Quantum theory: so precise, so astonishing

Juan Carlos Paniagua

jpaniagua@ub.edu

Departament de Ciència de Materials i Química Física &
Institut de Química Teòrica i Computacional (IQTC-UB)
Universitat de Barcelona

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Physics status prior to QM

- Albert Michelson (1899, USA first physics Nobel laureate in 1907):
  - *The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.*
- Planck, Nernst, Rubens and Warburg about Einstein (1913):
  - *That he might have in his speculations, occasionally, overshot the target, as for example in his light quantum hypothesis, should not be counted against him too much; because without taking a risk, even in the most exact science, one is not driven to real innovation.*

so precise...

- *Extraordinary coincidence between predictions and experiments*:

  - g_e = -2.002 319 304 361 82 (2014)

    $\Delta g_e/g_e = 2.6 \times 10^{-13}$

  - for $l = 1$m $\Delta l = 2.6 \times 10^{-13}$ m ≈ H diameter / 400

  - no known experiment (from cosmology to the smallest known scale) contradicts QT

so many predictions...

- *The quantum laws work so well that scientist blindly trust in every new discovered consequence of them*:
  - antimatter, induced emission, lasers, BEC (1924, 1995!),
  - high-density hard discs (GMR,TMR), flash memory,
  - nanotechnology, superconducting quantum interference devices (SQUIDS) ($B \sim 5\times10^{-18}$ T),
  - semiconductor electronics, chemical properties and reactivity, spectroscopy (NMR),
  - inviolable signal transmission, teleportation, quantum computing, weak measurements, …

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Some recent breakthroughs

- *Weak measurements* (1964-1988-2009) allow to detect:
  - $\Delta\varphi$ of a mirror = 4 x 10^{-13} rad (width of a hair in the moon)
  - Δx = 2 x 10^{-14} m (*U* nucleus diameter)
- *Optical lattice clock* (2018): 1s in 1.3 x 10^{11} years (≈ 4 x 10^{18}s). Universe age ≈ 1.4 x 10^{10} years!; gravitational time dilation for an elevation change of 0.2 mm!)
  - GPS atomic clocks: 50 ns/day ≈ 1s in 6 x 10^{13} s

…so astonishing

Classical physics:

- *continuous* properties

Quantum physics:

- *quantized* properties

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- light waves and matter particles

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- initial x and p of particles + forces

  → *trajectories*

- intuitive phase space (x-p)

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- locality, separability, (*isolability*)
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Classical physics (*intuition*): Quantum physics:

Classical physics	Quantum physics
• *continuous* properties	• *quantized* properties
• light waves and matter particles	• *photons, particle interferences*
• *determinism* (*cause&effect*→predictability)	• *indeterminism* (*tunnel* effect, …) (*)
• initial x and p of particles + forces → *trajectories*	• *incompatible* observables (x-p) → *wave functions*
• intuitive phase space (x-p)	• abstract *Hilbert space* (Ψ)
• negligible measurement effects	• *unavoidable* measurement effects
• a measurement *reveals a reality* indep. of the observer (*objectivity*)	• a measurement *creates the reality* (*subjectivity*) (*)
• locality, separability, (*isolability*)	• *non-locality*, non-separability (*) → entanglement, teleportation,…
• identical particles are *distinguishable*	• identical part. are *indistinguishable*

(*) an attack on the pillars of natural sciences!

Warning

- Some of the topics I will present pose *questions with no clear answer*... but don't worry about that: *the rules for applying QM to real problems are perfectly clear*, although the physical interpretation of some points is definitely not.

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- Bohr: *Anyone who is not shocked by quantum theory has not understood it.*
- Feynman (1918-1988), one of the twentieth-century physicists with a deeper understanding of QT:
  - *I think I can safely say that nobody understands quantum mechanics.*
  - *The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you accept Nature as She is — absurd.*

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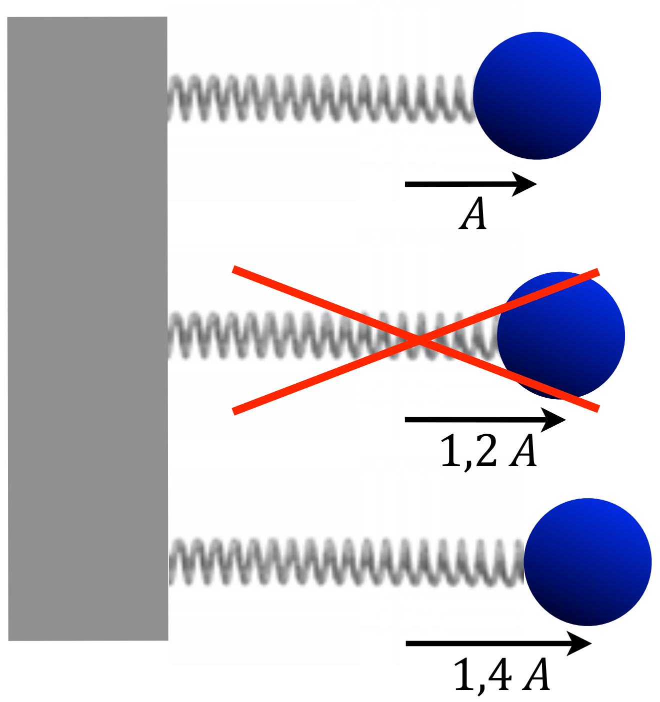
  - *The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you accept Nature as She is — absurd*.
- But *that's the wonder of QM*: it makes it a box of surprises that a century after it was founded still keeps unveiling surprising consequences (*teleportation, quantum computing*...) that will surely provide important and unexpected technological advances.

Quantization

- Classical harmonic oscillator:
  - $E \propto A^2$ (continuous)
  - minimum $E = 0$
- Quantum harmonic oscillator:
  - discontinuous E
  - minimum $E > 0$



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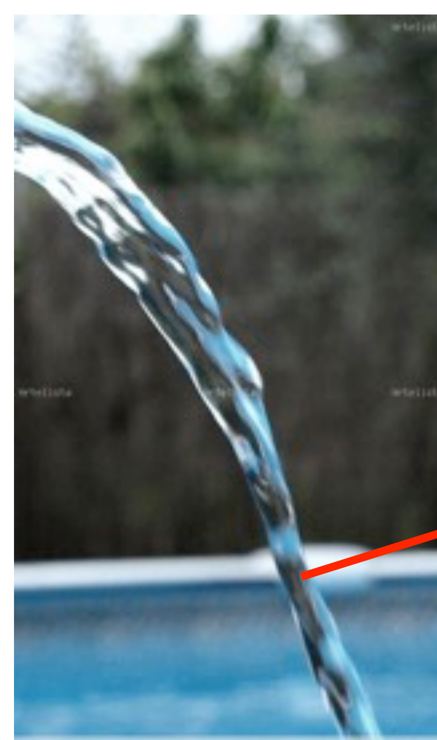
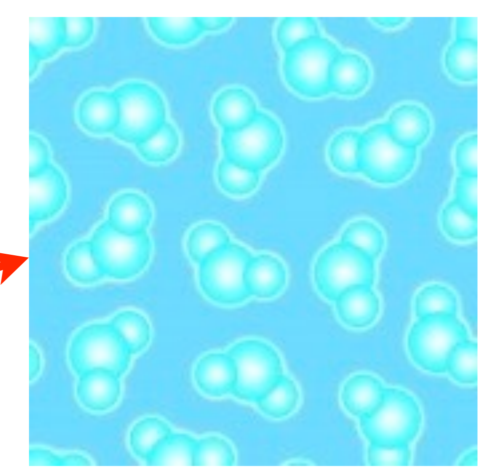
Quantization

- Quantization is not incompatible with daily observations, based on large objects.

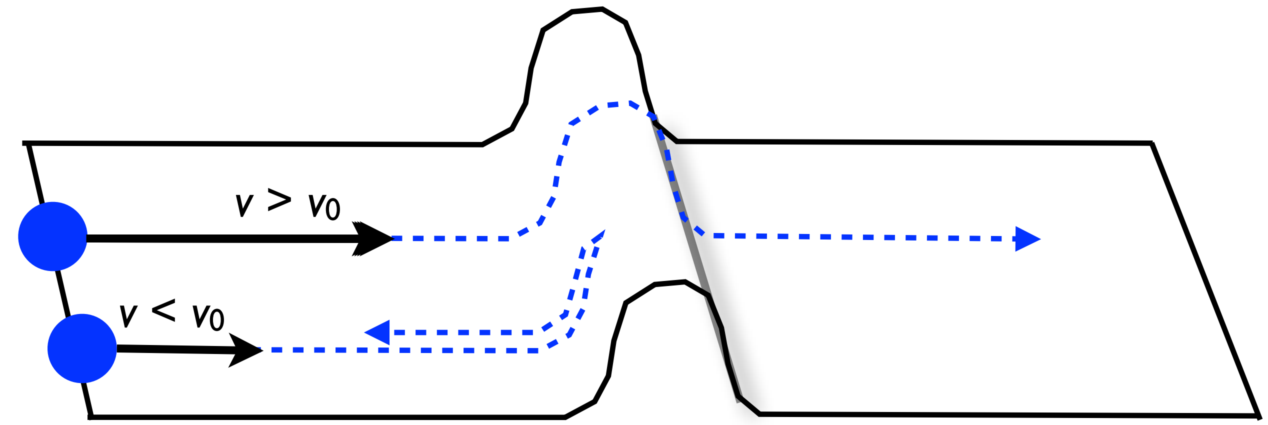
- Apparent continuities were already known in the classical world:
  - Egypt pyramids
  - a water jet
  - a light ray
  - a pendulum…



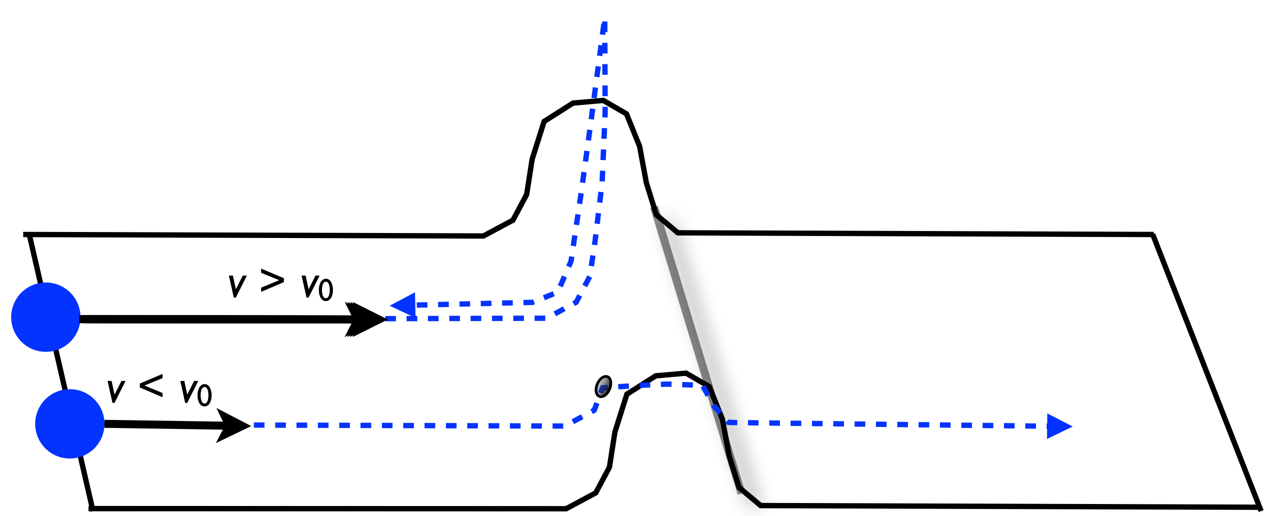


Indeterminism

- Classical mechanics:
  - $x(0), v(0), F \rightarrow x(t), v(t)$



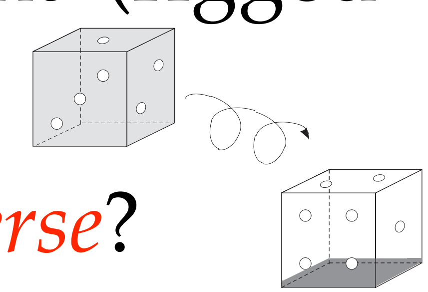
- Quantum mechanics:
  - several outcomes are possible for the same initial conditions
  - potential barrier
  - α particles existence??



Classical indeterminism?

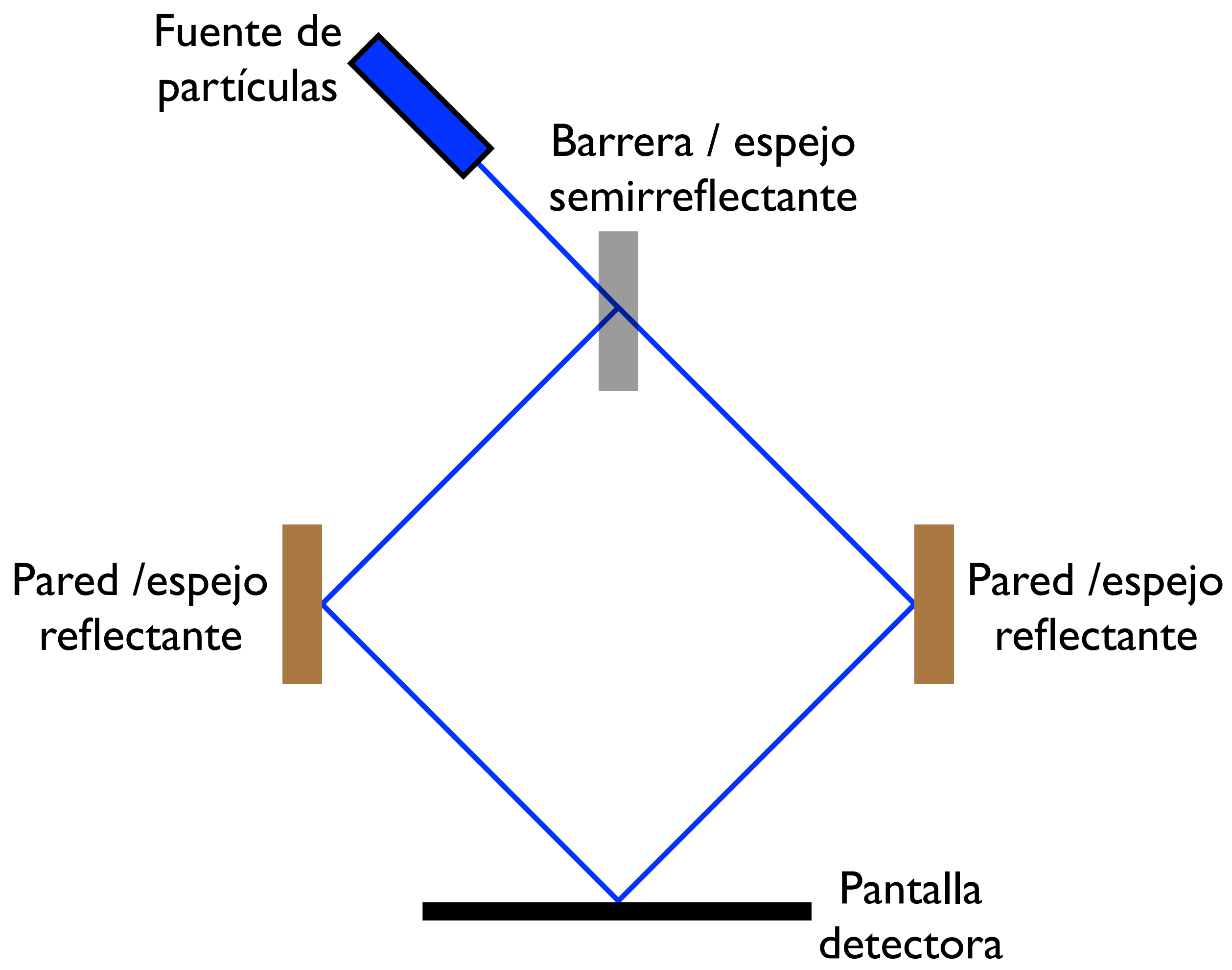
- Rolling the dice. 
- The length of the catalan coast ≈ 580 km.
  - *Fluctuations* as those due to water waves? Quantum uncertainty has nothing to do with fluctuations over time (stationary states).
  - The coast length is *intrinsically* indeterminate because it is ill-defined: even at a given time the edge of the land is diffuse:
    - wet sand? rain? rivers?

- Quantum indeterminacy *disappears* after a measurement (rigged die)
- Quantum uncertainty → *freewill*? *a predeterminate universe*?



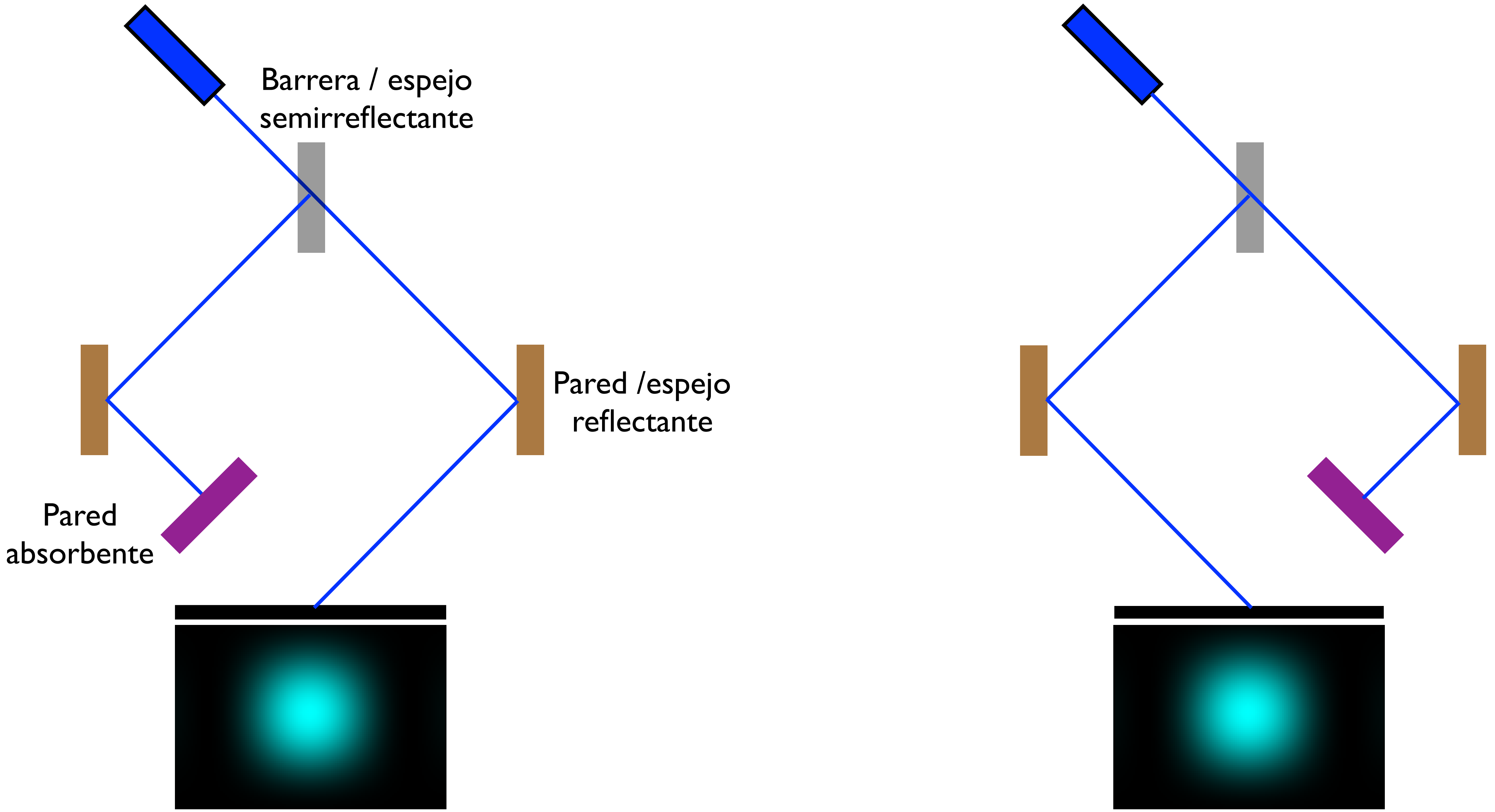
Is the position of a particle something *real* before it is measured?

- A modified potential barrier experiment:



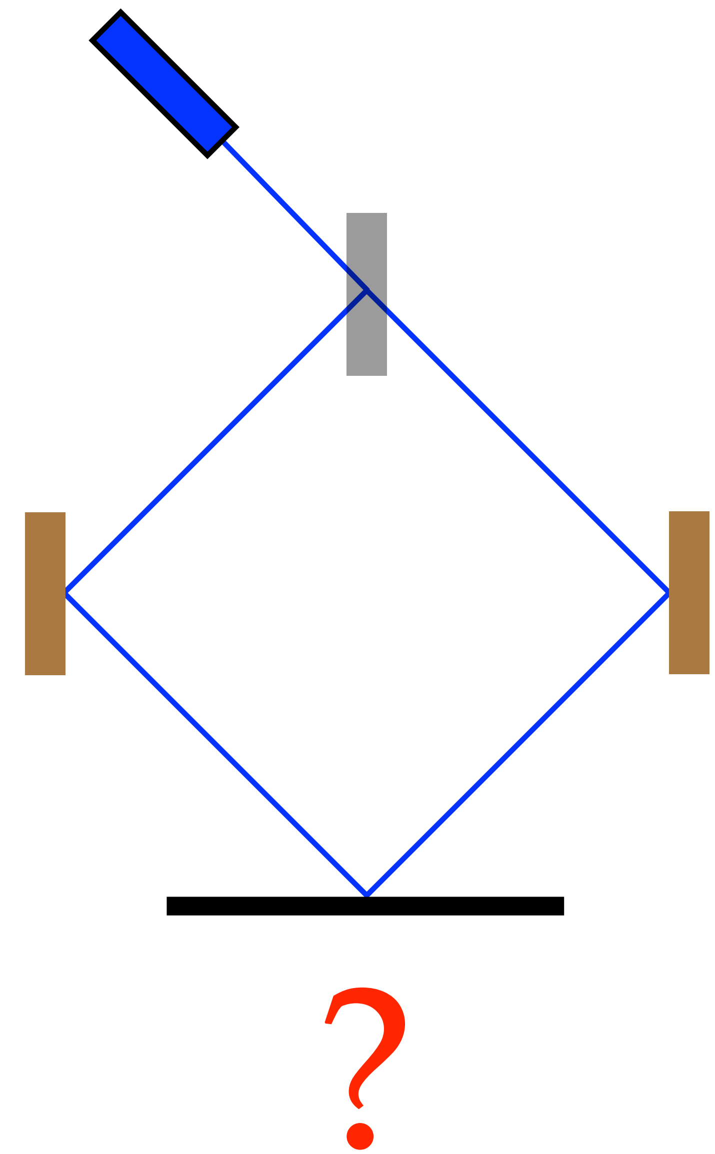
Is the position of a particle something *real* before it is measured?

- Let us block one of the 2 paths:



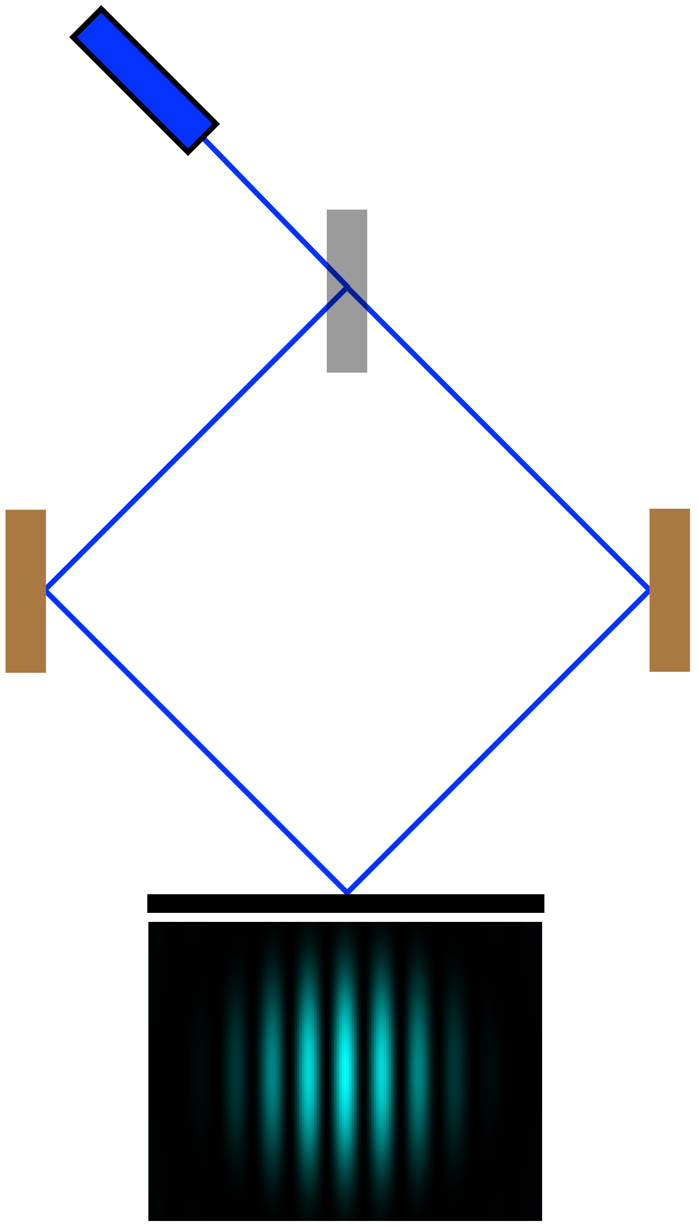
What should happen when the 2 walls are removed?

- The particle is *transmitted or reflected* by the potential barrier, so the screen pattern should be the sum of the 2 previous screen patterns...



What should happen when the 2 walls are removed?

- The particle is *transmitted or reflected* by the potential barrier, so the screen pattern should be the sum of the 2 previous screen patterns…
  - but the result is completely unexpected!
  - although not so unfamiliar …



