black holes' orbits assuming the star formation process as adiabatic. An enhanced density profile is obtained due to dark matter halo contraction in the innermost halo region. It is also discussed how this system might produce a LIGO event (10-100Hz). A long periodical signal is expected for $M_{PBH} \sim 10^{-7} M_{\odot}$ if this system is inside the Milky Way.

I. INTRODUCTION

Primordial black holes (PBHs) were theoretically predicted by Stephen Hawking in 1971 [3]. However, there is still no direct evidence of them as constituents of dark matter (DM). Despite of that, several constraints are applied on PBHs masses by studying the effects they should have in the Universe. For example, a peculiarity of these objects is that they evaporate via Hawking radiation so a constraint on PBHs would be whether they are evaporated or not at the present and the impact that would have this radiation in the observed γ-ray background. Taking into account all the present PBH constraints, a few mass range windows are left. We will focus our attention on the sublunar mass range ($10^{20}g - 10^{26}g$) [3].

A fascinating fact about PBHs is that they may account for at least part of the DM present in our Universe [3]. The cold dark matter (CDM) model predicts, in agreement with observational data, that fluctuations imprinted by the Cosmic Microwave Background (CMB) were very few, so that the Universe can be regarded as nearly homogeneous [5] at that time. Now it is quite natural to ask why the Universe has ended up looking this way; stars, planets, galaxies and so on. Even though in this work we will not answer this question, a deep understanding about DM's nature and its impact on astrophysical systems might help us on that. An important fact about DM is its presence in galactic halos. Several numerical simulations, such as [7], have been carried out in order to find a DM halo profile for observational data to match. Even though a universal DM halo profile has not been found at the present,there are several DM profiles in the literature.

Thus the aim of this work is the following. We want to find whether first star formation would be a suitable scenario in which PBHs, as CDM candidate, would be detected via a LIGO event . In order to do so, the plan of this work follows as: first of all, in section II we will describe and discuss the formation of the first stars inside a DM halo in a CDM cosmogony, in addition to that, we will select a DM halo profile. In section III we will consider that the growth of the star is an adiabatic process and a modified DM halo profile is presented due to halo compression. In section IV we will discuss the limits of the assumptions made through the project, how this work might be improved and the possibility that this system might produce a LIGO event.

[*]Electronic address: mcircudu14@alumnes.ub.edu

II. PBHS AROUND FIRST STARS

A. The birth of the first stars

After the CMB, nearly all the hydrogen left was neutral due to recombination. As DM was not damped, because DM doesn't electromagnetically interact, DM structures formed before than baryonic matter (BM) [5]. In addition, as DM weakly interacts with BM through gravity, DM halos acted as a host for stellar structure formation. Once the gas was able to follow the DM, the gas should have had some coolant mechanism in order to overcome its pressure and contract to higher densities. These stars should also have low or even zero metallicities, thus the only available coolant mechanism was H_2 [6]. Even though H_2 formation was very little, it led to a dramatic change in the gas behaviour because the lowest-energy rotational level of H_2 can be easily reached through collisions with atomic hydrogen. Then the emitted photons due to decay of a rotational level of H_2 carried away internal energy from the gas, leading to a further contraction. When the first star is formed, its own radiation prevents from further accretion or even the possibility of creating more primordial stars [1]. Formation of first stars put an end to the cosmic dark ages, they are the first luminous objects capable of reionising the interstellar medium. This generation of first stars are often called Population III stars and they are rather massive and self-destructive generation of stars. Even more these stars might have enriched the interstellar medium with metals through the first supernovae explosions [6].

However, how first stars formed is still a subject of discussion. Regarding the CDM model, numerical simulations about first stars formation argue that these luminous objects might have been formed inside a low mass DM halo , $10^6 M_\odot$, at redshift $z \sim 20$ [1]. A typical mass of $\sim 100 M_\odot$ was obtained, but its value seems to be affected by the accretion rate [10].

In this work we will regard this scenario; a low-mass DM halo, $M_h = 10^6 M_\odot$, composed by PBHs, where a primordial star formed at redshift $z = 20$. This election will be discussed in the following subsection.

B. DM NFW halo profile

In this subsection we will select a DM halo profile and discuss the scenario that was chosen in the former subsection. This profile will help us on finding an analytical solution in section III.

