Intergenerational mobility and unequal school opportunity*

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March 27, 2020

Abstract

We analyse the impact of unequal school opportunity on intergenerational income mobility and human capital accumulation. Building upon the classical Becker-Tomes-Solon framework, we use a regime-switch model allowing for differences in income transmission across groups. We find that unequal school opportunity raises average human capital because of assortative matching. However, because income dispersion tends to be higher at the top, in most cases unequal school opportunity decreases intergenerational mobility. Calibrating the model to the US, simulations suggest that school equalization and de-segregation policies have positive effects on mobility at relatively small efficiency costs.

JEL Classification: I24; I26; J62; J24.

Keywords: Intergenerational mobility; education; human capital; equality of opportunity; social segregation.

*We are grateful to the editor Estelle Cantillon, two anonymous reviewers, Roland Bénabou, Caterina Calsamiglia, David de la Croix, Gordon Dahl, Anne Gielen, Hubert Kempf, Mikael Lindahl, François Maniquet, Pierre Pestieau, Jan Stuhler and seminar participants at the 2nd Belgian-Japanese Public Finance Workshop at UCLouvain (March 2017, Belgium), LEER Workshop in Education Economics at KU Leuven (March 2017, Belgium), ECORES Summer School at UCLouvain (May-June 2017, Belgium), Journées LAGV 2017 at Aix-en-Provence (June 2017, France), the Workshop on New Developments in Intergenerational Mobility at ESE (September 2018, Netherlands) and the Leicester Economics Seminar (November 2019, England) for comments and suggestions. Andreu Arenas acknowledges financial support from the ERC Starting Grant 638893. Riccardo Fusari provided excellent research assistance.

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1 Introduction

There exist large disparities in intergenerational mobility across neighbourhoods in the US, with high mobility areas exhibiting less residential segregation, less income inequality, and better primary schools, among others (Chetty *et al.*, 2014). Differences in parental financial investment into children's education across high and low income families are large, especially in states with high income inequality (Schneider *et al.*, 2018). Importantly, these correlations are not only due to differences in family characteristics across neighbourhoods. Recent evidence from the Moving to Opportunity experiment, that offered randomly selected families housing vouchers to move from high-poverty housing projects to lower-poverty neighbourhoods, shows that children randomly growing up in less disadvantaged neighbourhoods benefit from an increase in college attendance and earnings (Chetty *et al.*, 2016). Evidence of neighbourhood effects has also been found in Europe (Goux and Maurin, 2007). These results suggest that besides self-selection and inheritability, neighbourhoods have a causal effect on outcomes and on intergenerational mobility.

Although neighbourhoods are heterogeneous across a wide range of dimensions, Card *et al.* (2018) find evidence that upward mobility in educational attainment in the US is significantly related to local public education policy. Their analysis suggests that black-white differences in school quality during the era of school segregation were a key precursor to large persistent black-white disparities in a range of socioeconomic outcomes. Likewise, Billings *et al.* (2013) provide evidence that school segregation has an important influence on inequality and educational attainment. Going forward, Rajan (2019) argues that technological change has increased the importance of good schooling, and that the resulting demand for education is leading to more socially segregated communities. Each parent wishes to move to the best community it can afford, where their kids will have the best chance of success in schooling and the best education that they need to compete in a global and digital economy -a secession of the successful-. Those left behind don't have the same opportunities as those who have moved into communities with the best schools, and this is reinforced by zoning laws.

The aim of this paper is to present a theoretical framework that incorporates these recent empirical insights. We build a model of parental investment into children's education and intergenerational mobility to understand how unequal school opportunity contributes to differences in social mobility and human capital, extending the standard parent-child transmission model à la Becker-Tomes-Solon. We introduce unequal opportunity as the combination of unequal school quality and an unequal probability of access to the best schools. Formally, we adopt a Markov bivariate switching model where the transition probabilities for having access to the best schools depend on the parental income rank. By permitting regime switches, our model is able to represent more complex (nonlinear) dynamic patterns of intergenerational income transmission, related to unequal school opportunity. We study the effects of unequal school opportunity on parental investment, human capital, and intergenerational mobility. School inequality relates to educational policies that allow for flexibility in school curriculum, funding, or management - policies that give room for heterogeneity in school quality across neighbourhoods.[1] Unequal access concerns the relationship between parental income and access to the best schools, and is related to educational policies that promote equal access to schools from any neighbourhood, such as priorities in school choice mechanisms, which lead to assortative matching of high-quality schools to high-income families across communities.[2] Throughout the analysis, we keep the average level of school quality and the (exogenous) education financing system constant.[3]

We find that the effect of unequal school opportunity on intergenerational persistence, as measured by the intergenerational elasticity of income, depends on the distribution of parental income. Unequal school opportunity increases the parenting gap: high income parents invest more and low income parents invest less. As a result, persistence decreases at the bottom and increases at the top. In most cases, when there is more income dispersion at the top, for instance when the parental income distribution is log-normal, unequal school opportunity increases intergenerational persistence overall.

At the same time, unequal school opportunity matches higher investment families with the best schools, producing efficiency gains. This increases average human capital (efficiency

[1]For instance, Eyles *et al.* (2016) and Brutti (2016) show that school autonomy can have very heterogeneous effects on school quality and pupil performance.

[2]For instance, Calsamiglia *et al.* (2015) show how the Deferred Acceptance and Boston mechanisms can lead to different segregation patterns depending on how schools set priorities and on the existence of informational asymmetries. Under segregation, local communities play the role of clubs in providing local public goods, such as education (Hindriks and Myles, 2013).

[3]A number of papers have studied the effects of different education financing schemes on similar questions, such as Fernández and Rogerson (2003), Fernández and Rogerson (1998), Bénabou (1996b), or Glomm and Ravikumar (1992).

gains due to positive assortative matching), although average parental investment may actually decrease, due to the decreasing returns to investment.

Hence, our model features an efficiency-mobility trade-off. This trade-off echoes the insights of Bénabou (1996a) that de-segregation policies may have a negative effect on growth and performance in the long run. To shed light on the magnitude of this trade-off, we calibrate the model to match the US parental income distribution, and simulate the effects of de-segregation and school equalization policies. The simulations reveal that equal opportunity policies produce high mobility gains and small efficiency losses.

The main contribution of this paper is to include unequal school opportunity -unequal access to unequal schools- within the classical parent-child model of intergenerational mobility (Solon (1999), Becker and Tomes (1986), Becker and Tomes (1979)). The closest paper to ours is Becker *et al.* (2018), that depart from the standard Becker-Tomes-Solon framework by allowing for complementarities in the production of children's human capital, considering the possibility that highly educated parents are better equipped to transmit their human capital to their children. This complementarity creates different persistence of economic status at the top and the middle of the income distribution. They do not consider efficiency issues in their model, but only intergenerational mobility. Our model highlights a different source of non-linearity which is related to unequal school opportunity.

Another related contribution is Cavalcanti and Giannitsarou (2017). They use networks to model local externalities in a human capital accumulation OLG model. In their model, human capital depends on parental investment and the average human capital of parent's neighbourhood. They show that inequality can persist in the long run when the network cohesion is low (local networks are weakly connected). However, their contribution is different from ours because they are interested in the impact of inequality on growth, whereas we study the efficiency-mobility trade-off.

Other contributions have studied alternative channels through which school inequality and segregation could affect social mobility and efficiency. For instance, Cremer *et al.* (2010) and Hare and Ulph (1979) study how the transmission of ability across generations induces a trade off between efficiency and equality based on the idea that concentrating resources on the most able individuals allows for an efficiency gain. Checchi *et al.* (1999) propose yet another mechanism, based on self-confidence about talent, where selective schooling can act as a sorting mechanism that allows talented students from disadvantaged families to reveal themselves.

Our model can also be related to the cultural transmission model à la Bisin and Verdier (2001). In the cultural transmission model, children can become educated either because parents have been successful in educating them (socialization inside the family), or because the neighbourhood where they live is of sufficiently high quality in terms of human capital (socialization outside the family). In different cases, the model can yield both cultural complementarity (higher neighbourhood quality leads to higher effort) and substitutability (higher neighbourhood quality leads to lower effort). One interpretation of our model is as a cultural transmission model with cultural complementarity between parental investment and the neighbourhood (school) quality. This is because in our case, higher school quality leads to higher parental investment in the human capital of their children. This is consistent with empirical evidence (Patacchini and Zenou, 2011; Gelber and Isen, 2013). Nonetheless, in the Appendix B we develop a more general version of the model that allows for parental investment to be a substitute or a complement of school quality, to see what the model predicts when parental investment decreases in communities with good schools. We find that substitutability reduces the parenting gap, and reverses the efficiency-mobility tradeoff: equal opportunity policies lead to more efficiency and less mobility. However, the efficiency-mobility tradeoff remains regardless of whether investment and school quality are complements or substitutes.