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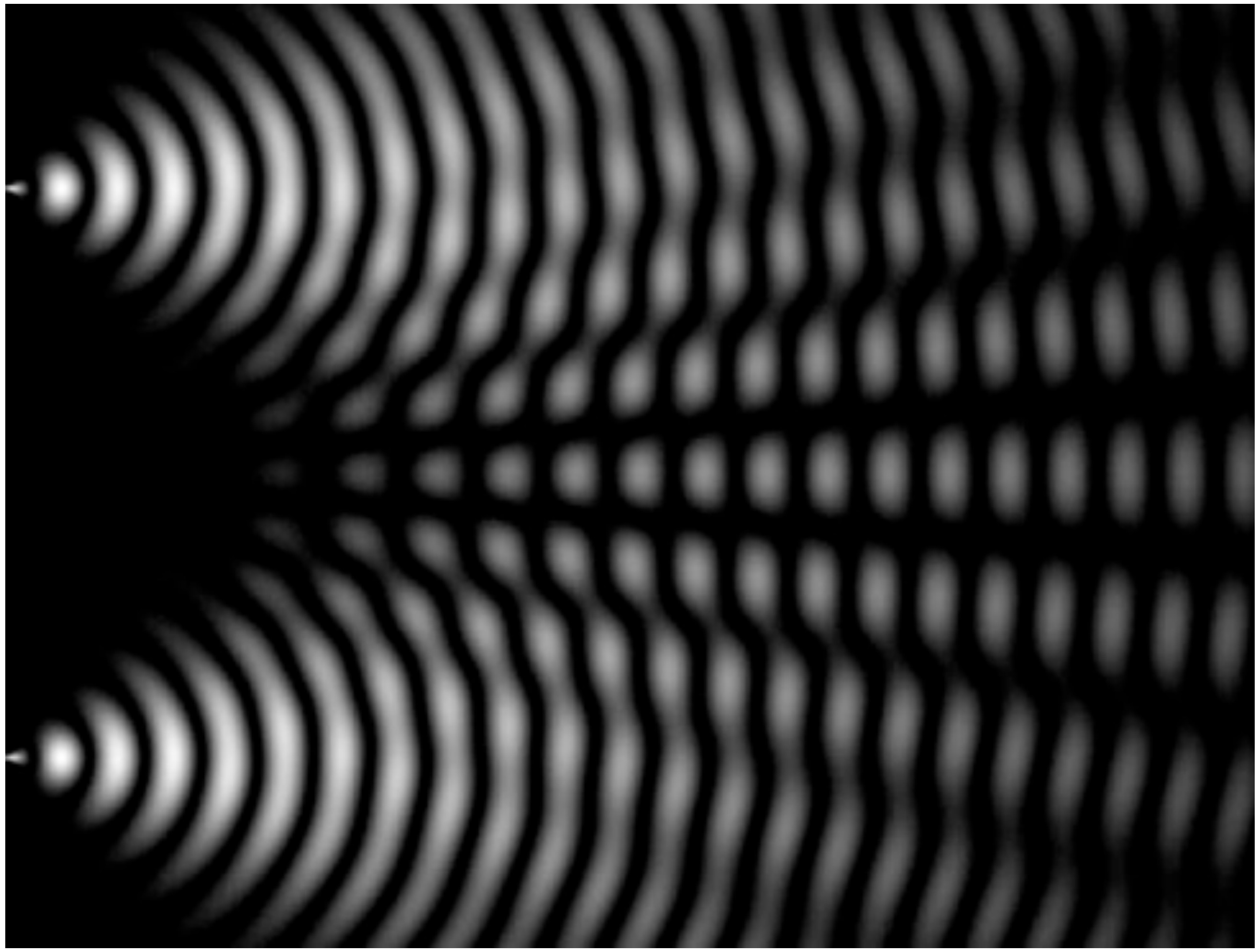
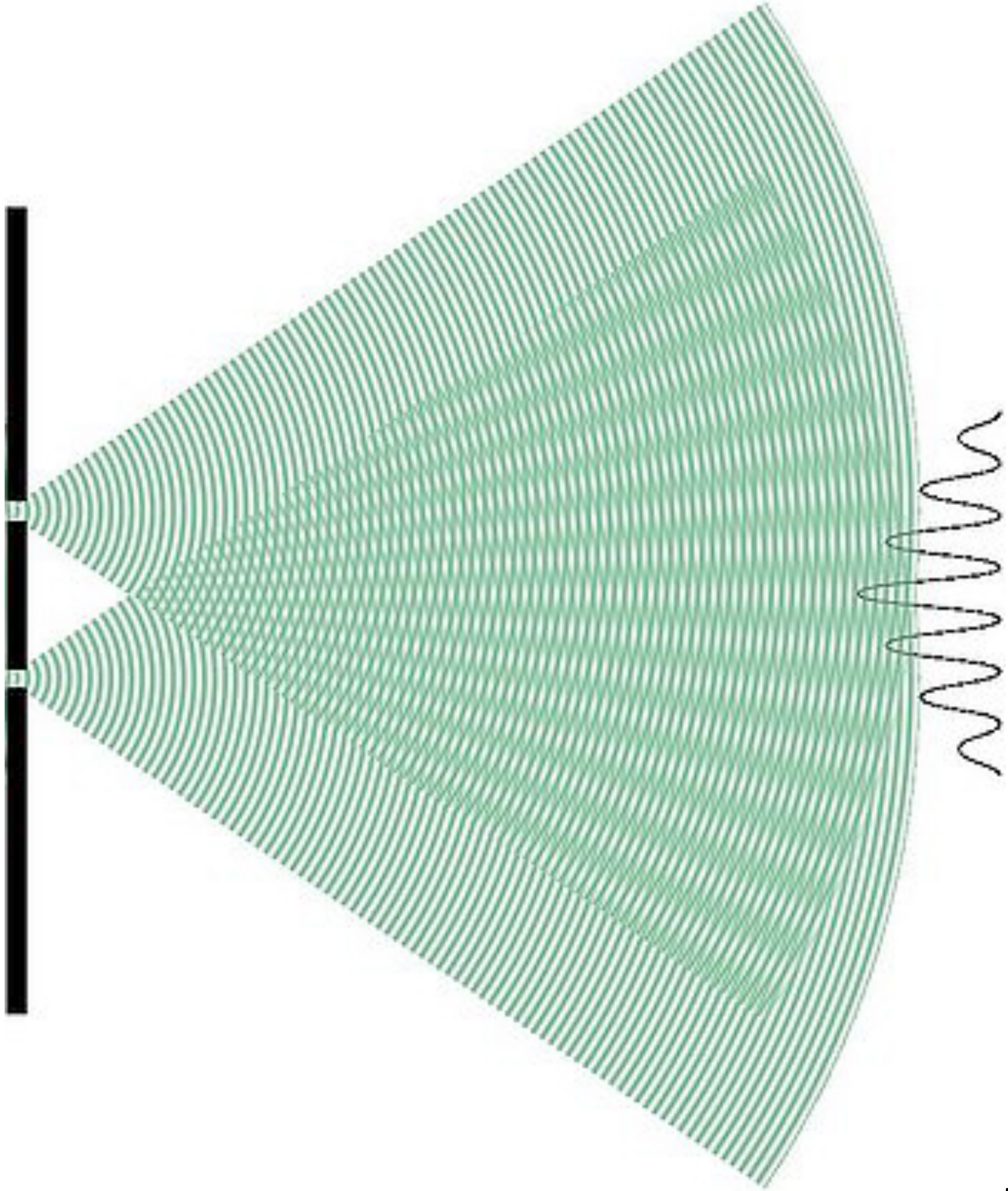
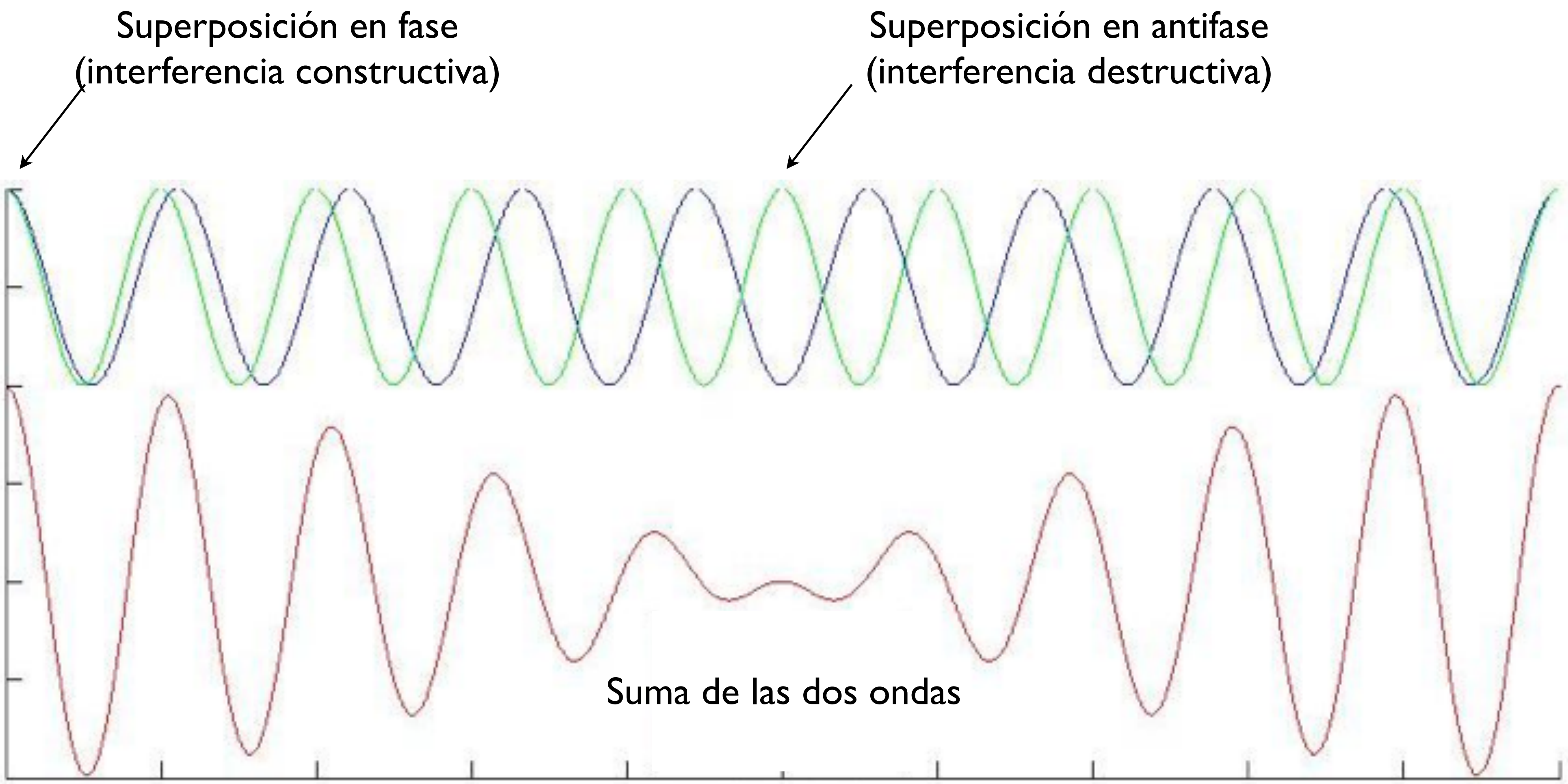
Wave interferences



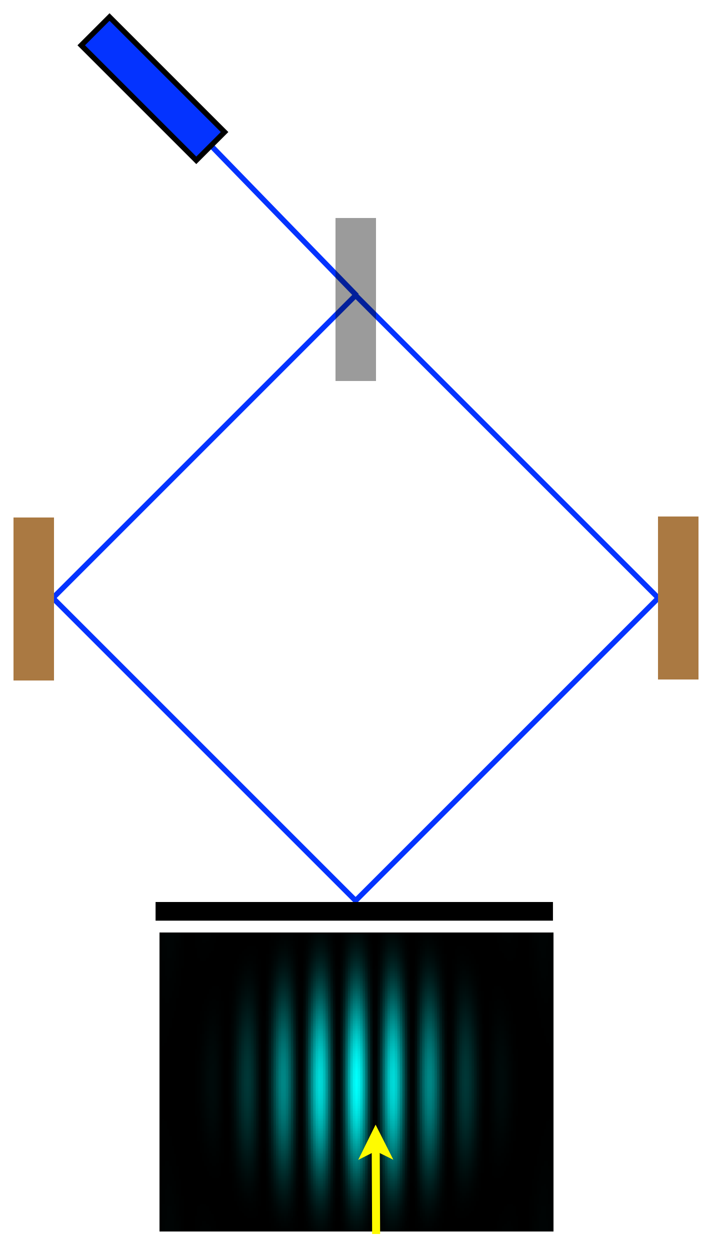


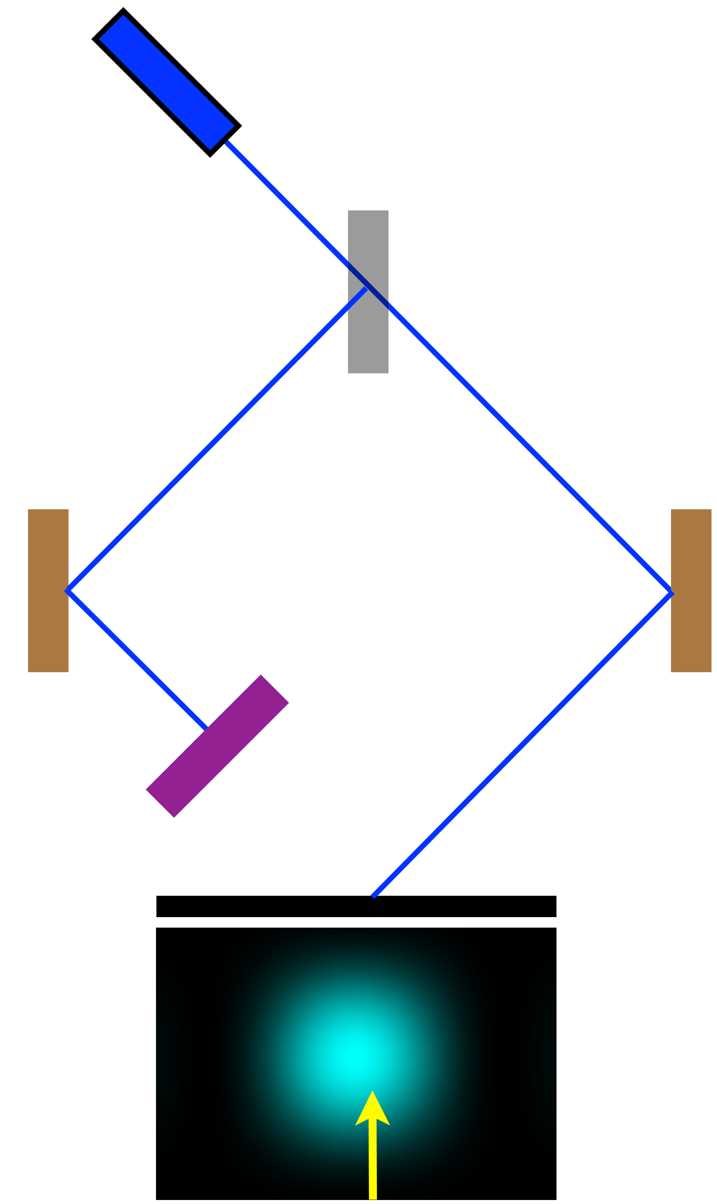
Wave interferences



Wave + wave can vanish, but …

NO particles arrive at some points that were frequently reached when one of the paths was blocked, then…





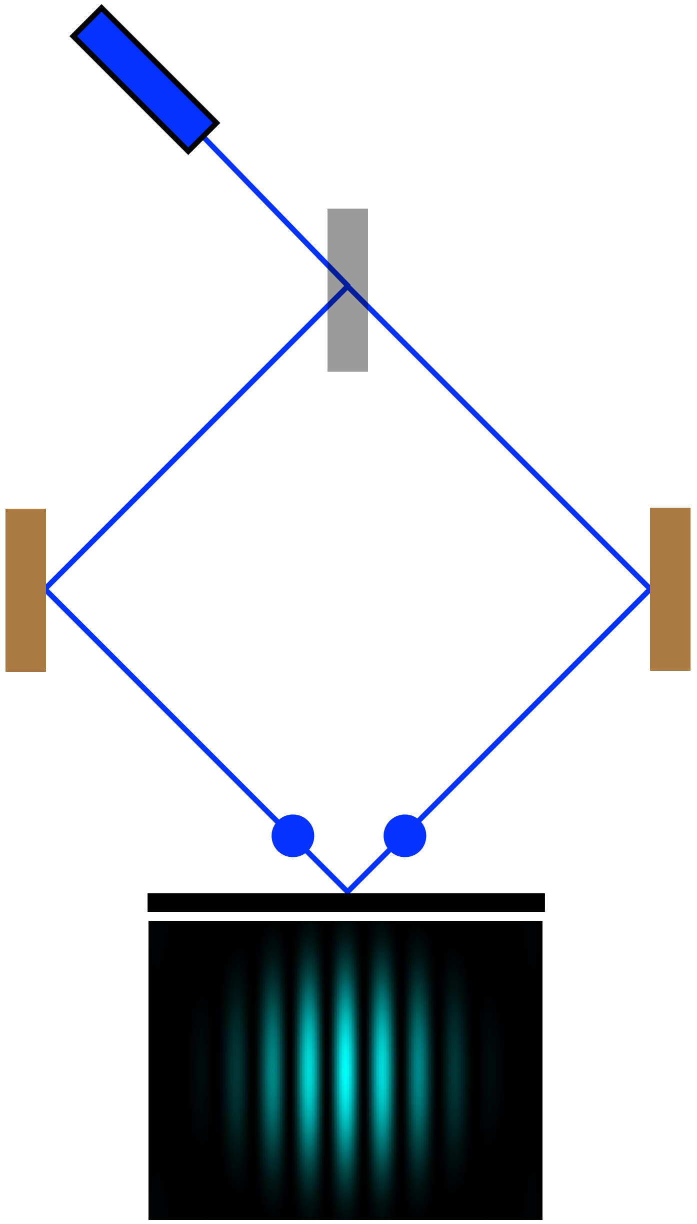
wave + wave can vanish, but …

particle + particle = no particle?

- Any explanation?

Particles may interact...

- The interference could be produced by some *interaction between particles* going through different paths and meeting at the screen (collisions?)

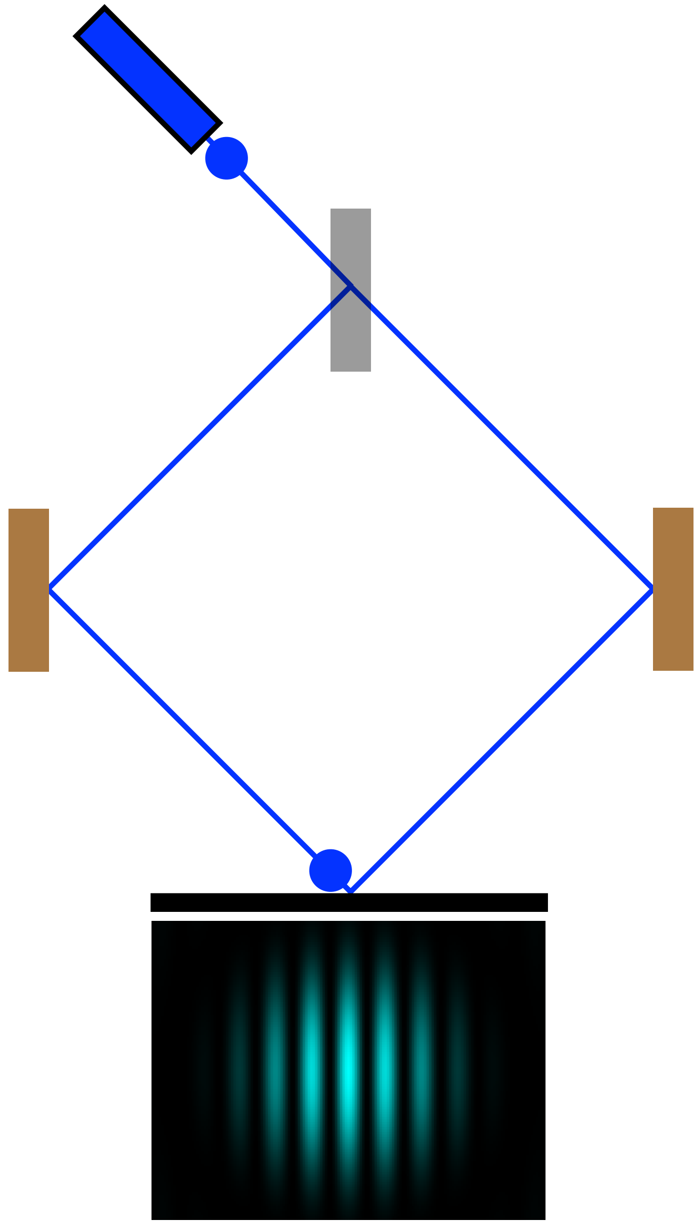


- How could we verify this?

Interactions do not explain it

- The same interference is obtained by accumulating the results of experiments done with individual particles (*one by one*).

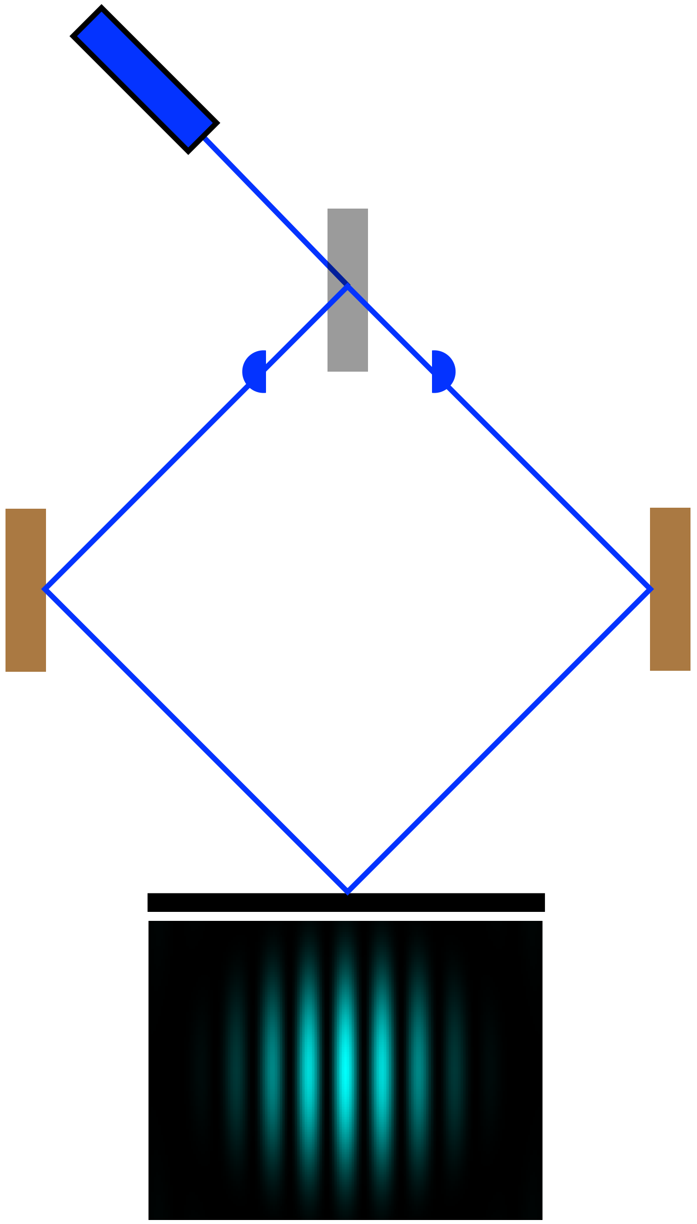
- Any other suggestion?



Particles split?

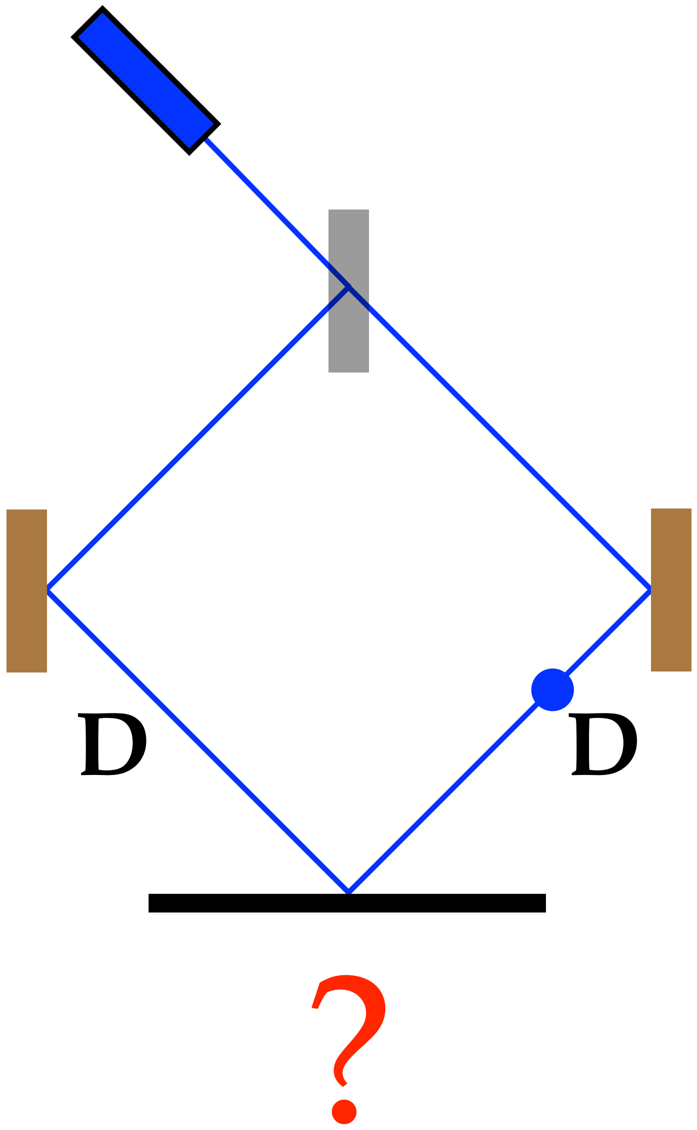
- Maybe each particle *splits* into two moieties that take different paths and interfere when they rejoin at the screen...

    - How could we verify this?



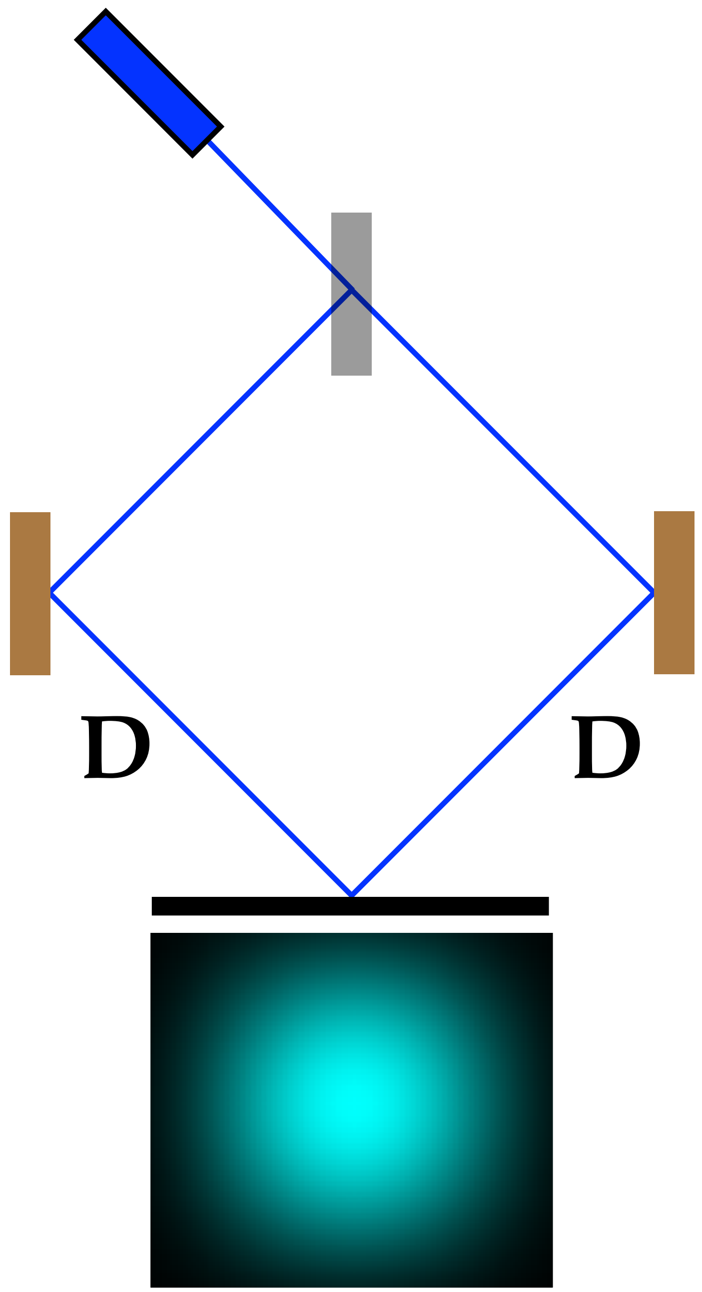
Splitting does not explain it

- If we put detectors that register the path or paths taken by each particle they always detect an *entire* particle in *one* of the paths, so that no splitting occurs.
- On the other hand, QM states that the detected particle should collapse into one of the 2 trajectories and stop behaving oddly …



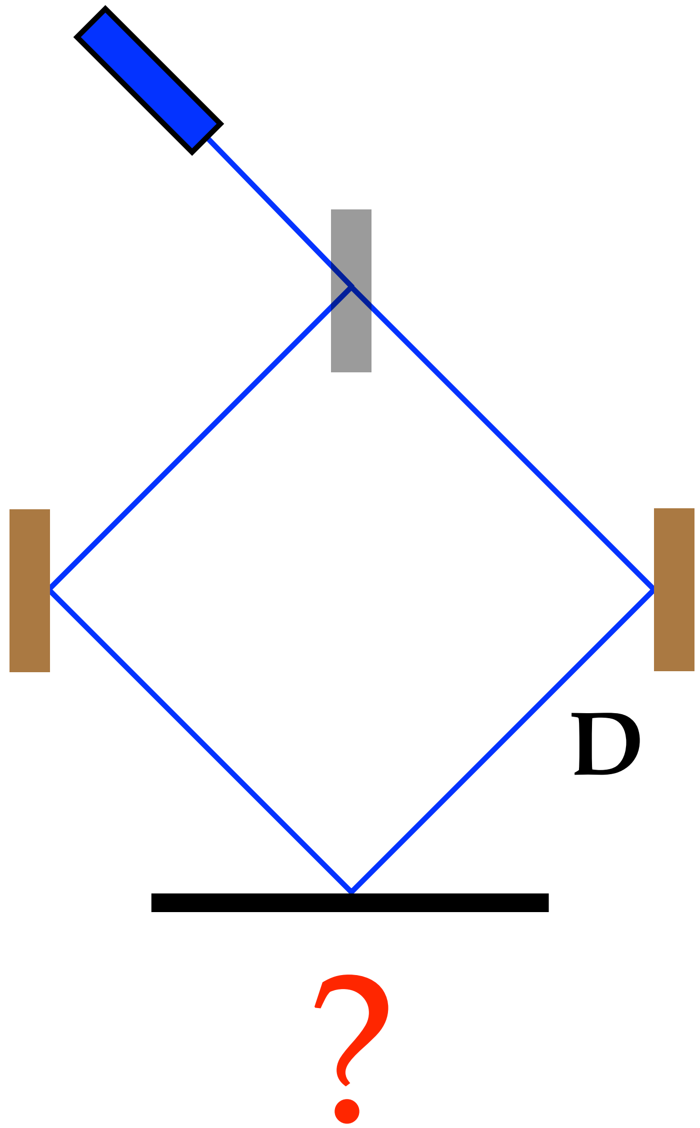
The strange behavior disappears!

- *Interferences disappear*!
- When the particles feel that they are observed they rectify their weird quantum behavior, and behave like reputable classical particles.
- Nature arranges things so that *the strange quantum behavior is kept inscrutable, disappearing when we try to directly observe it*.
- What about the disturbance produced by the detectors?



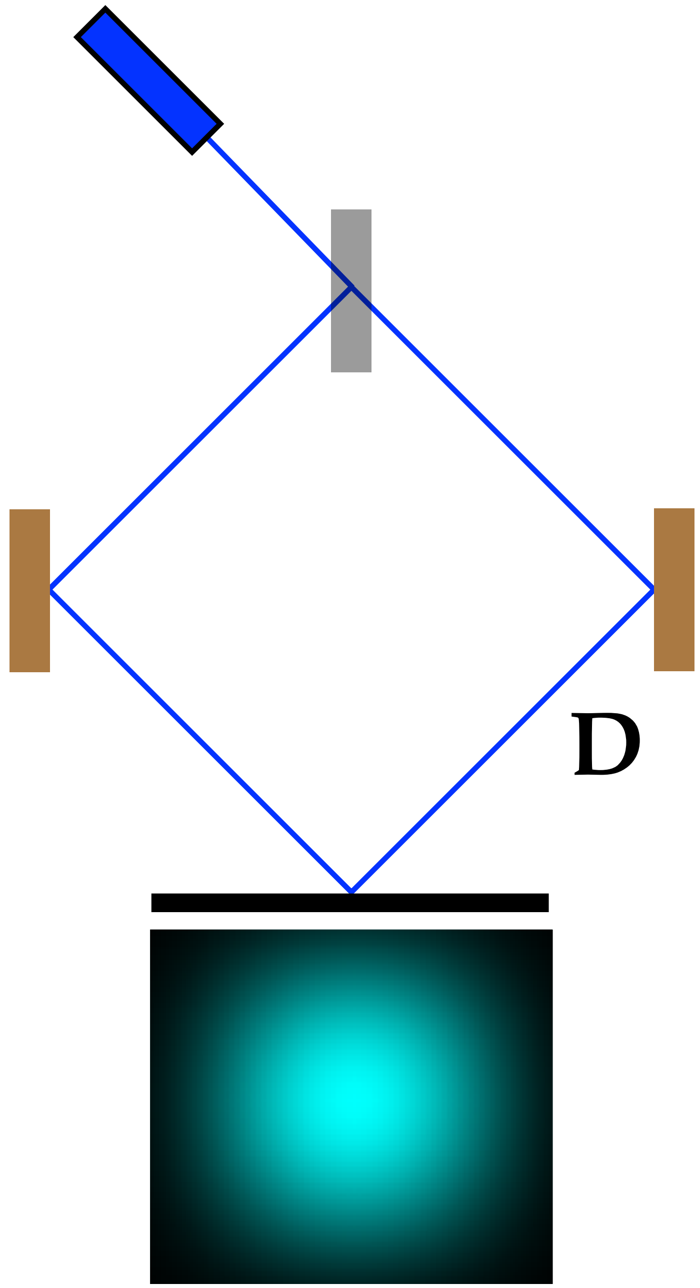
What if we put a single detector?