The Navarro-Frenk-White (NFW) profile, see (1), was first proposed in 1995 for X-ray cluster halos [7]. Even though this profile has singularities, its associated mass and potential converge in halo's center. This profile has the following limits: it is nearly an isothermal profile when the radius is larger than the scale radius r_s, thus the slope behaves as $dln\rho/dlnr \cong -2$. When its radius is smaller than r_s the profile slope behaves as $dln\rho/dlnr \cong -1$. In this work we will focus on the innermost halo regions so that $r << r_s$ and the profile slope is approximated as -1.The NFW halo profile has the following form:

$$\rho_{NFW}(r) = \rho_c \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2}, \tag{1}$$

where ρ_c, δ_c, r_s respectively are the critical density at redshift z, characteristic overdensity of the halo and scale radius. They are determined as follows:

$$\rho_c = \frac{3H_0^2}{8\pi G}(1+z)^3\Omega_m, \tag{2}$$

$$\delta_c = \frac{\Delta_c}{3}\frac{c^3}{ln(1+c) - c/(1+c)}, \tag{3}$$

$$c = r_{vir}/r_s. \tag{4}$$

These parameters (2), (3), (4) describe the DM halo. H_0 is the Hubble constant, G is Newton's gravitation constant, $\Omega_m = 0.3$ is the fraction of BM and DM now present in the Universe, $\Delta_c = 18\pi^2$ is the mean overdensity within the virial radius and r_{vir} is the virial radius. In the previous subsection we have presented the scenario of a low-mass DM halo where a first star is born at redshift $z = 20$. The purpose of choosing this scenario is because its high redshift. The higher the redshift of formation, the higher the critical density, see (2). As high redshifted DM halos are more concentrated one can expect that DM halo's central region will be even more denser. Thus a bigger amount of PBH are expected in a high redshifted DM halo than in a lower redshifted. Also the concentration parameter c has a value of 4 for newly collapsed halos [5].

III. ADIABATIC HALO COMPRESSION

A. Adiabatic invariant

The aim of this section is to study how a spherical density profile might be modified when BM is installed adiabatically into DM halo's center. In order to do so a NFW density halo profile has been selected. We will calculate the reshaping of DM particle's orbit, thus finding a relation between its initial and final orbital radius and eventually we will find its modified mass and density profile near the halo central region were a completely different behaviour of the DM halo profile it is expected. But first, we might take into account some considerations:

Firstly, BM is accumulated in the halo center through a dissipative process to form a star, but the DM will be adiabatically contracted in response to the change in the potential in a non-dissipative process, conserving therefore all the actions invariants.

Secondly, the BM gathered in the potential center is considered as point-like because its size is negligible in comparison to DM halo's size in the central region ($r << r_s$). This point-like mass is not orbiting but at rest at the potential center. The PBH particles are also considered as point-like, in addition to collisionless, and they are orbiting around the potential created by the point-like BM mass. We take the limit in which the number of PBHs tends to infinity and their individual mass to zero. This supposition is valid taking into account that PBH's mass ($10^{-10} M_\odot$) is very little compared to DM halo's mass ($10^6 M_\odot$).

Thirdly, the key supposition, is that PBHs will undergo through a slow evolution in time thanks to the potential generated by accretion of BM. That's due to the fact that the mass accretion rate in halo's center is negligible compared to PBH orbital period, so that we can regard the potential as slowly varying-in-time. When this occurs, the potential evolves adiabatically in time. Considering an adiabatic potential allows us to find an analytical solution to the problem. In addition, another strong consideration is supposing that every PBH has a circular orbit around the point-like BM mass. Then the action variable J_ϕ given by the angular momentum, p_ϕ, due to a circular orbit is: $J_\phi = \frac{1}{2\pi}\int_0^{2\pi} p_\phi d\phi$ and it will not change in a slowly varying-in-time spherical potential [2]. J_ϕ is then called an adiabatic invariant and its value for a spherically symmetric mass distribution is $\sqrt{rM(r)}$.

In conclusion, due to accretion of gas in DM halo's center, DM particles will experience a contraction and consequently they will reshape its orbits but leaving its actions unchanged because this quantity is conserved.

Taking all that into account , it can be found an analytical expression which relates the final orbital radius, r (once BM is fitted inside halo's center) with the initial orbital radius, r_i:

$$r_i M_i(r_i) = r[M_p + M_{halo}(r)], \quad (5)$$

where $M_i(r_i)$ is the halo mass enclosed inside r_i before mass gathering, M_p is the BM accreted and $M_{halo}(r)$ is the final modified DM profile that we are seeking. Equation (5) also show how the potential is modified between an initial halo state and a final halo state in which BM has been adiabatically accreted.