The rest of this paper is organized as follows. Section 2 presents the regime switch model of unequal school opportunity, while Section 3 analyses the impact of unequal school opportunity on the equilibrium investment. Section 4 studies the impact of unequal school opportunity on human capital accumulation. Section 5 characterizes the conditions under which unequal school opportunity reduces intergenerational mobility. Section 6 discusses sorting decisions. Section 7 provides some simulations for assessing the magnitude of the efficiency-mobility trade-off as well as the interaction between income inequality and unequal school opportunity. Section 8 concludes.

2 A regime switch model of unequal school opportunity

We consider a simplified version of the Solon (1999) model. Each parent (generation $t-1$) has one child (generation t). Parents must allocate lifetime income y_{t-1} between own consumption C_{t-1} and investment I_{t-1} in the child's human capital h_t. Parents cannot borrow against the child's future income and do not bequest income to the child. The resulting budget constraint of parents in generation $t-1$ is

$$y_{t-1} = C_{t-1} + I_{t-1} \tag{1}$$

Parental investment translates into the child's human capital according to the following human capital accumulation equation,

$$h_t = \theta log(I_{t-1}) + u_t, \qquad \theta > 0 \tag{2}$$

Where $\theta > 0$ represents a uniform school productivity parameter, and u_t represents a random child's ability component that is independent of the parental investment choice (child's endowed attributes, in the terminology of Becker and Tomes (1979)). u_t can be decomposed into the sum of a constant μ, a random *iid* mean zero component ϵ_t, and a random mean zero component τ_t positively correlated with parental human capital, that might capture endowment heritability (both luck and genetic):

$$u_t = \mu + \epsilon_t + \tau_t \tag{3}$$

where $\mu > 0$ and ϵ_t is a mean zero *iid* random term with variance σ^2_ϵ, and τ_t is a mean zero random term with $cov(h_{t-1}, \tau_t) \geq 0$.[4] Human capital translates into income on the labor market as follows:

$$log(y_t) = h_t \tag{4}$$

The specifications of the human capital production function and the earnings function are standard in the literature, and useful to derive the intergenerational income elasticity (Solon,

[4] This is a two-generation parent-child mobility, without grandparent effects. Recent empirical evidence has challenged the standard parent-child model because multi-generational measures of income persistence are greater than the repeated extrapolation from the parent-child transmission measure (Clark and Cummins, 2014; Lindahl *et al.*, 2015). However, Braun and Stuhler (2017) find no evidence of independent grand-parent effects. This could be because of indirect transmission through the inheritance of underlying latent factors, but also due to differential rates of transmission across groups (Solon, 2018). Our model is more consistent with the latter.

2018). Parents seek to maximize a utility function over own consumption and child's income:

$$U_{t-1} = log(C_{t-1}) + log(y_t) \tag{5}$$

This is a quasi-linear utility function in child's education attainment, since $log(y_t) = h_t$. Thus parents do care only about the expected value of h_t, which they correctly anticipate. For that reason, they do not need to know the child's ability while investing in her education.[5] Substituting the budget constraint, the human capital production function and the labor market transmission function into the utility function, parents maximize:

$$U_{t-1} \quad = \quad log(y_{t-1} - I_{t-1}) + h_t = log(y_{t-1} - I_{t-1}) + \theta log(I_{t-1}) + u_t \tag{6}$$

Solving the first order condition for I_{t-1} gives the optimal investment in child's human capital

$$I^*_{t-1} = \frac{\theta}{1+\theta} y_{t-1} \tag{7}$$

Hence, the school productivity parameter θ determines the proportion of parental income that is devoted to investment $\frac{\theta}{1+\theta}$. Becker and Tomes (1979) call this parameter $\frac{\theta}{1+\theta}$ the propensity to invest in children. In a uniform school system, as in the Becker-Tomes-Solon model, the propensity to invest in children is uniform, and parents are willing to invest more whenever schools are of higher quality. Hence, this specification of human capital and school quality, albeit standard, involves a complementarity between parental investment and school quality. Empirically, Patacchini and Zenou (2011) find that families in better neighbourhoods (including school quality) in the UK invest relatively more in the education of their children, and likewise (Gelber and Isen, 2013) show that parents of randomly-chosen children attending Head Start substantially increased their involvement with their children. On the other hand, Pop-Eleches and Urquiola (2013) find a reduction of parental effort after an increase in school quality in Romania's high schools. We present a more general model that allows for both substitutability and complementarity between school quality and parental investment in the Appendix B.

[5] If the utility function is not separable between consumption and child's income, and if parents do know the child ability while investing, then they would condition investment on the child ability. Parents would invest less in a high-ability child to expand their own consumption such as in Becker and Tomes (1979). See the Appendix B for further discussion.

2.1 Unequal school opportunity

We introduce unequal school opportunity, such that families above the median income have access, with probability $p \geq \frac{1}{2}$, to high-quality schools with productivity $\theta^H = \theta + \kappa$. On the other hand, families below the median income have access, with the same probability p, to low-quality schools with productivity $\theta^L = \theta - \kappa$. The average school productivity is constant ($E(\theta) = \theta$), and school inequality is given by the school productivity gap $\kappa = \frac{\theta^H - \theta^L}{2}$. Thus κ measures school inequality, and p measures school segregation. Hence "separate but equal schooling" is given by $p > \frac{1}{2}$ and $\kappa = 0$, and "integrated but unequal schooling" is given by $p = \frac{1}{2}$ and $\kappa > 0$. The two cases refer to different policies: school equalization policies (notably via equal public school funding and autonomy regulations) and school de-segregation policies (notably via busing and school assignment policies).[6] Note that p is directly related to the dissimilarity index of segregation D (Duncan and Duncan, 1955). The dissimilarity index is the average (absolute) difference across schools in the shares of classmates from two different income groups. In our case, defining the two income groups around the median, $D(p) = \frac{1}{2}|p - (1-p)| + \frac{1}{2}|(1-p) - p| = 2p - 1$, and therefore $D(p) \geq 0$ for $p \geq 1/2$.[7]

The human capital accumulation with unequal school opportunity is described as the following regime switch model where y^M_{t-1} is the median income of generation $t-1$.

For $y_{t-1} \leq y^M_{t-1}$ (i.e., below median income),

$$h_t = \begin{cases} \theta^L log(I_{t-1}) + u_t, & \text{with probability p.} \\ \theta^H log(I_{t-1}) + u_t, & \text{with probability 1-p.} \end{cases} \tag{8}$$

For $y_{t-1} > y^M_{t-1}$ (i.e., above median income),

$$h_t = \begin{cases} \theta^L log(I_{t-1}) + u_t, & \text{with probability 1-p.} \\ \theta^H log(I_{t-1}) + u_t, & \text{with probability p.} \end{cases} \tag{9}$$

[6]In fact, differences in the probability of attending a good school by parental income also do exist within postal codes, which also suggests a role for policies targeted at differences in information or in the ability to "work the system" (Burgess and Briggs, 2010)

[7]Gutiérrez *et al.* (2017) compute the dissimilarity index across OECD countries using the ESCS PISA index. They obtain indexes of 0.27 for Norway, 0.32 for Sweden, 0.33 for Canada, 0.35 for the Netherlands, 0.38 for England, and 0.39 for the US, among others.

Each parent in generation $t-1$ will choose I_{t-1} given the school quality available.

For $y_{t-1} \leq y^M_{t-1}$,

$$I^*_{t-1} = \begin{cases} \frac{\theta^L}{1+\theta^L} y_{t-1}, & \text{with probability p.} \\ \frac{\theta^H}{1+\theta^H} y_{t-1}, & \text{with probability 1-p.} \end{cases} \tag{10}$$

For $y_{t-1} > y^M_{t-1}$,

$$I^*_{t-1} = \begin{cases} \frac{\theta^L}{1+\theta^L} y_{t-1}, & \text{with probability 1-p.} \\ \frac{\theta^H}{1+\theta^H} y_{t-1}, & \text{with probability p.} \end{cases} \tag{11}$$

Let $\omega^H(\theta) = p\frac{\theta^H}{1+\theta^H} + (1-p)\frac{\theta^L}{1+\theta^L}$ and $\omega^L(\theta) = p\frac{\theta^L}{1+\theta^L} + (1-p)\frac{\theta^H}{1+\theta^H}$ denote the expected investment propensities above and below the median income, respectively. Then, $\omega^H(\theta) \geq \omega^L(\theta)$ for $p \geq 1/2$, and:

$$\frac{\partial \omega^H(\theta)}{\partial p} = -\frac{\partial \omega^L(\theta)}{\partial p} = \frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L} > 0$$

Hence, unequal school opportunity increases the parenting gap, because of the increased difference in the investment propensity across income groups. We can now use this regime switch model to focus on the mean behavior of the non-linear dynamic variables. We begin by analyzing the mean value of the dynamic pattern of parental investment.