- The detectors are macroscopic objects that interact with the particles in a complicated way, which could explain the change in their behavior ... but,
  - if we remove one of them, then a particle going through the non-detected path won't interact ⇒ there seems to be no mechanism that could produce the collapse ⇒ they should show interferences...



What if we put a single detector?

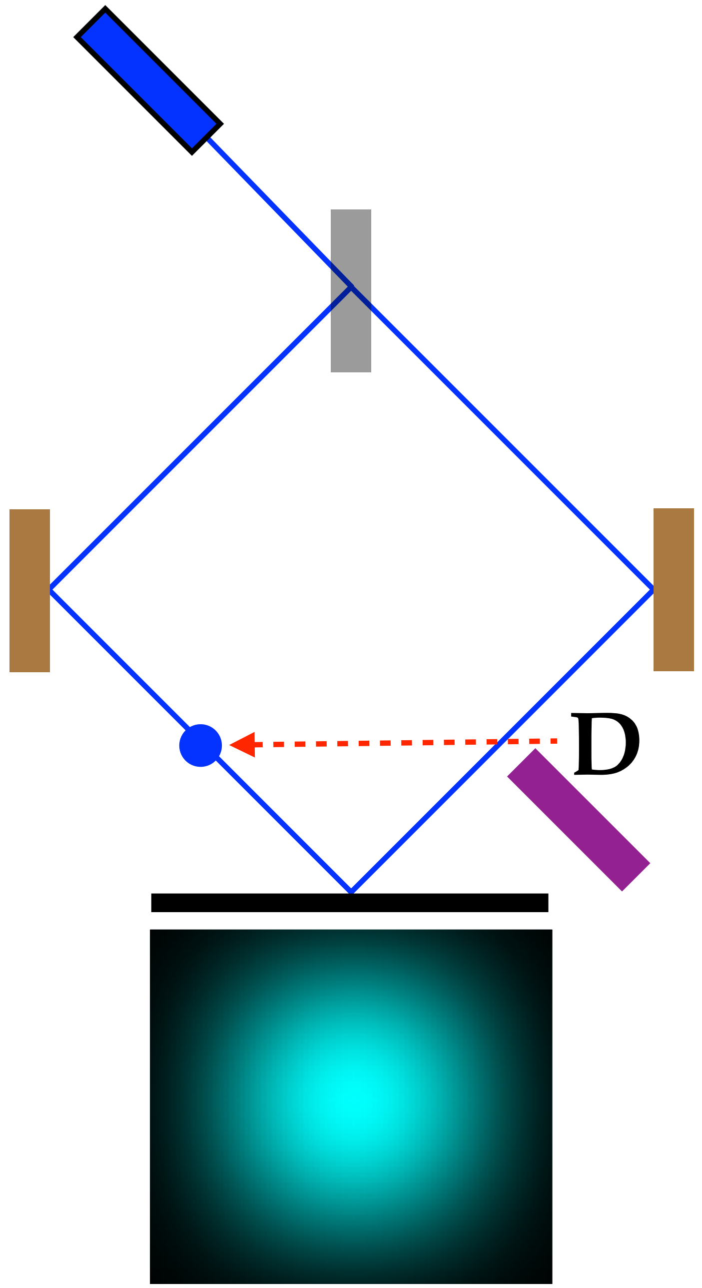
- No interferences are observed even with a single detector
  - QT: because we know the path taken by every particle
  - Any more plausible explanation?



Particle with eyes?

- Maybe the particles *have eyes* that let them see whether the other path is closed or a detector is present so as to adapt their behavior accordingly.

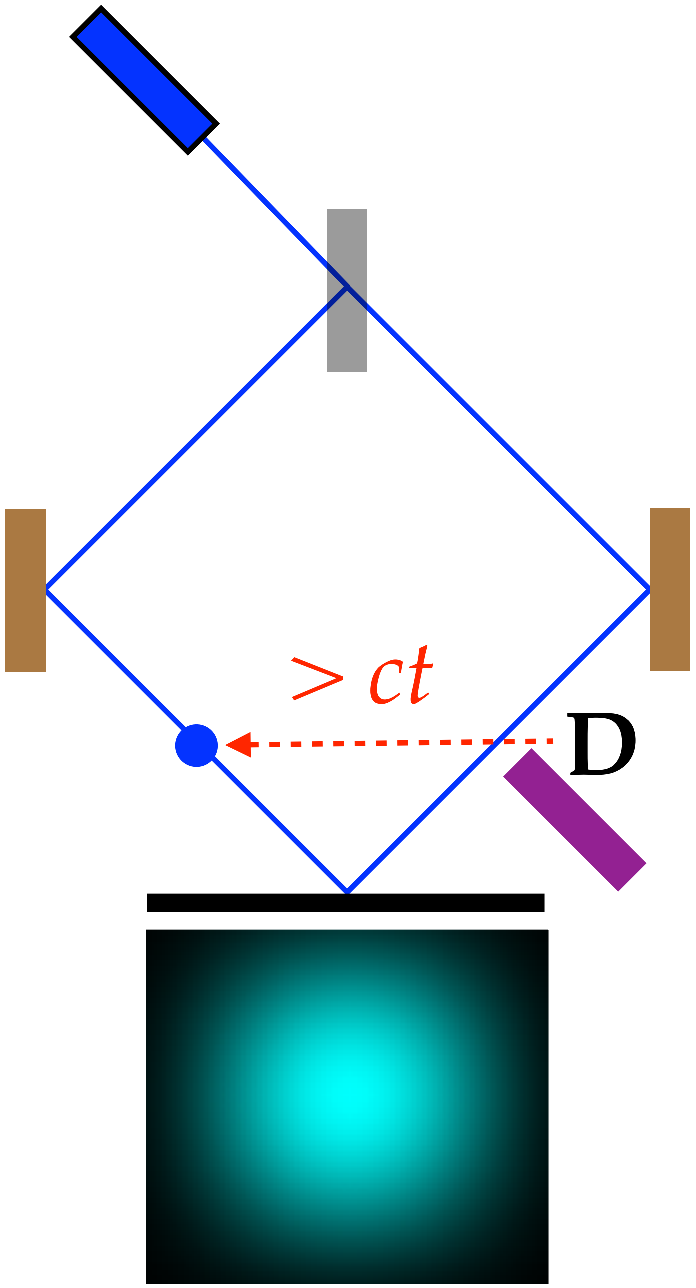
- Could we verify this?



Particle with eyes?

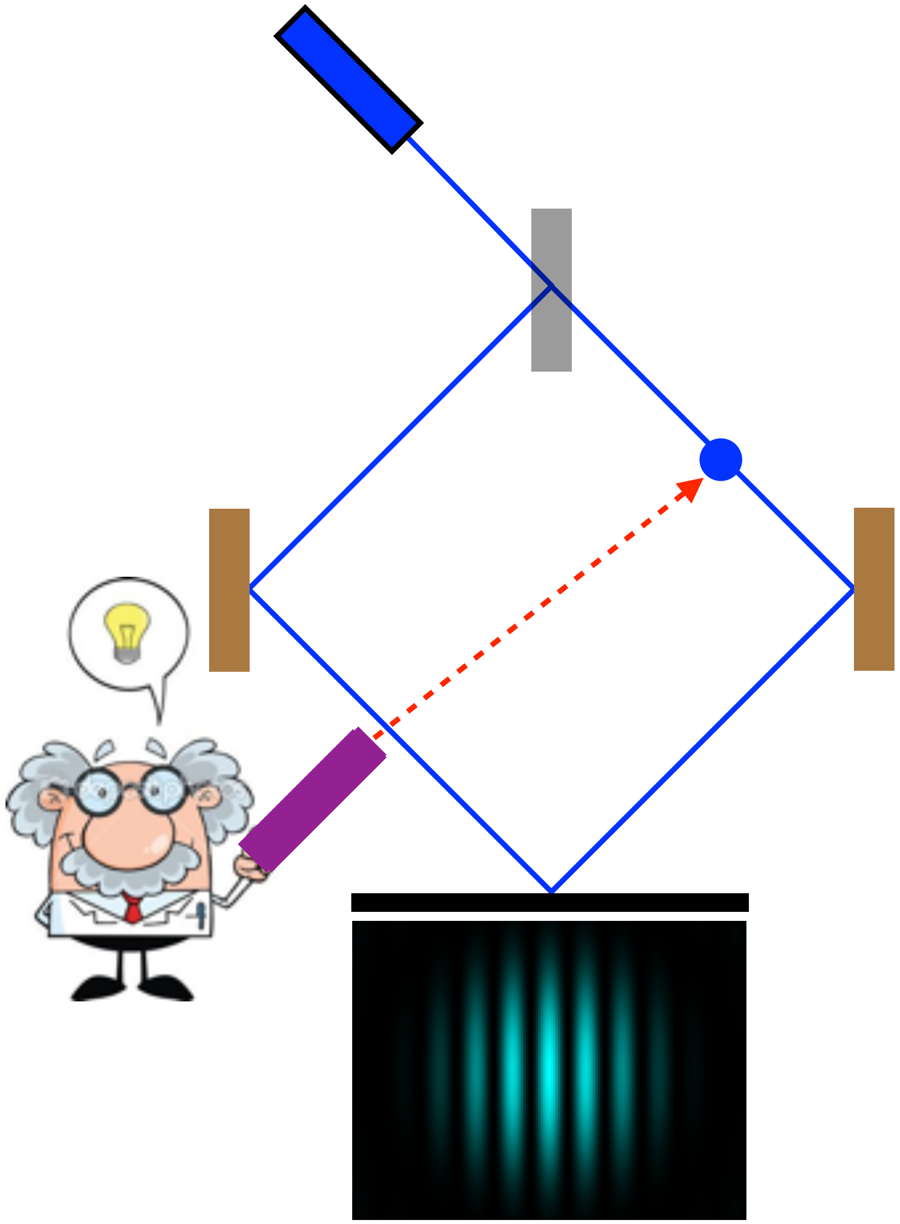
- *Yes, we can*: we will later see a different but related experiment in which the walls or detectors are put late enough for there to be no time for any information to be transmitted to the particle.

  - Any other suggestion?



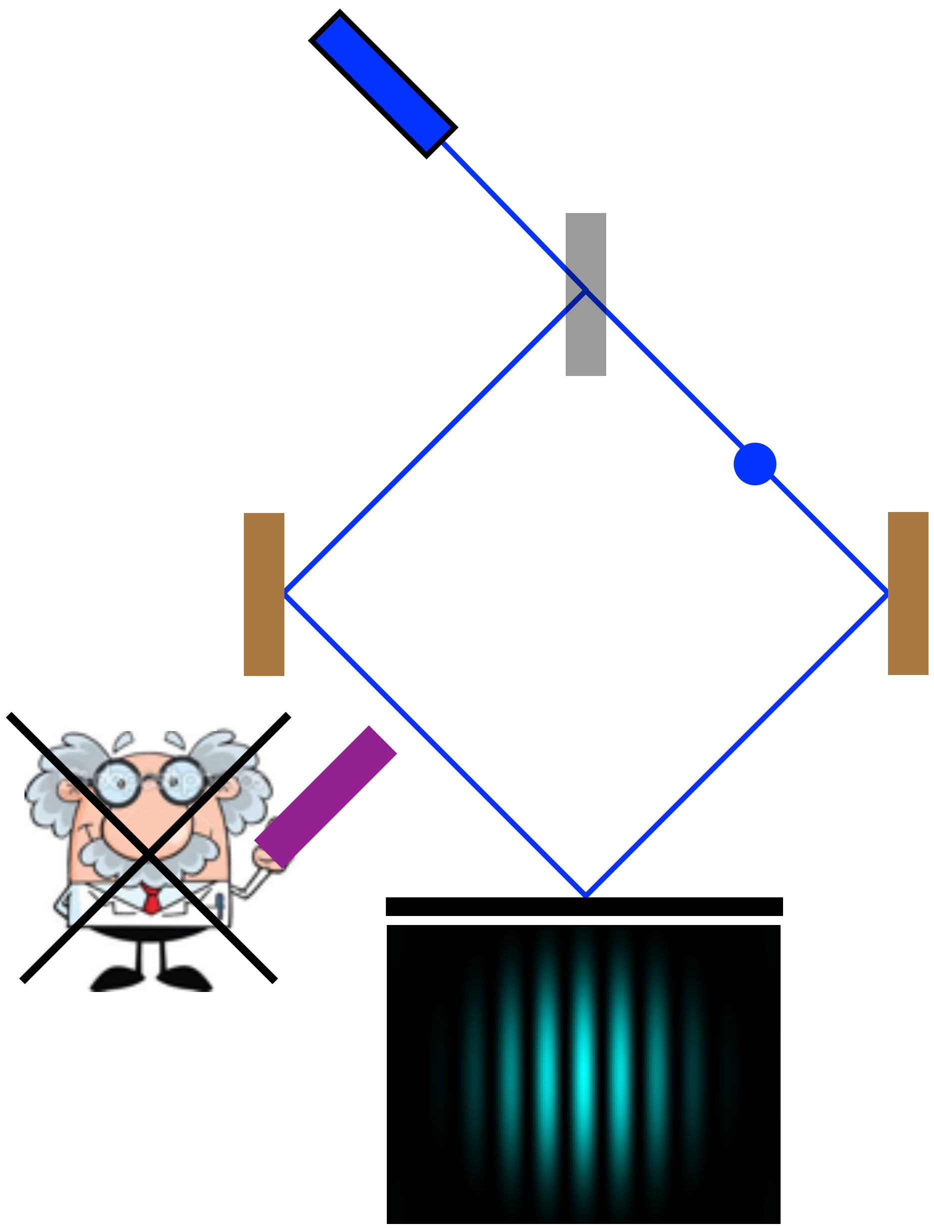
Particle telepathy?

- Maybe the particles can *read the mind* of the experimenter and know in advance whether he is planning to block one of the paths or putting a detector ...
  - Could we verify this?



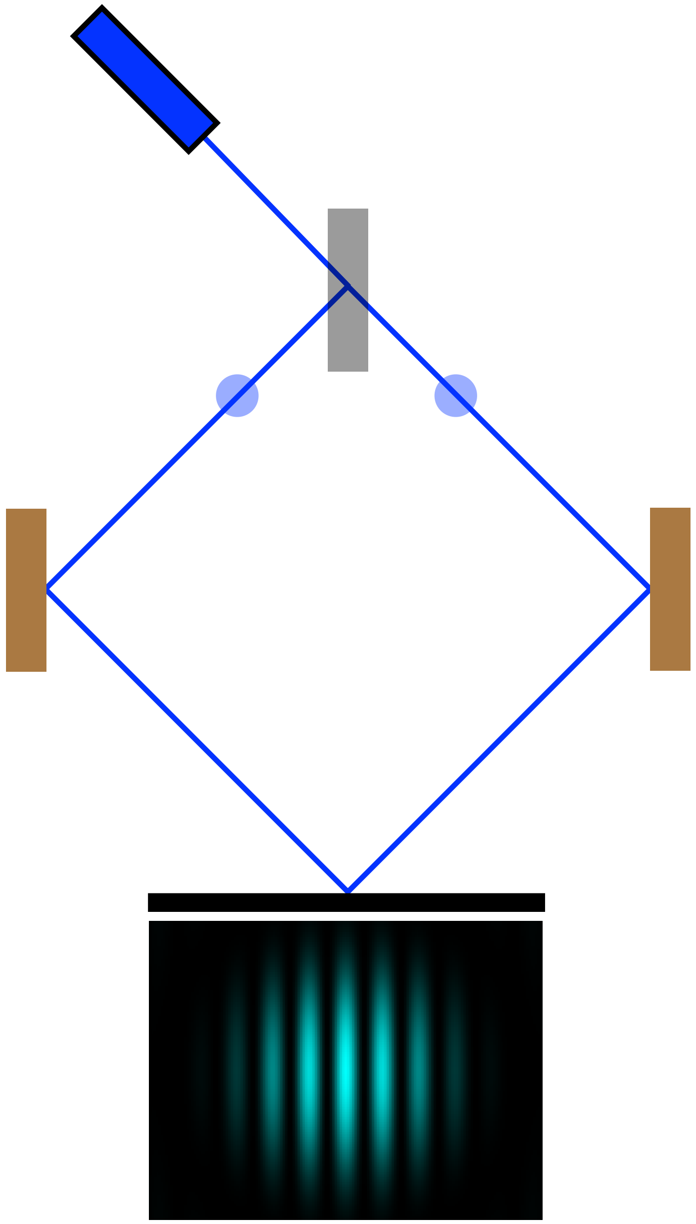
Particle telepathy?

- *Yes, we can*: in the above mentioned experiments a decision similar to putting or not detectors or walls is *randomly* taken by an appropriate experimental arrangement (Aspect, 1983).



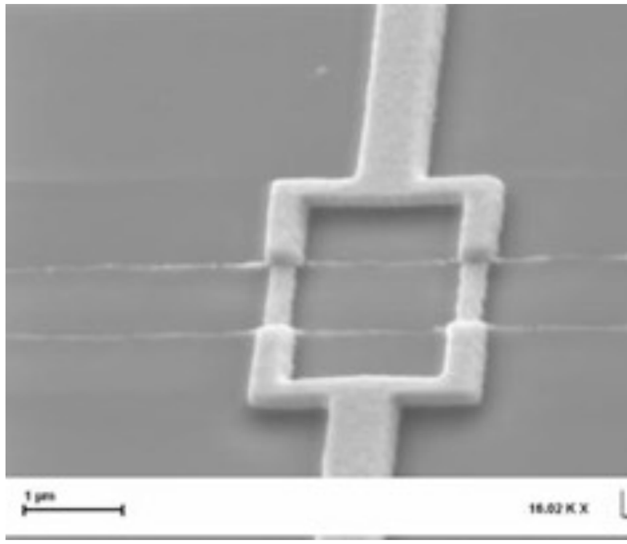
Is the particle position something *real* before we measure it?

- If both paths are open and no detectors are used *we cannot view the particles as localized entities*:
  - each particle seems to *explore both paths*.
  - this leads to an indefiniteness of its position that could be interpreted as an ability to *be in different places at the same time* (a *superposition* of the states corresponding to both paths), or ...

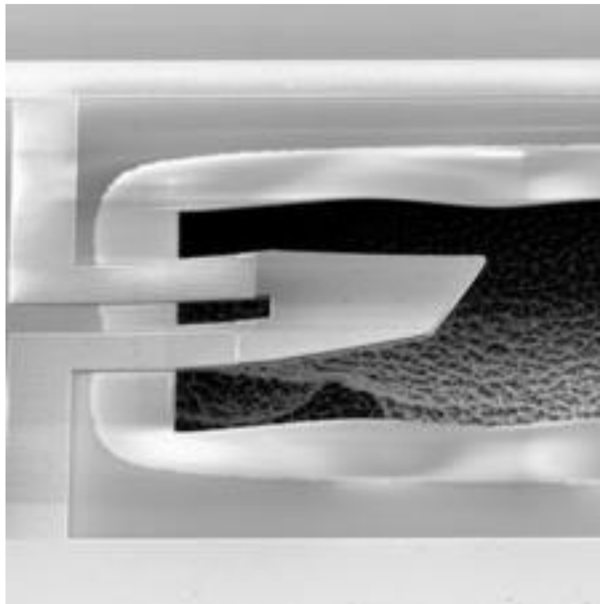


Can larger systems also be in superposition states?

- Large molecules, like C_{60} (1999), C_{70} (2001).
- SQUIDS: *Nature* **406**, 43 (2000): *Quantum superposition of distinct macroscopic states* (one corresponding to a small current flowing clockwise, the other corresponding to the same amount of current flowing anti-clockwise, even for a single-electron current!).



- *Scientific American* march 2010: *Macro-Weirdness: "Quantum Microphone" Puts Naked-Eye Object in 2 Places at Once.*
  - Zurek: *It confirms what many of us believe, but some continue to resist—that our universe is 'quantum to the core'.*
  - Cleland: *As to how the day-to-day reality of objects that we observe, such as furniture and fruit, emerges from such a different and exotic quantum world, that remains a mystery.*



- Viruses? Cats? …

Schrödinger's cat (1935)

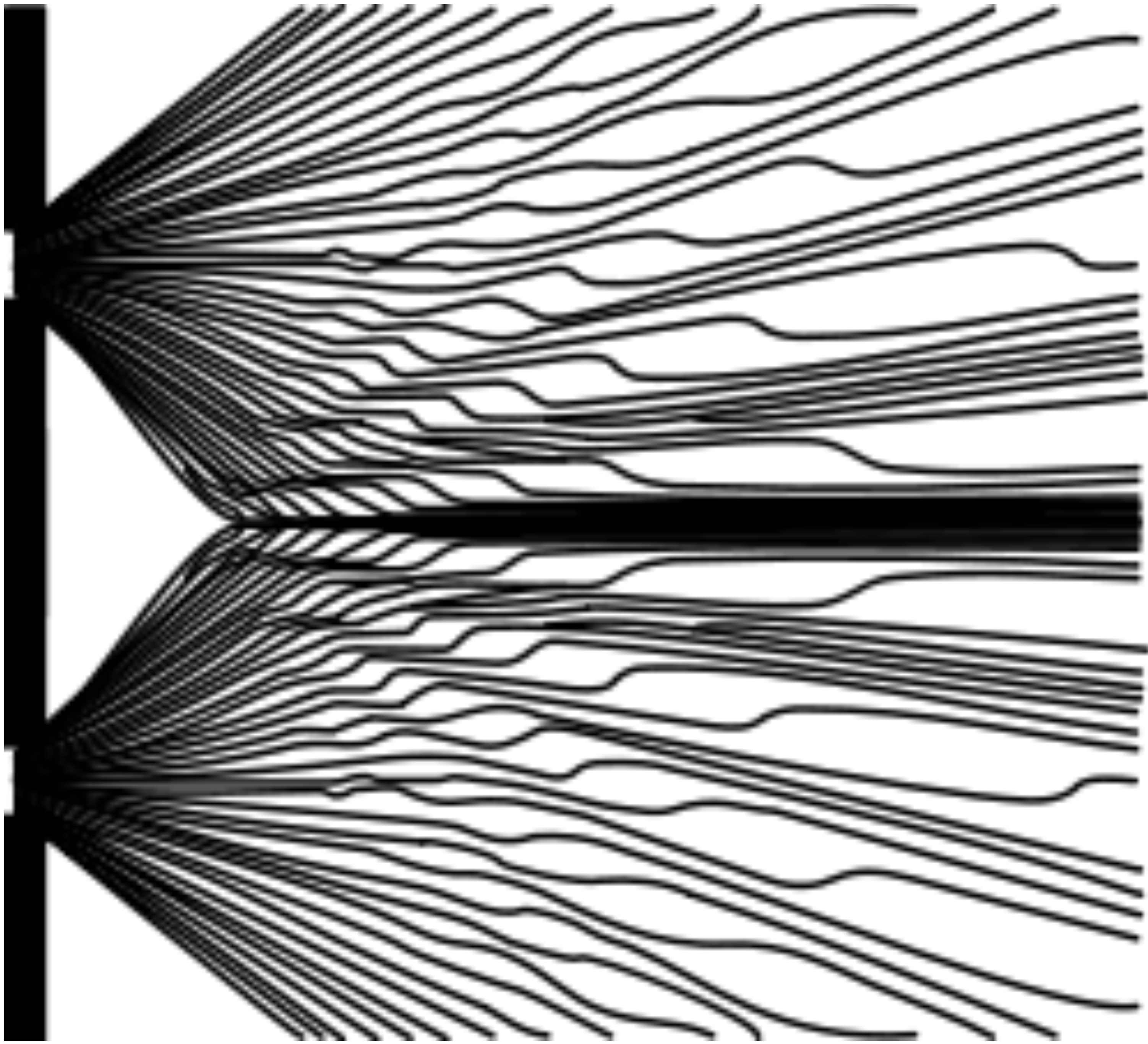
- If the α-particle is in a superposition state of *inside the nucleus - outside the nucleus* the cat should be in a superposition state of *alive-dead*.

- The "quantum spell" is broken when we open the box.

- We have never seen superposition states of large objects... *maybe only because of technical difficulties*? (superposition states are very fragile → they require isolation)

Hidden variable theories

- Is QT *incomplete*?
- *Bohm*: definite trajectories
  - *Non local forces* (difficult to reconcile with special relativity)
  - The *spin* cannot be considered a particle property existing before the measurement (see the discussion below for $s = 3/2$)
  - Virtual particles, QED, …
  - Weak measurements?

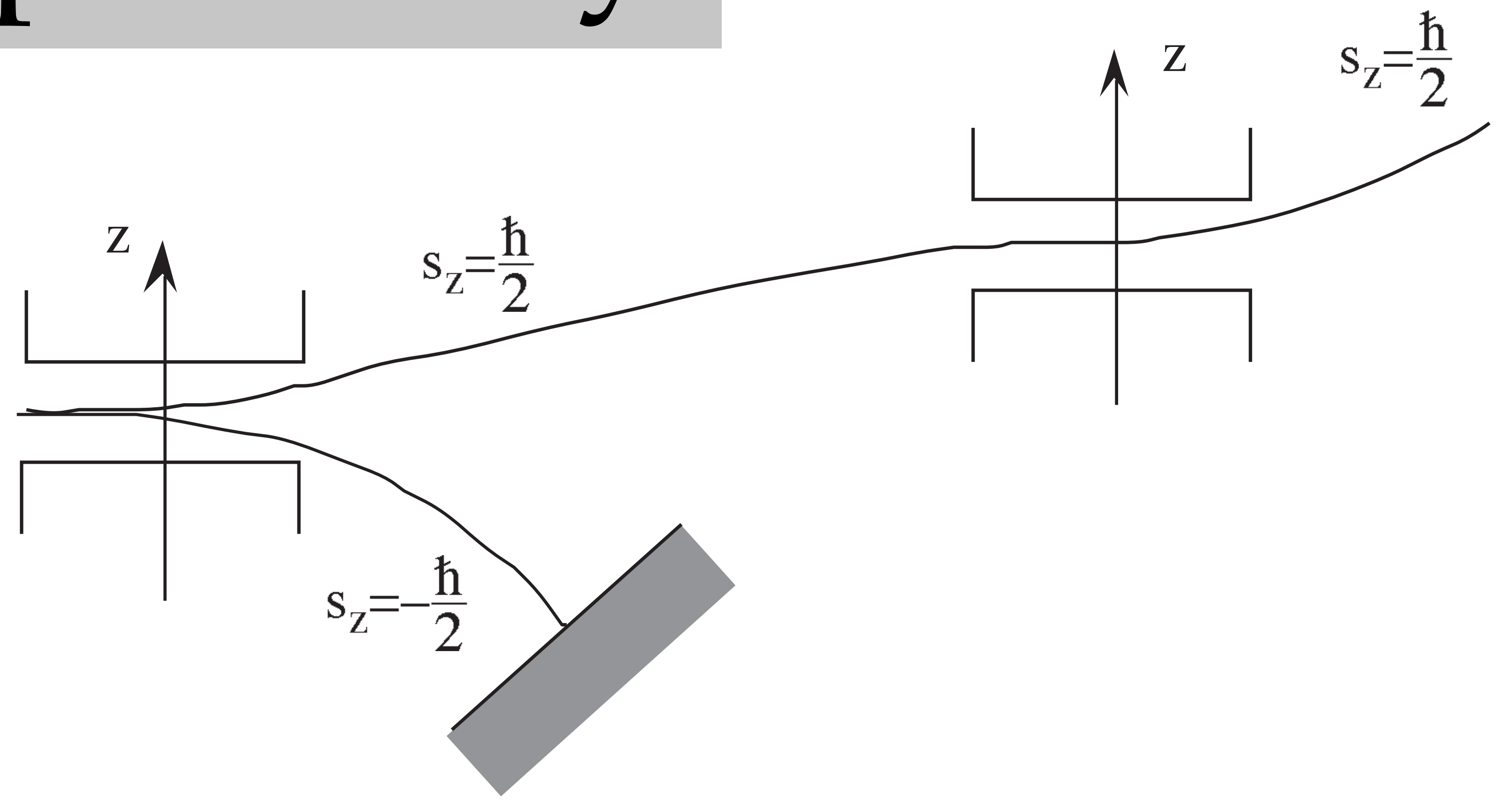


Incompatible properties

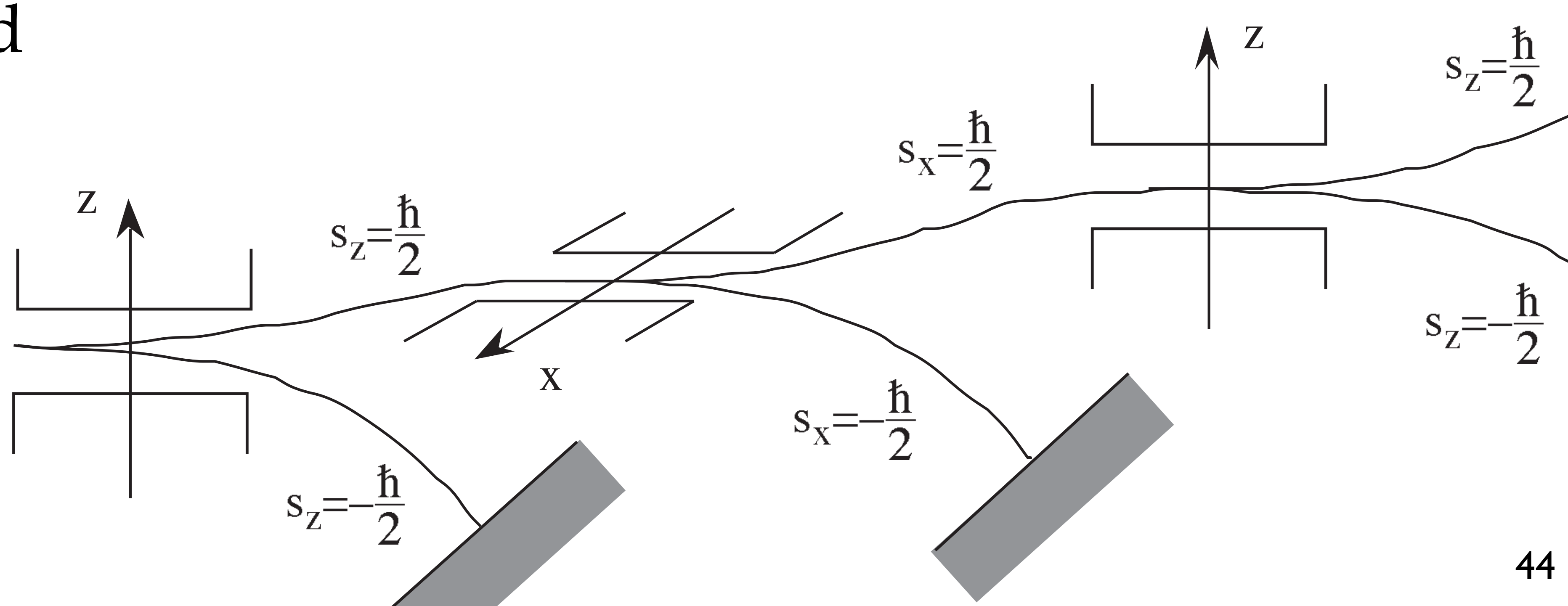
- *Classical mechanics*:
  - The accurate knowledge of one property does not prevent others to be known with unlimited accuracy.
- *Quantum mechanics*:
  - The accurate knowledge of some properties prevents others from being known or, more precisely, *from being defined*!
  - x and p are incompatible (no trajectories).
  - L_x, L_y and L_z are incompatible.
  - etc.