Finally, PBHs will reshape its orbits but the halo mass inside an arbitrary radius will be conserved as this will be a non-dissipative process due to the fact that DM particles are collisionless [5]. Then the total DM enclosed by a spherically symmetric shell of arbitrary radius will be conserved during the halo compression and is expressed as

$$M_i(r_i) = M_{halo}(r). \quad (6)$$

Moreover, putting (6) into (5) we explicitly find that the relation between r and r_i due to adiabatic halo compression is given by

$$r \ = \ \frac{r_i}{1 + M_p/M_i(r_i)}. \quad (7)$$

The only unknown term now is $M_i(r_i)$ and it can be obtained integrating all over the halo sphere with density ρ_{NFW}

$$\begin{aligned} M_i(r_i) \ &= \ 4\pi \int_0^{r_i} r'^2 \rho_{NFW}(r') dr' \\ &= \ 4\pi \rho_s r_s^3 \Big(ln(1 + r_i/r_s) - \frac{r_i/r_s}{1 + r_i/r_s} \Big). \quad (8) \end{aligned}$$

Although NFW density profile behaves as r^{-1} when $r << r_s$, see (1), we are focusing our attention in the innermost central regions of the DM halo where we expect that DM halo density profile will behave in another fashion. That's due to the fact that inside this radius range, the halo density profile will mostly obey the potential generated through adiabatic mass gathering. In other words, we are considering that inside this very central region, the adiabatic potential, characterised by M_p, dominates over the DM halo's potential, characterised by $M_i(r_i)$. Taking $M_p >> M_i(r_i)$ limit into account we can rewrite (8) as

$$M_i(r_i) \simeq 2\pi r_s \rho_s r_i^2, \quad (9)$$

moreover substituting (9) in (7) one obtains

$$r \simeq r_i \frac{M_i(r_i)}{M_p} = \Big(\frac{2\pi \rho_c \delta_c r_s}{M_p} \Big) r_i^3. \quad (10)$$

Substituting (10) into (6) and using $\rho(r) = \frac{1}{4\pi r^2} \frac{dM(r)}{dr}$ we obtain, respectively, the modified halo mass

$$M_{halo}(r) = (2\pi \rho_s r_s M_p^2)^{1/3} r^{2/3}, \quad (11)$$

and modified density halo profile due to halo compression

$$\rho_{halo}(r) = \Big(\frac{\rho_s r_s M_p^2}{108\pi^2} \Big)^{1/3} r^{-7/3}. \quad (12)$$

However, the result we have obtained is correct regarding PBHs orbit as circular and supposing a NFW halo profile. Comments on these main assumptions will be discussed in section IV.

B. Results

In this subsection the results of an adiabatic compression of a DM halo are presented and subsequently we evaluate the amount of DM inside a star formed at $z = 20$.

Equation (11) clearly show a steeper behaviour than NFW density profile does, see Figure 1. As a result of adiabatic mass accretion, DM particles close enough to the innermost halo center will experience a compression by cause of the growth of a slowly varying-in-time potential. Because of halo compression, PBHs located at $M_p >> M_i(r_i)$ are more concentrated than if no adiabatic invariant was considered. In Figure 1 it is represented the fact that due to halo compression, its density profile is enhanced in that limit.

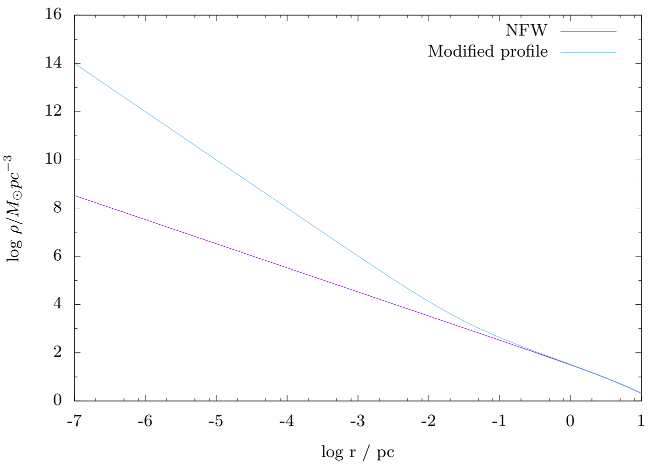

Figure 1: Plot of the DM halo density profile ρ vs radius r. Purple line corresponds to NFW density halo profile. Green line corresponds to NFW density halo profile taking into account the adiabatic invariant.

Another important feature is that related to the number of the nearest PBHs orbiting around the star. By means of Figure 1, it has been checked that the innermost central density region of the DM halo is enhanced as a result of halo compression, thus it is expected that the number of PBHs gathered around the star has increased. Figure 2 shows the number of PBHs within a radius r around the star, for a PBH mass $M_{PBH} = 10^{-10} M_{\odot}$. We observe that the number of PBHs inside this region is also enhanced.