3 Parental investment

Let $\gamma = \frac{E[y_{t-1}|y_{t-1}>y^M]}{E[y_{t-1}]}$ represent the between-group income inequality . For γ close to 1, between-group income inequality is minimum: both income groups have equal shares of total income. For γ close to 2, the between-group income inequality is maximum: total income is concentrated in the high income group. Hence $\gamma \in (1, 2)$. Using this between-group income inequality, we can rewrite the average investment under unequal school opportunity as

$$E[I_s] = [\gamma\omega^H(\theta) + (2-\gamma)\omega^L(\theta)]\frac{E[y_{t-1}]}{2}$$

Average investment with equal school opportunity is given by $E[I^*_{t-1}] = \frac{\theta}{1+\theta}E[y_{t-1}]$. The question is how unequal school opportunity affects the average level of investment in our regime switch model of human capital accumulation. In the following proposition, we derive the comparative statics of the separate impact of a change in school segregation p, and of a mean-preserving change in school inequality $\kappa = \frac{\theta^H - \theta^L}{2}$ for generation t, for any given initial condition of generation's $t-1$ income distribution. We also consider the impact of changes in between-group parental income inequality γ.

Proposition 1. *(i) For any $\kappa > 0$ and $\gamma > 1$, average investment increases with school segregation p; (ii) For any $\kappa > 0$ and $p > 1/2$, average investment is an increasing function of between-group income inequality γ; (iii) For any $\gamma > 1$, there exists $p^\circ = \frac{1}{2} + \frac{\kappa(1+\theta)}{(\gamma-1)((1+\theta)^2+\kappa^2)}$ such that average investment is an increasing function of school inequality κ if and only if $p \geq p^\circ$.*

The proof is provided in the Appendix A.[8] This proposition means that there exists a threshold value $p = p^\circ$ such that average investment is an increasing function of school inequality κ because high-quality schools are sufficiently concentrated among the high income group, which also invests more. This positive assortative matching effect compensates for the negative effect of decreasing returns of school quality on the propensity to invest in children ($\frac{\theta+\kappa}{1+\theta+\kappa} > \frac{\theta-\kappa}{1+\theta-\kappa}$).

In the polar case of full segregation ($p = 1$), for those below the median income ($y_{t-1} \leq y^M$), the average investment loss is given by:

$$\Delta I_L^* = \left(\frac{\theta^L}{1+\theta^L} - \frac{\theta}{1+\theta}\right) E[y_{t-1}|y_{t-1} \leq y^M] = \left(\frac{-\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} \leq y^M]}{1+\theta-\kappa}\right) < 0$$

For those above the median income ($y_{t-1} > y^M$), the average investment gain is:

$$\Delta I_H^* = \left(\frac{\theta^H}{1+\theta^H} - \frac{\theta}{1+\theta}\right) E[y_{t-1}|y_{t-1} > y^M] = \left(\frac{\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} > y^M]}{1+\theta+\kappa}\right) > 0$$

Therefore the net investment change is

$$\frac{1}{2}\left(\Delta I_H^* + \Delta I_L^*\right) = \frac{1}{2}\left(\frac{\kappa}{1+\theta}\right)\left(\frac{E[y_{t-1}|y_{t-1} > y^M]}{1+\theta+\kappa} - \frac{E[y_{t-1}|y_{t-1} \leq y^M]}{1+\theta-\kappa}\right) \lesseqgtr 0 \tag{12}$$

The intuition for this result is the following. High income families attend high-quality schools and invest a higher proportion of their income in education, whereas low-income families attending low-quality schools will invest a smaller proportion of their income. However, there exist decreasing returns: the propensity to invest in children is a concave function

[8] The assumption that $\theta + g(\theta)$ is increasing in θ simply rules out extreme values of $g(\theta)$ when θ is arbitrarily small.

of the school quality. Hence, the effect of school inequality, keeping average school quality constant, is to decrease the average propensity to invest in children. This effect tends to lower average investment for a sufficiently small level of between-group income inequality. However, investment is proportional to income, so the investment change is proportional to the income gap. This complementarity between income (and hence, capacity to invest) and school quality tends to increase average investment (positive assortative matching). Thus, with sufficiently high between-group income inequality, average investment is higher under unequal school opportunity. Note also that from the investment change within each group, it is straightforward to see that unequal school opportunity increases the investment gap (the parenting gap) between income groups, since $(\Delta I_H^* - \Delta I_L^*) > 0$.

4 Human capital

With equal school opportunity the average human capital level is:

$$E[h_t] \;=\; \theta E[log(I_{t-1}^*)] + E[u_t] = g(\theta) + \theta E[log(y_{t-1})] + \mu \tag{13}$$

where $g(\theta) = \theta log(\frac{\theta}{1+\theta}) < 0$. $g(\theta)$ is a decreasing and convex function, with $g'(\theta) < 0 < g''(\theta) > 0$ for all $\theta \in (0, 1)$.

We now study the impact of unequal school opportunity on the average level of human capital. We consider the short term impact of changes in school segregation and school inequality on the average human capital of generation t, given an initial income distribution for generation $t-1$, and also the impact of changes in between-group parental income inequality. Substituting for the optimal investment choices, the average human capital level is:

$$E[h_t, s] = \frac{1}{2}\Big(g(\theta^H)+g(\theta^L)+(p\theta^H+(1-p)\theta^L)\phi E[log(y_{t-1})]+(p\theta^L+(1-p)\theta^H)(2-\phi)E[log(y_{t-1})]\Big)+\mu$$

Where $\phi = \frac{E[\log(y_{t-1})|\log(y_{t-1})>\log(y^M)]}{E[\log(y_{t-1})]} \in (1, 2)$ is the between-group log-income inequality. The comparative static properties of school inequality, school segregation and between-group parental income inequality on average human capital are given in the following proposition:

Proposition 2. *(i) For any $\kappa > 0$ and $\phi > 1$, average human capital is an increasing function of school segregation p ; (ii) For any $\kappa > 0$ and $p > 1/2$, average human capital is an increasing function of between-group log-income inequality ϕ; (iii) For any $\phi > 1$ and $p > 1/2$, average human capital is an increasing function of school inequality κ.*

The proof is provided in the Appendix A. Note the difference between the investment result in the previous proposition and the human capital result in this proposition: while average investment may either increase or decrease under unequal school opportunity, average human capital always increases. The reason is the complementarity between school productivity and parental investment in the formation of human capital. This complementarity makes the human capital transmission a convex function of parental income. Indeed, school segregation boosts parental investment in the high income group, where the productivity of investment is also higher, and reduces investment where the productivity is lower. This endogenous response of parents to a change in school opportunities matters a lot for the intergenerational transmission of inequality.[9]

To understand the positive effect of unequal school opportunity on average human capital, we can focus on the polar case of $p = 1$. For those below the median income, whenever $p = 1$, the effect of unequal school opportunity on human capital (relative to the benchmark of equal opportunity) is:

$$\Delta h_t = h_t^s - h_t = g(\theta^L) - g(\theta) + (\theta^L - \theta)\log(y_{t-1})$$

The first term is positive since $g(\theta)$ is decreasing; the second term is negative and proportional to parental income. Hence those who lose most from unequal school opportunity are those with incomes closest to the median (from below). Those who lose the least are those with lowest incomes. For those above the median income, whenever $p = 1$, the effect of unequal school opportunity on human capital (relative to the benchmark of equal opportunity) is:

$$\Delta h_t = h_t^s - h_t = g(\theta^H) - g(\theta) + (\theta^H - \theta)log(y_{t-1})$$

The first term is negative since $g(\theta)$ is decreasing in θ; the second term is positive and increasing

[9]In fact, as put by Becker and Tomes (1979) "mechanical models of the intergenerational transmission of inequality that do not incorporate optimizing response of parents to their own or to their children's circumstances greatly understate the influence of family background on inequality" (p. 1165).

in parental income. This implies that those gaining more from segregation are those at the top, while those closer to the middle gain less. Hence the middle class is losing from unequal school opportunity while the rich benefits and the poor are less affected.