S_z-S_x incompatibility

- S_z becomes well-defined after it is measured (*collapse*)



- The measurement of S_x *destroys the information* previously obtained about S_z ... or should we say that *it destroys the property* S_z*?*

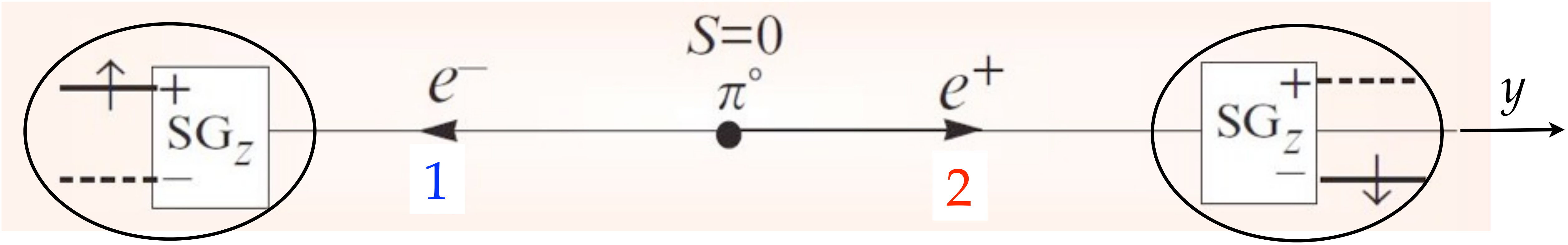


Einstein's concern

- Einstein thought that the decay of a nucleus occurs at a definite moment in time even though such a definite decay time is not implied by the wave function. Accordingly, quantum mechanics would not provide a complete description of reality.
- *Einstein–Podolsky–Rosen* 1935:
  - *Assumptions*:
    - if, without disturbing a system, we can predict with certainty the value of a physical quantity, then this quantity is something real,
    - no influence can travel faster than light (*locality*: the leitmotif of his successful theory of relativity).
  - *Conclusion*: quantum theory is, at least, incomplete.
  - They considered x and p, but we will apply their reasoning to S_z and S_x (*Bohm version*).

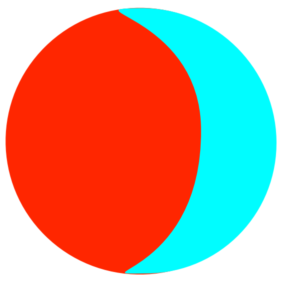
Let's go back to spin 1/2 particles

- EPRB "paradox" (*Einstein–Podolsky–Rosen* 1935, *Bohm* 1951)

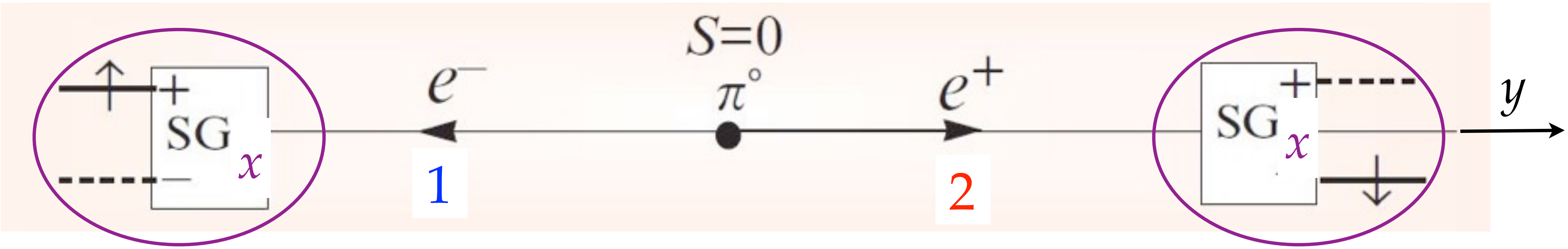


Einstein: If we measure $S_z(1)$ and obtain 1/2 we are certain that a measurement of $S_z(2)$ will give –1/2, and viceversa. If 1 and 2 are very far apart and both measurements are made very close in time, the one performed on particle 1 cannot influence particle 2 (*locality*). Therefore, $S_z(2)$ should have the value –1/2 before any measurement is performed.

Complementary-color rotating-ball analogue:



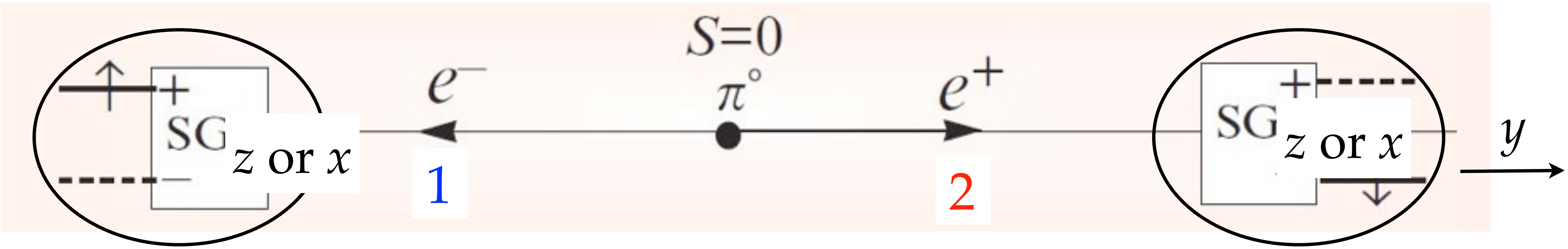
EPRB "paradox"



Einstein: We could have chosen to measure $S_x(1)$ instead of $S_z(1)$, so that both, $S_z(2)$ and $S_x(2)$, *should take definite values no matter they are measured or not* (*locality*: the decision of what to measure in 1 cannot instantaneously affect the state of 2).

⇒ QT is wrong or, at least, incomplete (*it does not admit states in which both components are well defined*).

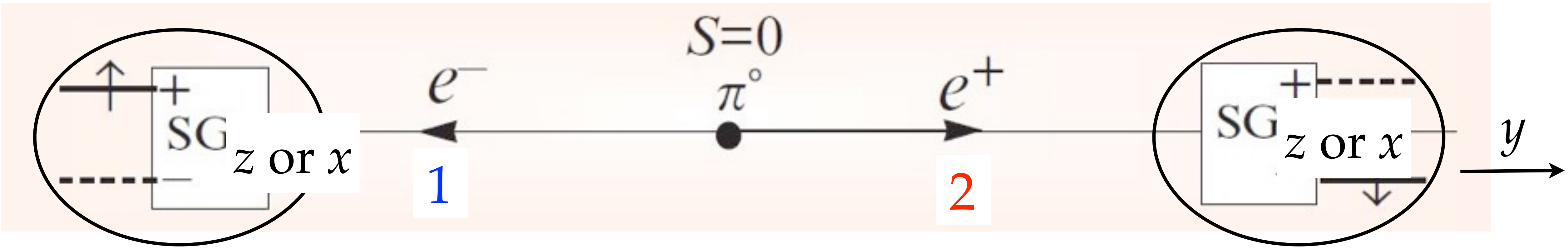
EPRB "paradox"



Bohr: the spin components are *undefined before measurements are made*, and the components of both particles along one axis become determined only when one of them is measured (*non-locality*).

If we chose to measure $S_z(1)$ then $S_z(2)$ *becomes instantaneously defined*, but if, at the last moment, we decide to measure $S_x(1)$ then $S_x(2)$ will become defined and $S_z(2)$ will remain undefined.

EPRB "paradox"



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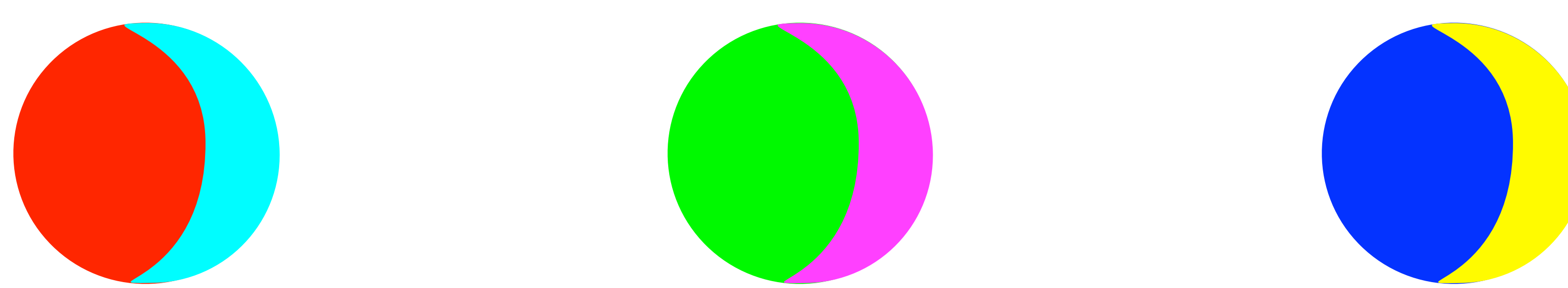
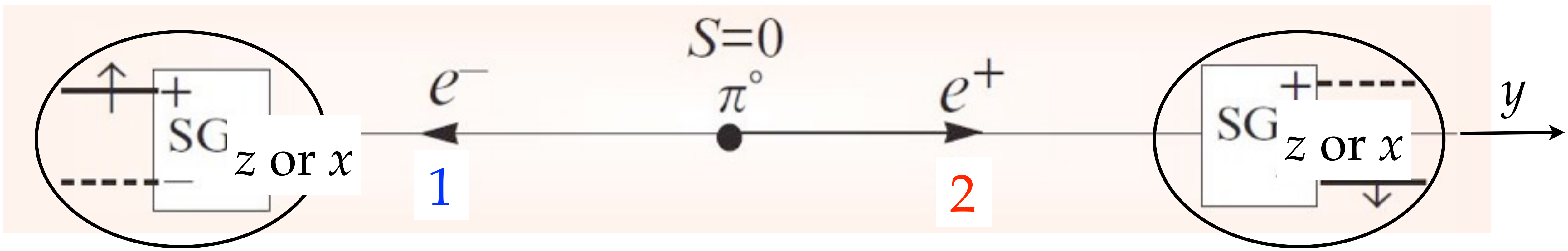


EPRB "paradox"

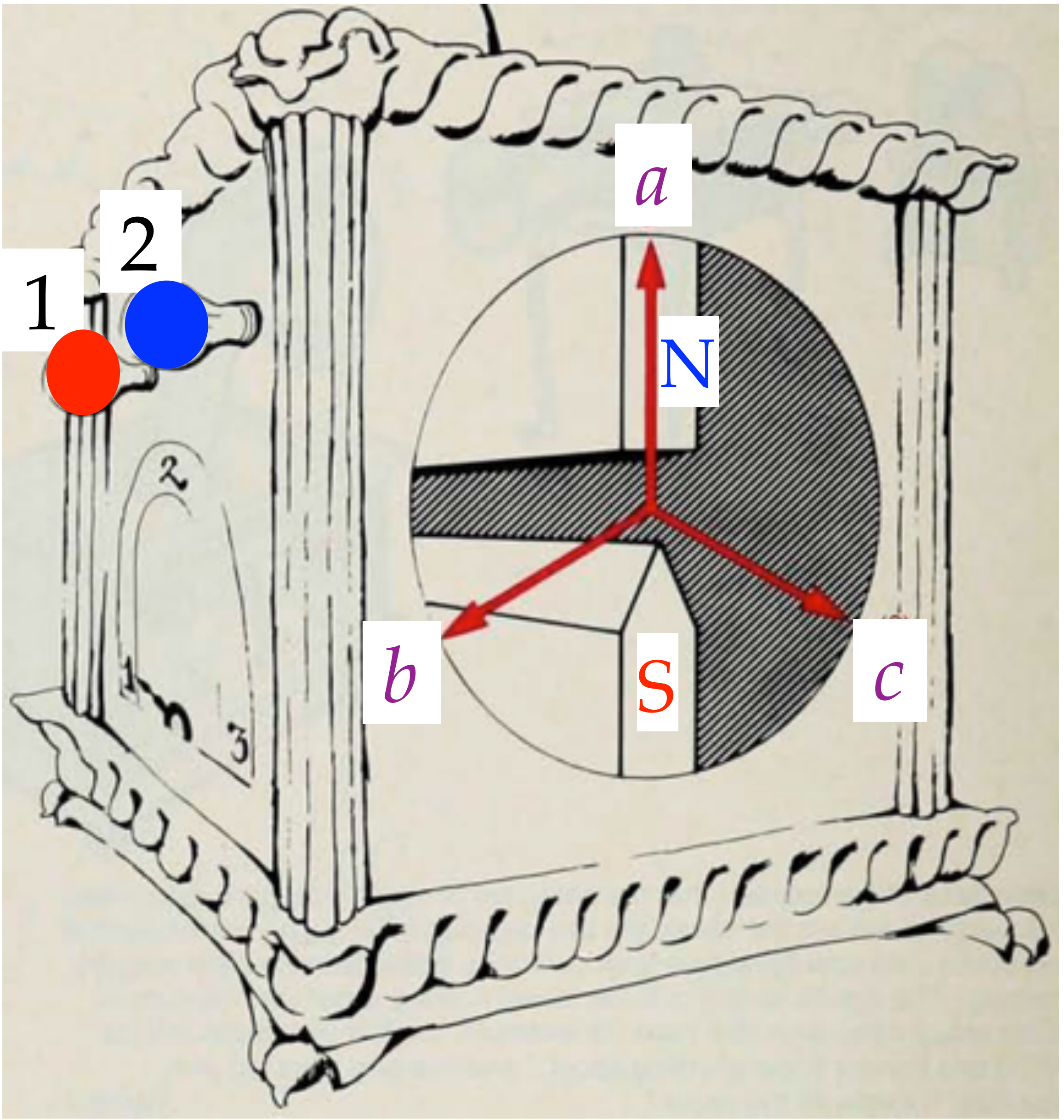


They disagreed about what happens *before* measurements are made, so they could not decide who was right, and Einstein died 20 years after, happily convinced that he was right, but…

Bell's great intuition (1964)

- They disagreed about what happened before measurements were made, so there seemed to be no way to discern who was right -a philosophical issue?-, *but 32 years later Bell found an ingenious experiment to decide who was right.*

- Let's consider a modified version of the EPRB experiment (Mermin's version, 1985)
  - We measure in each particle one component randomly chosen among 3 directions at 120º in a plane perpendicular to the trajectory (*a*, *b*, *c*).
  - The corresponding bulb flashes blue if the particle approaches N (say, comp. > 0) and red if it approaches S (comp. < 0).
  - There are 9 possible combinations of measurement directions: (*a*,*a*), (*a*,*b*), (*a*,*c*), (*b*,*a*), (*b*,*b*), (*b*,*c*), (*c*,*a*), (*c*,*b*) and (*c*,*c*).



Realistic view

- Let's assume that the particles have definite (*opposite*) spin components before they are measured.
  - If particle 1 has positive S_a and S_b and negative S_c (the other way around for particle 2) then the light will flash with different colors for 5 of the 9 cases (56%): (*a*,*a*), (*b*,*b*), (*c*,*c*), (*a*,*b*) and (*b*,*a*).
  - The same happens whenever one components has a different sign than the others.
  - If the particles have the 3 components with the same sign then the 2 measured values will be opposite in all of the 9 cases.
- Therefore, *the realistic view leads to > 56% (actually 2/3) of probability of observing different color flashes.*

Quantum description

- For the singlet state: $\Psi = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$
- For any arrangement of the detectors, say (*a*,*b*), the product of the measured components can take the values 1/4 or –1/4:

$$(\vec{a}\cdot\vec{S}_1)(\vec{b}\cdot\vec{S}_2) = \pm\frac{1}{4}$$

and its average value for many measurements will be:

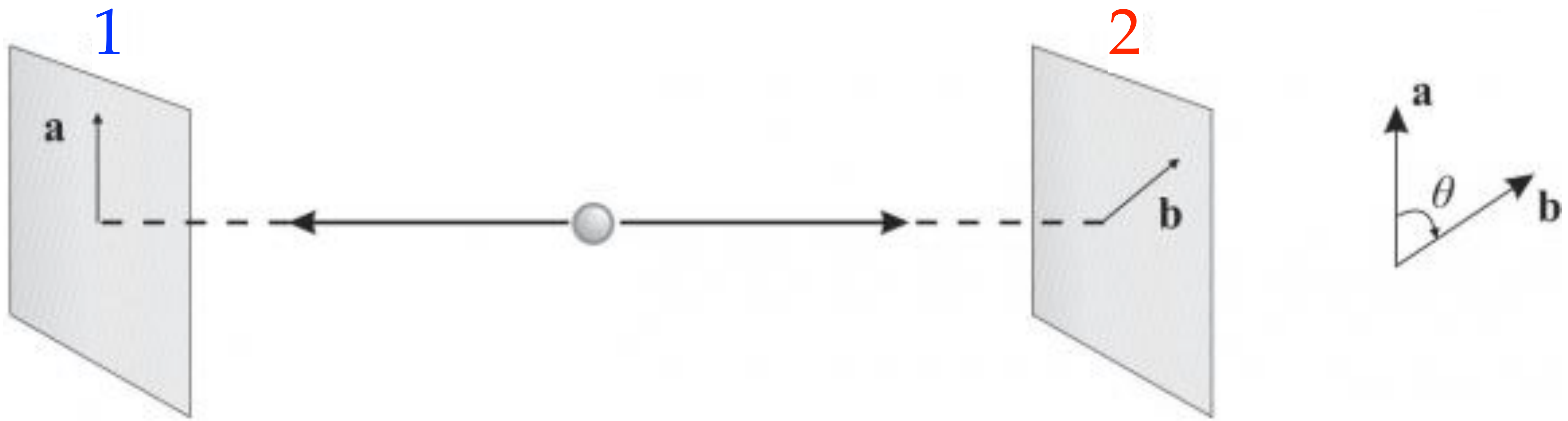
$$\langle\Psi|(\vec{a}\cdot\widehat{\vec{S}_1})(\vec{b}\cdot\widehat{\vec{S}_2})\Psi\rangle$$

- By summing over all the 9 arrangements:

$$\sum_i^{a,b,c}\sum_j^{a,b,c}\langle\Psi|(\vec{i}\cdot\widehat{\vec{S}_1})(\vec{j}\cdot\widehat{\vec{S}_2})\Psi\rangle = \langle\Psi|(\underbrace{\sum_i^{a,b,c}\vec{i}}_{=0}\cdot\widehat{\vec{S}_1})(\underbrace{\sum_j^{a,b,c}\vec{j}}_{=0}\cdot\widehat{\vec{S}_2}\Psi)\rangle = 0$$

so that *P*(¼, *same color*) = *P*(-¼, *different color*) = 50%

Bell's theorem (1964, Aspect 1981)

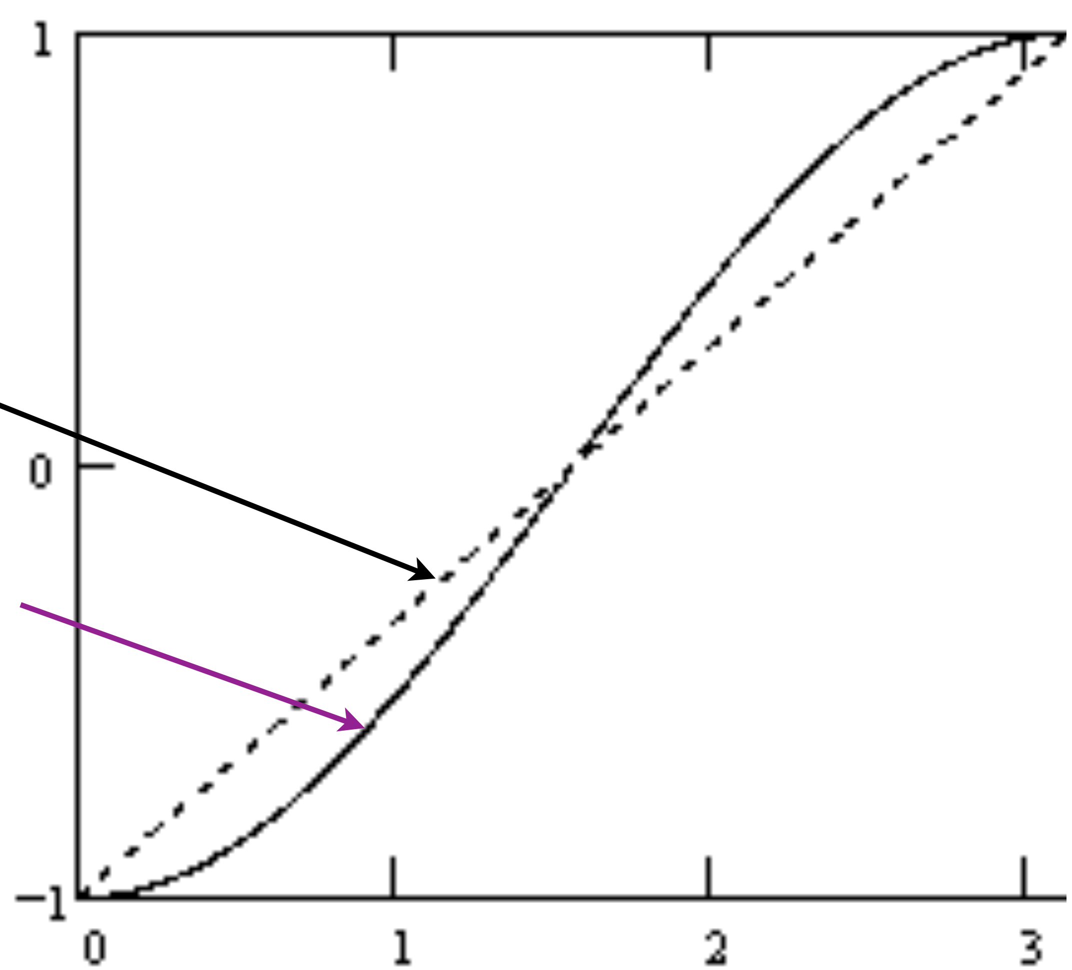


$$\sigma_a \equiv S_a \times 2 = \pm 1$$

$$\langle \sigma_a(1)\sigma_b(2)\rangle_{classic} = \frac{2}{\pi}\theta - 1$$

$$\langle \sigma_a(1)\sigma_b(2)\rangle_{quantic} = -\vec{a}\cdot\vec{b} = -\cos\theta$$

Quantum correlation is stronger!



More theorems

- *Bell* (1964), *Aspect* (1981, 13 m), *Gisin* (1997, 11 km!)

For *any Local Hidden Variable Theory*:

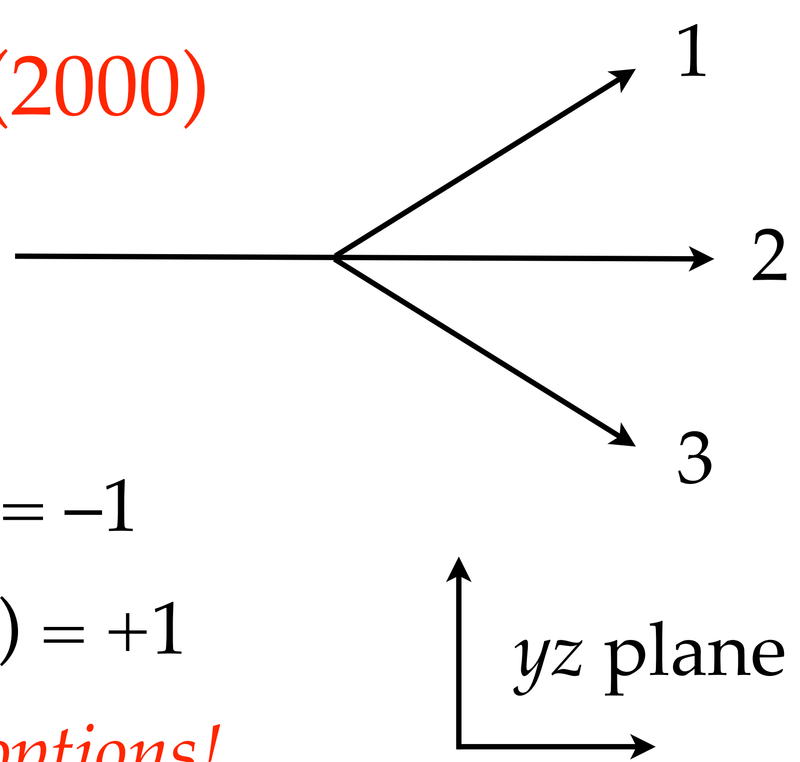
$$|\langle\sigma_a(1)\sigma_b(2)\rangle - \langle\sigma_a(1)\sigma_c(2)\rangle| \leq 1 + \langle\sigma_b(1)\sigma_c(2)\rangle$$

- According to QM this inequality only holds for certain directions.

- *Greenberger-Horne-Zeilinger* (1989), *Zeilinger* (2000)

Mermin (1990): 3 *s*=½-particles

$$\Psi = \frac{1}{\sqrt{2}}\left(\uparrow_1\uparrow_2\uparrow_3 - \downarrow_1\downarrow_2\downarrow_3\right)$$



- According to *quantum mechanics* $\sigma_x(1)\,\sigma_x(2)\,\sigma_x(3) = -1$
- According to *local realistic theories* $\sigma_x(1)\,\sigma_x(2)\,\sigma_x(3) = +1$
- *A single measurement allows to decide between both options!*

GHZ: quantum description

- Proof: $\widehat{\sigma_x} \uparrow = \downarrow \qquad \widehat{\sigma_x} \downarrow = \uparrow$

$$\widehat{\sigma_x}(1)\widehat{\sigma_x}(2)\widehat{\sigma_x}(3) \underbrace{\frac{1}{\sqrt{2}}(\uparrow_1\uparrow_2\uparrow_3 - \downarrow_1\downarrow_2\downarrow_3)}_{\Psi} = \frac{1}{\sqrt{2}}(\downarrow_1\downarrow_2\downarrow_3 - \uparrow_1\uparrow_2\uparrow_3) = -1 \times \Psi$$

- On the other hand $\widehat{\sigma_y} \uparrow = i \downarrow \qquad \widehat{\sigma_y} \downarrow = -i \uparrow$ so that

$$\widehat{\sigma_x}(1)\widehat{\sigma_y}(2)\widehat{\sigma_y}(3) \underbrace{\frac{1}{\sqrt{2}}(\uparrow_1\uparrow_2\uparrow_3 - \downarrow_1\downarrow_2\downarrow_3)}_{\Psi} = \frac{1}{\sqrt{2}}(-\downarrow_1\downarrow_2\downarrow_3 + \uparrow_1\uparrow_2\uparrow_3) = 1 \times \Psi$$

- Similarly: $\sigma_y(1)\sigma_x(2)\sigma_y(3)\,\Psi = \sigma_y(1)\sigma_y(2)\sigma_x(3)\,\Psi = 1 \times \Psi$

- Experiments confirm these predictions.

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GHZ: local realistic view

- Since $\sigma_x(1)\sigma_y(2)\sigma_y(3) = \sigma_y(1)\sigma_x(2)\sigma_y(3) = \sigma_y(1)\sigma_y(2)\sigma_x(3) = 1$ in Ψ
  - by measuring $\sigma_y(2)\ \sigma_y(3)$ we can predict $\sigma_x(1)$ no matter how far this is from those $\Rightarrow$ $\sigma_x(1)$ *must be well-defined* if local realism is admitted;
  - by measuring $\sigma_x(2)\ \sigma_y(3)$ we can predict $\sigma_y(1) \Rightarrow \sigma_y(1)$ *must be well-defined;*
  - same for 2 and 3 $\Rightarrow$ the 6 components $\sigma_x(1)$, $\sigma_y(1)$, $\sigma_x(2)$, $\sigma_y(2)$, $\sigma_x(3)$, $\sigma_y(3)$ *must be well-defined,* so
  - $\sigma_x(1)\sigma_y(2)\sigma_y(3) \times \sigma_y(1)\sigma_x(2)\sigma_y(3) \times \sigma_y(1)\sigma_y(2)\sigma_x(3) = 1 \times 1 \times 1 = \sigma_x(1)\sigma_x(2)\sigma_x(3)\ [\sigma_y(1)]^2\ [\sigma_y(2)]^2\ [\sigma_y(3)]^2$
  - that is $\sigma_x(1)\sigma_x(2)\sigma_x(3) = 1$.

