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Hydrodynamization and Attractors at Intermediate Coupling

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Abstract

The tremendous success of hydrodynamics in describing the Quark-Gluon Plasma poses many challenges to our understanding of collective phenomena in interacting systems out of equilibrium. Recently the concept of hydrodynamic attractors, which generalize the gradient expansion beyond local thermal equilibrium, has been put forward. In this short paper we present an analysis of this configuration at intermediate coupling. Using holography, we resum large orders in the gradient expansion to constrain the hydrodynamization dynamics and the attractor beyond the strong coupling limit. We find that independent of the coupling, hydrodynamization occurs at early times where the pressure anisotropies are large, and that the attractor is determined by first order hydrodynamics.

Keywords: Hydrodynamics, Attractors, Intermediate coupling, Holography, Resurgence

1. Introduction

Hydrodynamics is an effective description of a system via its stress tensor near local thermal equilibrium, characterised by its local energy density ϵ and the fluid velocity U^μ ,

$$T^{\mu\nu} = (\epsilon + p(\epsilon))\, U^\mu U^\nu + \eta^{\mu\nu} p(\epsilon) + \Pi^{\mu\nu}\,, \tag{1}$$

where $p(\epsilon)$ is the equation of state. The tensor $\Pi^{\mu\nu}$ encodes all deviations of the stress tensor in a given state from local thermal equilibrium. $\Pi^{\mu\nu}$ is expanded in space gradients[1] of the hydrodynamic fields as

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} - \zeta\nabla^\mu U_\nu \eta^{\mu\nu} + \dots\,, \tag{2}$$

where η and ζ are the first order transport coefficients, known as shear and bulk viscosities, and $\sigma^{\mu\nu}$ is the shear tensor, constructed from the symmetrised and traceless space gradient of the velocity field. Here the ellipses indicate additional gradient corrections which when trunctated will be refered to as *viscous hydrodynamics*.

[1] These are defined as the projection of the space-time gradient into the spatial components in the fluid rest frame, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$, where $\Delta^{\mu\nu} = \eta^{\mu\nu} - U^\mu U^\nu$.

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In the past decades, viscous hydrodynamics has been found to be a good description of the Quark Gluon Plasma as found in ultra-relavisitc heavy ion collisions [1, 2], even (surprisingly) in regimes where large higher order gradient corrections should render a low order trunctation of the expansion invalid. This begs the question, why does viscous hydrodynamics appear to work so well outside its naive regime of applicability?

We aim to provide insight on this discussion in the context of holography, where several recent numerical studies have suggested that in the infinitely strongly coupled limit of $\mathcal{N} = 4$ SYM, hydrodynamics will accurately describe a system's evolution even when "sub-leading" gradient terms are as large as leading order. We will further study the effects of finite coupling corrections to the infinite coupling limit via the gauge/gravity duality by introducing higher curvature terms in the dual gravitational theory. In particular we will consider Gauss-Bonnet gravity, an action including both quadratic and quartic curvature terms, governed by a single parameter λ_{GB}. While the dual field theory to Gauss-Bonnet gravity is unknown it is known to have qualitatively similar thermodynamic, transport and relaxation properties to finitely coupled $\mathcal{N} = 4$ SYM for negative values of λ_{GB}. A number of causality, positivity of energy and hyperbolicity considerations further constrain the range of values of this parameter to be $\frac{7}{36} \leq \lambda_{GB} \leq \frac{9}{100}$ [3]. Nevertheless, since these constraints concern the ultraviolet behaviour of the theory, we will still consider values of λ_{GB} outside this bound to explore infrared dynamics such as the approach of solutions towards hydrodynamics.

Under the assumptions of boost invariance and transverse homogeneity a solution to ideal hydrodynamics (when $\Pi^{\mu\nu} = 0$) can be found, known as Bjorken Flow. Both $\mathcal{N} = 4$ SYM and the dual theories of Gauss-Bonnet Gravity exhibit conformal symmetry which dictates that the local temperature and energy density will be given respectively by $T(\tau) \propto \tau^{-1/3}$, $\epsilon \propto \tau^{-4/3}$. Similarly all gradient corrections can be expressed in terms the temperature $T(\tau)$, the only scale that characterises the state at a given proper time τ. Therefore, independent of the microscopic behaviour of the underlying theory, the energy density $\epsilon(\tau)$ can be expressed as a series of negative fractional powers of the proper time. Further, with these particular symmetries, the stress tensor can be completely fixed in terms of the energy density [4]. Defining the longitudinal (P_L) and transverse (P_T) pressures as the diagonal components of the stress tensor in the fluid rest frame in the direction of expansion and perpendicular to it respectively, these are given by