Given that empirical papers increasingly rely on quasi-experimental methods such as Regression Discontinuity Designs to estimate causal effects and that our model features a discontinuity based on a running variable (parental income), it is worth pointing out that the causal effect of a better schooling on investment and human capital at the threshold will not be, in general, representative of the average effect. For simplicity, consider the $p = 1$ case. The effect of a better schooling on investment at the threshold, (i.e., the effect that we would estimate via RDD) is given by $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) y^M_{t-1}$. On the other hand, if we would run a Randomized Control Trial, the treatment effect that we would obtain, the Average Treatment Effect (ATE) is given by: $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) E[y_{t-1}]$. Finally, if we would estimate the effect by differences-in-differences, exploiting variation in individuals' change of schooling over time, we would estimate an Average Treatment Effect on the Treated (ATT) given by $\left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right) E[y_{t-1}|y_{t-1} > y^M]$. The ATT is the largest of the possibly estimate treatment effects, and the difference between the ATE and the RDD estimate will depend on whether the distribution of income is very asymmetric. In general, with a log-normal distribution for parental income, the RDD estimate will be the most conservative (since $y^M_{t-1} < E[y_{t-1}]$). Notice that similar expressions hold for the effects on Human Capital. For instance, if $g(\theta^L) - g(\theta^H) = (\theta^H - \theta^L)log(y^M)$, a RDD would estimate a zero effect of school segregation on human capital at the threshold, although the average treatment effect is positive.

5 Intergenerational elasticity

To measure intergenerational mobility, we will compute the intergenerational elasticity of income. Substituting optimal investment into the earnings equation:

$$log(y_t) = g(\theta) + \theta log(y_{t-1}) + u_t \tag{14}$$

The inter-generational income elasticity IGE (i.e. the structural interpretation of the OLS estimate $\hat{\beta}$) in the uniform school system without school segregation is given by $\beta = \theta +$

$\frac{cov(log(y_{t-1}),\tau_t)}{Var(log(y_{t-1}))}$: it depends both on school quality and on the persistence of endowments across generations. Under unequal school opportunity, the regime switch model is able to represent more complex (nonlinear) dynamic patterns of intergenerational income transmission.

Recall that $g(\theta) = \theta log(\frac{\theta}{1+\theta})$, and that ϕ is the between-group log income inequality. Let also δ be defined as $\delta = \frac{E[(log(y_{t-1}))^2|y>y^M]}{E[(log(y_{t-1}))^2]}$ implying that $E[(log(y_{t-1}))^2|y < y^M] = (2-\delta)E[(log(y_{t-1}))^2]$. Substituting optimal parental investment into the earnings equation for the regime switch model gives:

$$log(y_t) = pg(\theta^L) + (1-p)g(\theta^H) + (2p-1)(g(\theta^H) - g(\theta^L))Z_{t-1}$$

$$+(p\theta^L + (1-p)\theta^H)log(y_{t-1}) + (2p-1)(\theta^H - \theta^L)log(y_{t-1})Z_{t-1} + u_t$$

where the random variable $Z_{t-1} = 1$ with probability p if $y_{t-1} > y^M$ and zero otherwise, and $Z_{t-1} = 1$ with probability $1-p$ if $y_{t-1} < y^M$ and zero otherwise. This bivariate regime switch model implies both an heterogeneous intercept and an heterogeneous slope in the auto-regressive process of income transmission. We would like to compare the structural interpretation of the IGE arising from this non-linear regime switch model with the IGE arising from the (linear) benchmark of equal school opportunity, to know the impact of unequal school opportunity on intergenerational persistence. To this aim, it is useful to use the classical omitted variable bias results. We estimate:

$$log(y_t) = \alpha_s + \beta_s log(y_{t-1}) + \eta_t \tag{15}$$

where the omitted term is:

$$\eta_t = (2p-1)(g(\theta^H) - g(\theta^L))Z_{t-1} + (2p-1)(\theta^H - \theta^L)log(y_{t-1})Z_{t-1} + \epsilon_t + \tau_t$$

Recall that $\beta = \theta + \frac{cov(log(y_{t-1}),\tau_t)}{Var(log(y_{t-1}))}$ in the benchmark of equal school opportunity. The omitted term is the sum of the omitted change in the intercept (which is negative since $g(\theta^H) < g(\theta^L)$), the omitted change in the slope (which is positive, since $\theta^H > \theta^L$), and the omitted underlying latent factor (the inheritance term).

Let $D(p) = 2p - 1$ denote the dissimilarity index of segregation. Then we have

$$\begin{aligned} \beta_s &= D(p)(g(\theta^H) - g(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (p\theta^L + (1-p)\theta^H) \\ &+ D(p)(\theta^H - \theta^L)\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + \frac{cov(log(y_{t-1}), \tau_t)}{Var(log(y_{t-1}))} \end{aligned} \quad (16)$$

First, note that with equal opportunity $p = 1/2$, $D(\frac{1}{2}) = 2p - 1 = 0$, and $p\theta^L + (1-p)\theta^H = \theta$, so the regime switch model coincides to the benchmark model (random switching model) and $\beta_s = \theta + \frac{cov(log(y_{t-1}),\tau_t)}{Var(log(y_{t-1}))} = \frac{\theta^H+\theta^L}{2} + \frac{cov(log(y_{t-1}),\tau_t)}{Var(log(y_{t-1}))}$. Hence school inequality ($\kappa > 0$) does not affect intergenerational persistence if there is equal opportunity ($p = 1/2$).

Proposition 3. *Assume that $\theta + g(\theta)$ is increasing in θ. For any $\kappa > 0$, $p > 1/2$ and $\phi > 1$, the intergenerational elasticity of income β_s is an increasing function of school segregation p if $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. This (sufficient) condition is satisfied for a log-normal income distribution.*

The proof is provided in the Appendix A. The intuition for this result is the following. Since education is the channel through which parents pass their economic status to their offspring (besides endowment inheritance), an increase in θ increases the room for the influence of parental background. This room will be amplified if there is a lot of variation in parental income. The regression estimate of the IGE is a weighted average of the group specific persistence and such averaging gives more weight to the groups with the higher unexplained variance - like any regression coefficient. Hence, if there is more variation in parental income at the top than at the bottom of the income distribution, the increase in persistence at the top (due to higher school quality) has stronger effects than the decrease in persistence at the bottom (due to lower school quality).

Proposition 4. *Assume that $\theta + g(\theta)$ is increasing in θ. For any $p > 1/2$ and $\phi > 1$, the intergenerational elasticity of income β_s is an increasing function of school inequality κ if $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. This (sufficient) condition is satisfied for a log-normal income distribution. Under equal opportunity $p = \frac{1}{2}$, $\frac{\partial \beta_s}{\partial \kappa} = 0$.*

The intuition for this result is the same as the one for the previous proposition. The proof is provided in the Appendix A.

6 Unequal opportunity and sorting decisions

Our model does not explain how unequal school opportunity emerges. In this section we address the positive sorting of parents across different schools. We show that the complementarity between school quality and parental investment is a potential driver, among others, of the (positive) sorting decisions of high-income parents into high quality schools.

To see that, let us consider the value function of each household based on the school quality parameter θ. For simplicity we drop the time subscripts, with y here referring to parental income.

$$V(y;\theta) = U(I^*(\theta)) = \max_I \log(y - I) + \theta log(I) + u$$

with $V(y;\theta^L)$ for $\theta = \theta^L$ and $V(y;\theta^H)$ for $\theta = \theta^H$.

To sustain positive sorting decisions (i.e. $p > \frac{1}{2}$) as an equilibrium process, the marginal value for school quality must be increasing in income. For households with income y, differentiating the value function $V(y;\theta)$ with respect to school productivity parameter θ gives

$$\frac{\partial V(y;\theta)}{\partial \theta} = \left.\frac{\partial V(y;\theta)}{\partial \theta}\right|_{dI=0} + \frac{\partial V(y;\theta)}{\partial I}\frac{\partial I^*(\theta)}{\partial \theta} = \log(\frac{\theta}{1+\theta}y)$$

The marginal value of better school quality is increasing in income $\frac{\partial V(y;\theta)}{\partial \theta \partial y} = \frac{1}{y} > 0$. This is due to the complementarity between parental investment and school quality that creates positive assortative matching (PAM).[10] Therefore, differences in school quality are a complementary force of income sorting decisions across different schools. In turn, school quality differences can be related to average household income differences across communities. This model fits well with the observation that high income families work hard to give a leg up and segregate themselves in a high-income neighbourhoods with good schools, while poor families suffer diminishing opportunities. By leaving the poor out of the successful communities the efficiency gains of positive assortative matching are realized (proposition 2) but opportunity is closed down as well as social mobility (proposition 3). In the Appendix B we consider the case where investment and school quality are substitutes rather than complements, and show that it creates negative assortative matching (NAM) leading to efficiency loss from positive sorting of high income

[10] There is positive assortative matching (PAM) if the matching output function (the human capital function) is supemodular in both school quality and parental investment. That is for a differentiable output function $\frac{\partial h}{\partial I \partial \theta} > 0$. There is negative assortative matching (NAM) otherwise.

children into high quality schools. As a result, investment and school quality substitutability limits the scope for the positive sorting ($p > \frac{1}{2}$) that is observed in many countries, and which is the main focus of our paper.