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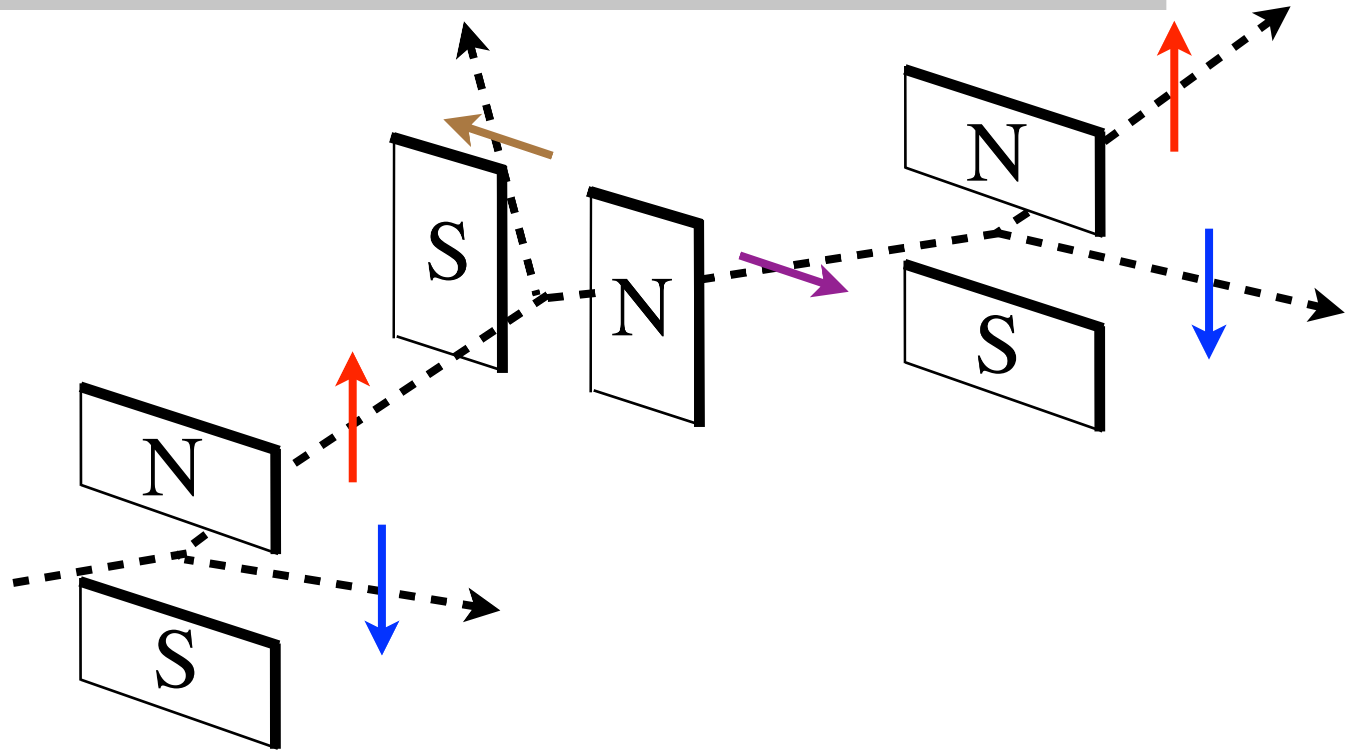
S_x-S_y-S_z for $S = 1/2$

- For any particle with spin quantum number $s = 1/2$ experiments and quantum theory show that:
  - the squared modulus of the spin vector takes the value $S^2 = s(s+1) = (1/2)(3/2) = 3/4$ (in a.u.)
  - the cartesian components of the spin vector can take the values $S_x = \pm 1/2$, $S_y = \pm 1/2$, $S_z = \pm 1/2$
  - so that, for any of the values that S_x, S_y and S_z can take, we can think of S^2 as being the squared modulus of a vector with well-defined cartesian components: $S^2 = S_x^2 + S_y^2 + S_z^2 = 1/4 + 1/4 + 1/4 = 3/4$
  - *i.e., there seems to be no reason to question the possibility of the three components having simultaneous well-defined values*.

S_x-S_y-S_z for $S = 3/2$

- ...but for a particle with spin quantum number $s = 3/2$ experiments and quantum theory show that:
  - the squared modulus of the spin vector takes the value $S^2 = s(s+1) = (3/2)(5/2) = 15/4$ (in a.u.)
  - the cartesian components of the spin vector can take the values $S_x = \pm 1/2, \pm 3/2$, $S_y = \pm 1/2, \pm 3/2$, $S_z = \pm 1/2, \pm 3/2$
  - so that $S_x^2 + S_y^2 + S_z^2$ should take one of the values
    - $1/4 + 1/4 + 1/4 = 3/4$
    - $1/4 + 1/4 + 9/4 = 11/4$
    - $1/4 + 9/4 + 9/4 = 19/4$
    - $9/4 + 9/4 + 9/4 = 27/4$
  - $\Rightarrow$ no combination of definite values of the 3 components gives the correct value for S^2
  - $\Rightarrow$ S_x, S_y and S_z *cannot simultaneously have well-defined values.*

S_z-S_x incompatibility



- We can say that the measurement *creates the measured property*:
  - S_z is *created* when it is measured, and it is *destroyed* when S_x is measured.
  - The so-called "quantum fluctuations" are not "fluctuations" between different existing values.
- By choosing which component to measure *we decide which of the 3 incompatible properties the particle will possess*.
  - Subjectivity: *we somewhat see what we want to see*!

No particles, only fields?

- Is the particle-like character of matter an illusion?
- ... that matter consists on (infinitely) extended fields that "concentrate" into a small region (collapse) when they are forced to by a detector, so that ...
- ... Born proposal that $|\Psi(x,t)|^2$ is the probability density of finding the particle at point x upon measurement at time t should be reinterpreted as the probability (density) for an interaction with, say, a detector screen, to take place at x ...
- ... as happens if we try to detect a big balloon in a dark room with a needle?
- But, what is this matter field made of? If it has some material consistency, can it be transmitted faster than light? What is able to produce the collapse? Why we never observe a measurement device in a superposition state?

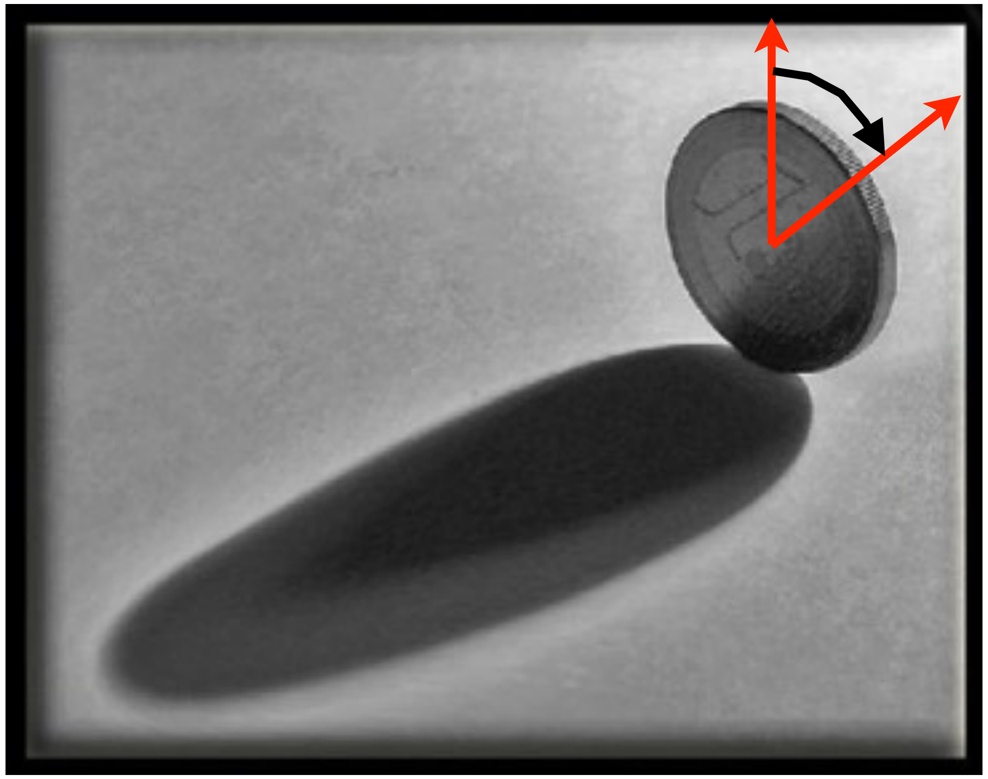
A. Hobson: There are no particles, there are only fields, *Am. J. Phys.* **81**, 211 (2013)

but then...

- It seems that properties are undefined before they are measured, but then...
  - an emitted α-particle does not have existence before it is detected?
  - an e^{-}-e^{+} pair created from a γ-ray does not exist before it is observed?
  - all the matter of the universe comes from high energy photons produced at the big bang; then, the stars did not exist before they were first observed?
- It's no surprising that Einstein and Schrödinger, among others, did not accept this point of view: Is the moon there when nobody looks?

Classical incompatibility?

- The head and tail when we toss a coin.
- The angle with the vertical after rolling it.



- The peculiarity of quantum incompatibility is that it affects to classically compatible observables (e.g., x and p).

Our world seems to be non-local!

- One important conclusion of the preceding results (besides their contribution to the *quantum theory - hidden variable theories* dilemma) is the revelation that *nature shows non-local behavior*!
- There exists *entangled states* in which the particles retain some interconnection no mater how far from each other they move.
  - Some influences can be transmitted *instantaneously*.
  - Does this *faster-than-light* influence violate relativity?
  - *No*: the values of, say, $S_z(1)$ (and $S_z(2)$) in EPRB are random (*contain no information*), and observers must communicate by ordinary means to notice the entanglement effects.

Quantum view of the past

- *Classical mechanics*: past, present and future are described with the same language and are equally real. The present is the consequence of a unique, well defined past.

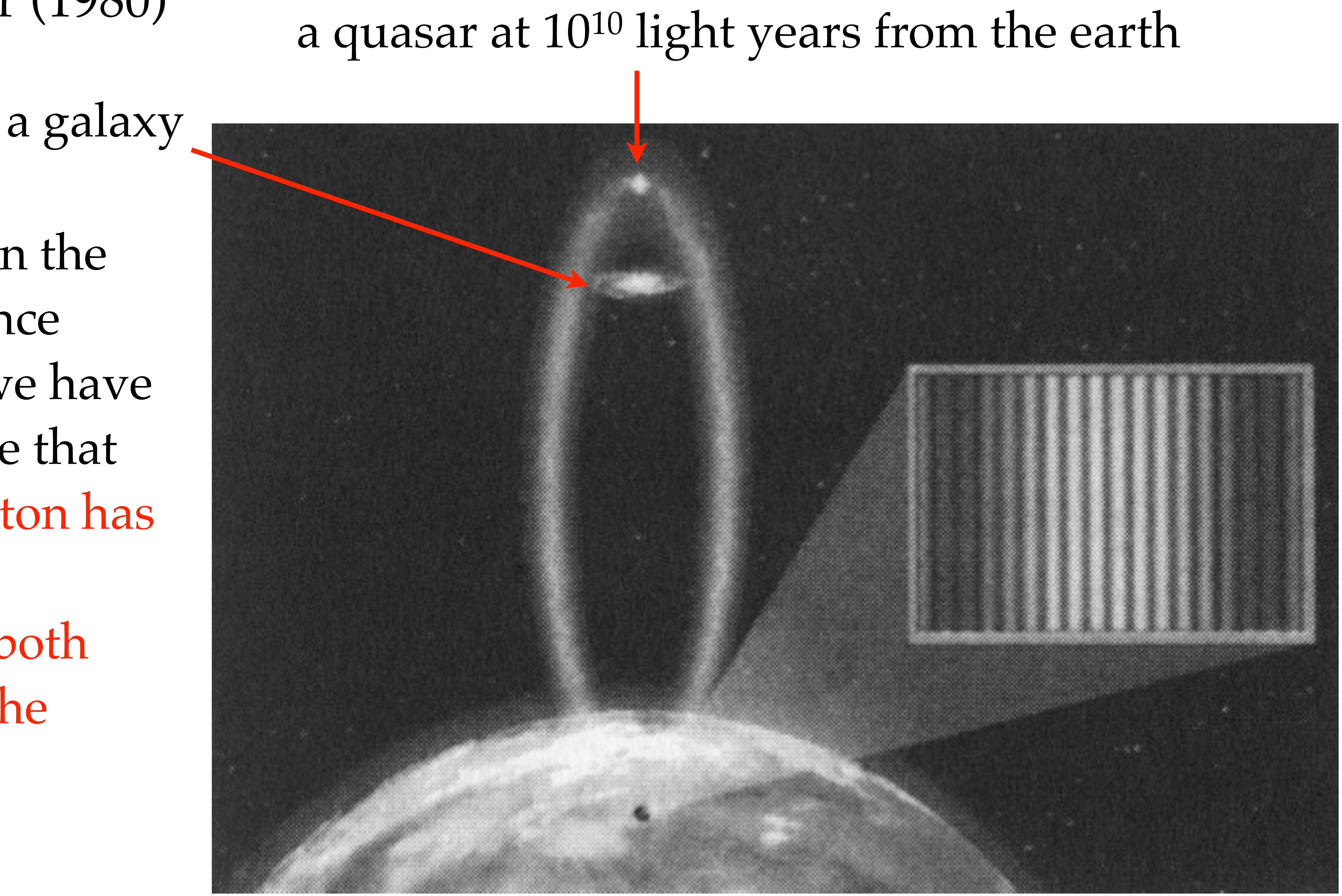
- *Quantum mechanics*: once observed, the properties are real and certain, but before the observation those properties had not uniquely defined values, and all the possible histories contribute to the present reality.

  - *Feynman*: if there are alternative ways of reaching some result all of them have taken place in a certain sense.

Ein Gedankenexperiment

- Wheeler (1980)

- To explain the interference pattern we have to assume that each photon has travelled through both sides of the galaxy.

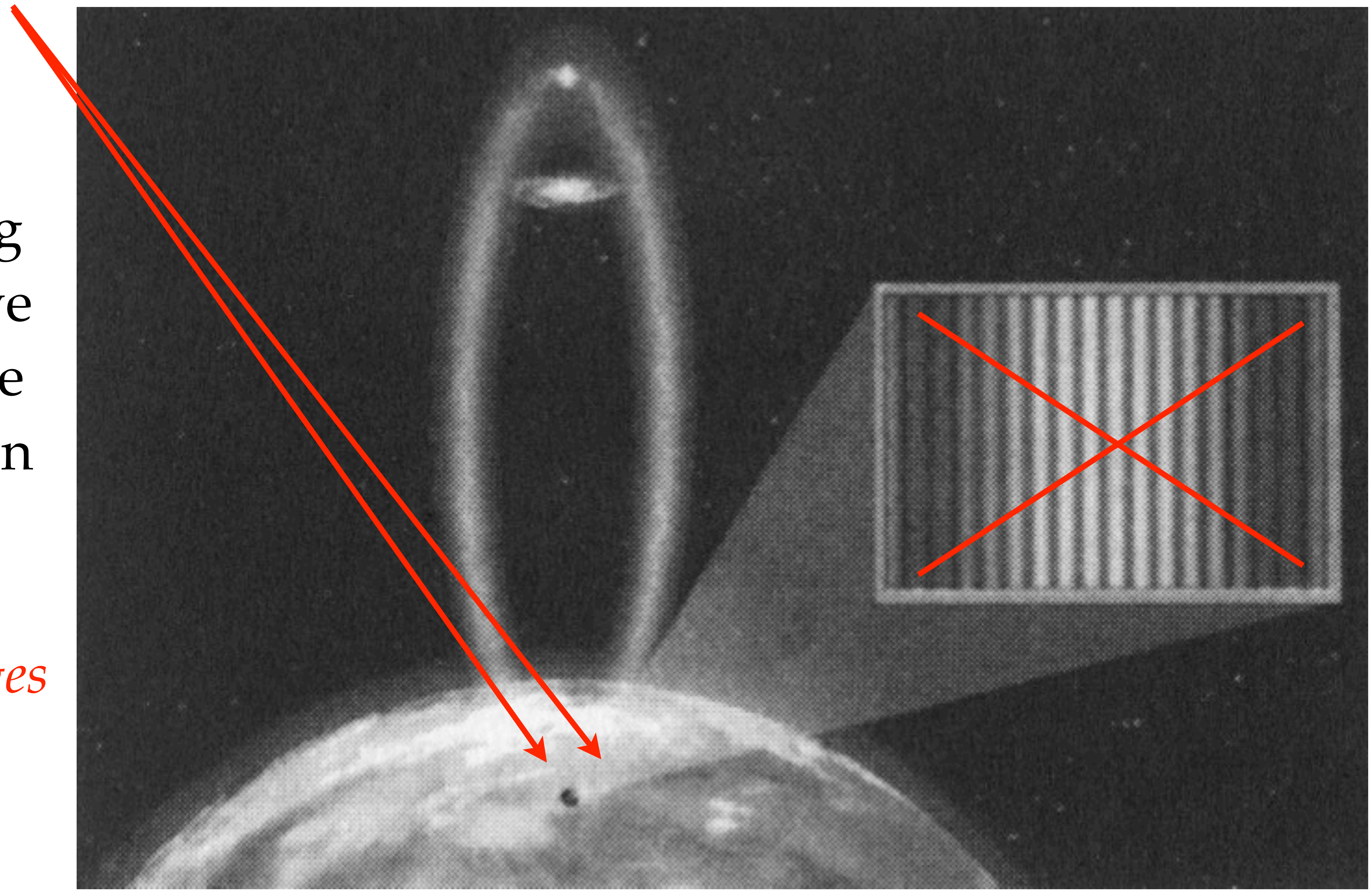


Original figure from *El tejido del cosmos*, Brian Greene, Ed. Crítica (2006)

Ein Gedankenexperiment

- The introduction of *detectors* pointing to either side of the galaxy makes the interference disappear.

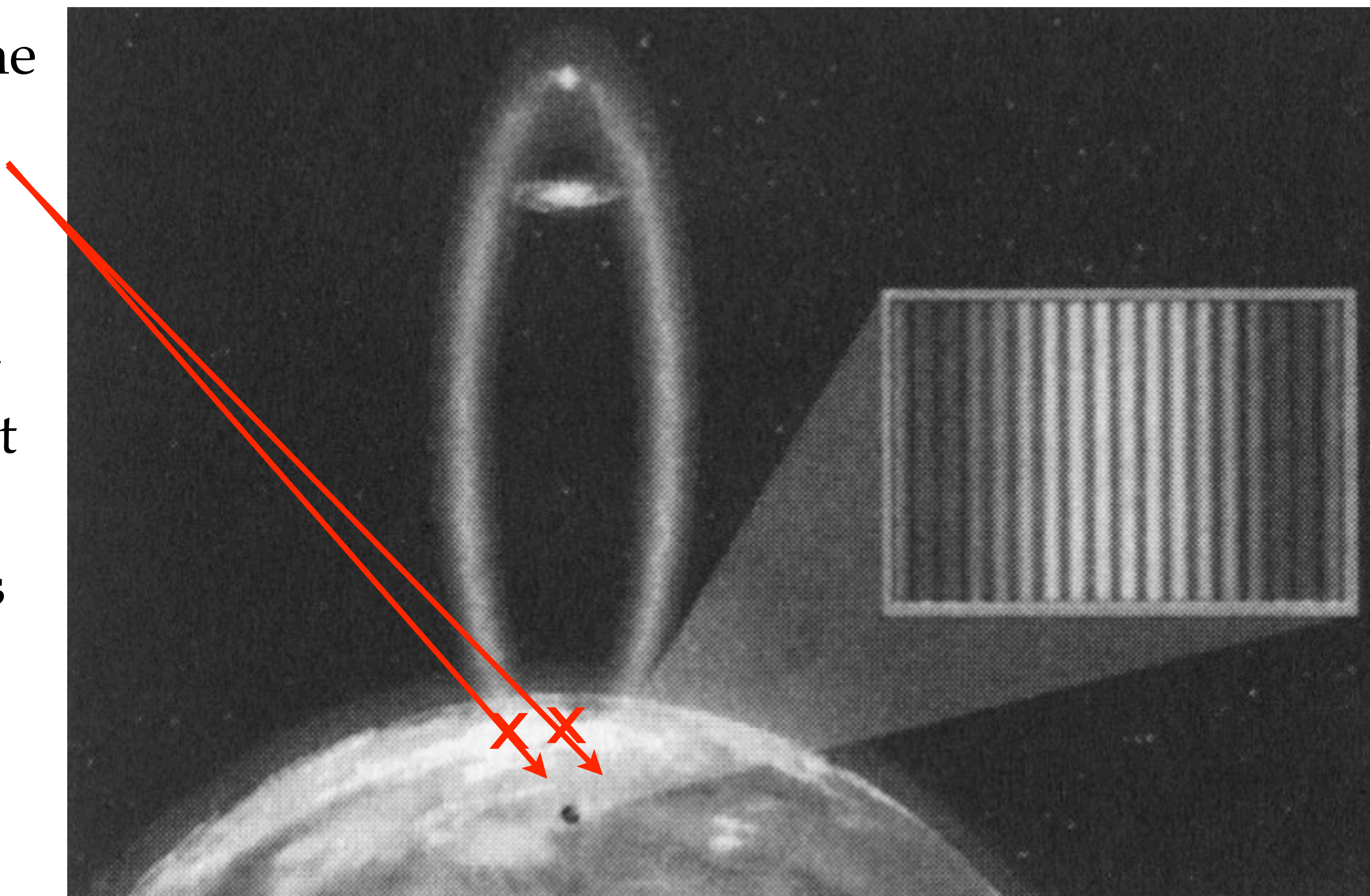
- If a photon is detected by the detector pointing to the left side we will infer that the photon had taken that path, not both paths $\Rightarrow$ *the detector changes a history lasting 10^{10} years!*



Original figure from *El tejido del cosmos*, Brian Greene, Ed. Crítica (2006)

Ein Gedankenexperiment

- and if 5 min later we disconnect the detectors, the interferences reappear immediately ⇒ we will infer that the photon had taken both paths ⇒ *we will have changed again a 10^{10} year history!*



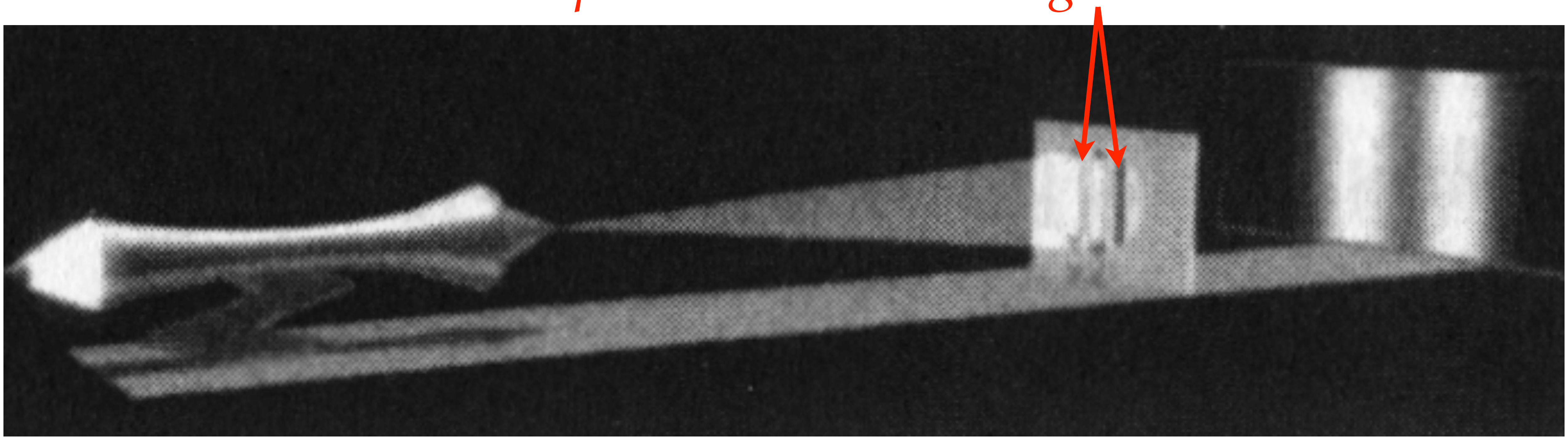
Original figure from *El tejido del cosmos*, Brian Greene, Ed. Crítica (2006)

Ein Gedankenexperiment

- Strictly speaking we cannot conclude that present observations change the past, what they change is *the history we have to tell to explain how the past leads to the present* (which is irrelevant for applying and verifying the theory). In a certain sense, *the (uncertain) past depends on the future!* (which does not imply that the past can be changed).
- According to *Bohm* hidden-variable theory the trajectories – lasting ≈ 10^{10} years– were real and dependent on a later decision of putting or not the detectors!
- The quasar+galaxy experiment has not been performed, but let's consider what has actually being done…

Labeling particles

A different polarization state is fixed for the photons passing through each slit so that we can learn the path taken by every photon *when it reaches a polarization-detecting screen*:

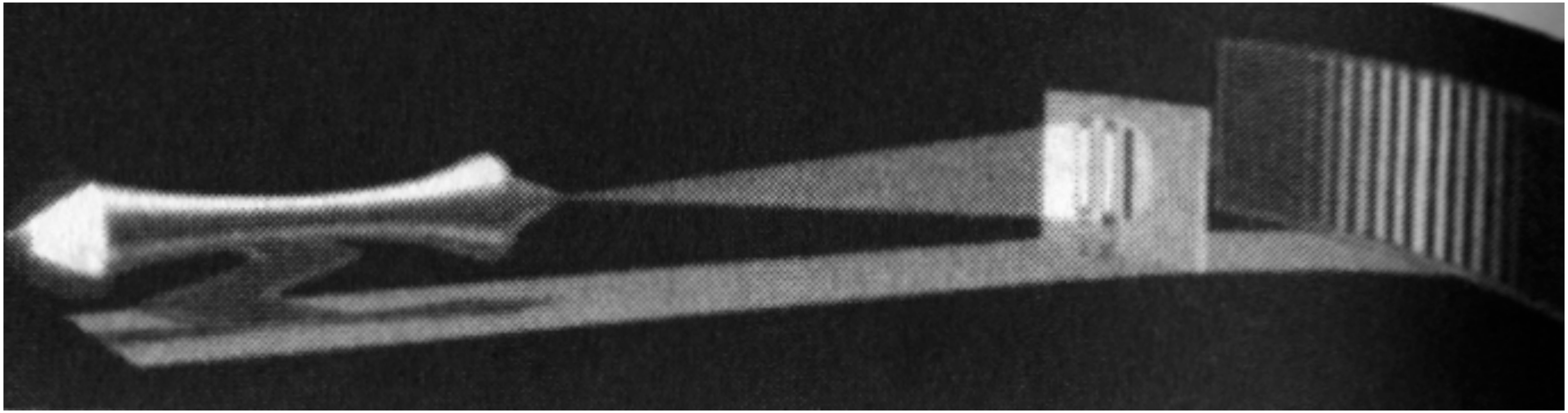


Original figure from *El tejido del cosmos*, Brian Greene, Ed. Crítica (2006)

Then *the interference disappears*. This suggest that *the interaction of the particles with the macroscopic labeling system fixes their path* –as would happen with 2 detectors– so they adjust their trajectories accordingly, *but*…

Quantum eraser

…if a quantum eraser (*Scully and Drühl* 1982; *Walborn, Terra Cunha, Pádua and Monken* 2000) resets the same polarization state for every photon when it is just about to reach the screen, the which-path information is lost and the interference reappears!



Original figure from *El tejido del cosmos*, Brian Greene, Ed. Crítica (2006)

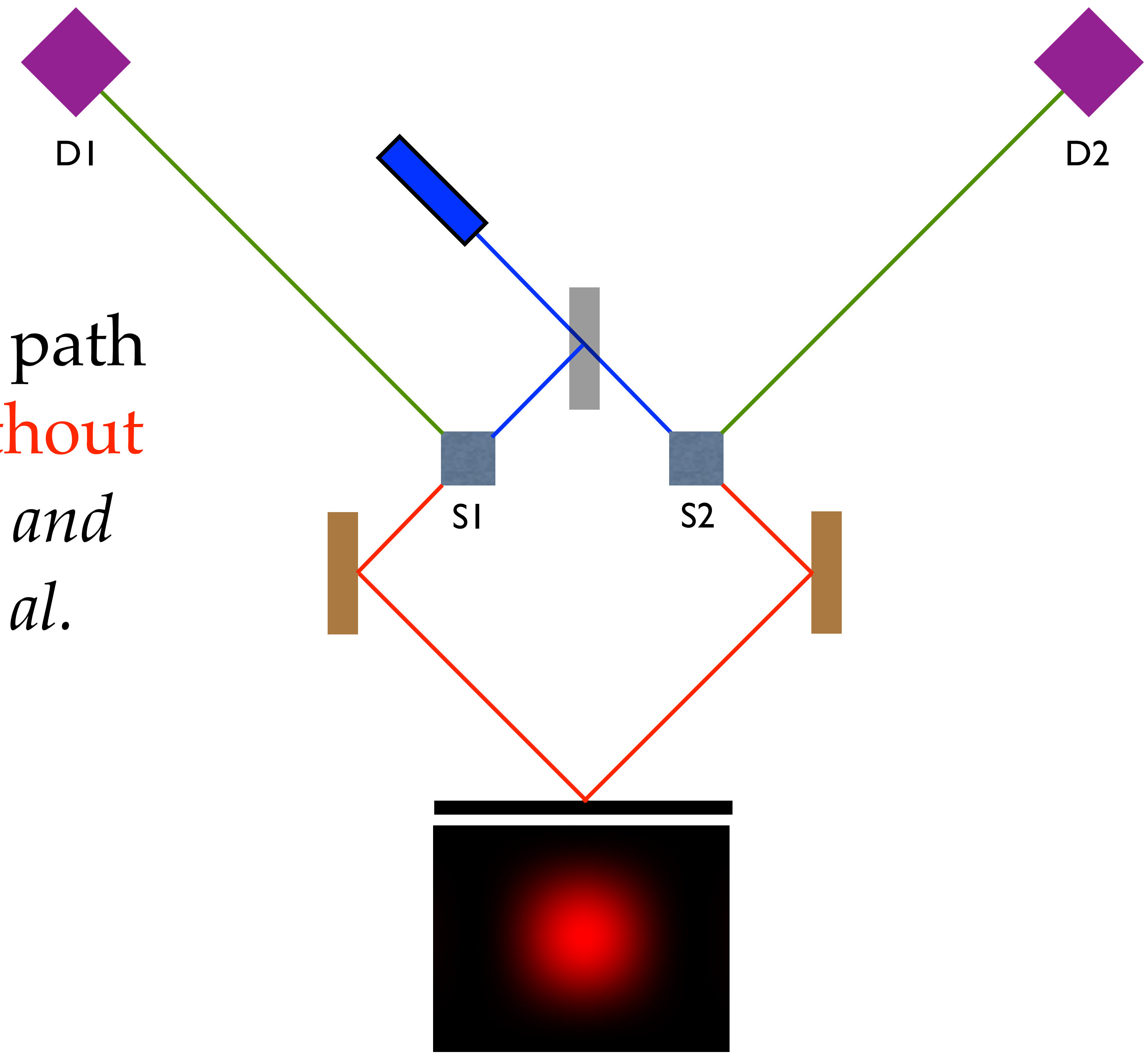
The photons that had gone through one of the two slits immediately adapt their trajectories to produce a pattern that can only be explained by assuming that they had gone through both slits!

Erasing the past?