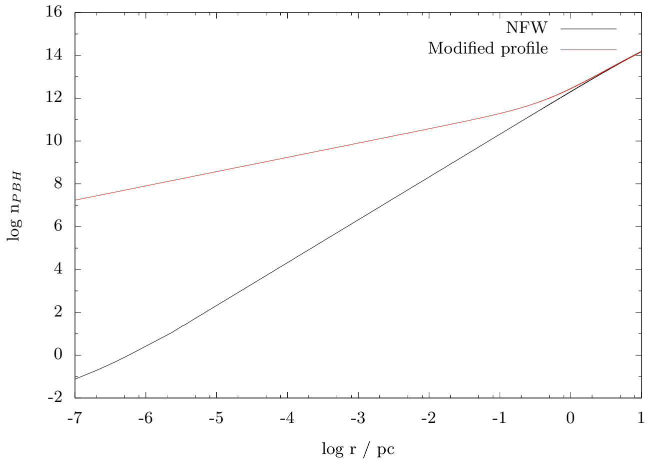

Figure 2: Plot of the number of PBHs n_{PBH} vs radius r. The number of PBHs has been defined as $n_{PBH} = M_{halo}(r)/M_{PBH}$, here both profiles are plotted with $M_{PBH} = 10^{-10} M_{\odot}$. Black line corresponds to NFW density halo profile. Red line corresponds to NFW density halo profile taking into account the adiabatic invariant.

Once the star has entered in the Zero Age Main Sequence (ZAMS), it has a typical radius of $r_{ZAMS} = 3 \cdot 10^{-7} pc$. Therefore, one should expect that some of those PBHs might end up inside the star. Regarding the chosen DM halo in this work, the total amount of DM inside the star can be estimated independently of M_{PBH} by (11). We therefore obtain a $0.0036 M_{\odot}$ inside the star. Also the number of PBHs, n_{PBH}, has been calculated, $36 \cdot 10^6$ and 36000, for two possible PBH mass values, $10^{-10} M_{\odot}$ and $10^{-7} M_{\odot}$, respectively. When the star ends its life, it may form a stellar black hole (SBH) as a result of core collapse. Then the PBH would end in orbit around a SBH. At a later time, the orbit of a PBH may be perturbed, for example by other PBH or any planetary objects or binary star that has formed around the most massive star. If these perturbations reduce the periastron to very small values, the PBH may lose enough energy by emission of a gravitational wave (GW) burst near periastron to reduce its semi-major axis enough, to make its timescale for orbital decay due to emission of GW to be less than the present age of the Universe. We could then see a LIGO event from this merger, which would have the same frequency as the observed LIGO events, but would be of much lower amplitude and last form any more periods because of the very large value of the mass ratio. This will be discussed in the next section.

IV. DISCUSSION

The analytical expressions we have obtained in section II, (12) and (11), are correct regarding PBHs orbit as circular and hence, no radial component has been considered. The supposition of circular orbit provides a steeper slope at the innermost regions of DM halo, see Figure 1. This is due to the fact that the softer the profile is at the central region, the greater velocity dispersion compared with a circular orbit. In essence, most of the PBHs shall not be orbiting the star with a circular orbit, but they shall be in an elliptical orbit, thus they are only temporarily close to the center. Numerical simulations of DM halo particles show that DM particles' velocity anisotropy varies with radius [4]. Also in [4], a parameter $\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$ is defined, where σ_t^2 and σ_r^2 are the tangential and radial velocity dispersion, respectively. It is found that in central regions β is harshly zero. Consequently, it would be more convenient considering velocity isotropy in order to improve our approximation. The assumption of velocity isotropy would then soften the halo profile after adiabatic halo compression. To do so, one should invoke the conservation of the distribution function of two action variables, assuming a spherical potential where orbits are confined within a plane [2].

Another way of reducing the central slope would be considering baryonic effects as spiral bars formation, disk or even planets. For example, the generation of spiral bars produces a time-dependant potential which eventually will led to a random kinetic energy contribution to DM particles (as happens with comets in the Solar system) and PBHs might even end up being expelled form the system. This process would also soften the slope at halo's central region. Nevertheless, it would also carry PBHs inside the star if the PBH suffered a reduction of its periapsis through some kind of perturbation. The moment when the periapsis enters inside the star, the PBH shall end up inside the star, captured.

Also in this work we have selected a NFW halo profile. This profile has a slope -1 when $r << r_s$. Another possibility would have been choosing another DM halo profile more up-to-date as the Sérsic profile which is more softened when DM particles approach to the halo central regions.