Another possible explanation for the positive sorting, as documented in Doepke and Zilibotti (2019) and Putnam (2015), is that communities are historically endowed with different housing and neighbourhood quality. Residential income segregation emerges as households sort out based on their demand for neighbourhood and housing quality. For instance, Loury *et al.* (1977), Durlauf (1996) or Bénabou (1996b) show how optimal segregation relates to complementarities in family and social attributes and to the decreasing returns to community quality, and De la Croix and Doepke (2009) emphasize the interaction between income inequality and the political economy of public school funding. In our model, school quality is a complementary force that may drive sorting decisions.

Last but not least, although our emphasis is on neighbourhood effects, our framework is also consistent with the empirical observation that unequal opportunities related to parental income arise even in the absence of residential segregation. For instance, Burgess and Briggs (2010) show that differences in the probability of attending a good school by family income in the UK are mainly explained by location differences, but that significant differences still remain within postal codes. This could be, for instance, due to differences in information or in the ability to "work the system", which expands the range of relevant policies related to unequal access beyond the aforementioned school choice mechanisms, busing, or transportation subsidies.

7 Model Simulations

With the aim of providing a quantitative assessment of the model predictions -most notably, the magnitude of the trade-off between intergenerational mobility and efficiency-, we calibrate the model to the US parental income distribution and simulate the effects on mobility and efficiency of changing unequal school opportunity by changing school segregation and school inequality, with the same starting conditions. In our calibration of the model, we assume that parental income is distributed as in the US, according to a Generalized Beta of the second kind, or Dagum I, with scale parameter $b = 41865$ and shape parameters $a = 3.008$ and $p = 0.592$ (this p is not to be confused with the school segregation parameter), which represent MLE coefficients for the US in 2013 (Clementi and Gallegati, 2016). For this particular distribution and calibration, the variance at the top is actually slightly smaller than the variance at the bottom. As in the proof of proposition 3, the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ does not necessarily require that the variance at the top is greater than the variance at the bottom. What is required is that $\pi \geq \frac{1}{2}\left(\frac{E[log(y_{t-1})]+1}{E[log(y_{t-1})]}\right)$. In our calibration, $\pi = 0.992$, and $\frac{1}{2}\left(\frac{E[log(y_{t-1})]+1}{E[log(y_{t-1})]}\right) = 0.548$, which means that the conditions for $\frac{\partial \beta_s}{\partial p} > 0$ and $\frac{\partial \beta_s}{\partial \kappa} > 0$ are satisfied for any $p > 1/2$ and $\phi > 1$.

As a benchmark, we set $\theta = \frac{1}{3}$, and $\mu = 7.9$, that deliver values for the average log-income that are similar to those in (Clementi and Gallegati, 2016). Moreover, setting $\theta = \frac{1}{3}$ yields intergenerational income elasticities that fall in the range of observed estimates across countries (Blanden, 2019), for the various combinations of school segregation and school inequality that we consider. The implied coefficient would go from 0.3 with equal school opportunity, to 0.4 or 0.5 under higher levels of unequal school opportunity.

Since endowment persistence $\frac{cov(log(y_{t-1}),\tau_t)}{Var(log(y_{t-1}))}$ is an additive term for the intergenerational elasticity that is identical both with and without segregation, we set it to zero. We then explore how aggregate human capital and intergenerational mobility, measured by the IGE, are affected by different segregation and school inequality (i.e, different values of p and κ). Since the unit of measure of Human Capital is not obvious, we use as a benchmark (normalized to be one) the case in which there is equality of opportunity (i.e., $p = \frac{1}{2}$) and an intermediate value of school inequality ($\kappa = 0.06 \times \theta$).[11] For every set of parameter values, we perform 10000 simulations. For each simulation, we draw the parental income distribution y_{t-1}, school quality θ (with different probabilities depending on the parental income rank), and the idiosyncratic shocks to human capital (u_t). We then compute children outcomes and the moments of interest (h_t, β). Finally, we take the averages of (h_t, β) in the 10000 simulations, for every set of parameter values.

Table 1 reports the results of increasing school segregation. The simulations are in line with the model predictions, but reveal an important insight on the magnitude of the efficiency-mobility tradeoff. The mobility gain of school opportunity equalization is much higher than the efficiency loss. For instance, reducing segregation from 0.7 to 0.633 (from the US to Norway,

[11] $\kappa = 0.06\theta$ implies that good schools are 11% better than bad schools. These is consistent, for instance, with the differences in spending-per-pupil documented in Jackson *et al.* (2015), given their estimate of the social segregation rate.

as measured by Gutiérrez *et al.* (2017)) would reduce the IGE from 0.366 to 0.344, and human capital by less than 0.1%. In a similar vein, table 2 reports the results of increasing school inequality, for different levels of school segregation. As predicted by the model, with no segregation ($p = 0.5$), school inequality does not affect intergenerational mobility (the small variations with κ are not systematic and due to randomness). Likewise, school inequality increases aggregate human capital, although this effect is very small. The second and third panels in table 2 reports the results of increasing school inequality, under $p = 0.7$ and $p = 0.9$, respectively. In this case, the effects of school inequality become larger due to the interaction with segregation. Moreover, we see again that the impact on mobility is much larger than the impact on efficiency. For instance, with $p = 0.7$, equalizing schools from $\kappa = 0.1 \times \theta$ reduces the IGE from 0.41 to 0.3 and reduces human capital by 0.08%. Hence, these results suggest that policies targeted at reducing either school segregation or school inequality would have a much bigger impact on intergenerational mobility than on efficiency.[12]

Table 1: Simulation: $\theta = 0.3$, $\kappa = \theta \times 0.06$

p	IGE	Aggregate HK
0.566	0.322	1.0001
0.6	0.333	1.0002
0.633	0.344	1.0003
0.666	0.356	1.0003
0.7	0.366	1.0004
0.733	0.377	1.0005

Table 2: Simulations for $\theta = 0.3$ and different values of κ (as a fraction of θ)

$p = 0.5$			$p = 0.7$			$p = 0.9$		
κ	IGE	Aggregate HK	κ	IGE	Aggregate HK	κ	IGE	Aggregate HK
0	0.3	1	0	0.3	1	0	0.3	1
0.02	0.3	1	0.02	0.322	1.0001	0.02	0.344	1.0003
0.04	0.3	1	0.04	0.344	1.0003	0.04	0.389	1.0006
0.06	0.3	1	0.06	0.366	1.0004	0.06	0.433	1.0008
0.08	0.3	1	0.08	0.388	1.0006	0.08	0.477	1.0010
0.1	0.3	1	0.1	0.411	1.0008	0.1	0.522	1.0013

[12] A simplifying assumption of our model is the bivariate regime switch around the median, which makes it easier to present the results and the intuition. Nonetheless, we obtain similar results in analogous simulations modeling p as a continuous function of the parental income rank. These results are available upon request.

7.1 Interactions between inequalities

We can use the model simulations to study the possible interaction between school inequality and school segregation and income inequality. We do that by increasing between-group log-income inequality (ϕ), first independently of κ and p, and later allowing these increases in income inequality to feed back on school inequality and school segregation with more or less intensity. Hence, this relates income inequality to income persistence under different scenarios. Across countries, there is a positive correlation between income inequality and the intergenerational income elasticity -*The Great Gatsby Curve*- (Corak, 2013).[13]

Proposition 2 shows that average human capital is an increasing function of between-group log-income inequality ϕ. If on top of that, log-income inequality exacerbates school segregation and inequality, for instance because differences in returns to parental investment across schools are related to the compositional pattern of students in the school, it follows that the effect on human capital is even larger. To see that, let $\kappa = \kappa(\phi)$, and $p = p(\phi)$. Then,

$$\begin{aligned}\frac{\partial E[h_t, s]}{\partial \phi} &= \kappa E[log(y_{t-1})]D(p) + \frac{\partial p}{\partial \phi}\left(\kappa E[log(y_{t-1})]2(\phi - 1)\right) \\ &\quad + \frac{\partial \kappa}{\partial \phi}\left(\frac{1}{2}\left(g'(\theta^H) - g'(\theta^H)\right) + \left(D(p)(\phi - 1)E[log(y_{t-1})]\right)\right)\end{aligned}$$

where $D(p) = 2p - 1 > 0$. Thus $\frac{\partial E[h_t,s]}{\partial \phi} > 0$ if $\frac{\partial \kappa}{\partial \phi} > 0$ and $\frac{\partial p}{\delta \phi} > 0$.