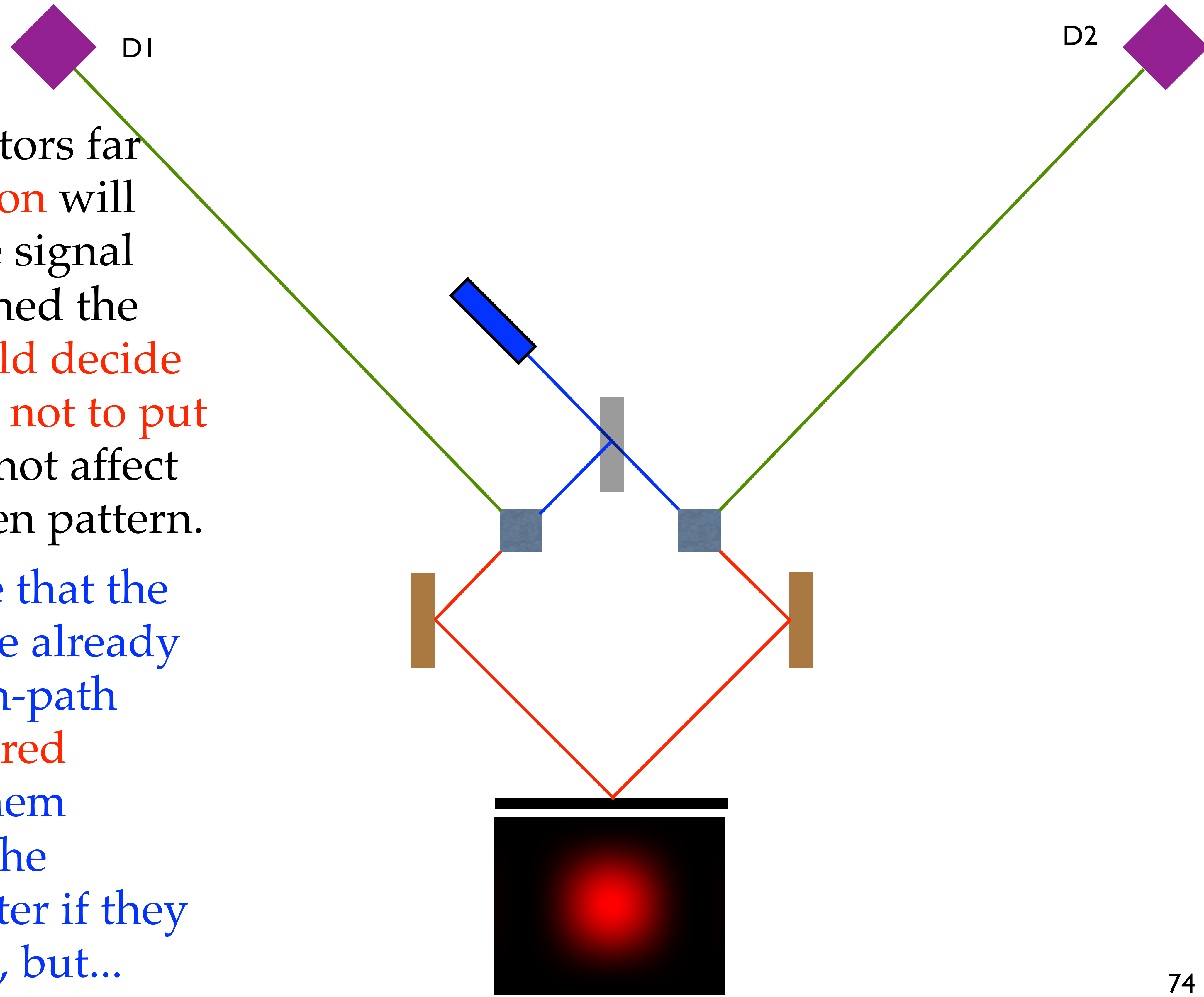
- The quantum eraser wipes out the photon path-history provided by the labeling system!
- If the labeling is used to find out the slit through which each photon passes no interference appears, but the information about the availability of the other path seems to remain latent in the photons until the last moment to make the photons disappear from the dark fringes if needed:
  - consider a labelled photon (with a definite trajectory) that is about to reach a point on the screen that would be dark if there were no labeling. If the eraser is connected at the last moment the photon should immediately jump to a light point… or maybe it should change its past trajectory (it should know that both slits were open, implying that it went through both slits)?

Raising the bar even higher

A system of knowing the path taken by each photon without interacting with it (*Scully and Drühl* 1982; *Kim, Scully et al.* 2000):



Raising the bar even higher



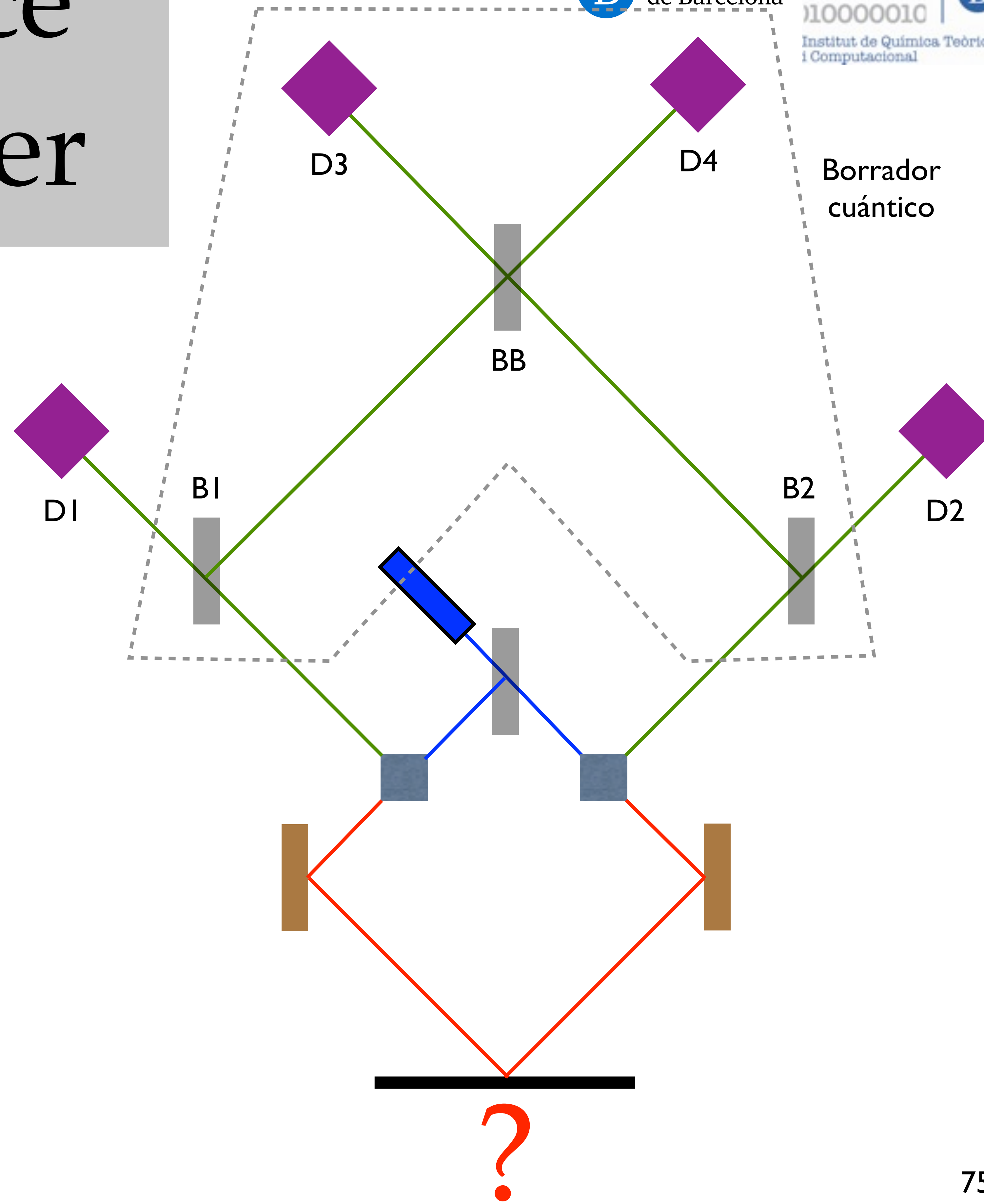
If we put the detectors far enough the detection will take place after the signal photons have reached the screen. But we could decide in the last moment not to put them, and this cannot affect the registered screen pattern.

We could conclude that the green photons have already collected the which-path information of the red photons making them collapse in one of the trajectories no matter if they are detected or not, but...

Delayed choice quantum eraser

Green photons reaching D1 or D2 give path information ⇒ the corresponding red photons should not interfere.

Green photons reaching D3 or D4 cannot give path information ⇒ the corresponding red photons should interfere.



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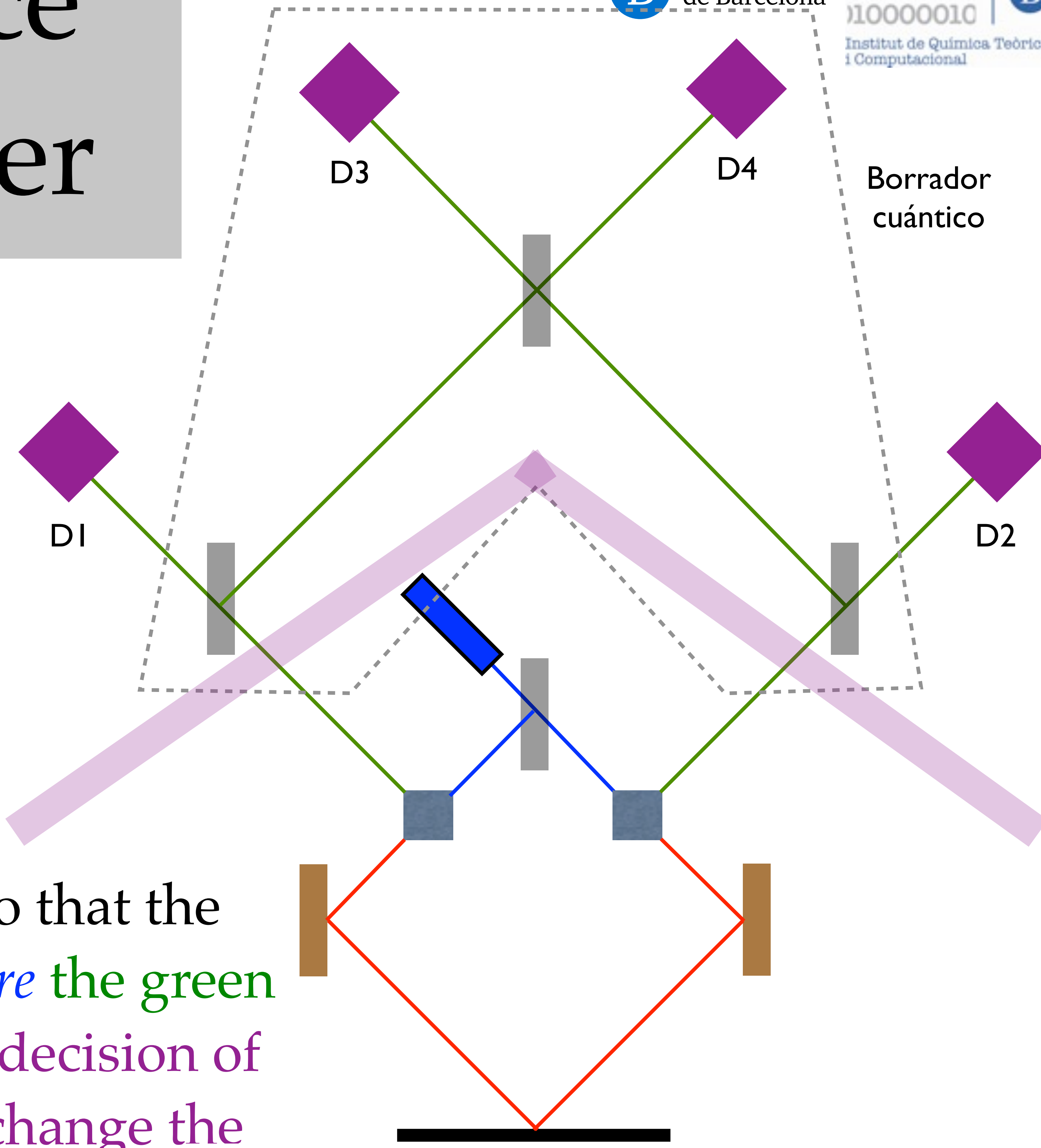
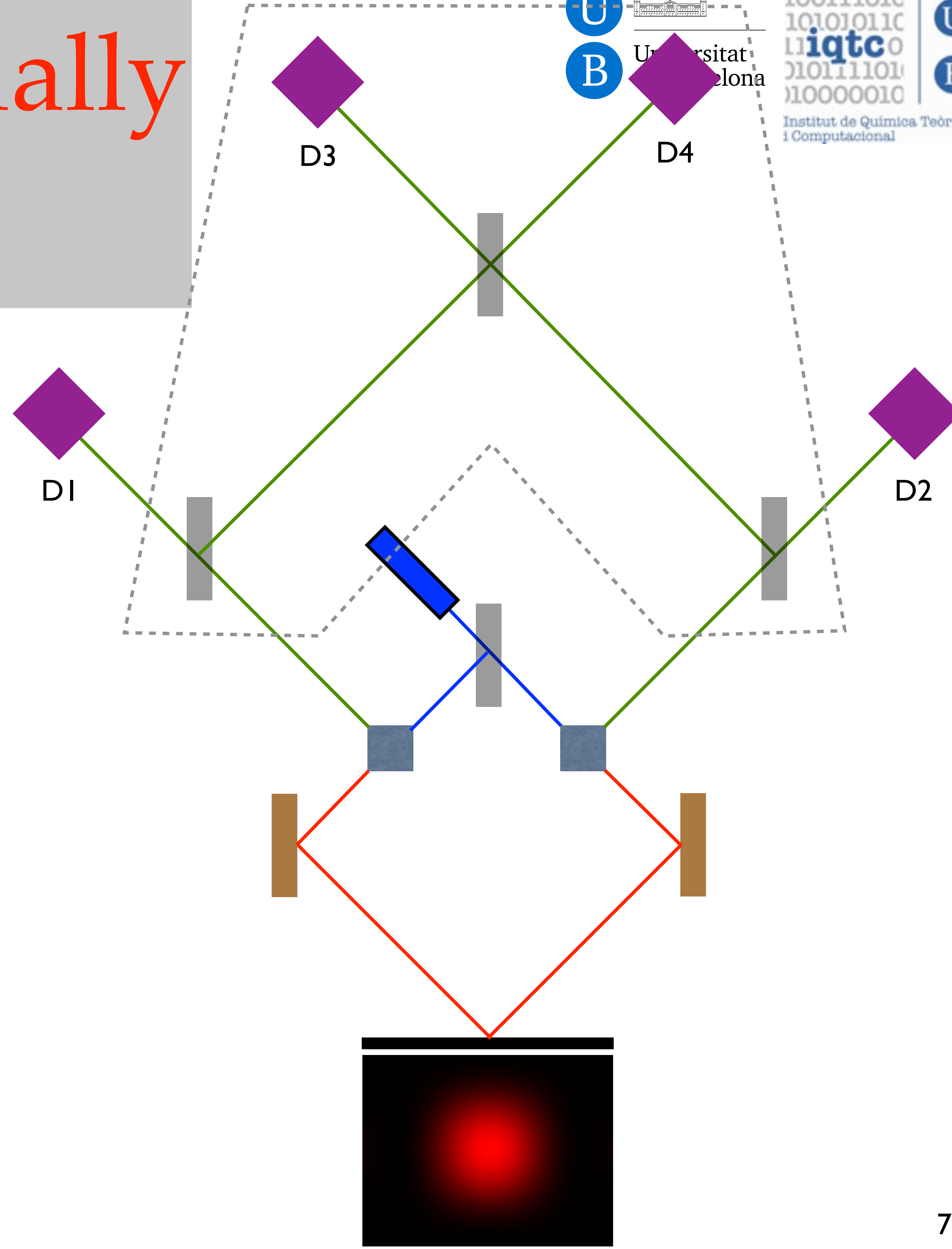


We can arrange the experiment so that the red photons reach the screen *before* the green ones reach the detectors. But the decision of putting or not the eraser cannot change the previously observed screen pattern!

A wrong quantum prediction?

What does actually happen?

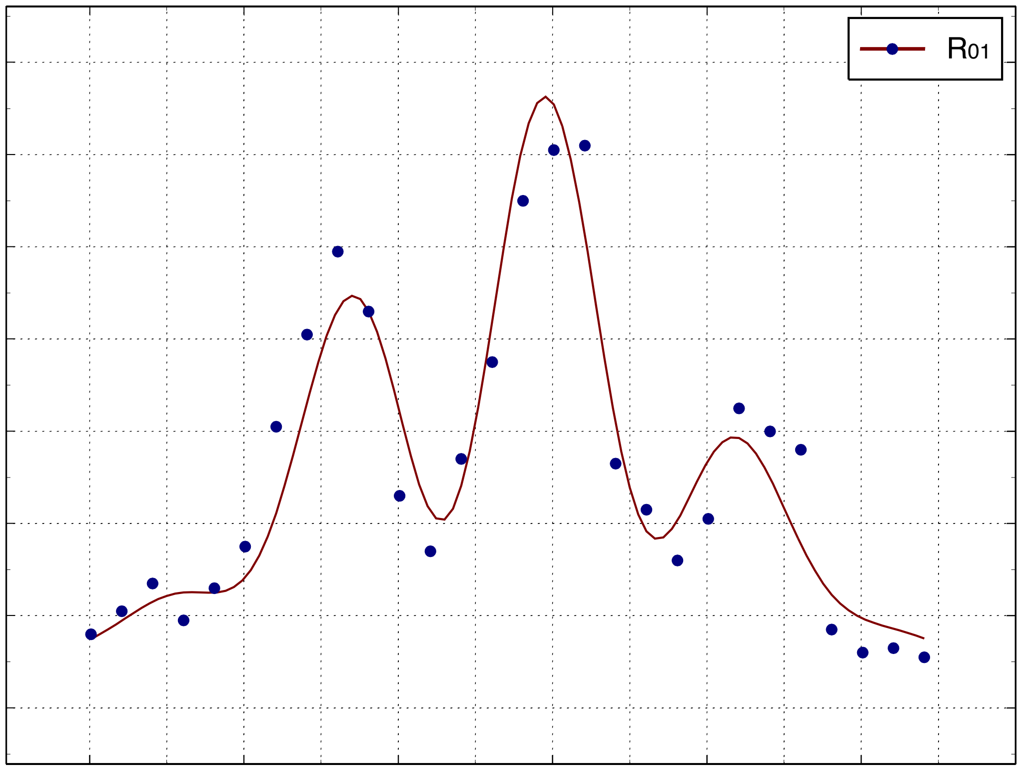
The overall screen pattern shows no interferences, no matter the quantum eraser being installed or not, but a careful application of QT predicts that red photons whose green partner is detected by D3 or D4 should show interferences ...



QT gets again saved!

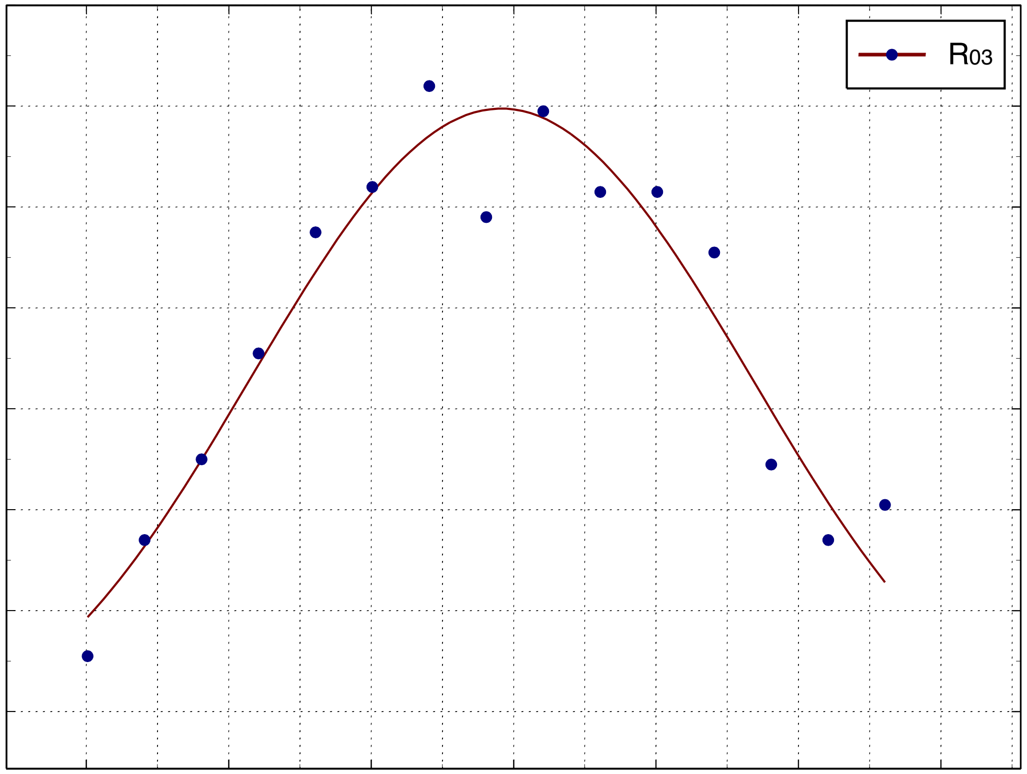
... when represented separately, and this is just what is observed!

D3

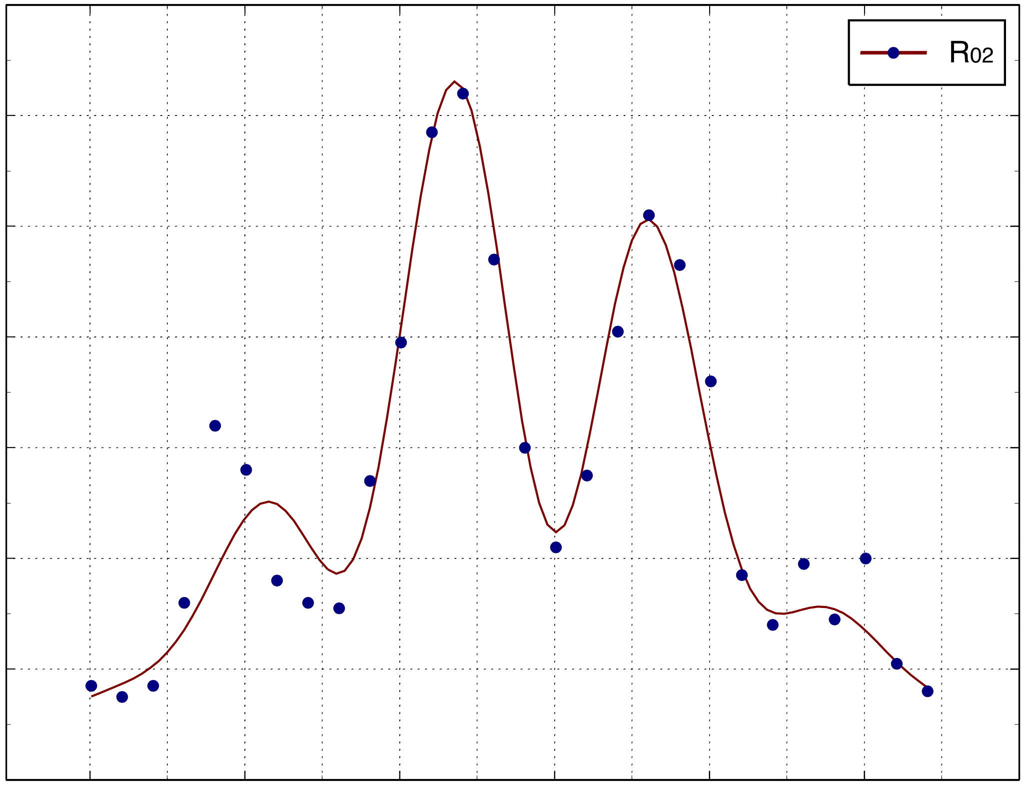


D3 + D4

(≈ 1 or 2)



D4



- To observe the interferences we have to wait until the results of D3 and D4 have been collected, so that the interference only shows up *after* the quantum eraser has been applied (absurd retrocausality is avoided).
- Now again, nature arrange things so that quantum laws are respected!

Figures from arXiv:quant-ph/9903047v1 (1999)

Shaping the past…

- If we don't include the erasing system we find no sign of interferences, and we will say that these is because each red particle has followed a definite path.

- But, if we decide to put it interferences can be revealed by representing separately the registers of D3 and D4, and we will say that the corresponding red particles have gone through both paths.

- Actual past facts (the screen image) are not changed by future decisions, but the story we have to tell to explain the whole experiment –the photon history– does change.

An inverted magic trick

- The nature behavior seems an inverted magic trick:
  - magics make us believe that amazing things happen, while a simple explanation is kept hidden,
  - the preceding experiment makes us think that a familiar classical evolution is followed, while a complex quantum interference remains hidden, to be revealed in the future only if we make sure that the which-path information carried by the green photons is destroyed by the quantum eraser.

Let’s go back to the cat

9GAG.COM/GAG/3388095

- If we don’t look into the box *the particle existence is not defined*, nor it is the living/dead state of the cat,
  - so, if we open the box and find the cat dead WE will have killed it! and if we find it alive WE will have saved his life!
- Maybe the cat acts as an observer of the setup and only dies if he observes the process triggered by the emission,
  - but what happens if it doesn’t look at the detector?
- Maybe the detector “creates” the α-particle independently of any living observer being present.
- This rises an important question…

What's a measuring device?

- The free evolution is *deterministic* (*Schrödinger equation*: *SE*).
- The collapse is *indeterministic* ⇒ *system* + *measurement device* + *observer* do not evolve according to the *SE*.
- Which of these parts escapes the *SE*?
  - The ability to collapse a quantum system is restricted to *conscious beings*
    - but… are *animals* conscious beings? and *plants*?
    - was the universe real before conscious beings appeared?
  - It is restricted to *live beings*
    - but there is not a sharp frontier between live beings and inanimate objects
  - It is restricted to *large and complicated objects*
    - but then, which is the *size threshold* that separates quantum systems from classically behaving measuring devices?

The measurement problem

- The standard formulation of quantum theory relies on the existence of measurement devices that do not conform to the quantum laws!
- This has for long been considered a banal question by most theoreticians, but…
- *measurements are the only means to connect theory and experiment in all of the natural sciences*!

Interpretations of Q M

http://en.wikipedia.org/wiki/Interpretation_of_quantum_mechanics

Interpretation	Author(s)	Deterministic?	Wavefunction real?	Unique history?	Hidden variables?	Collapsing wavefunctions?	Observer role?	Local?	Counterfactual definiteness?
Ensemble interpretation	Max Born, 1926	Agnostic	No	Yes	Agnostic	No	None	No	No
Copenhagen interpretation	Niels Bohr, Werner Heisenberg, 1927	No	No[1]	Yes	No	Yes[2]	None	No	No
de Broglie-Bohm theory	Louis de Broglie, 1927, David Bohm, 1952	Yes	Yes[3]	Yes[4]	Yes	No	None	No	Yes
von Neumann interpretation	von Neumann, 1932, Wheeler, Wigner	No	Yes	Yes	No	Yes	Causal	No	No
Quantum logic	Garrett Birkhoff, 1936	Agnostic	Agnostic	Yes[5]	No	No	Interpretational[6]	Agnostic	No
Many-worlds interpretation	Hugh Everett, 1957	Yes	Yes	No	No	No	None	Yes	No
Popper's interpretation[36]	Karl Popper, 1957[37]	No	Yes	Yes	Yes	No	None	Yes	Yes[13]
Time-symmetric theories	Yakir Aharonov, 1964	Yes	Yes	Yes	Yes	No	No	Yes	No
Stochastic interpretation	Edward Nelson, 1966	No	No	Yes	No	No	None	No	No
Many-minds interpretation	H. Dieter Zeh, 1970	Yes	Yes	No	No	No	Interpretational[7]	Yes	No
Consistent histories	Robert B. Griffiths, 1984	Agnostic[8]	Agnostic[8]	No	No	No	Interpretational[6]	Yes	No
Objective collapse theories	Ghirardi-Rimini-Weber, 1986	No	Yes	Yes	No	Yes	None	No	No
Transactional interpretation	John G. Cramer, 1986	No	Yes	Yes	No	Yes[9]	None	No	No
Relational interpretation	Carlo Rovelli, 1994	No	No	Agnostic[10]	No	Yes[11]	Intrinsic[12]	Yes	No

Interpretations of Q M

- *Copenhagen interpretation*: stick to the facts (measurements).
- *Hidden variables*.
- The Schrödinger equation does not apply to macroscopic systems (small stochastic *corrections*).
- The wave function represents *our knowledge* about the system.
  - No collapse without conscious beings?
  - Is the moon there when nobody looks?
- *Many worlds* interpretation (*Everett*, 1957) (freewill?).
- *Decoherence*: a quantum description of collapse?
  - measurement device coupled to a large environment
  - collapse is only apparent?
  - how is one of the potential values selected? (intellectual avarice?)

Quantum information/computing: the next revolution

20th century

Science: quantum theory

Technology: computers

→

21th century

Quantum information / computing

Quantum information/computing: the next revolution

20th century	21th century
Science: quantum theory	Quantum information / computing
Technology: computers	

Science: quantum theory → Quantum information / computing

Technology: computers → Quantum information / computing

Quantum theory is applied to understand and manipulate *matter: particles and fields*

→

Quantum theory is also applied to understand and manipulation *information*

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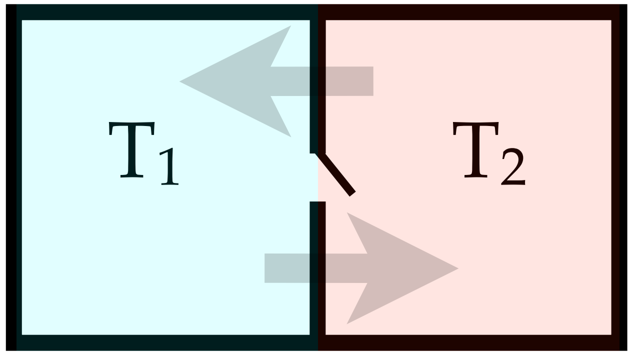
Quantum information

- *Information* is a physical quantity, on an equal footing with energy and intimately connected with *entropy* (Maxwell's demon (1867), Szilard 1929, Landauer 1961, Bennett 1982)
  - Experimental realization: Toyabe 2010.



- Ekert (1995):
  - Computations are *physical* processes.
  - Fundamental questions regarding computability and computational complexity are questions that belong to *physics* rather than *deterministic computation* and *abstract mathematics*.

- It is likely that the study of quantum information will help to deal with the most basic problems of QT.