$$P_L = -\epsilon(\tau) - \tau\dot{\epsilon}(\tau)\,, \quad P_T = \epsilon(\tau) + \frac{1}{2}\tau\dot{\epsilon}(\tau)\,, \tag{3}$$

which are valid independent of whether the system is far from local thermal equilibrium. We can gauge the system's approach to local thermal equilibrium by computing the anisotropy parameter $R \equiv 3\frac{P_T - P_L}{\epsilon}$, where the equilibrium pressure has been used to normalise the pressure anisotropy. We can write R as a function of the dimensionless gradient, $w = \tau T$ where out of equilibrium we can identify the effective temperature as $T(\tau) \sim \epsilon(\tau)^{1/4}$. In the hydrodynamic limit, R admits an expansion in inverse powers of w as

$$R = \sum_{n=1} \frac{r_n}{w^n}\,. \tag{4}$$

r_i are dimensionless constants which are solely dependent on the degrees of freedom and transport coefficients, and are therefore a property of the theory.

2. The gravity dual of Bjorken Flow

One can search for solutions to the Gauss Bonnet field equations with negative cosmological constant $\Lambda = -6$ using an ansatz of the form,

$$ds^2 = -r^2 A(\tau, r)d\tau^2 + 2drd\tau + (r\tau + 1)^2 e^{b(\tau,r)}dy^2 + r^2 e^{c(\tau,r)}dx_\perp^2 \tag{5}$$

where $\tau = \sqrt{t^2 - z^2}$ and $y = \operatorname{arctanh}(z/t)$ are the standard proper time and rapidity coordinates and the asymptotically AdS boundary is located at $r \to \infty$. Motivated by the late time behaviour of the local temperature we scale our radial co-ordinate as $s = \frac{1}{r}\tau^{-1/3}$, and fix $u = \tau^{-2/3}$ with the expected form of gradient corrections at each order in the expansion. We assume the metric coefficients have a power series expansion of the form,

$$A(\tau, r) = \sum_{i=0} u^i A_i(s), \quad b(\tau, r) = \sum_{i=0} u^i b_i(s), \quad c(\tau, r) = \sum_{i=0} u^i c_i(s). \tag{6}$$

With this assumption, the problem simplifies to solving a coupled set of ordinary differential equations (ODE's) in s subject to AdS asymptotics at $s \to 0$ and regularity at the horizon, which can be fixed at $s = 1$ via a choice of radial gauge. The energy density can be found from evaluating the solution at the boundary $\epsilon(\tau) \propto \lim_{s\to 0} u^2 \sum_i u^i \partial_s^4 A_i(s)$ (where the normalization factor will not matter for our analysis), from which the coefficients r_i can be computated for a variety of coupling strengths. This analysis follows a similar approach to [5] which first computed this series for $\lambda_{GB} = 0$, and a more detailed description of our calculations are contained in [6]. Similar approaches have been made towards studying the large order gradient expansion at both strong and weak coupling in [7, 8, 9], M. Strickland, these proceedings, and J. Noronha, these proceedings. One finds that for $\lambda_{GB} = 0, -0.1, -0.2, -0.5$ and -1 that the coefficients r_n grow as $n!$ so that the series Eq. (4) will diverge.

3. Resurgence and the Attractor

A standard technique known as Borel summation can be invoked to make sense of these formally divergent series'. The Borel Transform of R, defined as $R_B(\xi) = \sum_{n=1} \frac{r_n}{n!}\xi^n$ will typically have a finite radius of convergence that can be analytically continued through the complex plane. One can recover a sensible resummation of the anistropy function through the Inverse Borel Transform,

$$R_{resum}(w) = w \int_C d\xi \, e^{-w\xi} R_B(\xi) \,, \tag{7}$$

where C is a contour in complex plane which connects $\xi = 0$ and $\xi = \infty$. The presence of singularities in R_B will result in different resummations for different choices of C. The residues of these singularities can be used to recover linearized non-hydrodynamic (non-perturbative) solutions which are not described by our power series ansatz in Eq. (6). In Figure [1] we have picked some choice of resummation to give the dark grey curve in every subfigure. To this curve we add different linear combinations of all non-hydrodynamic modes (given by the colourful curves) which we have recovered from the Borel Transform of each theory. Each one of these curves can be interpreted as the evolution of the full system given by different choices of initial conditions, and the typical spread of the collection of curves gives an indication as to when the system will have typically relaxed to a common curve.