Regarding intergenerational persistence, however, the expression for $\frac{\partial \beta_s}{\partial \phi}$, does not have a closed form. In fact, from equation (16) we can see how the effect of between-group inequality on intergenerational persistence depends on two factors. On the one hand, on how much it comes along with a higher covariance between parental income at being at the top, which increases persistence. On the other hand, on how much it increases the overall variance of parental log-income, which attenuates the importance of unequal school opportunity for intergenerational persistence. The net effect, hence, is ambiguous, and it depends on how the inequality increase arises. In the following simulations, to increase inequality without changing the average parental log-income, we proceed as follows. We draw the initial parental income distribution, subtract the mean, multiply the centered distribution by a certain factor (0.7, 0.8, 0.9, 1, ..., 1.3), and add the mean back into the distribution.

[13]Likewise, Godin and Hindriks (2018) provide evidence of a Great Gatsby Curve in PISA math test scores.

The top panel in table 3 reports the results of this exercise, when p and κ remain constant. The results show that introducing parental income inequality in this way slightly reduces income persistence, and very slightly increases human capital. Hence the positive correlation between inequality and persistence of *The Great Gatsby Curve* does not emerge in this case. This is convenient because it makes sure that when income inequality interacts with p and κ, the direct effects of income inequality do not affect the trade-off between efficiency and mobility.

We then study the effect of simultaneous changes in ϕ, κ, and p, which can be interpreted as ϕ feeding back with more or less intensity into κ and p. The bottom left panel reports results with a small elasticity of κ and p with respect to ϕ; the bottom right panel reports results for a higher elasticity. More precisely, in the bottom left panel, the income inequality elasticity of school segregation is around 0.5, while the income inequality elasticity of school inequality is around 0.75.[14] In the bottom right panel, the income inequality elasticity of school segregation is around 1.4, while the income inequality elasticity of school inequality is around 1.9. Hence, the elasticities in the bottom right panel are around 2.5 times larger than those in the bottom left panel. The results show that in both cases the results are similar to the baseline, when those elasticities are zero, with unequal school opportunity leading to a trade-off between efficiency and mobility, where the efficiency loss of equalizing school opportunity is very small compared to the mobility gain. Another interesting result is that by interacting income inequality with school inequality and segregation, we can recover the positive correlation of *The Great Gatsby Curve*: more inequality is now associated with more intergenerational income persistence.

One limitation of this exercise is that we model the interaction between unequal school opportunity and income inequality as a black-box. However, our finding is consistent with earlier work with a full-fledged equilibrium sorting of heterogenous individuals across different communities (Bénabou, 1996a; Durlauf, 1996; Fernández and Rogerson, 1998). The equilibrium sorting outcomes display a negative income inequality-mobility relationship under two key conditions: first, more segregated communities have less mobility; second, more unequal communities are more segregated (Durlauf and Seshadri, 2018). Interestingly Chetty *et al.* (2014, 2020) empirically validate those key predictions. Our contribution is to pin down an important mechanism underlying these facts.

[14] These are computed as the table averages of $\epsilon_\phi^p = \frac{\%\Delta p}{\%\Delta(\phi-1)}$ and $\epsilon_\phi^\kappa = \frac{\%\Delta \kappa}{\%\Delta(\phi-1)}$.

Table 3: Simulations for simultaneous changes in κ, p and ϕ.

Independent changes in ϕ

κ	p	ϕ	IGE	Aggregate HK
0.06	0.7	1.0469	0.380	1.00035
0.06	0.7	1.0501	0.375	1.00036
0.06	0.7	1.0532	0.370	1.00038
0.06	0.7	1.056	0.367	1.00040
0.06	0.7	1.0588	0.364	1.00040
0.06	0.7	1.0614	0.361	1.00044
0.06	0.7	1.0639	0.359	1.00045

Changes in ϕ, small elasticity of p and κ

κ	p	ϕ	IGE	Aggregate HK
0.054	0.64	1.0469	0.350	1.00021
0.056	0.66	1.0501	0.355	1.00026
0.058	0.68	1.0532	0.361	1.00034
0.06	0.7	1.056	0.366	1.00041
0.062	0.72	1.0588	0.372	1.00048
0.064	0.74	1.0614	0.378	1.00057
0.066	0.76	1.0639	0.383	1.00065

Changes in ϕ, large elasticity of p and κ

κ	p	ϕ	IGE	Aggregate HK
0.045	0.55	1.0469	0.315	1.00004
0.05	0.6	1.0501	0.331	1.00013
0.055	0.65	1.0532	0.348	1.00027
0.06	0.7	1.0561	0.366	1.00040
0.065	0.75	1.0588	0.386	1.00057
0.07	0.8	1.0614	0.406	1.00078
0.075	0.85	1.0639	0.427	1.00100

8 Conclusion

Motivated by recent evidence on the causal effects of neighbourhoods and social mobility, we analyse the effect of unequal school opportunity on intergenerational income persistence and human capital accumulation, building upon the classical Becker-Tomes-Solon framework. Formally, we use a regime switch model where the transition probabilities for having access to a high quality school depend on the parental income rank. In this framework, unequal school opportunity is the combination of school inequality -which relates to school autonomy, funding, and other equalization policies- and school segregation -which relates to school choice mechanisms, busing, and de-segregation policies-.

In our model, the effect of unequal school opportunity on parental investment can go either way, because of the diminishing returns to parental investment. Unequal school opportunity produces a shifting of parental investment towards the richer families, exacerbating the parenting gap. Because high income families on average can attend better schools, this increases average human capital -an efficiency gain due to positive assortative matching-. At the same time, this increases income persistence within the top and decreases it within the bottom. Because income dispersion tends to be higher at the top, in most cases unequal school opportunity reduces intergenerational income mobility overall. We calibrate and simulate the model to assess the magnitude of this mobility-efficiency trade-off. The simulations indicate that school equalization and de-segregation policies have larger effects on mobility than on efficiency.

We further use simulations to study the interaction between income inequality and unequal school opportunity, by studying the effect of simultaneous changes in school inequality, segregation, and income inequality. These results are interesting because in our analysis, school quality and its inequality have a multi-faceted interpretation. Differences in school productivity could arise because of a variety of reasons, among them differences in school inputs, peers, the family, and the surrounding community. The latter are more consistent with an interpretation where school inequality is a natural result of school segregation, or a broad neighbourhood effect. The results of simultaneous changes in income inequality, school segregation and school inequality can fit this interpretation, while retaining the intuition of the main results of the paper. These simulation results also show that independent increases in income inequality do not necessarily lead to more intergenerational persistence; while increases in income inequality accompanied by increases in unequal school opportunity do lead to a positive correlation between inequality and persistence. This suggests an important role of unequal school opportunity in explaining *The Great Gatsby Curve.*

Our model combines unequal school quality and an unequal access to high quality schools linked to social segregation. Both parameters are taken as exogenous policy choices, whereas in a full-fledge equilibrium model of endogenous sorting they should be endogenous. Hence, although we test the importance of sorting via simulations and obtain similar results, a word of caution on policy implications is in order. Further investigating to what extent the relatively low efficiency cost of reducing school opportunities is sensitive to endogenous sorting and whether policies should focus on neighbourhoods or schools is left for future work.

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Appendix A

Proposition 1

Proof. Part (i).

$$\frac{\partial E[I_s]}{\partial p} = \left(\frac{\theta^H}{1+\theta^H} - \frac{\theta^L}{1+\theta^L}\right)(\gamma-1)E[y_{t-1}] = \left(\frac{\theta^H-\theta^L}{(1+\theta^H)(1+\theta^L)}\right)(\gamma-1)E[y_{t-1}] > 0$$

Which means that aggregate investment is an increasing function of school segregation p.