Cryptography

- *Public-key system RSA* (Rivest , Shamir and Adleman 1979)
  - Bob publicly announces a very large number $n = pq$, which is the product of two prime numbers p and q.
    - Alice uses n to encode a message.
    - Bob uses p and q to decipher it.
  - If Eve intercepts the message she cannot decrypt it, since she may know n, but this number only allows to encrypt the message, not to decrypt it. Only Bob knows p and q…
    - …unless she manages to decompose n in factors…
    - …but this is a difficult task, that becomes impossible in practice if n is very large ($2^{1000} \approx 10^{300}$ to $2^{2000} \approx 10^{600}$ are currently used, but 10^{300} keys might become breakable in the near future; a 795 bits long number ($\approx 10^{240}$) was factored in November 2019).

Cryptography

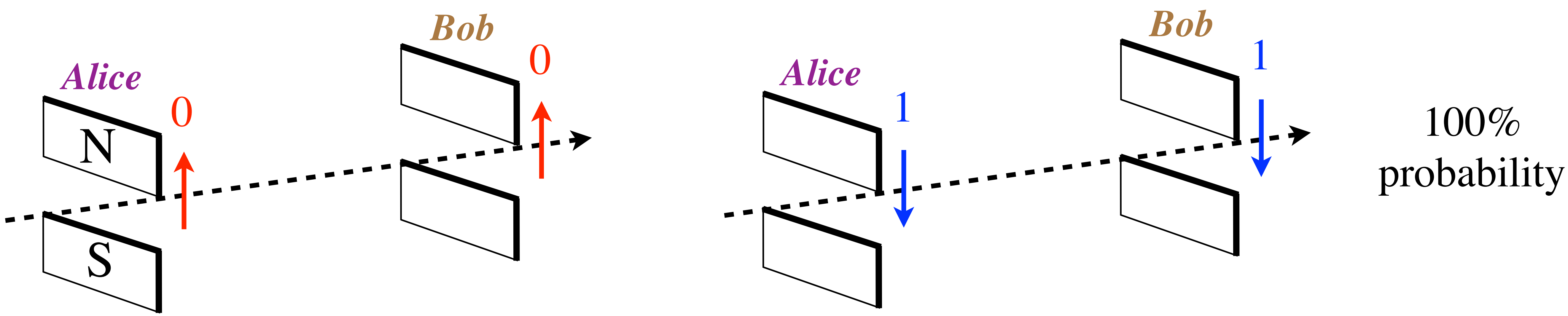
- *Secret-key cryptography*
  - Message: $m_1, \ldots m_n$ (1=A, 2=B, ... 27=Z, 28=A, ...)
  - Key: $k_1, \ldots k_n$ (*e.g.*: random numbers)
  - Encrypted message: $m_1+k_1, \ldots m_n+k_n$
  - Very secure (absolutely secure if the key is used only once).
  - The sender and the receiver must have a way of sharing the key:
    - they could have 2 unique copies of a key book and communicate to choose a different key for every message (bank key cards)
    - keys must be changed frequently , specially for large numbers of messages
    - if they are far apart this is a risky process that raises the same problem as exchanging messages: a spy could intercept the connection.

What does QT has to do with this?

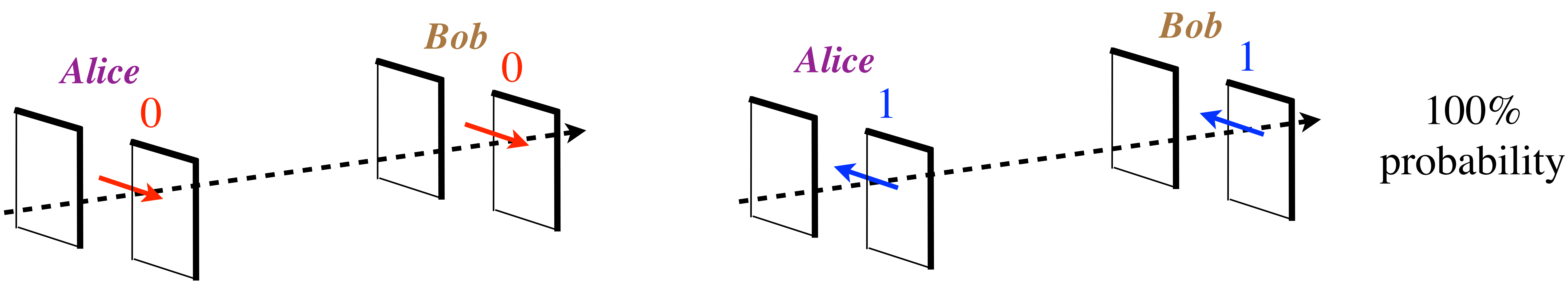
- *Public-key system RSA*
  - Quantum computers might be able (in the future) to decompose large numbers in a much shorter time: polynomial ($\sim n^2$) *vs* exponential (2^n) time
    - $400^2 = 1.6 \times 10^5$; $2^{400} = 2.6 \times 10^{120}$ (the age of the universe is 4×10^{17} s!!!)
- *Secret-key cryptography*
  - Quantum key distribution systems are, in principle, absolutely secure.

Quantum key distribution BB84

Both measurements in the vertical axis: ↑ means 0, ↓ means 1

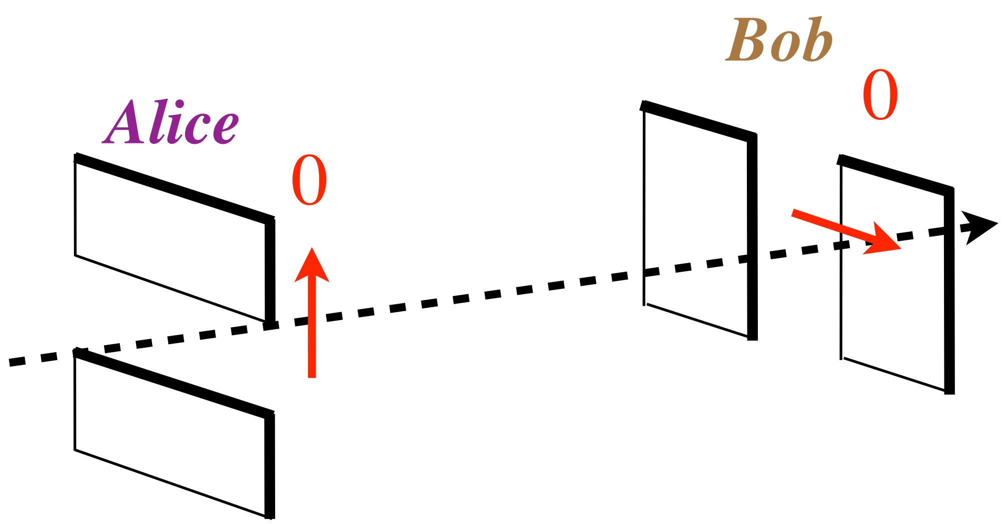


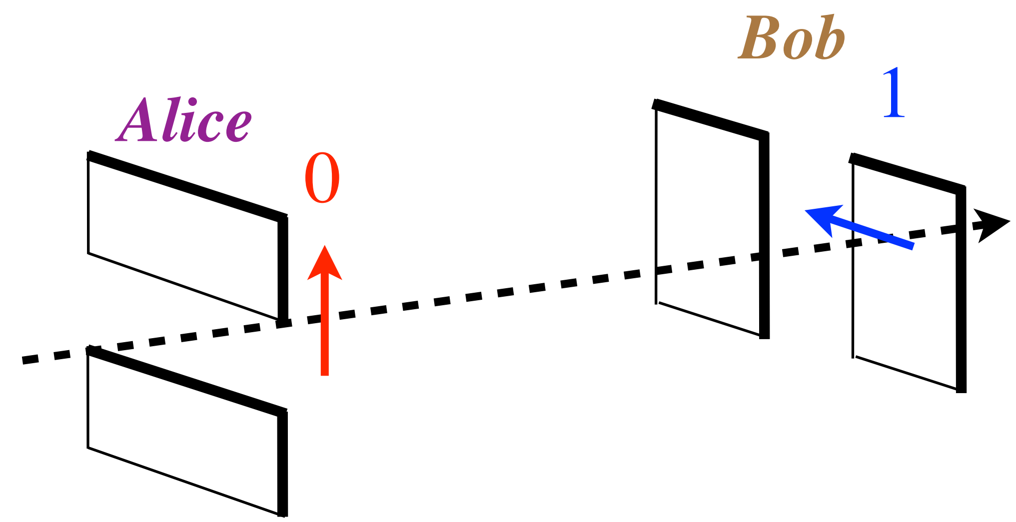
Same for measurements in the horizontal axis: → means 0, ← means 1



Quantum key distribution BB84

Measurements in different axes:

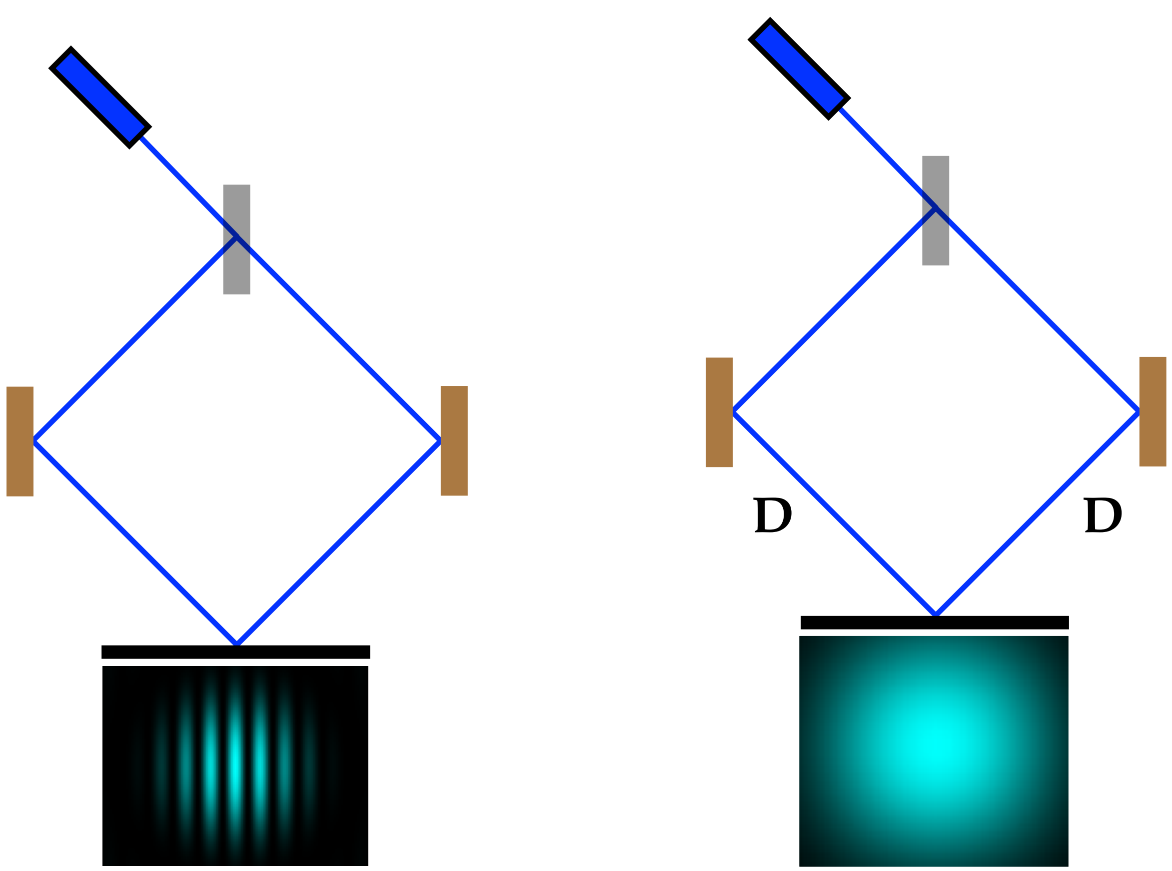




$\uparrow$ is a superposition of $\rightarrow$ and $\leftarrow$:

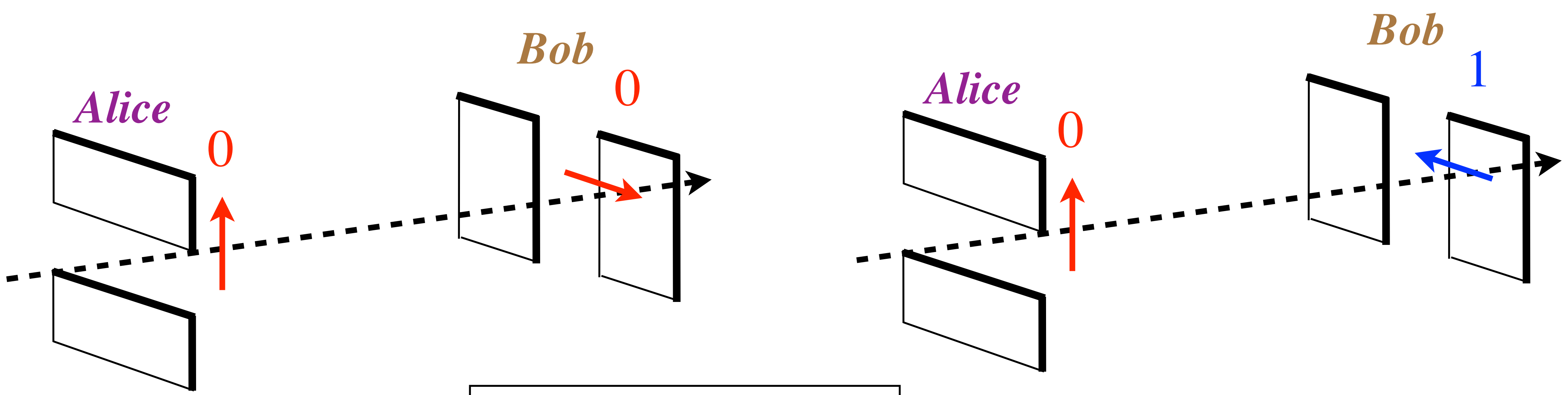
$\uparrow = (\rightarrow + \leftarrow)/\sqrt{2}$

50% probability for each result



Quantum key distribution BB84

Measurements in different axes:



$\uparrow = (\rightarrow + \leftarrow)/\sqrt{2}$ 50% probability for each result

Similarly: $\downarrow = (\rightarrow - \leftarrow)/\sqrt{2}$ 50% probability for each result

$\rightarrow = (\uparrow + \downarrow)/\sqrt{2}$

$\leftarrow = (\uparrow - \downarrow)/\sqrt{2}$

50% probability for each result

Therefore all of these particle sendings are useless.

Quantum key distribution BB84

Particle number	*Alice*	*Bob*	*bit sent*
1	↑	↑	0
2	←	↑	
3	←	←	
4	↓	→	
5	↓	↓	1
6	↑	←	
7	→	→	
8	←	↓	
9	←	←	1
10	↑	→	
11	→	→	
12	↓	←	

↑ and → mean 0
↓ and ← mean 1

- *Alice* and *Bob* randomly choose their measurement axes and publicly tell the axes used for every measurement. The chosen axes will coincide in half of the cases (colored rows). The rest are discarded (white rows).
- *A* selects half of the coincident measurements (in pink) and publicly tells the corresponding results to *B*. If all the results obtained by *B* for these cases match those obtained by *A* the transmission has not been intercepted, and they can use the other half of matching bits (in green) as a *reliable shared information*.
- This could be the key, or could be used to decide which of a number of previously agreed keys should be used. Also, *A* could reorder the shared bits to form a short message and publicly tell *B* the reordering.
- How can they be sure that the signal has not been intercepted?

Quantum key distribution BB84

Particle number	*Alice*	*Bob*	*bit sent*	*Eve*	*Bob*
1	↑	↑	0		
2	←	↑			
3	←	←		←	←
4	↓	→			
5	↓	↓	1		
6	↑	←			
7	→	→		↓	→
8	←	↓			
9	←	←	1		
10	↑	→			
11	→	→		↓	←
12	↓	←			

Eve has 50% of probability of choosing the same axis as *Alice*

No sign of *Eve*'s presence if her axis happens to coincide with *Alice*'s.

Eve's choice will *not* coincide with that of *Alice* in half of the cases. Then *Eve* forwards the particle with its spin in the wrong axis:

1) No sign of *Eve*'s presence.

2) *Eve*'s presence is revealed.

- A matching failure in a quarter of the publicized measurements indicates that *the transmission has been intercepted*.

Quantum information processing

Classical computers

- 1 *bit* can be in 2 states, say (0) and (1):

- *n bits* can be in 2^n states: *E.g.*: (0100 0001) ≡ *A* (1 ASCII char. out of 2^8)
- Each state requires *n* binary digits to be specified.
- Computations are applied to one set of data; e.g.: x=3 or x=5 or ...
- ≠ bits are always *uncorrelated*.
- *Limited* miniaturization (Moore's law): *Heisenberg uncertainty principle*.
- Pseudo-random numbers.

Quantum computers

- 1 *qubit* can be in ∞ *superposition* states: $|0\rangle$ and $|1\rangle$ or $c_0|0\rangle+c_1|1\rangle$

- *n qubits* can be in any linear comb. of 2^n states: $c_A|0100\ 0001\rangle+c_B|0100\ 0010\rangle-\ldots \equiv c_A A+c_B B-\ldots$
- To represent in a classical computer the state of 100 qubits one needs $2^{100}=10^{30}$ complex numbers!
- Computations can be applied to a linear comb. of several sets of data (*quantum parallelism*).
- ≠ qubits can be in *entangled* states.
- *Unlimited* miniaturization
  - quantum laws do not entail a limitation, but an opportunity for new algorithms.
- True random numbers (http://qrng.anu.edu.au/)

Quantum computing

- Some important problems that are dealt with *exponentially* complex classical algorithms have a *polynomial* complexity for quantum computers.
  - *Benioff* 1980: formulated a quantum reversible Turing machine.
  - *Manin* 1980; *Feynman* 1982: quantum computers could be exponentially more powerful than classical ones for simulating other quantum systems (universal quantum simulator; *Barreiro-Lanyon* 2011).
  - *Deutsch* 1985-1992: developed a (theoretically) physically realizable model of quantum computer; superposition → parallelism.
  - *Bernstein*; *Yao*; *Berthiaume*; *Simon*; …
  - *Coppersmith* 1994: discrete Fast FT $O(n2^n)$ steps; quantum FT $O(n^2)$ steps
  - *Shor* 1994: decomposition of a number in factors
    - theoretical curiosity → practical and political interest (hackers!)
  - *Grover* 1996: search of a number in a non-ordered list (telephone list), cryptanalysis, satisfiability problem (useful in AI, QC circuit design, automatic theorem proving, …)
  - *Lloyd* 1996-1997: simulating many-body Fermi systems.
  - Quantum biology?…

Entanglement

$$\Psi_{triplet+} = \uparrow_1\uparrow_2$$

Both spins point up.

A measurements of $S_z(1)$ does not affect $S_z(2)$, as with classical bits $\rightarrow$ they are not entangled.

Entanglement

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$$\Psi_{singlet} = \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)$$

A superposition state: both spins point in opposite z-directions, but their individual orientations are not defined.

Entanglement

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A superposition state: both spins point in opposite z-directions, but their individual orientations are not defined.

A measurement of $S_z(1)$ with upwards/downwards result changes

$\Psi_{singlet}$ to $\Psi_{after} = \uparrow_1\downarrow_2$ / $\Psi_{after} = \downarrow_1\uparrow_2$

$S_z(2)$ becomes down/up, no matter the distance between 1 and 2!!!

→ $S_z(1)$ and $S_z(2)$ are entangled. No classical analog.

Quantum computing

Classical gates

- They implement *boolean operations*
  - some of them are *irreversible* (the input cannot be recovered from the output) ⇒ dissipation
  - same gates as in the 1940's.

- *Examples:*

NOT

input	output
(0)	(1)
(1)	(0)

AND

input		output
(0)	(0)	(0)
(0)	(1)	(0)
(1)	(0)	(0)
(1)	(1)	(1)

Quantum gates

- They implement *unitary transformations* (Schrödinger equation)
  - they are *reversible* ⇒ *no* dissipation
  - the amount of information is conserved
  - new gates (Hadamard...)

- *Examples:*

NOT (Pauli-X)

input	output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Hadamard

input	output
$\|0\rangle$	$(\|0\rangle + \|1\rangle)/\sqrt{2}$
$\|1\rangle$	$(\|0\rangle - \|1\rangle)/\sqrt{2}$

Quantum computing

- The square root of any quantum gate can be designed; for instance:

$$|0\rangle \xrightarrow{\sqrt{NOT}} \tfrac{1}{2}[(1+i)|0\rangle + (1-i)|1\rangle] \xrightarrow{\sqrt{NOT}} \cdots |1\rangle$$

$$|1\rangle \xrightarrow{\sqrt{NOT}} \tfrac{1}{2}[(1-i)|0\rangle + (1+i)|1\rangle) \xrightarrow{\sqrt{NOT}} \cdots |0\rangle$$

- Some quantum gates can entangle qubits:

CNOT

control qubit → (first input qubit)

input		*output*	
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

non-entangled qubits states
(products of one-qubit states)

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|0\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle)$$

entangled qubits

- The 3-qubit CCNOT (Toffoli) gate is universal!

Simulating a coin

- *Deutsch 1985 - Cleve et al. 1998*:
  - A computer program simulates a coin by means of a function $f(x)$ that tell us whether each face A/B of the coin is head or tail.
  - In order to test whether the simulated coin is fair or fake we have to run 2 times the program, one with the input x=0 (A) and the other with x=1 (B), so as to find out which of these 4 functions has been programmed:

x (side)	$f_{00}(x)$	$f_{01}(x)$	$f_{10}(x)$	$f_{11}(x)$
0 (A)	0 (head)	0 (head)	1 (tail)	1 (tail)
1 (B)	0 (head)	1 (tail)	0 (head)	1 (tail)
The coin is	fake	fair	fair	fake

Simulating a coin

- *Deutsch 1985 - Cleve et al. 1998*:
  - A quantum program allows to know if the simulated coin is fair or fake in a single run. It uses un operation (f-CNOT) that gives the unknown built-in function $f(x)$ in an ancillary qubit y, previously initialized to 0:

$$|x\rangle\,|y\rangle \xrightarrow{\hat{U}_{fCNOT}} |x\rangle\,|y \oplus f(x)\rangle \qquad |x\rangle\,|0\rangle \xrightarrow{\hat{U}_{fCNOT}} |x\rangle\,|f(x)\rangle$$

addition modulo 2

y	$f(x)$	$y\oplus f(x)$
0	0	0
0	1	1
1	0	1
1	1	0

Simulating a coin

- **The quantum trick**: to obtain $f(0)\oplus f(1)$ (=**1** for a **fair** coin and =**0** for a **fake** coin) without knowing $f(0)$ and $f(1)$ individually:
  - Note first that

$$|x\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}\xrightarrow{\hat{U}_{fCNOT}}|x\rangle\frac{|0\oplus f(x)\rangle-|1\oplus f(x)\rangle}{\sqrt{2}}=\begin{cases}|x\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}} & \text{if } f(x)=0\\ |x\rangle\frac{|1\rangle-|0\rangle}{\sqrt{2}} & \text{if } f(x)=1\end{cases}=(-1)^{f(x)}|x\rangle\frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

$$|x\rangle|y\rangle\xrightarrow{\hat{U}_{fCNOT}}|x\rangle|y\oplus f(x)\rangle$$

  - then

$$(-1)^{f(0)}(-1)^{f(0)}=1$$

$$|0\rangle\xrightarrow{\hat{H}_x}\frac{|0\rangle+|1\rangle}{\sqrt{2}}\xrightarrow{\hat{U}_{fCNOT}}\frac{(-1)^{f(0)}|0\rangle+(-1)^{f(1)}|1\rangle}{\sqrt{2}}=(-1)^{f(0)}\frac{|0\rangle+(-1)^{f(0)\oplus f(1)}|1\rangle}{\sqrt{2}}$$

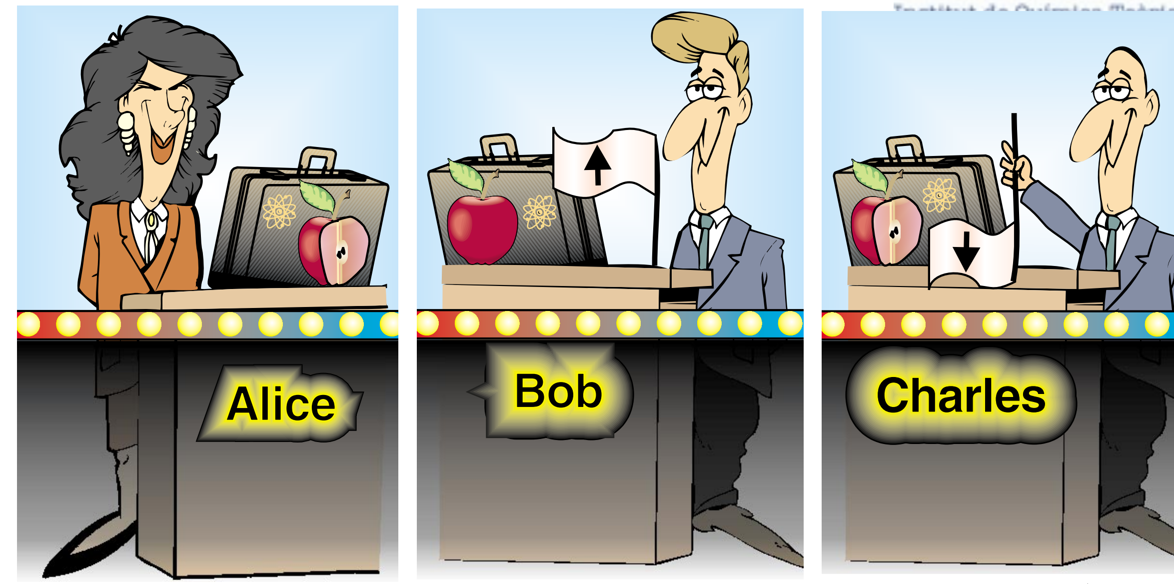
$$\xrightarrow{\hat{H}_x}(-1)^{f(0)}\frac{|0\rangle+|1\rangle+(-1)^{f(0)\oplus f(1)}(|0\rangle-|1\rangle)}{2}$$

$$=\begin{cases}(-1)^{f(0)}|0\rangle & \text{if } f(0)\oplus f(1)=0 \text{ (fake)}\\ (-1)^{f(0)}|1\rangle & \text{if } f(0)\oplus f(1)=1 \text{ (fair)}\end{cases}$$

Deutsch problem

- *Deutsch-Jozsa 1992 - Cleve et al. 1998*:
  - We have n coins that can be in face or tail position over a table, and an unknown computer program implementing a function f of this collection of faces and tails that can only be one of these:
    - either a constant function giving the same result –0 or 1– for any of the 2^n possible combinations of faces and tails,
    - or a function taking the value 0 for half of the combinations (2^{n-1}) and 1 for the other half.
  - The number of times we have to evaluate f in a classical computer to know which of those 2 functions has been implemented is $\leq 2^{n-1}+1 \Rightarrow$ grows exponentially with n.
  - For 100 coins and 10^{-12} s/eval, $t = (2^{99}+1)\text{x}10^{-12}$ s $\approx 6\text{x}10^{17}$ s
  - The age of the universe is $\approx 4\text{x}10^{17}$ s
  - The Deutsch-Jozsa-Cleeve quantum algorithm produces an answer that is always correct with a single evaluation of f.

A TV competition: Guess my number



Figs. from *Physics Today* **53**-2, 35 (2000)

- Entanglement → telepathy?

- Alice, Bob and Charles play *Guess my number*:
  - The moderator divides 0, 1, 2, 3 or 4 apples between the 3 isolated contestants.
  - Each of them receives 0, 1/2, 1, 3/2,… or 4 apples.
  - Then Bob and Charles can hold a flag either up or down.
  - The moderator informs Alice of the positions of the flags and she has to guess whether the total number of distributed apples is even or odd.
  - Any (classical) strategy leads, at most, to a 75% chance of success, and Alice needs 10 consecutive hits to win the price…
  - …but Alice always gives the right answer!

Quantum strategy

- Before the show Alice, Bob and Charles prepare a triplet of qubits in the entangled state $|000\rangle+|111\rangle$. Each contestant carries one of the qubits.

- The moderator gives x_A, x_B and x_C apples to the contestants, where $x_j = 0, 1/2, 1$ or $3/2$.
  - x_j can be $> 3/2$ as long as $x_A + x_B + x_C \leq 4$, but adding 2 apples to any contestant doesn't affect the result.

- Then they apply to their qubit the operation $|0\rangle\langle 0| + e^{i\pi x_j}|1\rangle\langle 1|$
  - $|000\rangle + |111\rangle \rightarrow |000\rangle + |111\rangle$ if $x_A+x_B+x_C$ is *even*
  - $|000\rangle + |111\rangle \rightarrow |000\rangle - |111\rangle$ if $x_A+x_B+x_C$ is *odd*.

- Each contestant then applies the Hadamard operation to his qubit:
  - if $x_A+x_B+x_C$ is *even*: $|000\rangle + |111\rangle \rightarrow |000\rangle + |011\rangle + |\underline{101}\rangle + |110\rangle$
  - if $x_A+x_B+x_C$ is *odd*: $|000\rangle - |111\rangle \rightarrow |000\rangle + |100\rangle + |010\rangle + |001\rangle$

input	*output*
$\|0\rangle$	$\|0\rangle + \|1\rangle$
$\|1\rangle$	$\|0\rangle - \|1\rangle$

- They measure their qubits, and Bob and Charles tell Alice their result (↑=0 and ↓=1)

- If, say, Alice has obtained (1), Bob (0) and Charles (1) this results can only be produced by the joint state $|000\rangle + |011\rangle + |\underline{101}\rangle + |110\rangle$, so $x_A+x_B+x_C$ is *even*, and so on.

Quantum computing has limitations...

- However QC also has important limitations:
  - Classical information can be *read* and *duplicated* (intermediate backups), while quantum information cannot (no-cloning theorem).
  - The *final state* of a classical system can be faithfully read, while that of a quantum system *cannot* in general *be simply inferred* from measurement outcomes; a good dose of ingenuity is needed to extract the desired properties. As a consequence *quantum parallelism* is difficult to exploit.
  - Accuracy: unlike classical discrete gates (digital), unitary operations form a continuum (analogic) ⇒ error accumulation (bit-flip + phase errors should be corrected without reading the encoded information).
  - The interaction of a quantum system with its environment produces a *decoherence* of quantum states that introduces random errors and information losses (≈ measurements). A trade-off between isolation and qubit manipulability.
  - So far, a small number of problems have been found for which quantum algorithms beat classical ones. Quantum coprocessor?