4. Conclusion

Fascinatingly, in all cases presented the typical relaxation time of the transient non-hydrodynamic modes occurs at time scales where $\frac{w}{4\pi\frac{\eta}{s}} < 1$, and for anisotropies $R(w) > 1$, when the gradients of the system are large. Despite this fact, in each case this common curve (which we will call an estimate of an attractor solution in each theory) is comparible to viscous hydrodynamics, given by a red dashed curve. In the cases of $\lambda_{GB} = 0$ and $\lambda_{GB} = -0.2$, which have the most phenomenologically relevant (for the QGP) choices of $\frac{\eta}{s}$, first order hydrodynamics barely deviates from this attractor in the region after the transient non-hydrodynamic solutions have relaxed to it.

A lesson to draw from this analysis is that the estimate of the naive regime of applicability of truncated hydrodynamics relies implicitly on the convergence of the gradient expansion. In the absense of this fact the applicability of hydrodynamics can only really be determined by comparing tunctations of the gradient expansion to the fully resummed theory. This explicit comparison indicates that if one were to use low order hydrodynamics at early times to model a boost invariant, transversely unifrom and approximately conformal medium of $\mathcal{N} = 4$ SYM or Gauss Bonnet plasma, (say, at the center of the fireball), the error made due to neglecting higher order gradients would be surprisingly small.

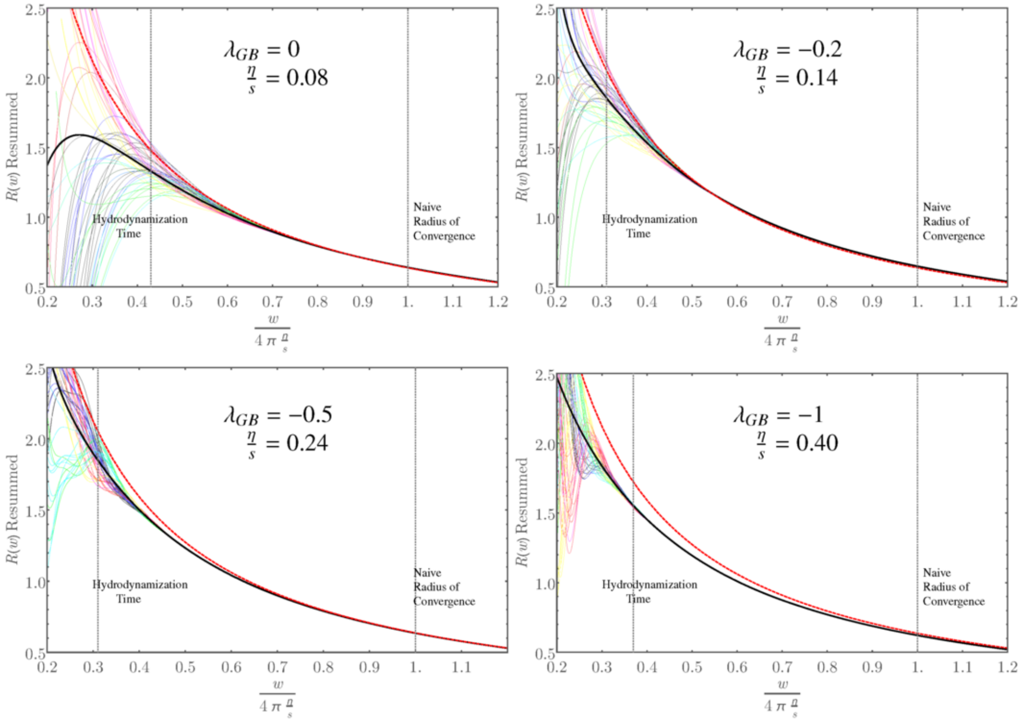

Fig. 1. Resummations of the anisotropy $R_{resum}(w)$ for selected choices of λ_{GB}. The dark gray line corresponds to a particular choice of resummation contour C with other possible choices being encoded in the linear fluctuations around this solution given by the colourful curves that lie on top of it. The spread of these solutions indicates the typical timescale at which the evolution of the system relaxes to a common curve. On each subfigure we include a red dashed curve given by viscous hydrodynamics, $R_{Hydro}(w)$, the first order t/runctation of Eq. (4). Dotted gray vertical lines indicate the *naive hydrodynamic radius of convergence* placed at $\frac{w}{4\pi\frac{\eta}{s}} = 1$ (before which we expect higher order gradient terms to be non-negligible in the hydrodynamic expansion) and the *Hydrodynamization time* which is given as the value of $\frac{w}{4\pi\frac{\eta}{s}}$ at which $R_{resum}(w)$ and $R_{Hydro}(w)$ deviate by 10%.

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