Part (ii). It is straightforward to see that average investment is increasing in γ since $\omega^H(\theta) > \omega^L(\theta)$ for $\kappa > 0$ and $p > 1/2$:

$$\frac{\partial E[I_s]}{\partial \gamma} = \frac{1}{2}\omega^H(\theta)E[y_{t-1}] - \frac{1}{2}\omega^L(\theta)E[y_{t-1}] > 0$$

Part (iii). The effect of school inequality κ on average investment is given by :

$$\frac{\partial E[I_s]}{\partial \kappa} = \frac{1}{2}\left(\frac{\partial \omega^H(\theta)}{\partial \kappa}\right)\gamma E[y_{t-1}] + \frac{1}{2}\left(\frac{\partial \omega^L(\theta)}{\partial \kappa}\right)(2-\gamma)E[y_{t-1}]$$

where the first term $\frac{\partial \omega^H(\theta)}{\partial \kappa} = p\frac{1}{(1+\theta^H)^2} - (1-p)\frac{1}{(1+\theta^L)^2}$ is negative for $p = 1/2$ and positive for $p = 1$. So there exists intermediate value $1/2 < p < 1$ such that this expression is equal to zero. The second term is,

$$\frac{\partial \omega^L(\theta)}{\partial \kappa} = -p\frac{1}{(1+\theta^L)^2} + (1-p)\frac{1}{(1+\theta^H)^2} < 0$$

for all p and $\kappa > 0$. Hence, we can define a threshold for p solving

$$p\left(\gamma(1+\theta^L)^2 + \gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2 - (2-\gamma)(1+\theta^H)^2\right) - \gamma(1+\theta^H)^2 + (2-\gamma)(1+\theta^L)^2 = 0$$

such that $\frac{\partial E[I_s]}{\partial \kappa} > 0$ iff:

$$p > \frac{\gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2}{\left(\gamma(1+\theta^L)^2 + \gamma(1+\theta^H)^2 - (2-\gamma)(1+\theta^L)^2 - (2-\gamma)(1+\theta^H)^2\right)} = \frac{1}{2} + \frac{\kappa(1+\theta)}{(\gamma-1)\left((1+\theta)^2+\kappa^2\right)}$$

□

Proposition 2

Proof.

$$\frac{\partial E[h_t, s]}{\partial p} = \kappa\phi E[log\, y_{t-1}] - \kappa(2-\phi)E[log\, y_{t-1}] = \kappa E[log\, y_{t-1}]2(\phi - 1) > 0$$

$$\frac{\partial E[h_t, s]}{\partial \phi} = \frac{1}{2}E[log\, y_{t-1}]\left(p\theta^H + (1-p)\theta^L - p\theta^L - (1-p)\theta^H\right) = \kappa E[log\, y_{t-1}]\,(2p-1) > 0$$

$$\frac{\partial E[h_t, s]}{\partial \kappa} = \frac{1}{2}\left(g'(\theta^H) - g'(\theta^L)\right) + ((2p-1)(\phi-1)E[log\, y_{t-1}]) > 0$$

where $g'(\theta^H) - g'(\theta^L) > 0$ by the convexity of $g(\theta)$. $\square$

Proposition 3

Proof. Note that

$$\frac{\partial \beta_s}{\partial p} = 2(g(\theta^H) - g(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (\theta^L - \theta^H)$$

$$+2(\theta^H - \theta^L)\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))}$$

With $\theta + g(\theta)$ increasing in θ, $g(\theta^H) - g(\theta^L) > (\theta^L - \theta^H)$ a sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ is:

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1})) + \frac{Var(log(y_{t-1}))}{2}$$

First, note that as long as $E[log(y_t) \geq 1]$,

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) - cov(Z_{t-1}, log(y_{t-1})) =$$

$$\frac{1}{2}\left(Var(log(y_{t-1})|y > y^M) + (\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})]]\right)$$

For $\frac{\partial \beta_s}{\partial p} > 0$, the difference in the covariances above must be larger than $\frac{1}{2}Var(log(y_{t-1}))$:

$$\frac{1}{2}\left((Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})) + (\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})]]\right) > 0$$

To see why $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$ is sufficient, note that:

$$Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})|y < y^M)$$

$$= \delta E[(log(y_{t-1})^2] - \phi^2 E[log(y_{t-1})]^2 - (2-\delta)E[(log(y_{t-1})^2] + (2-\phi)^2 E[log(y_{t-1})]^2$$

$$= 2(\delta - 1)E[(log(y_{t-1})^2] - 4(\phi - 1)E[log(y_{t-1})]^2$$

Now, define $\pi = \frac{2(\delta-1)E[(log(y_{t-1})^2]}{4(\phi-1)E[log(y_{t-1})]^2}$, such that $\pi \geq 1$ iff $Var(log(y_{t-1})|y > y^M) \geq Var(log(y_{t-1})|y < y^M)$. Using this, we can rewrite the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ as:

$$\frac{1}{2}(Var(log(y_{t-1})|y > y^M) - Var(log(y_{t-1})) + \frac{1}{2}(\phi E[log(y_{t-1})] - 1)(\phi - 1)E[log(y_{t-1})] > 0$$

$$\frac{1}{2}\left((\delta-1)E[log(y_{t-1})^2] - (\phi-1)E[log(y_{t-1})]^2 + (1-\phi)E[log(y_{t-1})]\right) > 0$$

$$\frac{1}{2}\left(2\pi(\phi-1)E[log(y_{t-1})]^2 - (\phi-1)E[log(y_{t-1})]^2 + (1-\phi)E[log(y_{t-1})]\right) > 0$$

$$\frac{1}{2}\left((\phi-1)E[log(y_{t-1})]\left((2\pi-1)E[log(y_{t-1})] - 1\right)\right) > 0$$

Hence, the sufficient condition for $\frac{\partial \beta_s}{\partial p} > 0$ will be satisfied whenever $\pi \geq \frac{1}{2}\left(\frac{E[log(y_{t-1})]+1}{E[log(y_{t-1})]}\right)$.

Given that $E[log(y_{t-1})] \geq 1$, $\pi \geq 1$ is a sufficient condition. Note however that with more plausible values of $E[log(y_{t-1})]$, with π just a little larger than $\frac{1}{2}$ (i.e., for a wide range of values such that $Var(log(y_{t-1})|y > y^M) < Var(log(y_{t-1})|y < y^M)$), the condition will still be satisfied. Finally, note that whenever y is log-normally distributed, $log(y)$ follows a symmetric distribution, such that $\pi = 1$, and hence the condition will always be satisfied. □

Proposition 4

Proof.

$$\frac{\partial \beta_s}{\partial \kappa} = (2p-1)(g'(\theta^H) + g'(\theta^L))\frac{cov(Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))} + (1-2p)$$

$$+(2p-1)2\frac{cov(log(y_{t-1})Z_{t-1}, log(y_{t-1}))}{Var(log(y_{t-1}))}$$

Where the first two terms are negative, and the last one is positive. First, note that $\theta + g(\theta)$ increasing in θ implies $(g'(\theta^H) + g'(\theta^L)) + 2 \geq 0$.

Second, note that as long as $E[log(y_t) \geq 1]$, $cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1}))$. Then it follows that a sufficient condition for $\frac{\partial \beta_s}{\partial \kappa} > 0$ is given by

$$cov(log(y_{t-1})Z_{t-1}, log(y_{t-1})) > cov(Z_{t-1}, log(y_{t-1})) + \frac{Var(log(y_{t-1}))}{2}$$

Which is exactly the same sufficient condition that we obtained for $\frac{\partial \beta_s}{\partial p} > 0$. □

Appendix B

Substitutability of investment and school quality

Human capital depends on the child initial endowment u, parental investment I, and school quality θ according to a general function which maps these inputs into a human capital outcome

$$h(I, \theta, u)$$

Parents choose investment I knowing school quality θ. The cross-derivatives of the production function determine the degree of complementarity/substitutability between investment and school quality.

$$\frac{\partial^2 h}{\partial I \partial \theta} \lesseqgtr 0$$

When the cross-derivative is positive, parental investment and school quality are complements in the production of human capital; when the cross-derivative is negative, parental investment and school quality are substitutes. Consider the following human capital production function:

$$h_t = \alpha_0 \theta + \alpha_1 I_{t-1} + \alpha_2 \theta I_{t-1} + u_t, \qquad \theta, \alpha_0, \alpha_1 > 0.$$

where $\frac{\partial^2 h}{\partial I \partial \theta} = \alpha_2 \lesseqgtr 0$ is the cross-derivative between investment and school quality (i.e., when $\alpha_2 < 0$ parents perceive investment as a substitute rather than as a complement to school quality).

Note that if $\alpha_2 > 0$, parents perceive investment as a complement to school quality and we are back to the previous baseline setting. We now check the implication of $\alpha_2 < 0$ for the main results on efficiency (proposition 2) and mobility (propositions 3 and 4).