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Simulation is more feasible

- Thousands of qubits and billions of quantum gates are needed to solve classically difficult factoring problems.
- Tens of qubits and a few thousand operations are needed to perform simulations that would be classically intractable (≈ Avogadro's number of memory sites and operations).
  - particularly useful when little information needs to be extracted from a very complex wave function (energy, rate constant...),
  - the number of basis states (Slater determinants) grows exponentially with the system size ($\approx m^n/n!$), there are more efficient quantum simulation algorithms
  - time evolution can be simulated by combining quantum gates:

$$\Psi(t) = \widehat{U}(t)\Psi(0) \approx \widehat{U}_1 \cdots \widehat{U}_k \Psi(0) \qquad \text{(trotterization of } U\text{)}$$

  - the frontier between theory and experiment blurs: a physical system is simulated by another physical system ⇒ easier to extract physical intuition.

Quantum simulation of *n*-electron systems

- How can we represent an *n*-electron wave function?
- In an *m*-dimensional 1-electron space:
  - Hartree-Fock determinant of *n*-electrons that occupy *n* of a set of *m* spin-orbitals we need *m* qubits:

$$\Phi^{HF} = \frac{1}{\sqrt{n!}} |\psi_1 \cdots \psi_n| = |\overbrace{\underbrace{1, \cdots 1}_{n}, 0, 0, \cdots 0}^{m}\rangle$$

  - any determinant:

$$|n_1, \cdots n_i, \cdots, n_m\rangle \quad \text{with } n_i = 0 \text{ or } 1 \text{ and } \sum_{i=1}^{m} n_i = n$$

Phase estimation algorithm

$$\widehat{U}(t)\Psi = e^{-i\widehat{H}t/\hbar}\Psi = e^{-iEt/\hbar}\Psi = e^{-i2\pi\phi}\Psi$$

stationary state

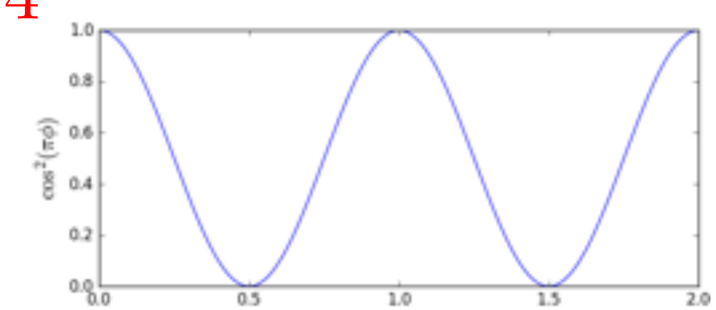
control qubit

$$|0\rangle|\Psi\rangle \xrightarrow{\widehat{H}_x} \frac{|0\rangle + |1\rangle}{\sqrt{2}}|\Psi\rangle \xrightarrow{\widehat{cU}} \frac{|0\rangle + e^{i2\pi\phi}|1\rangle}{\sqrt{2}}|\Psi\rangle \xrightarrow{\widehat{H}_x}$$

$$\frac{|0\rangle + |1\rangle + e^{i2\pi\phi}(|0\rangle - |1\rangle)}{2}|\Psi\rangle = \frac{(1+e^{i2\pi\phi})|0\rangle + (1-e^{i2\pi\phi})|1\rangle}{2}|\Psi\rangle$$

$0<\phi\leq 1/2$

$$P(x\rightarrow 0) = \frac{|1+e^{i2\pi\phi}|^2}{4} = \frac{(1+e^{i2\pi\phi})(1+e^{-i2\pi\phi})}{4} = \frac{2+2\cos(2\pi\phi)}{4} = \cos^2(\pi\phi)$$



Probability ⇒ ~ 2^{2d} rounds to obtain d accurate binary digits of ϕ

Iterative PEA

- Let's first assume that the binary expansion of ϕ is $0.\phi_1\,\phi_2\ldots\phi_d\,0\ldots$
  - By applying $\widehat{cU}^{2^{d-1}}$ instead of $\widehat{cU}$ we obtain ϕ_d in a single round:

$$|0\rangle|\Psi\rangle \xrightarrow{\widehat{H}_x} \xrightarrow{\widehat{cU}^{2^{d-1}}} \xrightarrow{\widehat{H}_x} \frac{(1+e^{i2\pi 2^{d-1}\phi})\,|0\rangle + (1-e^{i2\pi 2^{d-1}\phi})\,|1\rangle}{2}|\Psi\rangle$$

$$P\,(x\to 0) = \cos^2(\pi\,\underbrace{2^{d-1}\phi}_{\phi_1\phi_2\cdots.\phi_d 0\cdots}) = \cos^2(\pi\,0.\phi_d 0\ldots) = \begin{cases} \cos^2(0) = 1 & \text{if } \phi_d = 0 \\ \cos^2(\pi/2) = 0 & \text{if } \phi_d = 1 \end{cases}$$

$$\cos^2\left(\pi(n+\alpha)\right) = \cos^2(\pi\alpha)$$

Universitat de Barcelona

iqtc Institut de Química Teòrica i Computacional

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  - we use ϕ_d to obtain ϕ_{d-1}:

$$|0\rangle|\Psi\rangle \xrightarrow{\widehat{H}_x} \xrightarrow{\widehat{cU}^{2^{d-2}}} \frac{|0\rangle + e^{i2\pi 2^{d-2}\phi}|1\rangle}{\sqrt{2}}|\Psi\rangle \xrightarrow{\widehat{R}_z(-2\pi 0.0\phi_d 0\cdots)} \frac{|0\rangle + e^{i2\pi 0.\phi_{d-1}0\cdots}|1\rangle}{\sqrt{2}}|\Psi\rangle \xrightarrow{\widehat{H}_x}$$

($2^{d-2}\phi$: $0.\phi_{d-1}\phi_d0\cdots$)

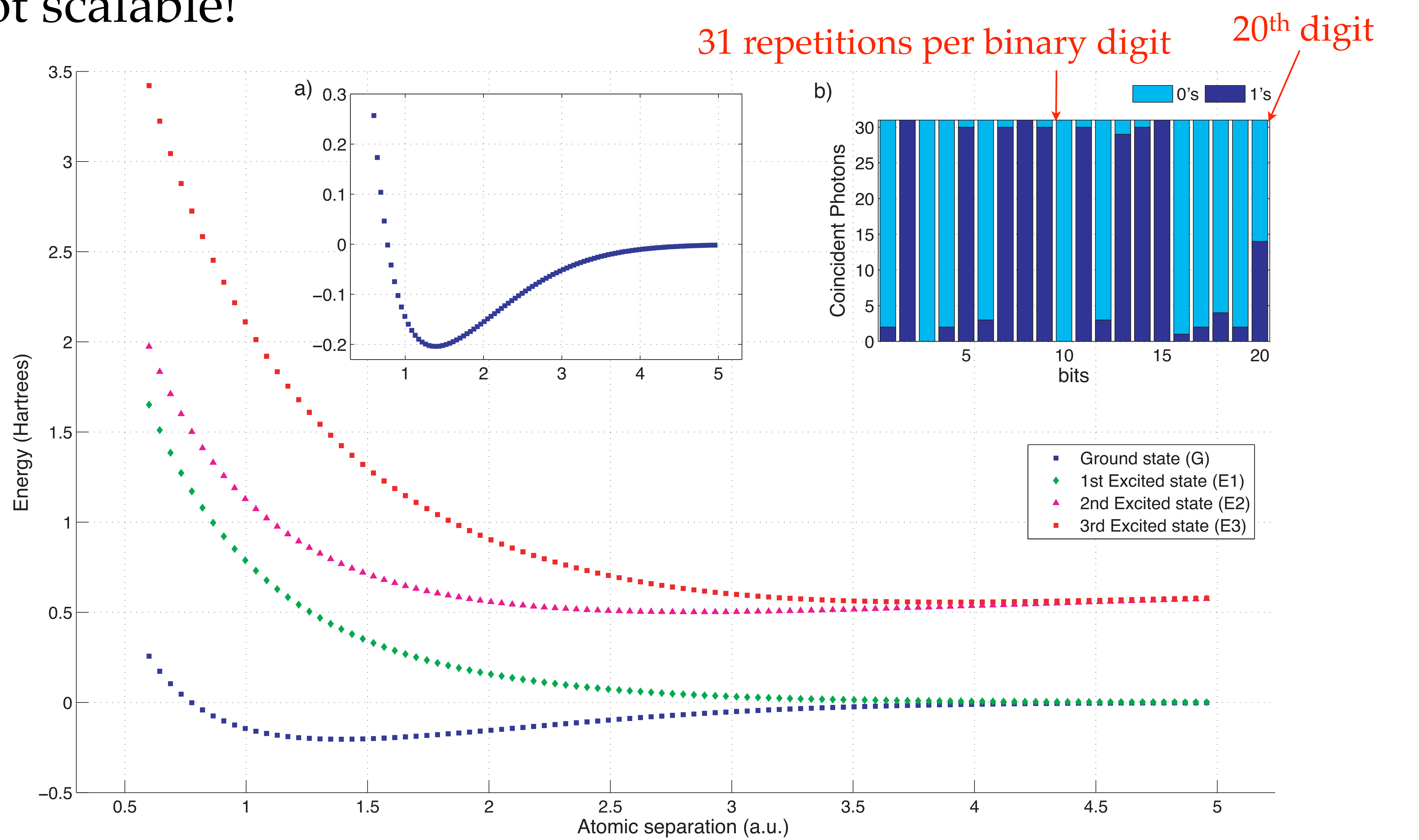
$$\frac{\left(1+e^{i2\pi 0.\phi_{d-1}0\cdots}\right)|0\rangle + \left(1-e^{i2\pi 0.\phi_{d-1}0\cdots}\right)|1\rangle}{2}|\Psi\rangle$$

$$P(x\to 0) = \cos^2(\pi\, 0.\phi_{d-1}0\ldots) = \begin{cases} 1 & \text{if } \phi_{d-1} = 0 \\ 0 & \text{if } \phi_{d-1} = 1 \end{cases}$$

  - etc.
  - Repetitions to deal with imperfect gates.

FCI of H_2 (minimal basis)

- Aspuru-Guzik (2010): Ψ^{FCI} has to be provided!
- Not scalable!



Figures from Aspuru-Guzik *et al.*, *Nature Chemistry* **2**, 106 (2010), DOI: 10.1038/NCHEM.483

Quantum simulation of *n*-electron systems

- Coupled Cluster: $\Psi = e^{\widehat{T}}\Psi^{HF}$
  - intractable for large strongly correlated (multireferent) systems
  - non-variational

- Unitary Coupled Cluster: $\Psi = e^{\widehat{T}-\widehat{T}^\dagger}\Psi^{HF}$
  - variational
  - intractable on classical computers
  - tractable with quantum computers: variational quantum eigensolver (optimization in a classical computer): $e^{\widehat{T}-\widehat{T}^\dagger}\Psi^{HF} \approx \widehat{U_1}\cdots\widehat{U_k}\Psi^{HF}$

Quantum teleportation

- It has been predicted (Bennett 1993) and experimentally demonstrated (Zeilinger, Popescu 1997; Jian-Wei Pan 2017: 1400km) that quantum entanglement can be used to teleportate the state of a particle (e.g., a qubit), not the particle itself, to a distant particle.

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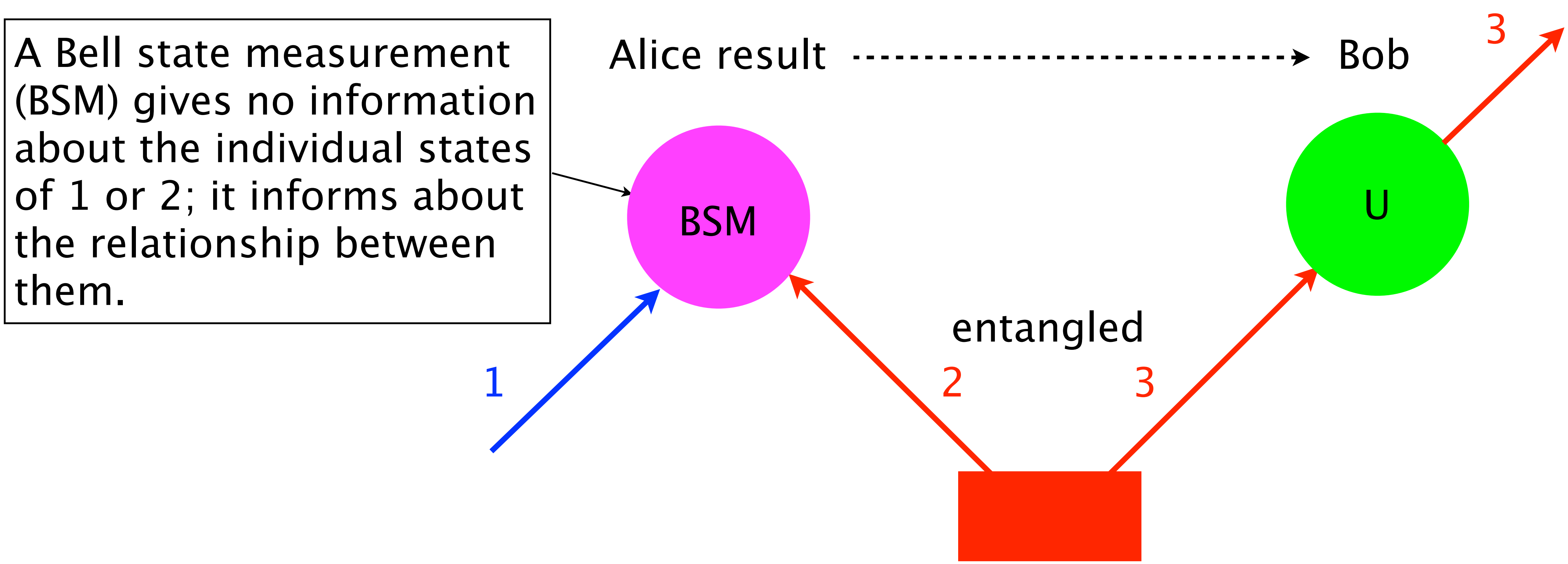
- If it were possible to extend this procedure to a complex system, *e. g.* a human being, the second system would catch the exact state of the first one: *his appearance, his memories, his character, his behavior, his life…*
- More realistic applications:
  - To transfer the state of short-lived particles to more stable systems.
  - *Entanglement purification*: entanglement transmission through noisy channels or long distances could be improved by teleporting the states instead of the particles (*quantum repeater*).
  - Preserving quantum states in an hostile environment by transferring them to better isolated particles (*quantum memories*?).
  - Links between quantum computers (communications).

Quantum teleportation

- Teleportation of the *spin state* of particle 1 to particle 3 (Zellinger setup):



Teleportation (Zellinger)

- Alice wants to transmit to Bob the state of particle 1: $\Psi_1 = c_+(\uparrow_1) + c_-(\downarrow_1)$

- They share particles 2 and 3 in the state $\Psi_{23}^- = \frac{1}{\sqrt{2}}(\uparrow_2\downarrow_3 - \downarrow_2\uparrow_3)$

$$\Psi_{123} = \Psi_1\Psi_{23}^- = [c_+(\uparrow_1) + c_-(\downarrow_1)]\,\frac{1}{\sqrt{2}}(\uparrow_2\downarrow_3 - \downarrow_2\uparrow_3)$$
$$= \frac{c_+}{\sqrt{2}}(\uparrow_1\uparrow_2\downarrow_3 - \uparrow_1\downarrow_2\uparrow_3) + \frac{c_-}{\sqrt{2}}(\downarrow_1\uparrow_2\downarrow_3 - \downarrow_1\downarrow_2\uparrow_3)$$

- Alice measures the total spin ($S_{12}{}^2$) of particles 1 and 2 (a joint property). If she obtains $S_{12}{}^2 = 0$, Ψ_{123} collapses to the corresponding $S_{12}{}^2$ eigenstate:

$$\Psi_{12}^- = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

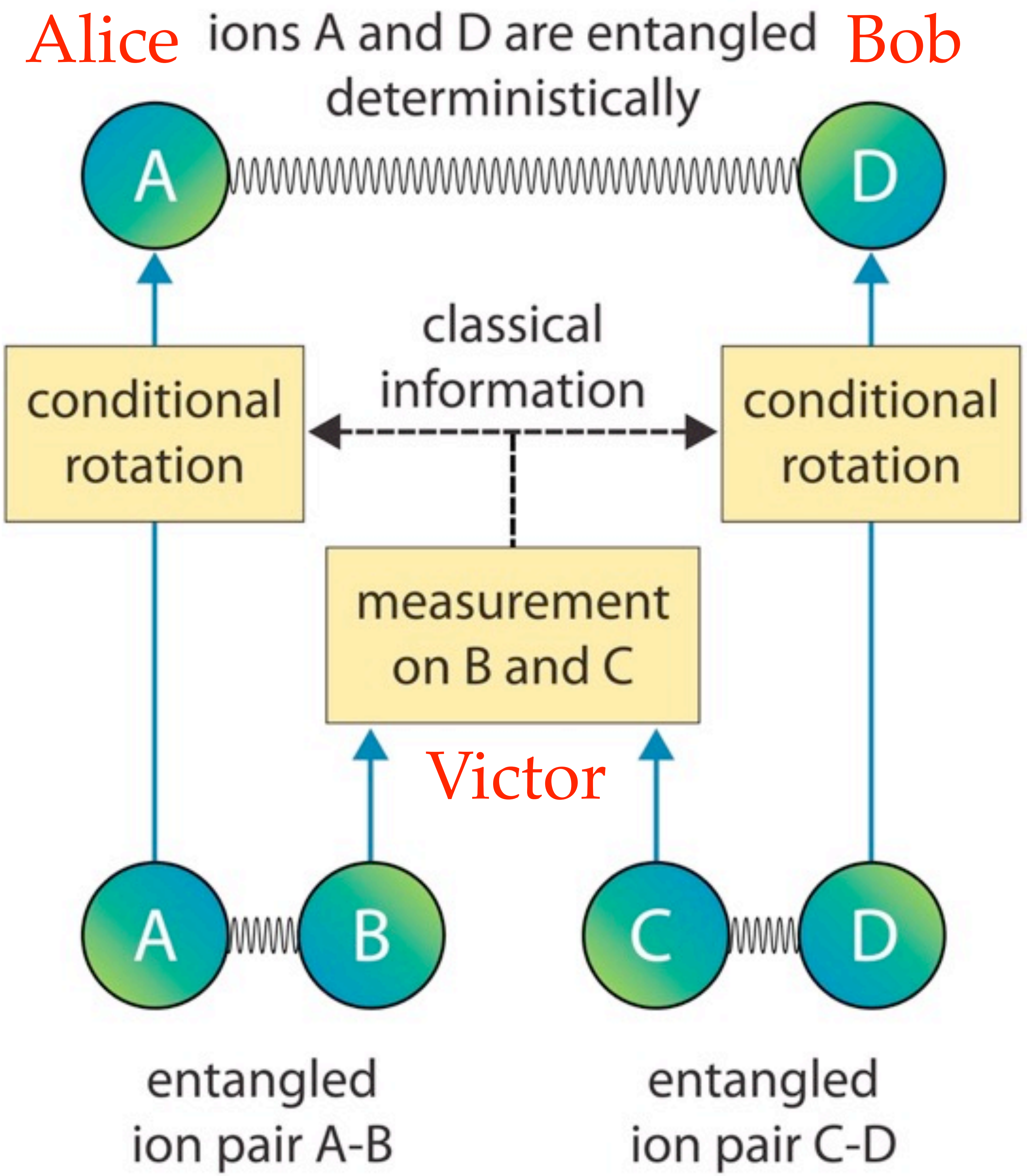
$$\left|\Psi_{12}^-\right\rangle\left\langle\Psi_{12}^-\right|\Psi_{123}\rangle$$
$$= \frac{1}{2}\left|\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right\rangle\left\langle\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right|\frac{c_+}{\sqrt{2}}(\uparrow_1\uparrow_2\downarrow_3 - \uparrow_1\downarrow_2\uparrow_3) + \frac{c_-}{\sqrt{2}}(\downarrow_1\uparrow_2\downarrow_3 - \downarrow_1\downarrow_2\uparrow_3)\Big\rangle$$
$$= \frac{1}{2}\left|\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2\right\rangle\left|\frac{-1}{\sqrt{2}}\{c_+(\uparrow_3) + c_-(\downarrow_3)\}\right\rangle$$

Teleportation (Zellinger)

- If Alice obtains $S_{12}^2 = 2$ teleportations does not work with Zellinger's setup (it works in Popescu setup).
- Particle 3 resumes the evolution of particle 1, which is no longer available in its original state (≠ cloning).
- Neither the sender nor the receiver need to know the transmitted state.
- The sender does not even need to know where the receiver is.
- The state of particle 1 is completely arbitrary. Alice's original particle 1 can even be entangled with another particle 0, which may be far away from Alice and Bob. Then teleportation transfers the entanglement to particle 3, which eventually becomes entangled with particle 0. (*entanglement swapping*, *correction of errors developed during propagation,…*).
- This can be generalized to establish *multiparticle entanglement* between particles belonging to distant users (*cryptographic conferencing*, …).

Entanglement swapping

- *Theory*: Zeilinger *et al.* 1993
- *Experiment*: Zeilinger *et al.* 1998
  - Photons A and D become entangled, although they *have never interacted* nor shared any common past.
- *Deterministic ES*: Riebe *et al.* 2008
- Delayed-choice entanglement swapping with random choice: Zeilinger 2012



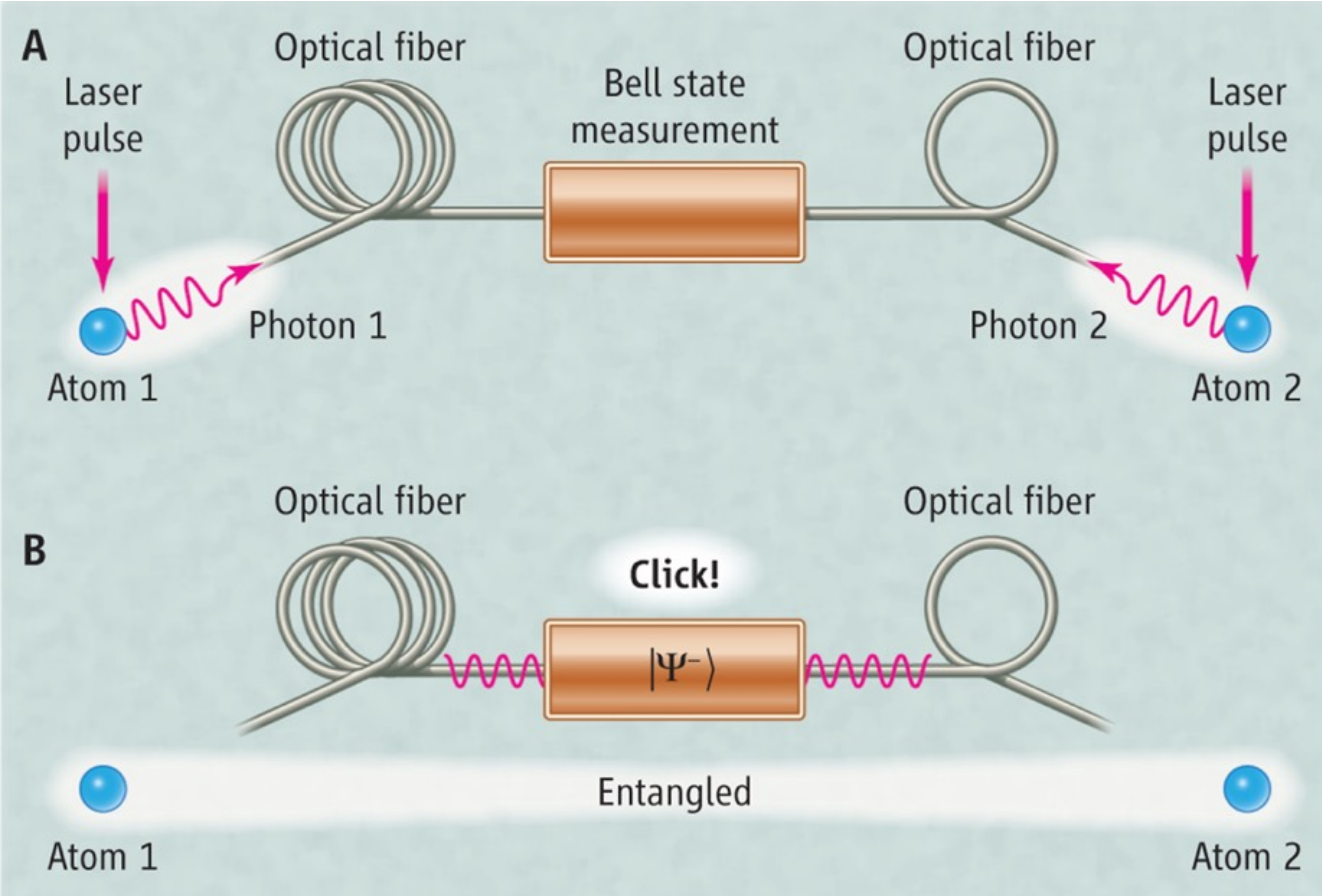
Delayed-choice entanglement swapping

- *Theory*: Peres 2000, Cohen 1999
- *Experiment*: Zeilinger 2001, De Martni 2002
  - Victor can choose either to project his 2 photons into an entangled state or to measure them individually
  - Alice and Bob measure their photons polarization *before* Victor makes his choice, so they should find entanglement or not *depending on Victor's future decision*

Delayed-choice entanglement swapping with random choice

- *Theory*: Zeilinger 2005
- *Experiment*: Zeilinger 2012 (*Catalunya informació* 4/5/12):
  - Victor's choice is taken by a *quantum random-number generator* to avoid the possibility that the photons can know in advance Victor's choice.

Heralded entanglement

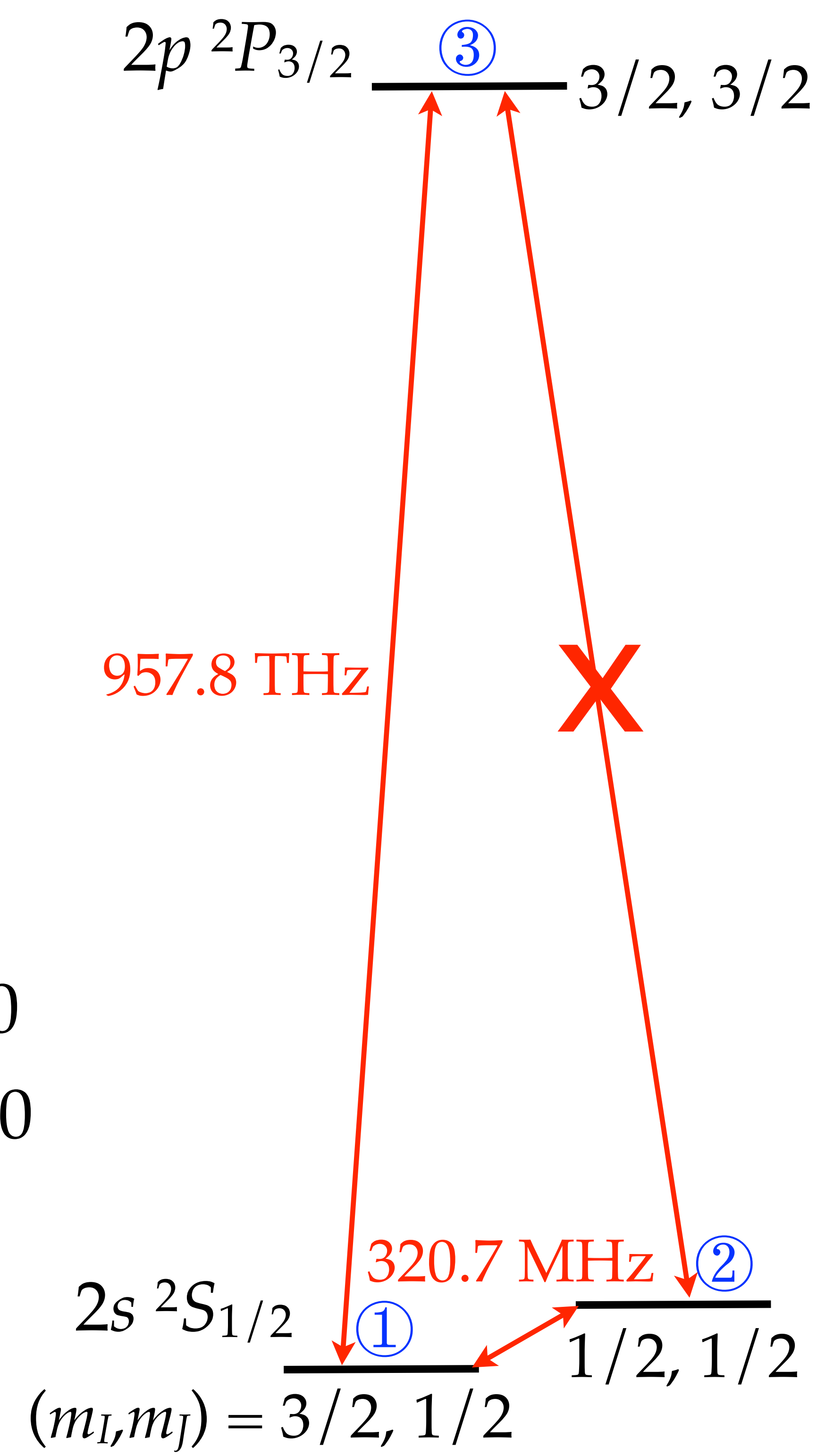


Quantum Zeno effect

- *Zeno* "paradox": motion is impossible?
- *Misra-Sudarshan* (1977): a continuously observed excited state can never decay (*a watched pot never boils*).
  - Difficult to observe in spontaneous decay.
  - *Itano et al.* (1990): inhibition of induced transitions in $^{9}Be^{+}$.

Quantum Zeno effect

- *Itano et al.* (1990): $^9Be^+$ ($I = 3/2$) at $B_0 = 0.8194$ T
  - The ion is irradiated at $\omega_{21} = (E_2 - E_1)/\hbar$ and oscillates between ① and ② with $\tau = 2\pi/\gamma B_1$
  - For $t << \tau/2$, $P_1 \approx 1$; for $t \approx \tau/2$, $P_2 \approx 1$
  - A pulse at $\omega_{31} = (E_3 - E_1)/\hbar$ produces spontaneous emission if the atom is in ① but not if it is in ② $\Rightarrow$
  - if the emission is produced the atom collapses to ①
  - if the emission is not produced the atom collapses to ②
  - As the number of pulses in $(0,\tau/2)$ increases $P_{2\leftarrow 1} \rightarrow 0$
  - As the number of pulses in $(\tau/2, \tau)$ increases $P_{2\rightarrow 1} \rightarrow 0$



Energy teleportation?

- *Latest news (January 16, 2014)*: Energy Teleportation Overcomes Distance Limit
- *The news*: http://www.technologyreview.com/view/523716/energy-teleportation-overcomes-distance-limit/
- *The paper*: http://arxiv.org/pdf/1305.3955v2.pdf

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- Brian Greene, *El Tejido del cosmos : espacio, tiempo y la textura de la realidad*, Crítica, Barcelona, 2010.