Now we shall discuss the fact that possibly a LIGO event would be detected. First of all PBHs orbits suffer a strong contraction due to adiabatic BM accretion. This compression could led to a significant number of PBHs being trapped inside the central star. Friction of the PBH inside the star may reduce its semi-major axis during the star lifetime, helping to reduce the orbital radius, but this is not necessary to cause a LIGO event because in the end, the LIGO orbit can shrink due to emission of GW when the periastron reaches a very small value. When GW are emitted, if there is a notorious mass difference between both BHs, the luminosity in which energy is emitted as GW is proportional to

μ^2, where $\mu \equiv M_{PBH}/M_{SBH}$ and M_{SBH} is the SBH mass. Then the amplitude of the GW is proportional to μ [9]. Regarding the stellar mass discussed in section II and $M_{PBH} = 10^{-7}M_{\odot}$, $\mu \sim 10^{-9}$, for example. The GW emission period would be equal to the PBH period around the SBH and the PBH will lose energy as GW and its orbit will be reshaped. The rate in which PBH gets closer to the SBH is obtained contrasting the orbital energy (proportional to μ) and the luminosity emitted as GW (proportional to μ^2) until the PBH reaches the Innermost Stable Circular Orbit (ISCO). This process should led to a LIGO event ,about 10-100Hz oscillation [8] depending on the SBH mass, because the frequency only depends on the SBH mass. However, the amplitude of these oscillations would be smaller than the GW detected in 2015 by a merge of massive BH binary at a distance of $1Gpc$ [8]. Putting our system at the same distance, the amplitude would be strongly diminished due to its dependence with μ. One possibility would be finding this system inside the Milky Way (MW) sufficiently close to us in order to compensate the small amplitude of the GW. In this situation this system shall be emitting GW periodically thus observing more periods than in the merge of massive BH binary, because of the energy releasing as GW in the last occurs more violently and rapidly. In conclusion, a long periodic signal is expected to arise from the merge of PBHs around the stellar BH inside the MW in this case.

V. CONCLUSION

The conclusions of this work can be summarised as follows:

- We have solved the reshaping of a DM halo profile due to BM accretion assuming this process as adiabatic and spherically symmetric.

- We have realised that the assumption of circular orbits of DM particles should be improved. In order to do that, we must assume velocity isotropy of DM particles near the halo center, $r << r_s$. This would soften the halo density profile after compression, thus the expected number of PBHs would be reduced and therefore the expected merger rate would be also reduced. Furthermore, taking into account baryonic effects such as spiral bars would also soften the slope in those regions through kinetic energy transfer to DM particles.

- We have discussed that in this scenario, a LIGO event (10-100Hz[8]) is plausible if a low-mass DM halo formed at redshift $z = 20$ is found inside the MW when PBHs mass is $\sim 10^{-7}M_{\odot}$. A long signal period is expected in this situation.

Acknowledgments

I would like to thank my advisor Dr. Jordi Miralda Escudé for his advertisements and patience during this process. I would also like to thank my family for their unconditional support.

[1] Tom Abel, Greg L Bryan, and Michael L Norman. The formation of the first star in the universe. *science*, 295(5552):93–98, 2002.

[2] James Binney and Scott Tremaine. *Galactic dynamics*. Princeton university press, 2011.

[3] Bernard Carr, Kazunori Kohri, Yuuiti Sendouda, and Jun'ichi Yokoyama. Constraints on primordial black holes. *arXiv preprint arXiv:2002.12778*, 2020.

[4] Steen H Hansen. Might we eventually understand the origin of the dark matter velocity anisotropy? *The Astrophysical Journal*, 694(2):1250, 2009.

[5] Abraham Loeb. *How did the first stars and galaxies form?*, volume 216. Princeton University Press Princeton, 2010.

[6] Jordi Miralda-Escudé. The dark age of the universe. *Science*, 300(5627):1904–1909, 2003.

[7] Julio F. Navarro and Carlos S. Frenk aSimon D. M. White. A universal density profile from hierarchical clustering. *The Astrophysical Journal*, 490(2):493–508, dec 1997.

[8] A Pisarski, LIGO Scientific Collaboration, Virgo Collaboration, et al. All-sky search for continuous gravitational waves from isolated neutron stars using advanced ligo o2 data. *arXiv preprint arXiv:1903.01901*, 2019.

[9] Bernard Schutz. *A first course in general relativity*. Cambridge university press, 2009.

[10] Naoki Yoshida, Kazuyuki Omukai, Lars Hernquist, and Tom Abel. Formation of primordial stars in a λcdm universe. *The Astrophysical Journal*, 652(1):6, 2006.