Given our regime switch model, for $y_{t-1} \leq y_{t-1}^{M}$,

$$h_t = \begin{cases} \alpha_0\theta^L + \alpha_1 I_{t-1} + \alpha_2\theta^L I_{t-1} + u_t, & \text{with probability p.} \\ \alpha_0\theta^H + \alpha_1 I_{t-1} + \alpha_2\theta^H I_{t-1} + u_t, & \text{with probability 1-p.} \end{cases}$$

While the reverse holds for $y_{t-1} > y_{t-1}^{M}$. The distribution of parental investment conditional on school quality for $y_{t-1} \leq y_{t-1}^{M}$ is given by:

$$I_{t-1}^{*} = \begin{cases} y_{t-1} - \frac{1}{\alpha_1+\alpha_2\theta^L}, & \text{with probability p.} \\ y_{t-1} - \frac{1}{\alpha_1+\alpha_2\theta^H}, & \text{with probability 1-p.} \end{cases}$$

While the reverse holds for $y_{t-1} > y_{t-1}^{M}$. Note that for $\theta^L = \theta^H = \theta$ (equal schooling) the average investment is

$$E[I] = E[y_{t-1}] - \frac{1}{\alpha_1 + \alpha_2\theta}$$

Note that for $\alpha_2 < 0$, $E[I]$ is a decreasing concave function of θ, which means that any mean preserving spread in θ (unequal schooling) will reduce average investment. Substituting for optimal investment in the human capital production function, we obtain, for equal schooling:

$$E[h_t] = (\alpha_0\theta - 1) + (\alpha_1 + \alpha_2\theta)\,E[y_{t-1}] + E[u_t]$$

Under unequal school opportunity:

$$E[h_t, s] = E[h_t] + \alpha_2\kappa D(p)(\gamma - 1)E[y_{t-1}]$$

where $D(p) = (2p - 1)$ is the dissimilarity index of segregation. Thus $E[h_t, s] - E[h_t] = \alpha_2\kappa D(p)(\gamma - 1)E[y_{t-1}] < 0$, since $\alpha_2 < 0$, $D(p) > 0$ for $p > 1/2$ and $\gamma > 1$. Hence when investment is a substitute to school quality, unequal school opportunities reduce average human capital (efficiency loss). Alternatively, when investment is a complement to school quality, unequal school opportunities increase human capital (as in Proposition 2). The reason is that positive assortative matching of high income children to high quality schools is efficiency enhancing when the matching function (the human capital outcome) is supemodular, $\frac{\partial^2 h}{\partial I \partial\theta} > 0$.

That is, positive assortative matching ($p > 1/2$) is efficient when school quality and investment are complements ($\alpha_2 > 0$). From the expression above we can derive the following proposition:

Proposition 2b. *Given the human capital production function H with $\alpha_2 < 0$ (respectively $\alpha_2 > 0$), (i) for any $\kappa > 0$, $p > 1/2$ and $\gamma > 1$, average human capital is a decreasing (increasing) function of school segregation p ; (ii) for any $\kappa > 0$, $p > 1/2$, and $\gamma > 1$, average human capital is a decreasing (increasing) function of between-group income inequality γ; (iii) for any $\kappa > 0$, $p > 1/2$, and $\gamma > 1$, average human capital is a decreasing (increasing) function of school inequality κ.*

Proof.

$$\frac{\partial E[h_t, s] - E[h_t]}{\partial p} = \alpha_2 \kappa 2(\gamma - 1)E[y_{t-1}] \propto \alpha_2; \; \frac{\partial E[h_t, s] - E[h_t]}{\partial \gamma} = \alpha_2 \kappa (2p - 1)E[y_{t-1}] \propto \alpha_2$$

$$\frac{\partial E[h_t, s] - E[h_t]}{\partial \kappa} = \alpha_2 (2p - 1)(\gamma - 1)E[y_{t-1}] \propto \alpha_2$$

□

Turning to the intergenerational elasticity, substituting optimal investment into the earnings equation with equal school opportunity we obtain:[15]

$$log(y_t) = (\alpha_0 \theta - 1) + (\alpha_1 + \alpha_2 \theta) y_{t-1} + u_t$$

The intergenerational elasticity with equal school opportunity is

$$\beta = (\alpha_1 + \alpha_2 \theta) \frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))}$$

Under unequal school opportunity, the regime switch equation is :

$$log(y_t) = (\alpha_0 \theta - 1) + \left(\alpha_1 + \alpha_2 (p\theta^L + (1-p)\theta^H)\right) y_{t-1} + \alpha_2 \kappa D(p) y_{t-1} Z_{t-1} + u_t$$

[15]We assume no skill inheritability: $cov(u_t, y_{t-1}) = 0$.

where the regime switch variable $Z_{t-1} = 1$ with probability p if $y_{t-1} > y_{t-1}^M$and zero otherwise, and $Z_{t-1} = 1$ with probability $1 - p$ if $y_{t-1} < y_{t-1}^M$and zero otherwise.

The intergenerational elasticity is:

$$\beta_s = (\alpha_1 + \alpha_2(p\theta^L + (1-p)\theta^H))\frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} + (\alpha_2 \kappa D(p))\frac{cov(y_{t-1}Z_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))}$$

Which can be rewritten as:

$$\beta_s = \beta + (\alpha_2 \kappa D(p)) \left(\frac{cov(y_{t-1}Z_{t-1}, log(y_{t-1})) - cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} \right)$$

Note that for $p = 1/2$, we have $D(p) = 0$ and so $\beta_s = \beta$. For any $p > 1/2$ we have $D(p) > 0$ and for $\alpha_2 \leq 0$, $\alpha_2 \kappa D(p) \leq 0$ so that $\beta_s \leq \beta$ if and only if:

$$cov(y_{t-1}Z_{t-1}, log(y_{t-1})) \geq cov(y_{t-1}, log(y_{t-1}))$$

A stronger (but only sufficient) condition is given by:

$$cov(y_{t-1}, log(y_{t-1})|y > y^M) > cov(y_{t-1}, log(y_{t-1}))$$

Note that the condition above is reminiscent to the condition in propositions 3-4, where we obtained that under complementarity between investment and school quality, the intergenerational elasticity is an increasing function of school segregation $p > 1/2$ and school inequality $\kappa > 0$ if the variance of parental income is higher in the high income group than in the low income group. Here, with substitutability between investment and school quality we have a reverse result.

Propositions 3b and 4b. *Given the human capital production function H with $\alpha_2 < 0$ (respectively $\alpha_2 > 0$), for any $\kappa > 0$, $p > 1/2$, the intergenerational elasticity of income β_s is a decreasing (increasing) function of school segregation p and school inequality κ if $cov(y_{t-1}, log(y_{t-1})|y > y^M) > cov(y_{t-1}, log(y_{t-1}))$. This condition is satisfied for a log-normal income distribution.*

Proof.

$$\frac{\partial \beta_s}{\partial p} = \alpha_2 \kappa 2 \left(\frac{cov(y_{t-1} Z_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} - \frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} \right)$$

$$\propto \left(\frac{cov(y_{t-1} Z_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} - \frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} \right)$$

$$\frac{\partial \beta_s}{\partial \kappa} = \alpha_2 (2p-1) \left(\frac{cov(y_{t-1} Z_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} - \frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} \right)$$

$$\propto \left(\frac{cov(y_{t-1} Z_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} - \frac{cov(y_{t-1}, log(y_{t-1}))}{var(log(y_{t-1}))} \right)$$

The variance of a log-normal random variable $Y = e^X$, where X is normally distributed with mean μ and variance σ^2, is given by $e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}$. The variance of a log-normal truncated from below at the median is given by:

$$e^{2\mu+2\sigma^2} \left(\frac{\Phi(2\sigma)}{\Phi(0)} \right) - e^{2\mu+\sigma^2} \left(\frac{\Phi(\sigma)}{\Phi(0)} \right)^2$$

The difference is given by:

$$e^{2\mu+\sigma^2} \left(e^{\sigma^2} \left(\frac{\Phi(2\sigma)}{\Phi(0)} - 1 \right) - \left(\left(\frac{\Phi(\sigma)}{\Phi(0)} \right)^2 - 1 \right) \right)$$

The term in parenthesis is equal to zero whenever $\sigma = 0$, and is positive for any $\sigma > 0$. Hence, the variance of any log-normal truncated from below at the median is larger than the unconditional variance of the log-normal. □

To sum up, when investment and school quality are complements ($\alpha_2 > 0$), both school inequality ($\kappa > 0$) and the assignment of high-quality schools to high income families ($p > 1/2$) increase average human capital, but reduces mobility. This is just the inverse trade-off when investment and school quality are substitutes (unequal school opportunities reduce average human capital but increase mobility). Therefore, in both cases the trade-off between efficiency and mobility arises.